An investigation of the anisotropic stress-strain-strength characteristics of an Eocene clay

By

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December 2006

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
IMPERIAL COLLEGE LONDON
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Thesis submitted to the University of London in partial fulfilment for the degree of Doctor of Philosophy and for the Diploma of the Imperial College

By

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IMPERIAL COLLEGE LONDON
To my dearest Bống (Thu Nga), Bee (Minh Nhật) and Đỗ Quyên

Đầy la anh Bi, đầy la em Bống, Ông Ngoại
13-09-2004

Khi mẹ về, Bi Bống đã đi vào giấc ngủ
Không có tiếng khóc
không có ai nói gì
ma nghe như có ai đang kể,
gióng ngóng liều ngóng lồ
trong một câu chuyện cổ tích
rúc ra rúc rích
hai anh em cùng cười.

(This is brother Bee, this is young sister Bông, by their maternal grandpa)

Bee and Bông are sleeping when Mom comes back home
No more crying
also no words that she could hear
but there seems to be murmuring
in a very young child's voice
of a fairy story
secretly and happily
they are both smiling.
Abstract

It is widely acknowledged that soils may exhibit anisotropic mechanical properties, resulting both from their initial anisotropic fabric and from applied stress changes. Recent improvements in numerical analysis have allowed the anisotropy in shear strength and stiffness to be addressed in certain constitutive models to improve the prediction of the ground’s response, and this development requires a comprehensive experimental database to be built. However, rigorous laboratory investigations into the anisotropic behaviour of many soil types, including stiff clays have been limited.

This study investigated the mechanical anisotropy of natural London Clay, an Eocene deposit widely distributed in the London and Hampshire basins. The study was conducted by using a recently developed Hollow Cylinder Apparatus (HCA) (Jardine, 1996) and was part of a comprehensive London Clay characterization programme focused on the Terminal 5 project at Heathrow Airport. The programme was carried out by a team of researchers at Imperial College, deploying a range of equipment including large triaxial stress path cells equipped with local strain measurements and bender elements, and high pressure triaxial cells (Gasparre, 2005) and a resonant column HCA (Nishimura, 2006).

The author carried out two HCA test series on samples obtained from the same depth (5.2 m BGL) at the T5 site. The first series investigated the clay’s anisotropic drained stiffness properties by conducting small uniaxial drained probing tests on samples that were re-consolidated to the estimated in situ stresses. It was shown that the clay deformation characteristics at very small strains could be reasonably linked to those of a cross-anisotropic elastic material. Uniaxial drained shear tests to failure were also carried out and the non-linear stress-strain responses from small to moderate strains were monitored with local strain measurements. The clay’s pre-failure yielding characteristics were interpreted by reference to a kinematic sub-yield framework (Jardine, 1992). The second series studied the stress-strain-strength and deformation characteristics of the clay from undrained multi-axial stress path tests involving controlled principal stress axis directions and intermediate principal stress ratios. Both factors were found to influence the clay’s shear strength significantly. These multi-axial shear tests were also incorporated with undrained HCA shear tests on samples from a lower elevation at the same site to construct a general view of shear strength anisotropy of the clay at depths up to 11 m below ground surface. This thesis summarises the author’s findings, as well as synthesizes the author’s work with the other advanced laboratory and in situ tests and lithological research (Hight et al., 2007).
Declaration

The work presented in this thesis was carried out at the Soil Mechanics Group at the Department of Civil and Environmental Engineering, Imperial College London from October 2002.

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except where specifically indicated in the text.

This thesis is not substantially the same as any that I have submitted for any degree, diploma or other qualification at any other University. No part of this thesis has been or is being concurrently submitted for any such degree, diploma or other qualification.

This dissertation is available online at www.imperial.ac.uk/geotechnics/publications. It is less than 80,000 words long and contains less than 35 tables and 280 figures.

Nguyen Anh Minh

12/12/2006
I would like to thank my supervisor, Prof. Richard J. Jardine for his wisdom and guidance, his consistent support and firm interest on this research. It has been a long route to produce the thesis in its present form and his patience and understanding are paramount. Equally precious is the welcome that he and his family have warmly given to my family. Working with him is a pleasure and it is a cooperation that I look forward to maintain.

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My mother and father have raised me up and taught me to appreciate life and work. Their joy and happiness are the best gifts to me and ones that I wish to see more often. I thank also my grandmothers, brothers and sisters for their unlimited support and understanding. Special thanks are due to my parents-in-law who helped to take care of my children and shared with us the fantastic holidays in UK and Europe, in particular the photos and poems sent from my father-in-law that reduced my homesickness.

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### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>iv</td>
</tr>
<tr>
<td>Declaration</td>
<td>v</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>vii</td>
</tr>
<tr>
<td>Contents</td>
<td>viii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xiii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xxi</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>xxiii</td>
</tr>
<tr>
<td>Abbreviations</td>
<td>xxiv</td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objective and scope of the research</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Layout of the thesis</td>
<td>3</td>
</tr>
<tr>
<td><strong>2 The mechanical anisotropy of soils</strong></td>
<td>5</td>
</tr>
<tr>
<td>2.1 The components of anisotropy</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Laboratory testing for the investigation of anisotropy</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Generalised parameters for stresses</td>
<td>6</td>
</tr>
<tr>
<td>2.2.2 Suitability of hollow cylinder device to study soil anisotropy</td>
<td>7</td>
</tr>
<tr>
<td>2.3 Stress and strain non-uniformities in HC testing</td>
<td>9</td>
</tr>
<tr>
<td>2.4 Review of HC tests on cohesive soils</td>
<td>17</td>
</tr>
<tr>
<td>2.4.1 Tests on silt and reconstituted clays</td>
<td>17</td>
</tr>
<tr>
<td>2.4.2 Tests on natural clays</td>
<td>20</td>
</tr>
<tr>
<td>2.5 Cross-anisotropic elasticity theory</td>
<td>27</td>
</tr>
<tr>
<td>2.5.1 Theoretical background</td>
<td>27</td>
</tr>
<tr>
<td>2.5.2 Determination of elastic parameters from laboratory shear tests</td>
<td>28</td>
</tr>
<tr>
<td>2.5.3 Application of the cross-anisotropy elasticity theory in undrained multi-axial shear tests</td>
<td>29</td>
</tr>
<tr>
<td>2.6 Summary</td>
<td>30</td>
</tr>
<tr>
<td><strong>3 Review on the geology and engineering properties of London Clay</strong></td>
<td>31</td>
</tr>
<tr>
<td>3.1 Geology of London Clay</td>
<td>31</td>
</tr>
<tr>
<td>3.1.1 Depositional environment and post depositional processes</td>
<td>31</td>
</tr>
<tr>
<td>3.1.2 Lithological units</td>
<td>32</td>
</tr>
<tr>
<td>3.1.3 Discontinuities</td>
<td>33</td>
</tr>
<tr>
<td>3.1.4 Microfabric</td>
<td>34</td>
</tr>
<tr>
<td>3.1.5 Hydrogeology</td>
<td>34</td>
</tr>
<tr>
<td>3.2 Laboratory shear strength and stiffness</td>
<td>38</td>
</tr>
<tr>
<td>3.2.1 Shear strength</td>
<td>38</td>
</tr>
</tbody>
</table>
3.2.2 Shear stiffness .................................................. 41
3.3 Summary ......................................................... 53

4 Laboratory equipment ............................................. 54
  4.1 HCA Mark II .................................................. 54
    4.1.1 System components ........................................ 54
    4.1.2 Instrumentation ........................................... 59
    4.1.3 Computer control and data acquisition system ......... 69
  4.2 Calculation of stresses and strains in HCA ................. 72
    4.2.1 Individual stress and strain components ............... 72
    4.2.2 Principal stress and strain components ............... 73
  4.3 Analysis of data from HC tests ................................ 73
    4.3.1 Deformation analysis ...................................... 75
    4.3.2 Components of global and local strains ............... 76
    4.3.3 Other parameters ........................................... 77
  4.4 Comparisons between global and local strain measurements ........................................ 77
    4.4.1 In the calculation of individual strain components .... 77
    4.4.2 In the calculation of the deviatoric shear strain .... 78

5 Descriptions of site, material and testing procedures .......... 83
  5.1 Site condition and block sampling .......................... 83
    5.1.1 Site condition ............................................ 83
    5.1.2 Block sampling ............................................ 84
  5.2 In situ stresses ............................................... 88
  5.3 Soil description ............................................... 89
    5.3.1 Macrofabric, microfabric and mineralogy ............... 90
    5.3.2 Index properties ......................................... 91
    5.3.3 Initial effective stress from laboratory tests .......... 91
  5.4 Specimen preparation and set-up ................................ 95
    5.4.1 Hollow cylinder specimen preparation .................... 95
    5.4.2 Specimen set-up in the HCA Mark II ..................... 96
  5.5 Saturation ..................................................... 99
  5.6 Procedure after test completion .............................. 99

6 Behaviour of London Clay during re-consolidation stages .... 100
  6.1 Re-consolidation stress path .................................. 101
  6.2 Isotropically re-consolidation stages ......................... 102
    6.2.1 Changes in void ratio and normal strains ............. 102
    6.2.2 Bulk modulus ............................................. 103
  6.3 Anisotropic re-consolidation stages .......................... 108
    6.3.1 Developments of strains ................................... 108
    6.3.2 Relationships for stress ratio with induced strain .... 108

7 Stiffness anisotropy of London Clay from uniaxial drained shear tests ..................................... 113
  7.1 Shearing programme and methods of measurement .......... 114
    7.1.1 Small stress probing uniaxial tests ...................... 114
    7.1.2 Uniaxial tests with large stress increments .......... 115
    7.1.3 Methods of measurement and evaluations of errors ...... 117
  7.2 The clay's cross-anisotropy elasticity at small strain .... 118
    7.2.1 Anisotropy in Poisson's ratios and stiffness moduli .... 118

ix
# List of Tables

2.1 Errors in stresses and strains from numerical simulations for HC undrained shear tests (Rolo [2003]) ........................................... 14
2.2 Specimen dimensions used in the FE simulations by Zdravkovic and Potts (2005) .................................................. 14
2.3 Reconstituted clayey soils and silt tested in the large IC HCA .......................................................... 19

4.1 Influences of averaging scheme to local transducers’ precision in HCA Mark II .......................................................... 62
4.2 Degrees of precision in strain measurements in HCA Mark II .......................................................... 62
4.3 Resolution and precision of transducers in HCA Mark II ........................................................................... 63
4.4 Order of data analysis for a testing stage in HCA Mark II ........................................................................... 75

5.1 Locations and ground levels of boreholes and block sampling area .......................................................... 84
5.2 Index properties of 100mm hollow cylinder specimens ........................................................................... 92

6.1 Values of mean pressure and void ratio of samples tested in HCA Mark II .......................................................... 103
6.2 State of samples at the end of re-consolidation path ........................................................................... 109

7.1 Testing conditions of uniaxial drained shear probes ........................................................................... 116
7.2 Testing conditions of uniaxial drained shear with large stress increments ........................................... 116
7.3 Degrees of accuracy in the determination of stiffness moduli at small strains ........................................... 118
7.4 Elastic parameters from uniaxial drained shear probes at initial effective stress ........................................... 123
7.5 Elastic parameters from uniaxial drained shear probes at isotropic $p' = 280$ kPa ........................................... 123
7.6 Elastic parameters from uniaxial drained shear probes at anisotropic stress ........................................... 123
7.7 Drained shear strength parameters at peak of CAD uniaxial tests ........................................................................... 152

8.1 Conditions for drained $b$-change stages ........................................................................... 162
8.2 Conditions of tests under undrained conditions ........................................................................... 167
8.3 Strains developed during $b$-change stages in AMα-05 series ........................................................................... 170
8.4 Strains developed during $b$-change stages in AMα-03 series ........................................................................... 170
8.5 Direction of undrained multi-axial shear tests to failure ........................................................................... 183
8.6 Shear strength from multi-axial undrained shear tests ........................................................................... 189
8.7 Observed and predicted slopes of ESPs in multi-axial shear tests ........................................................................... 189
8.8 Comparisons with predictions by cross-anisotropic elastic theory ........................................................................... 235
8.9 Patterns of shear planes in multi-axial undrained shear tests ........................................................................... 244

9.1 Scheme of drained and undrained tests used in the identification of small strain yield loci .......................................................... 254
9.2 Identification of small strain yield loci $Y_1$ and $Y_2$ from drained tests ........................................................................... 258
9.3 Listings of reference figures for the undrained tests ........................................................................... 266
9.4 Yielding limits identified from undrained tests ........................................................................... 269
9.5 $Y_1$ yielding limits in terms of (average) incremental stress components ........................................................................... 283
9.6 Yielding limits in terms of incremental stress components .................................................. 284
A.1 HCA Mark II transducer calibration characteristics ............................................................ 316
List of Figures

1.1 Recent investigations of London Clay in the London and Hampshire Basins described by Hight et al. [2003] ................................................................. 2
2.1 General state of stress acting on a soil specimen ......................................................... 9
2.2 Projection in the deviatoric $\pi$-plane of principal stresses ........................................... 10
2.3 Directional shear cell – plane strain element stress state ............................................. 10
2.4 Hollow cylinder testing: boundary loads and element stress state ............................... 11
2.5 Definition of the major principal stress rotation angle ................................................. 11
2.6 Applicable stress states at failure in some advanced laboratory strength devices .............. 12
2.7 Regions of ‘no-go’ stress paths (Symes [1983]) ......................................................... 15
2.8 Effective stress paths for simulations of shearing at $\alpha = 30^\circ$, $b = 0.5$ (Zdravkovic and Potts [2005]) ................................................................. 15
2.9 Stress-strain relationship during shear for simulations at $\alpha = 30^\circ$, $b = 0.5$ ........................ 15
2.10 Shear stiffness $G_{oct}$ degradation curves, simulations at $\alpha = 30^\circ$, $b = 0.5$ (Zdravkovic and Potts [2005]) ................................................................. 16
2.11 Lade’s isotropic failure criterion applied to HCA data on EPK specimens (Hong and Lade [1989a]) ................................................................. 18
2.12 Undrained strength anisotropy at $b=0.5$ of $K_0$-consolidated soils at $OCR = 1$ from IC database (Jardine et al. [1997]) ......................................................... 19
2.13 The influence of soil structure on $G_{oct}^0$ and damping ratio of two Italian stiff clays (d’Onofrio and Magistris [1998]) ......................................................... 21
2.14 Strength anisotropy of SFB Mud (Lade and Kirkgard [2000]) .................................... 22
2.15 Torsional tests on London and Thanet clay samples from Sizewell site (Hight et al. [1997]) ................................................................. 23
2.16 Effective stress paths from torsional shear tests on clay specimens from Sizewell site (Hight et al. [1997]) ................................................................. 23
2.17 Shear modulus $G_{zb} - \gamma_{zb}$ from undrained RC-TS tests on London clay samples (Hight et al. [1997]) ................................................................. 24
2.18 Normalized modulus decay curves from resonant column and monotonic torsional shear tests (Hight et al. [1997]) ................................................................. 24
2.19 Profiles of peak $S_u$ from tests on samples at Bothkennar soft clay site (Albert et al. 2003) ................................................................. 25
2.20 Shear strength envelopes of Louiseville clay from HC tests (Leroueil et al. [2003]) .... 26
2.21 Relations between failure surface inclination to $\alpha$ in shear tests on Louiseville clay .... 26
3.1 The distribution of sedimentary rocks, including London Clay, in London Basin ............. 35
3.2 Typical weathering profile in London Clay (Ellison et al. [2004]) ................................. 36
3.3 Informal stratigraphical divisions of London Clay (King [1981]) ................................. 37
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>Shear strength–water content relationships for intact and remoulded samples of London Clay at Ashford Common, Level E at 34.8m BGL (Bishop et al., 1965)</td>
<td>43</td>
</tr>
<tr>
<td>3.5</td>
<td>Normalised effective stress paths from CIU compression tests of undisturbed London Clay samples at Ashford Common (Burland, 1990)</td>
<td>43</td>
</tr>
<tr>
<td>3.6</td>
<td>Drained ring shear tests on undisturbed and remoulded blue London Clay from Wraysbury (Garga, 1970)</td>
<td>44</td>
</tr>
<tr>
<td>3.7</td>
<td>Post-rupture strength envelopes for intact samples from Ashford Common, levels C and E (Burland, 1990)</td>
<td>45</td>
</tr>
<tr>
<td>3.8</td>
<td>Triaxial CID and CIU compression tests on intact vertically and horizontally cut samples from Ashford Common, level E (34.8m) (Burland, 1990)</td>
<td>45</td>
</tr>
<tr>
<td>3.9</td>
<td>Anistropy in stress-strain response of A2 London Clay unit from Triaxial CIU tests (Yimsiri, 2002)</td>
<td>46</td>
</tr>
<tr>
<td>3.10</td>
<td>Anisotropic effective strength envelope at ( b = 0.5 ) of London Clay, samples at 7.2mOD and anisotropically consolidated (Nishimura, 2006)</td>
<td>46</td>
</tr>
<tr>
<td>3.11</td>
<td>Peak strength ratio of London Clay at different combinations of ( \alpha - b ), samples at depth of 7.2mOD (Nishimura, 2006)</td>
<td>47</td>
</tr>
<tr>
<td>3.12</td>
<td>Effective strength envelopes of undisturbed and remoulded blue London Clay from triaxial tests (Bishop and Garga, 1969)</td>
<td>48</td>
</tr>
<tr>
<td>3.14</td>
<td>Loss of structure from swelling, samples of London Clay at Level E, Ashford Common (Bishop et al., 1965)</td>
<td>50</td>
</tr>
<tr>
<td>3.15</td>
<td>Effect of sample size on undrained strength of undisturbed stiff-fissured London Clay (Bishop, 1971b)</td>
<td>50</td>
</tr>
<tr>
<td>3.16</td>
<td>Anistropy in undrained Young modulus of A2 London Clay unit from triaxial CIU tests (Yimsiri, 2002)</td>
<td>51</td>
</tr>
<tr>
<td>3.17</td>
<td>Degradation of shear stiffness from triaxial tests on samples of London Clay at Heathrow T5 (Gasparre, 2005)</td>
<td>52</td>
</tr>
<tr>
<td>4.1</td>
<td>General layout of HCA Mark II</td>
<td>56</td>
</tr>
<tr>
<td>4.2</td>
<td>The HCA Mark II in Bishop Laboratory, Imperial College</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>Modified rings with vanes for clay testing</td>
<td>57</td>
</tr>
<tr>
<td>4.4</td>
<td>HCA Mark II loading system</td>
<td>58</td>
</tr>
<tr>
<td>4.5</td>
<td>Control panel for HCA</td>
<td>58</td>
</tr>
<tr>
<td>4.6</td>
<td>The local strain transducer system for HCA Mark II</td>
<td>64</td>
</tr>
<tr>
<td>4.7</td>
<td>Improvements to regression characteristics of Kaman KDM8200/6U1 proximeter</td>
<td>65</td>
</tr>
<tr>
<td>4.8</td>
<td>Scatters in local strain sensor readings w.r.t scanning scheme</td>
<td>66</td>
</tr>
<tr>
<td>4.9</td>
<td>Degrees of precision w.r.t averaging scheme for local strain measurements</td>
<td>67</td>
</tr>
<tr>
<td>4.10</td>
<td>Temperature sensitivity of local sensors</td>
<td>68</td>
</tr>
<tr>
<td>4.11</td>
<td>Schematic diagram of automatic control system for HCA Mark II</td>
<td>71</td>
</tr>
<tr>
<td>4.12</td>
<td>Displacement and strain components</td>
<td>74</td>
</tr>
<tr>
<td>4.13</td>
<td>Mohr circle of strain</td>
<td>74</td>
</tr>
<tr>
<td>4.14</td>
<td>Geometry relationship in the determination of local torsional shear strain</td>
<td>76</td>
</tr>
<tr>
<td>4.15</td>
<td>Comparison between global and local ( \varepsilon_z ), shearing stage in tests AM00-00 and AM30-03</td>
<td>79</td>
</tr>
<tr>
<td>4.16</td>
<td>Comparison between global and local ( \gamma_{2\theta} ), shearing stage in tests AM30-00 and AM45-10</td>
<td>80</td>
</tr>
<tr>
<td>4.17</td>
<td>Comparison between global and local ( \varepsilon_{\theta} ), shearing stage in tests AM45-00 and AM00-03</td>
<td>81</td>
</tr>
</tbody>
</table>
4.18 Comparison between global and local \( \varepsilon_d \) during shearing stage in tests AM30-03 and AM00-03

5.1 Plan of Heathrow T5 site and sampling locations

5.2 Geotechnical profiles for BH404 and BH406 at T5

5.3 A simple geological model for London Clay

5.4 Assumed distribution of vertical stress at T5 site

5.5 Reconstruction of stress history for samples at 12.5mOD

5.6 SEM images of London Clay sample from 7m BGL

5.7 Hydrometer test results for samples at 12.5mOD

5.8 Profiles of Atterberg’s limits and initial effective stress measured in HCA and triaxial tests

5.9 Tools used for the preparation of the London Clay hollow cylinder specimen

5.10 Stages of preparation for 100mm OD hollow cylinder specimen

6.1 Scheme of reconsolidation

6.2 Typical development of strains during isotropic re-consolidation to \( p' = 280 \text{kPa} \), test AM45-00

6.3 Typical development of strains during isotropic re-consolidation to \( p' = 312 \text{kPa} \), test AM50-05

6.4 Volume and effective stress state for 12.5mOD (5.2mBGL) natural London Clay samples

6.5 Incremental final volumetric strain and mean effective stress for 12.5mOD (5.2mBGL) natural London Clay samples

6.6 Ranges of secant bulk modulus during isotropic re-consolidation

6.7 Ranges of normalised \( K'/p' \) during isotropic re-consolidation

6.8 Typical evolution of strains during anisotropic re-consolidation, test AM00-00

6.9 Rates of creep at the end of anisotropic re-consolidation stage in test AM00-00

6.10 Typical development of strains during anisotropic re-consolidation path in test AM00-00

6.11 Stress ratio \( q/p' \) and strain \( (\varepsilon_1 - \varepsilon_3) = \varepsilon_1 - \varepsilon_3 \) relationship during anisotropic re-consolidation stage

6.12 Observed \( \varepsilon_{13} \sim \varepsilon_e \) relationships at early stages (shear strain < 0.2 %) during anisotropic re-consolidation stage

7.1 HCA uniaxial test series

7.2 Fitting void ratio functions for clayey soils within \( 0.4 < e < 1.2 \)

7.3 Fitting method for small uniaxial stress probes

7.4 Drained small stress probes at initial stress state, test HCDQ-p420

7.5 Drained small stress probes at initial stress state, test HCDT-p405

7.6 Drained small stress probes at initial stress state, test AM4500-p380

7.7 Drained small stress probes at \( p' = 280 \text{kPa} \), test HCDQ-p280

7.8 Drained small stress probes at \( p' = 280 \text{kPa} \), test HCDT-p280

7.9 Drained small stress probes at \( p' = 280 \text{kPa} \), test HCDZ-p280

7.10 Drained small stress probes at \( p' = 280 \text{kPa} \), test AM4500-p280

7.11 Drained small stress probes at \( p' = 280 \text{kPa} \), test IM9005-p280

7.12 Drained small stress probes at anisotropic stress state, test HCDQ-ani

7.13 Drained small stress probes at anisotropic stress state, test HCDT-ani

7.14 Drained small stress probes at anisotropic stress state, test HCDZ-ani

7.15 Drained small stress probes at anisotropic stress state, test AM4500-ani
8.21 Strains developed during $d\alpha = 45^\circ$, test AM00-03
8.22 Normalised $q/p' \sim \gamma_{13}$ relationship during unloading stages
8.23 Unloading stages $p' \sim \gamma_{13}$ relationship
8.24 Strains developed during $\alpha$—rotation stage, test AM50-05
8.25 Strains developed during $\alpha$—rotation stage, test AM30-05
8.26 Strains developed during $\alpha$—rotation stage, test AM00-05
8.27 Development of shear strain $\gamma_{13}$ w.r.t $\alpha$
8.28 Changes in $p'$ during $\alpha$—rotation stages
8.29 Normalised stress-strain relationship $q/p' \sim \gamma_{13}$
8.30 Effective stress paths in test series AM$\alpha$-00 with constant $b = 0.0$
8.31 Developments of strains during undrained shear in series AM$\alpha$-00
8.32 Stress-strain response in test series AM$\alpha$-00
8.33 Effective stress paths in test series AM$\alpha$-10 with constant $b = 1.0$
8.34 Developments of strains during undrained shear in series AM$\alpha$-10
8.35 Stress-strain response in test series AM$\alpha$-10
8.36 Effective stress paths in test series AM$\alpha$-03 with constant $b = 0.3$
8.37 Developments of strains during undrained shear in series AM$\alpha$-03
8.38 Stress-strain response in test series AM$\alpha$-03
8.39 Effective stress paths in test series IM$\alpha$-05 with constant $b = 0.5$
8.40 Developments of strains during undrained shear in series IM$\alpha$-05
8.41 Stress-strain response in test series IM$\alpha$-05
8.42 Effective stress paths in test series IM$\alpha$-05 with constant $b = 0.5$
8.43 Developments of strains during undrained shear in series IM$\alpha$-05
8.44 Developments of strains during undrained shear in series IM$\alpha$-05
8.45 Stress-strain response in test series IM$\alpha$-05
8.46 Definition of angle $\beta$ in TC test on diagonally cut sample
8.47 Experimental-based fitting curve for shear strength anisotropy by Bishop (1966)
8.48 Anisotropy of stress ratio at peak $q/p'$ at $b = 0$
8.49 Anisotropy of stress ratio at peak $q/p'$ at $b = 0.3$
8.50 Anisotropy of stress ratio at peak $q/p'$ at $b = 0.5$
8.51 Anisotropy of stress ratio at peak $q/p'$ at $b = 1$
8.52 Anisotropy of undrained shear strength $S_u$ at $b < 0.5$
8.53 Anisotropy of undrained shear strength $S_u$ at $b = 0.5$
8.54 Anisotropy of undrained shear strength $S_u$ at $b = 1$
8.55 Anisotropy of undrained shear strength ratio $S_u/S_u(\alpha = 0)$ under near plane strain condition ($b = 0.5$) of natural London Clay from hollow cylinder tests
8.56 Shear strength anisotropy of natural London Clay and low OCR soils under $b = 0.5$
8.57 Comparison of $S_{u,b=0.5}/S_{u,(\alpha=0,b=0.5)}$ ratio at $b = 0.5$ with Bishop (1966) expression
8.58 Comparison of $S_{u,b=0.3}/S_{u,(\alpha=0,b=0.3)}$ ratio at $b = 0.3$ with Bishop (1966) expression
8.59 Comparison of $S_{u,b=0}/S_{u,(\alpha=0)}$ ratio at $b = 0$ and 1 with Bishop (1966) expression
8.60 Development of strains during undrained shear in series AM$\alpha$-05
8.61 Failure criteria in $\pi$-plane, fitting to $\phi' = 30^\circ$ at $b = 0$ or 1
8.62 Peak shear strength on $\pi$-plane of $\alpha = 45^\circ$ for London Clay samples at 5.2 mBGL
8.63 Peak shear strength on $\pi$-planes for London Clay samples at 5.2 mBGL
8.64 Peak shear strength for $\alpha = 45^\circ$ for London Clay at 5.2 and 10.5 mBGL
8.65 Peak $q/p'$ at section of $\alpha = 45^\circ$ for London Clay samples at 5.2 mBGL
8.66 Peak $q/p'$ at sections of $\alpha < 45^\circ$
8.67 Peak \( q/p' \) at sections of \( \alpha > 45^\circ \) ................................. 229
8.68 Changes in stress ratio \( q/p' \) at peak with \( \alpha \) and \( b \) London Clay samples at 5.2mBGL 230
8.69 Comparison of TE and SS shear strengths with Hight and Jardine (1993) TC envelopes (after Nishimura et al. [2007]) ................................. 231
8.70 Undrained shear strength profiles of three different shear modes (after Nishimura et al. [2007]) ................................. 231
8.71 Variation of elastic \( G_{oct} \) values with \( \alpha \) and \( b \) based on cross-anisotropic elastic theory ................................. 233
8.72 Torsional shear stiffness from CAU tests at \( b = 0, 0.3 \) and 1 ................................. 236
8.73 Octahedral shear stiffness in series AM\( \alpha \)-00 and AM\( \alpha \)-10 ................................. 236
8.74 Octahedral shear stiffness in series AM\( \alpha \)-03 ................................. 237
8.75 Octahedral shear stiffness from CIU tests (IM\( \alpha \)-05) ................................. 237
8.76 Normalised 3\( G_{oct} \)/\( p' \) under \( b = 0 \) condition ................................. 238
8.77 Normalised 3\( G_{oct} \)/\( p' \) under \( b = 1 \) condition ................................. 238
8.78 Normalised 3\( G_{oct} \)/\( p' \) under \( b = 0.3 \) condition ................................. 239
8.79 Normalised 3\( G_{oct} \)/\( p' \) under \( b = 0.5 \), CIU tests ................................. 239
8.80 Normalised stiffness data of triaxial CAU compression tests, Heathrow T5 site, after Hight et al. (2002) ................................. 240
8.81 Normalised stiffness data of triaxial CAU extension tests, Heathrow T5 site, after Hight et al. (2007) ................................. 240
8.82 Modes of failure in torsional shear tests ................................. 242

9.1 Scheme of kinematic multiple yield surfaces (Jardine 1992) ................................. 250
9.2 Schematic diagram illustrating the calculation of the incremental strain energy ................................. 252
9.3 Schematic presentation of the initial effective stress origins and outgoing stress paths performed in this study ................................. 254
9.4 Identification of \( Y_2 \), anisotropic reconsolidation stage ................................. 259
9.5 Limits of shear stress \( t \) for \( Y_2 \), anisotropic reconsolidation stage ................................. 259
9.6 Incremental strain energy, anisotropic reconsolidation stage ................................. 260
9.7 Identification of \( Y_2 \) from \( \epsilon_v \sim \epsilon_d \), CID uniaxial tests ................................. 260
9.8 Stress limits of \( Y_2 \), CID uniaxial tests ................................. 261
9.9 Incremental strain energy \( \Delta U \), CID uniaxial tests ................................. 261
9.10 Identification of \( Y_2 \) in \( \epsilon_v \sim \epsilon_d \) or \( \epsilon_v \sim \gamma_e \), CAD uniaxial tests ................................. 262
9.11 Stress limits of \( Y_2 \), CAD uniaxial tests ................................. 263
9.12 Incremental strain energy \( \Delta U \), CAD uniaxial tests ................................. 263
9.13 Identification of \( Y_2 \) limits, drained \( b = 1 \rightarrow 0.3 \) stages ................................. 264
9.14 Limits of \( Y_2 \) in terms of deviatoric shear strain as established in Figure 9.13 from drained \( b = 1 \rightarrow 0.3 \) stages ................................. 264
9.15 Incremental strain energy, drained \( b = 1 \rightarrow 0.3 \) stages ................................. 265
9.16 Corresponding \( Y_2 \) limits as established in Figure 9.13 from drained \( b = 1 \rightarrow 0.3 \) stages ................................. 265
9.17 Identification of \( Y_2 \) limits in undrained unloading stages using strain data ................................. 269
9.18 Identification of \( Y_2 \) limit in undrained unloading stages using pwp measurement ................................. 270
9.19 Limits of \( Y_2 \) in \( \epsilon_d \), undrained unloading stages ................................. 270
9.20 Incremental strain energy during undrained unloading stages ................................. 271
9.21 Comparison of \( \Delta U \) calculated by local and semi-local strain measurements ................................. 271
9.22 Identification of \( Y_2 \) limit in series AM\( \alpha \)-00 (at \( b = 0.0 \)) using strain data, multi-axial probes from point D ................................. 272
9.23 Identification of \( Y_2 \) limit in series AM\( \alpha \)-00 (at \( b = 0.0 \)) using pwp measurement, multi-axial probes from point D ................................. 272

xix
9.24 Limits of $Y_2$ in $c_d$, series AM$\alpha$-00 (at $b = 0.0$), multi-axial probes from point D ........................................... 273
9.25 Incremental strain energy in series AM$\alpha$-00 (at $b = 0.0$), multi-axial probes from point D ........................................... 273
9.26 Identification of $Y_2$ limit in series AM$\alpha$-03 (at $b = 0.3$) using strain data, multi-axial probes from point D ................................. 274
9.27 Identification of $Y_2$ limit in series AM$\alpha$-03 (at $b = 0.3$) using pwp measurement, multi-axial probes from point D ........................................... 274
9.28 Limits of $Y_2$ in $c_d$, series AM$\alpha$-03 (at $b = 0.3$), multi-axial probes from point D ........................................... 275
9.29 Incremental strain energy in series AM$\alpha$-03 (at $b = 0.3$), multi-axial probes from point D ........................................... 275
9.30 Identification of $Y_2$ limit in series AM$\alpha$-10 (at $b = 1.0$) using strain data, multi-axial probes from point D ........................................... 276
9.31 Identification of $Y_2$ limit in series AM$\alpha$-10 (at $b = 1.0$) using pwp measurement, multi-axial probes from point D ........................................... 276
9.32 Limits of $Y_2$ in $c_d$, series AM$\alpha$-10 (at $b = 1.0$), multi-axial probes from point D ........................................... 277
9.33 Incremental strain energy in series AM$\alpha$-10 (at $b = 1.0$), multi-axial probes from point D ........................................... 277
9.34 Identification of $Y_2$ limit in series IM$\alpha$-05 (at $b = 0.5$) using strain data, multi-axial probes from point A ........................................... 278
9.35 Identification of $Y_2$ limit in series IM$\alpha$-05 (at $b = 0.5$) using pwp measurement, multi-axial probes from point A ........................................... 278
9.36 Limits of $Y_2$ in $c_d$, series IM$\alpha$-05 (at $b = 0.5$), multi-axial probes from point A ........................................... 279
9.37 Incremental strain energy in series IM$\alpha$-05 (at $b = 0.5$), multi-axial probes from point A ........................................... 279
9.38 Experimental $Y_2$ surface in $\Delta p^' / \Delta q$ space, triaxial tests on London Clay samples from unit B2 (Gasparre 2005). Sample from 11.0 and 12.5 mOD, $P_0^' = 260$, $q_o = 86$ kPa ........................................... 285
9.39 Experimental $Y_2$ surface in $\Delta p^' / \Delta q$ space, triaxial tests on London Clay samples from unit B2 (Gasparre 2005). Sample from 11.0 and 12.5 mOD, $P_0^' = 260$, $q_o = 86$ kPa ........................................... 285
9.40 Normalised $Y_1$ locus in $\Delta p^' / P_0^' \sim \Delta q / P_0^'$ space, triaxial testing results after Gasparre (2005) ........................................... 286
9.41 Normalised $Y_2$ locus in $\Delta p^' / P_0^' \sim \Delta q / P_0^'$ space, triaxial testing results after Gasparre (2005) ........................................... 286
9.42 Yield $Y_1$ in the incremental effective stress space, (uniaxial, b-change and anisotropic-reconsolidation) drained tests ........................................... 287
9.43 $Y_1$ in the normalised incremental effective stress space ........................................... 287
9.44 Yield $Y_2$ in the incremental effective stress space, (uniaxial, b-change and anisotropic-reconsolidation) drained tests ........................................... 288
9.45 Yield $Y_2$ in the incremental effective stress space, (unloading and multi-axial stress path) undrained tests ........................................... 289
9.46 $Y_2$ in the normalised incremental effective stress space ........................................... 289

A.1 Calibration of axial load ........................................... 317
A.2 Calibration of torque component ........................................... 317
A.3 Calibration map for combined effects in load cell ........................................... 317
A.4 Calibration of the LVDT ........................................... 318
A.5 Calibration of the bi-axes electrolevel ........................................... 318
A.6 Calibration of the single-axis shear electrolevel ........................................... 318
B.1 Bulk modulus during isotropic consolidation, series AM\(\alpha\)-00 .......................... 320
B.2 Bulk modulus during isotropic consolidation, series AM\(\alpha\)-03 .......................... 321
B.3 Bulk modulus during isotropic consolidation, series IM\(\alpha\)-05 .......................... 322
B.4 Bulk modulus during isotropic consolidation, series AM\(\alpha\)-10 .......................... 323
B.5 Bulk modulus during isotropic consolidation, series HC-uniaxial ......................... 324
B.6 Development of strains during anisotropic re-consolidation, series AM\(\alpha\)-00 ........ 325
B.7 Development of strains during anisotropic re-consolidation, series AM\(\alpha\)-03 ........ 326
B.8 Development of strains during anisotropic re-consolidation, series AM\(\alpha\)-00 ........ 327
B.9 Development of strains during anisotropic re-consolidation, series AM\(\alpha\)-10 ........ 328
B.10 Development of strains during anisotropic re-consolidation, series HC-uniaxial ...... 329
B.11 Stress-strain during anisotropic re-consolidation, series AM\(\alpha\)-00 ...................... 330
B.12 Stress-strain during anisotropic re-consolidation, series AM\(\alpha\)-03 ...................... 330
B.13 Stress-strain during anisotropic re-consolidation, series AM\(\alpha\)-10 ...................... 331
B.14 Stress-strain during anisotropic re-consolidation, series HC-uniaxial .................. 331

C.1 Torsional shear stiffness in series AM\(\alpha\)-05 (semi-local strain measurement) ....... 333
C.2 Octahedral shear stiffness in series AM\(\alpha\)-05 (semi-local strain measurement) ...... 333

D.1 Failure pattern of sample in axi-symmetric triaxial compression condition .......... 335
D.2 Failure pattern of sample in axi-symmetric triaxial extension condition .......... 335
D.3 Failure pattern of sample in true triaxial condition with \(\alpha = 0^\circ, b = 0.3\) .......... 336
D.4 Failure pattern of sample in true triaxial condition with \(\alpha = 0^\circ, b = 0.3\) .......... 336
D.5 Failure pattern of sample in true triaxial condition with \(\alpha = 0^\circ, b = 0.5\) .......... 337
D.6 Failure pattern of sample in true triaxial condition with \(\alpha = 0^\circ, b = 0.5\) .......... 337
D.7 Failure pattern of sample in true triaxial condition with \(\alpha = 90^\circ, b = 0.5\) .......... 338
D.8 Failure pattern of sample in true triaxial condition with \(\alpha = 90^\circ, b = 0.5\) .......... 338
D.9 Failure pattern of sample in torsional shear condition with \(\alpha = 30^\circ, b = 0.3\) .......... 339
D.10 Failure pattern of sample in torsional shear condition with \(\alpha = 30^\circ, b = 0.5\) ........ 339
D.11 Failure pattern of sample in torsional shear condition with \(\alpha = 45^\circ, b = 0\) .......... 340
D.12 Failure pattern of sample in torsional shear condition with \(\alpha = 50^\circ, b = 0.5\) ........ 340
D.13 Failure pattern of sample in torsional shear condition with \(\alpha = 45^\circ, b = 1\) .......... 341
D.14 Failure pattern of sample in torsional shear condition with \(\alpha = 60^\circ, b = 0.5\) ........ 341
D.15 Failure pattern of sample in torsional shear condition with \(\alpha = 60^\circ, b = 1\) .......... 342
List of Symbols

Roman symbols

\( A \)  
clay activity
\( A, B \)  
Skempton's pore pressure coefficients
\( b \)  
= \( (\sigma'_2 - \sigma'_3)/(\sigma'_1 - \sigma'_3) \) intermediate principal stress ratio
\( e \)  
void ratio
\( E' \)  
Drained Young's modulus
\( E_u \)  
Undrained stiffness modulus
\( F \)  
axial load applied to hollow cylindrical specimen
\( G_{vh} \)  
shear modulus
\( G'_{oct} \)  
octahedral shear stiffness
\( G_s \)  
specific gravity
\( H \)  
specimen height
\( J \)  
= \( 1/\sqrt[6]{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2} \) second stress invariant
\( k \)  
permeability coefficient
\( K' \)  
drained bulk modulus
\( K_{0v} \)  
= \( \sigma'_{0v}/\sigma'_v \) coefficient of earth pressure at rest
\( M \)  
torque applied to hollow cylindrical specimen
\( OCR \)  
= \( \sigma'_{yield}/\sigma'_{v0} \) over consolidation ratio
\( p' \)  
= \( (\sigma'_1 + \sigma'_2 + \sigma'_3)/3 \) mean effective stress
\( p_i, p_o \)  
ininner and outer cell pressures (in HCA testing)
\( q \)  
= \( \sigma'_1 - \sigma'_3 \) deviatoric shear stress
\( r \)  
radial direction
\( r_i, r_o \)  
ininner and outer radii (for HCA specimen)
\( s' \)  
= \( (\sigma'_1 + \sigma'_3)/2 \) effective mean stress parameter (MIT notation)
\( S_u \)  
undrained shear strength
\( t \)  
= \( (\sigma'_1 - \sigma'_3)/2 \) maximum shear stress (MIT notation)
\( U \)  
strain energy
\( V_i, V_s \)  
ininner and soil specimen's volumes, respectively
\( Y_1, Y_2, Y_3 \)  
multiple sub-yielding surfaces (after [Jardine, 1992])
\( z \)  
vertical direction

Greek symbols

\( \alpha = \alpha_{\sigma} \)  
the angle between \( \sigma_1 \) direction and the vertical axis
\[ \alpha_{da} \] the angle between \( d\sigma_1 \) direction and the vertical axis
\[ \alpha_{de} \] the angle between \( d\epsilon_1 \) direction and the vertical axis
\[ \Delta h, \Delta H \] vertical displacement
\[ \Delta r, \Delta R \] radial displacement
\[ \Delta \theta, \Delta \Theta \] angular circumferential displacement (in HCA testing)
\[ \epsilon_1, \epsilon_2, \epsilon_3 \] major, intermediate and minor principal strains
\[ \epsilon_d = 2 \sqrt{\frac{1}{6} \cdot \left( (\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2 \right)} \] deviatoric shear strain
\[ \epsilon_r, \epsilon_z, \epsilon_\theta \] radial, axial and circumferential strain, respectively
\[ \epsilon_v \] volumetric strain
\[ \gamma_{13} = \epsilon_1 - \epsilon_3 \] maximum engineering shear strain
\[ \gamma_t \] total unit weight
\[ \gamma_{z\theta} = 2\epsilon_{z\theta} \] engineering torsional shear strain
\[ \nu' \] Poisson's ratio
\[ \phi' \] effective angle of shearing resistance
\[ \sigma'_1, \sigma'_2, \sigma'_3 \] major, intermediate and minor effective principal stresses
\[ \sigma'_r, \sigma'_z, \sigma'_\theta \] radial, vertical and circumferential effective stresses
\[ \tau_{z\theta} \] torsional shear stress (in HCA testing)
\[ \theta = \arctan \left( \frac{2b - 1}{3} \right) \] Lode's angle in the deviatoric \( \pi \)-plane
\[ w_n \] water content

**Prefixes**

\[ \mu \] micro \( \left( \times 10^{-6} \right) \)
\[ k \] kilo \( \left( \times 10^3 \right) \)
\[ M \] mega \( \left( \times 10^6 \right) \)

**Superscripts and subscripts**

\[ ' \] effective stress component
\[ p \] peak value

xxiii
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D, 3D</td>
<td>two- and three-dimensional</td>
</tr>
<tr>
<td>Avg.</td>
<td>Average value</td>
</tr>
<tr>
<td>A/D</td>
<td>analogue-digital converter</td>
</tr>
<tr>
<td>A/W</td>
<td>air-water interface</td>
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<tr>
<td>BS</td>
<td>British Standard</td>
</tr>
<tr>
<td>BE</td>
<td>bender element</td>
</tr>
<tr>
<td>CAD, CID</td>
<td>drained shear test from anisotropic or isotropic consolidation state</td>
</tr>
<tr>
<td>CAU, CIU</td>
<td>undrained shear test from anisotropic or isotropic consolidation state</td>
</tr>
<tr>
<td>DSC</td>
<td>directional shear cell</td>
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<tr>
<td>HCA</td>
<td>hollow cylinder apparatus</td>
</tr>
<tr>
<td>ID</td>
<td>inside diameter</td>
</tr>
<tr>
<td>LL</td>
<td>liquid limit</td>
</tr>
<tr>
<td>LVDT</td>
<td>linear variable differential transformer</td>
</tr>
<tr>
<td>max</td>
<td>maximum</td>
</tr>
<tr>
<td>min</td>
<td>minimum</td>
</tr>
<tr>
<td>mOD</td>
<td>Ordinance Survey elevation (in meter)</td>
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<tr>
<td>NC</td>
<td>normally consolidated state</td>
</tr>
<tr>
<td>OC</td>
<td>over consolidated state</td>
</tr>
<tr>
<td>OD</td>
<td>outside diameter</td>
</tr>
<tr>
<td>PL</td>
<td>plastic limit</td>
</tr>
<tr>
<td>PI</td>
<td>plasticity index</td>
</tr>
<tr>
<td>PSA</td>
<td>plane strain apparatus</td>
</tr>
<tr>
<td>RC</td>
<td>resonant column (dynamic testing)</td>
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<tr>
<td>Std. dev.</td>
<td>standard deviation</td>
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<tr>
<td>SBS</td>
<td>state bounding surface</td>
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<tr>
<td>SSA</td>
<td>simple shear apparatus</td>
</tr>
<tr>
<td>TC/TE</td>
<td>triaxial compression/extension test</td>
</tr>
<tr>
<td>TTA</td>
<td>true triaxial apparatus</td>
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<tr>
<td>VG</td>
<td>volume gauge</td>
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</table>
Chapter 1

Introduction

1.1 Background

In the design of foundations for off-shore structures, or for deep excavations, or for dams and multi-stage construction embankments there are critical conditions in which not only the magnitudes but also the directions of principal stress axes must be considered. Anisotropy refers to the directional dependence of material properties. The influence of anisotropy on soil behaviour has been widely acknowledged since the early 1970s (see Bjerrum [1973]; Arthur et al. [1980]; Saada and Townsend [1981]; Jamiolkowski et al. [1985]) and several reports addressing the soil mechanical anisotropic behaviour drawn from experimental research have been published (Saada and Puccini [1987]; Saada [1988]; Saada et al. [1993]; Jardine et al. [1997]; Tatsuoka et al. [1997]; Jardine et al. [2001]; Leroueil and Hight [2003]; Jardine et al. [2004]). However, there is still a gap in the available database for natural stiff clays.

Recent developments in numerical analysis have allowed the deployment of constitutive models that can reproduce some aspects of the shear strength anisotropy, and several researchers (e.g. Whittle et al. [1993]; Zdravkovic et al. [2001, 2002]) have illustrated this benefit in predicting the failures of excavation, embankment and foundation on soft clays. In addition, constitutive models that take into account stiffness anisotropy at small strains have attracted more attention in recent years (e.g. Lee and Rowe [1989a, b]; Gunn [1993]; Addenbrooke et al. [1997]; Simpson et al. [1996]; Hird and Pierpoint [1997]; Puzrin et al. [2001]; Potts and Zdravkovic [2001]; Franzius [2003]; Grammatikopoulou [2004]). Although there are different views as to the controlling factors that enhance the predictions of ground movements due to tunnelling and excavation, the stiffness anisotropy at small strains and its degradation with shear strain remain as sources of uncertainty and need to be investigated.

London Clay is one of the few natural soils that have been studied extensively (see Figure 1.1) and a substantial effort has been made to understand the clay’s anisotropy in shear strength and stiffness at small strain. However, reviewing its behaviour with data collected until 2003, Hight et al. [2003] noted that the understanding of London Clay anisotropy had relied principally on triaxial tests in which only limited loading directions could be studied. Apart from a limited number of tests on clay samples from Sizewell site reported by Porovic [1995], the database considering more generalized loading conditions was sparse. Considering the recent advance in numerical analysis and laboratory testing, it is now desirable to improve our understanding of the clay’s shear strength...
1.2. OBJECTIVE AND SCOPE OF THE RESEARCH

In view of the current lack in the database concerning the mechanical anisotropy of London Clay, the present study was carried out to investigate its anisotropic stress-strain-strength characteristics through laboratory shear tests. The clay's behaviour was studied by stress paths that involved four-dimensional stress conditions in a Hollow Cylinder Apparatus (HCA), in which the principal stresses ($\sigma_1, \sigma_2, \sigma_3$) along with the direction of the major principal stress axis in the vertical plane ($\alpha$) could be controlled independently. The samples were obtained at shallow depth by block sampling technique from the same horizontal level (12.5 mOD, i.e. about 5.2 m below ground level) at Heathrow Terminal 5 (T5) construction site.

This research was performed in parallel with the work of Gasparre (2005), Nishimura (2006) and Mannion (2007) who contributed to a comprehensive programme of oedometer, triaxial, simple shear, resonant column, HCA tests and lithological studies. Fellow researchers mainly used rotary-cored and block samples obtained at deeper elevations at the same site. In addition to this, an industrial programme of field and laboratory tests has also been conducted (Hight et al., 2003). Some of the results from these tests are mentioned in this thesis to assist the author’s interpretations.

The author’s study has the following objectives and scope:

and stiffness anisotropy.

1.2 Objective and scope of the research

In view of the current lack in the database concerning the mechanical anisotropy of London Clay, the present study was carried out to investigate its anisotropic stress-strain-strength characteristics through laboratory shear tests. The clay's behaviour was studied by stress paths that involved four-dimensional stress conditions in a Hollow Cylinder Apparatus (HCA), in which the principal stresses ($\sigma_1, \sigma_2, \sigma_3$) along with the direction of the major principal stress axis in the vertical plane ($\alpha$) could be controlled independently. The samples were obtained at shallow depth by block sampling technique from the same horizontal level (12.5 mOD, i.e. about 5.2 m below ground level) at Heathrow Terminal 5 (T5) construction site.

This research was performed in parallel with the work of Gasparre (2005), Nishimura (2006) and Mannion (2007) who contributed to a comprehensive programme of oedometer, triaxial, simple shear, resonant column, HCA tests and lithological studies. Fellow researchers mainly used rotary-cored and block samples obtained at deeper elevations at the same site. In addition to this, an industrial programme of field and laboratory tests has also been conducted (Hight et al., 2003). Some of the results from these tests are mentioned in this thesis to assist the author’s interpretations.

The author’s study has the following objectives and scope:
1.3 LAYOUT OF THE THESIS

- Using the new HCA Mark II \cite{jardine1996} to study the mechanical anisotropy of the clay from very small strains up to failure in various shearing modes;

- Investigating the degrees of anisotropy in shear strength and deformation properties of the clay in a particular shallow horizon (5.2 mBGL);

- Evaluating the influences of the intermediate principal stress ratio $b$ on the clay’s mechanical behaviour, and accessing the relevance of some classic plastic failure criteria (Mohr-Coulomb, Lade-Duncan and Matsuoka-Nakai) in describing the shear strength of London Clay in the deviatoric stress space;

- Determining the full sets of drained elastic parameters (at very small strains, around 0.001\% ) of the clay at the expected in situ stress conditions and evaluating the possible application of cross-anisotropic elasticity theory to predict the stress-strain response within this elastic zone;

- Studying the pre-failure nonlinear and anisotropic stress-strain responses of the clay and using a multiple sub-yield surface framework proposed by \cite{jardine1992} to interpret this behaviour.

1.3 Layout of the thesis

Overall the thesis consists of ten chapters, including this introductory chapter. Chapter 2 introduces the definition of anisotropy, its components and causes. The potential advantages of using an HCA to investigate the mechanical anisotropic behaviour of soils are then explained with due consideration to the possible stress and strain non-uniformities involved in hollow cylinder tests. The following part of the chapter discusses previous HCA research, most of which has concentrated on low OCR sediments and reconstituted clays from research laboratories worldwide, as well as on experience gained at Imperial College. The chapter also includes a review on the theory of elasticity for cross-anisotropic material and the application of this theory to predict the directions of effective stress paths and the elastic shear stiffness moduli with changes in $\alpha$ and $b$.

Chapter 3 presents an overview of the geology and geotechnical properties of London Clay. The applications of a London Clay stratigraphical classification \cite{king1981} to characterise the clay’s engineering behaviour are highlighted, drawing on site characterisation data from construction projects in and around the London area. The latter part of the chapter presents a review and discussions on the clay’s mechanical properties, with particular emphasis on the anisotropy of strength and stiffness of London Clay.

Chapter 4 describes the HCA testing apparatus used in this research. The local strain measurement system and the data acquisition and controlling programme for the HCA Mark II are also presented, including the developments undertaken to achieve the desired resolution and accuracy at very small strain (0.001\% ) in static HCA tests. The next part explains the method of calculation for strain components based on local strain transducers and highlights the importance of using local strain measurements if the pre-failure stress-strain response is investigated.

Chapter 5 details the geotechnical conditions at the site (Heathrow Terminal 5) where the London Clay specimens were obtained and explains the method used in the preparation of the hollow cylinder specimens. The testing procedures for the two main test series, carried out on London Clay
samples obtained from 5.2m below ground level, are then explained. The first series investigates the clay’s pre-failure yielding and deformation characteristics through HCA tests involving monotonic uniaxial stress changes under drained conditions. The second series investigates the strength and stress–strain anisotropy of London Clay using HCA tests with controlled values of $\alpha$ and $b$, termed as multi-axial shear tests.

Chapter 6 summarises the responses of the London Clay during the isotropic and anisotropic re-consolidation stages. The re-consolidation stress paths were conducted to bring the samples to the estimated in situ anisotropic stress state. The changes in states of the samples are reported. There is evidence of the inherent anisotropy of the London Clay based on the observed response of strains at early stages when strains were less than 0.2%.

Chapter 7 presents the results of the investigation into the clay’s anisotropic deformation properties in drained conditions. The chapter places particular emphasis on parameters derived at very small strains for the quasi–elastic compliance matrix, assuming the clay behaves as a cross-anisotropic elastic material. The effects of void ratio, effective stress level and re-consolidation stress history on stiffness moduli are also examined. Overall the author’s uniaxial shear test series comprises the first static hollow cylinder experimental database considering the drained stiffness anisotropy of the natural London Clay. The drained elastic parameters are compared with those obtained from other techniques, including resonant column, bender element and static triaxial stress probing tests as reported by Gasparre et al. (2007a).

Chapter 8 discusses the strength and yielding behaviour of London Clay as established by undrained multi-axial shear test series, in which $\alpha$ and $b$ were controlled independently. The results are reported in the anisotropic stress space and combined with those from other depths and tested in different shearing modes (triaxial and simple shear) as reported by Nishimura et al. (2007). The effective stress path directions, the non-linear stress–strain relationships, the effective and total shear strength parameters and the secant octahedral shear stiffness characteristics under undrained conditions are also discussed. In addition, the clay’s shear strengths are compared with an empirical expression for the undrained strength anisotropy suggested by Bishop (1966), as well as being examined in relation to several plastic failure criteria.

Chapter 9 investigates the progressive yielding characteristics of the London Clay observed from small to large strains during drained and undrained testing stages. The multiple yield surface framework proposed by Jardine (1992) was employed to explain the states of the clay in the pre-failure strain range. The kinematic sub-yield loci were constructed in the incremental stress planes. It was observed that the yield points identified from the drained and undrained tests appeared to be compatible in terms of the deviatoric shear strains and of the shear stress increments.

Finally Chapter 10 summarizes the findings of the research and their potential applications to engineering practice. Recommendations are made for future research work.

Supplementary information is presented in four appendices. Appendix A explains the calibration procedure for the HCA stress and strain measuring systems. Appendix B provides the results from the re-consolidation stages reported in Chapter 6. Appendix C presents the stiffness calculated using a semi-local strain approach of the test series AMn-05 introduced in Chapter 8, in which the full local strain sensor set had not been available. Appendix D presents sketches of the failure patterns observed in the HCA tests.
Chapter 2

The mechanical anisotropy of soils

The Chapter first describes the components of soil anisotropy, considering both its origin and fabric. Next 2.2 discusses the relevance of using hollow cylinder apparatus (HCA) for the investigation of anisotropy. The stress and strain non-uniformity problems in HC testing are presented in 2.3 followed by proposals for countermeasures to limit these effects. Next 2.4 reviews previous hollow cylinder tests on the mechanical anisotropy of soils, with particular reference to cohesive soils and natural clays. The theory of elasticity for cross-anisotropic material is reviewed in 2.5. This section also describes the method to determine elastic parameters and the application of the theory to predict the directions of effective stress paths and the elastic shear stiffness moduli with changes in $\alpha$ and $b$. Finally 2.6 summarises the main points of interest.

2.1 The components of anisotropy

Following the definitions of Casagrande and Carillo (1944), there are two components of material anisotropy: inherent and induced anisotropy. The latter due exclusively to strains arising from applied stresses, whereas inherent anisotropy can be considered as a material characteristic that is entirely independent of applied stress.

In the field laminated and bedded soils are obvious examples of soils with inherent anisotropic fabric. Other examples are sediments having large non-spherical grains as their long axes tend to align perpendicularly to the direction of deposition. Experiments also indicate that samples from spherical glass balls pluviated under gravity develop anisotropic structures that cannot arise from the grain orientations (e.g. Oda, 1972a; Shibuya, 1985). Moreover, tests on reconstituted granular samples that are of similar particles and densities but prepared by different methods (air pluviation, water pluviation and dry rodding) have shown different degrees of anisotropy in their stress-strain and volumetric behaviour under the same testing conditions (e.g. Tatsuoka et al., 1979; Georgiannou et al., 1990; Vaid et al., 1990; Chaudhary et al., 2002). It has been recognised that the inherent anisotropy of granular soils depends on several factors: the grain shapes and sizes, the vertical variation in grain-size distribution and the nature of the depositional environment. For cohesive soils the sources are likely from the geological processes and stresses occurred during sedimentation.

The stress-induced anisotropy is developed as soon as loads are applied: the particles assemble, even if initially of isotropic fabric, instantly form interparticle structure which is anisotropic. As
the loads are changing then new sets of interparticle contacts are established and deformations are accumulated. The pattern of the load carrying network can be visualized from the distributions of interparticle contact forces. These forces tend to concentrate into internal load-carrying structures, in which the force chains align roughly with the direction of the applied major principal stress. Such re-orientations of particles have been observed in laboratory experiments (see Oda [1972b]; Drescher and De Josselin De Jong [1972], and in numerical simulations using discrete element method (e.g. Dobry and Ng [1989]; Oda and Iwashita [1999]). It is also recognised that any rotation of principal axes requires re-alignment of the load carrying contacts and is likely to imply softer material response than from a loading condition in which the direction of major principal stress is kept unchanged. Thus the resulting induced-anisotropic responses are dependent on stress states.

In addition to this is the strain-induced anisotropy that commonly associated with processes of one-dimensional straining history like sedimentation, gravitational compaction and erosion. As a result, the yield loci of natural soils are often aligned with the $K_0$ consolidation line (Leroueil and Hight [2003]). Other post-depositional processes – aging, chemical bonding, biological activity – can further modify the anisotropic characteristics of soils. Due to these complex imposed stress and strain histories most natural soils show remarkable anisotropic behaviour.

Although the inherent and induced anisotropy are of different origins but it has been acknowledged that separating their influences in experiments is difficult. In reality, most natural soils possess both aspects that combine to define their initial anisotropy. Hereafter, the term anisotropy is referred to this combination, except where it is necessary to make a distinction between the two components. From practical viewpoints it is also of most interest to the engineers.

### 2.2 Laboratory testing for the investigation of anisotropy

#### 2.2.1 Generalised parameters for stresses

The generalised six independent stress components ($\sigma_{ij}$ with $i, j = 1 \sim 3$ and $\sigma_{ij} = \sigma_{ji}$ for $i \neq j$) acting on an soil element can be equally represented by the magnitudes of the major, intermediate and minor principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) and the corresponding orientations of the planes in which they act [Figure 2.1].

The magnitudes of the three effective principal stresses can be expressed by:

$$
\begin{align*}
\sigma'_1 &= \rho' + 2J / \sqrt{3} \sin(\theta + 2\pi/3) \\
\sigma'_2 &= \rho' + 2J / \sqrt{3} \sin(\theta) \\
\sigma'_3 &= \rho' + 2J / \sqrt{3} \sin(\theta - 2\pi/3)
\end{align*}
$$

(2.1)

Note here that the normal stresses are in terms of effective stress, $\sigma' = \sigma - u$, and:

$$
\begin{align*}
\rho' &= (\sigma'_1 + \sigma'_2 + \sigma'_3)/3 \\
J &= \frac{1}{6} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2} \\
\theta &= \arctan \left[ (2b - 1) / \sqrt{3} \right]
\end{align*}
$$

(2.2)
and $b$ is the intermediate principal stress ratio which relates the magnitude of $\sigma'_2$ to $\sigma'_1$ and $\sigma'_3$:

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3}$$

(2.3)

therefore $0 \leq b \leq 1$. For symmetric loading $b$ equals to either 0 (triaxial compression) or 1 (triaxial extension). In plane strain condition ($\epsilon_2 = 0$) $b = 0.3 \sim 0.5$.

As seen in Figure 2.2, the mean effective stress $p'$ is related to the distance along the space diagonal of the current deviatoric plane from the origin $(OA)$. In the deviatoric plane, the second invariant of the stress deviator $J$ is a measure of the current stress state from the space diagonal $(AN)$ and the Lode's angle $\theta \left(-\pi/6 \sim \pi/6\right)$ indicates the orientation of the stress state. None of these parameters, however, indicates the directions of the principal stresses.

For tests that involve or permit the rotations of principal axes, additional variables in the forms of either the shear stresses or the directions of the principal axes are needed. As a consequence of the one-dimensional straining depositional history, most soils posses cross-anisotropy characteristics, for which the horizontal plane (parallel to the bedding plane) is commonly associated with isotropic response and the axis of symmetry is vertical. For the particular cases of loading conditions in which $\sigma'_2$ acts parallelly to the bedding plane only one stress parameter, angle $\alpha$ of the $\sigma'_1$ direction with respect to vertical axis, is required to describe the inclinations of the major and minor principal stresses. There is, as the discussions in the next section reveal, only a few laboratory devices can perform tests in which this angle is controlled at will. So far the maximum degrees of freedom for stress state that can be achieved are four ($J, p', \alpha$ and $b$) on current laboratory strength testing devices.

### 2.2.2 Suitability of hollow cylinder device to study soil anisotropy

There have been a number of comprehensive reviews (e.g. [Saada and Townsend, 1981; Airey and Wood, 1988; Arthur, 1988; Tatsuoka, 1988]) on the relevance of laboratory devices for the study of soil strength. Several criteria have been addressed to achieve a suitable device. First, it is desirable to have uniformities in the stress and strain distributions within the specimen. This and the capability of reliable determinations of boundary loads and displacements are vital for any device as only then the resulting stress and strain states within the specimen can be accurately defined. Secondly, the ability to control both the magnitudes and directions of the principal stresses during shear is ideal to understand soil anisotropy. As a result, the choices of testing devices become rather limited.

The common triaxial apparatus has widespread use in laboratory testing but it offers only two restricted stress states at failure, in which a sudden change from $\alpha = 0^\circ$ (compression) to $90^\circ$ (extension) is coupled with a jump of $b$ (either 0 or 1). This combination inevitably complicates the assessment of anisotropy. The limitation in $\alpha$ rotation is still remained in tests using plane strain (PSA) or true triaxial (TTA) apparatus although $b$ can be controlled in the latter condition. For simulation of a rotation, some studies used specimens cut at various inclinations to the direction of deposition and subjected them to compression tests. However such practice, except for the case of horizontally cut specimen, results to significant undesirable shears and bending moments at the specimen’s ends hence creates doubts in these results (Saada, 1970).

A better approach is to keep the specimens vertical while subjected to inclinations of the principal stresses. Under these conditions and providing stresses are uniform and axially symmetric, no
tendency for out-of-plane shears will be generated when changes in $\alpha$ are imposed. Well-designed directional shear cell (DSC) and hollow cylinder apparatus (HCA) are recognised to be the suitable devices to achieve these requirements.

The DSC is originally developed by Prof. Arthur and his co-workers at University College London (Arthur et al., 1977, 1979, 1980). It is essentially a plane strain device which can impose subjectively normal and shear stresses on the sides (flexible boundaries) of a cubical specimen and therefore three degrees of freedom ($\sigma_1$, $\sigma_3$ and $\alpha$) can be controlled (Figure 2.3). The original design for the testing of sands was later modified at MIT for tests on reconstituted Boston clay specimens (Germain, 1982; Seah, 1990). However, the DSC has some inherent problems. Firstly, the presence of the edges leads to non-uniformities in the stress and strain distributions. The normal stresses at edges can be either too high if using fixed edge space frame or too low if membrane deficiency occurring after a certain sample strain. In addition, the incorrect edge stresses can induce progressive failure at the corners. Although modifications to corner design helped to reduce these effects but the amount of errors with respect to strains that should be corrected has been difficult to establish (Arthur, 1988). Secondly, the side membranes have limited strength which in turn do not allow large differences in stress. Finally, the application of rapid shearing to mimic undrained condition results to very undesirable rheological effects.

Thus the DSC offers attractive capabilities for the study of strength anisotropy, but there are problems that hamper its wider applications. In comparison, the HCA brings more advantages for its versatility to control at will not only $\alpha$ but also $b$ changes, its similarities to the triaxial stress path cell and its popularity in applications on advanced soil and rock mechanics research.

Figure 2.4 shows the boundary loads imposed on a hollow cylinder specimen and the stress state that can be stimulated on an imaginary element at the specimen wall. The axial load generates a vertical stress; the torque develops complementary torsional shear stresses in the vertical and horizontal planes; and the inner and outer cell pressures create gradient distributions of both radial stress and circumferential stress in the horizontal plane. If the considered element is free from end effects then there are no shear stresses in the radial circumferential plane. The radial stress is a principal stress and the other three stress components define the remaining two principal stresses. As the boundary loads can be applied independently, it is possible to control the magnitudes of the three principal stresses and the inclination of the major principal stress as seen in Figure 2.5. In case where the inner and outer cell pressures are equal ($p_i = p_o$) it can be shown that the $\alpha$ and $b$ are related via $b = \sin^2 \alpha$, otherwise this restriction is removed. The versatility in applying different stress states at failure in HC testing is therefore obvious.

Apart from the capability to generate torsional shear stresses and its different construction to accommodate hollow cylinder specimens, the HCA resembles very closely a stress-path triaxial cell. Accordingly it can do as much as a triaxial apparatus: applying back pressure, performing either undrained or drained tests and imposing anisotropic consolidation schemes. Furthermore similar sample preparation techniques, loading arrangements and instruments that have been developed for triaxial tests can also apply to HC tests. The popularity of HCA for soil testing has been shown by Saada and Townsend (1981); Saada et al. (1983); Tatsuoka et al. (1986); Saada (1988) and Jardine et al. (1997), among others. Recent developments in apparatus design has made it become even more attractive. For example, Frost and Drnevich (1994) introduced a HCA system with resonant column capability and Jardine et al. (2004) reported a new HCA that had been equipped with local strain transducers which facilitate the investigation of stress-strain response at small strains.

The simple shear apparatus (SSA) can also apply normal and shear stresses to the boundary of
2.3 STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

Figure 2.1: Generalised stress state: a) Stress components; b) Principal stresses and directions

To summarise the picture, Figure 2.6 indicates the applicable stress states at failure by some of the advanced laboratory strength devices. By far the HCA is the most versatile device as four degrees of freedom can be subjectively controlled. Its problems with stress and strain non-uniformities, as presented in the following section, can be practically limited by proper selections on specimen dimensions and applied stress paths.

2.3 Stress and strain non-uniformities in HC testing and proposal for countermeasure

The stresses and strains on a hollow cylinder specimen are not uniform due to the combined effects of wall curvature and end restraint. On the one hand, the difference between inner and outer cell pressures leads to gradient distributions across the wall of both $\sigma_r$ and $\sigma_\theta$. Moreover, once torque is applied then non-uniform distributions of stresses are unavoidable even if the cell pressures are equal. On the other hand, due to the end restraint the distribution of $\sigma_z$ is non-uniform and radial shear stresses $\tau_{zr}$ are developed when radial straining occurs. As for wall curvature, its influences can be reduced if specimen of thin wall and large diameter is used. To limit the end effect having a tall specimen can help since $\tau_{zr}$ is self-equilibrating and its effects vanish as one moves away from the ends according to St. Venant’s principal.
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

\[ \sigma_1^' = \frac{1}{2} (\sigma_1 + \sigma_3) \]

\[ \sigma_2^' = \frac{1}{2} (\sigma_1 + \sigma_3) \]

\[ \sigma_3^' = \frac{1}{2} (\sigma_1 + \sigma_3) \]

Space diagonal \( (\sigma_1^' = \sigma_2^' = \sigma_3^') \)

Figure 2.2: Generalised stress state: a) Principal stresses; b) Deviatoric plane

\[ \tan 2\alpha = \frac{2\tau_{xz}}{\sigma_z - \sigma_x} \]

Plane strain condition \( \epsilon_y = \epsilon_z = 0 \)

Figure 2.3: Directional shear cell: a) Stress components; b) Principal stresses and their directions
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

Figure 2.4: Hollow cylinder testing: a) Boundary loads; b) Individual stress components; and c) Principal stresses and their directions

Figure 2.5: Definition of the major principal stress rotation angle $\alpha$

$$\tan 2\alpha = \frac{2\tau_{z\theta}}{\sigma_z - \sigma_\theta}$$
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

Legends:
TC, TE: Triaxial compression ($\alpha = 0^\circ, b = 0$) and extension ($\alpha = 90^\circ, b = 1$)
PSC, PSE: Plane strain compression and extension (under condition of $\epsilon_2 = 0$)
TTC, TTE: True triaxial compression and extension (Range $0 \leq b \leq 1; \alpha = 0$ or $90^\circ$)
DSC: Directional shear cell ($0^\circ \leq \alpha \leq 90^\circ$ under condition of $\epsilon_2 = 0$)
HCA: Hollow cylinder apparatus at all possible regions ($p_o, p_i$ are outer and inner cell pressures).

Figure 2.6: Applicable stress states at failure in some advanced laboratory strength devices

Two important questions should be addressed in the assessment of non-uniformity. First, is how to evaluate the deviations of the averaged values from the analytical distributions. Several evaluations in terms of stress variations along the height (Lade, 1976; Saada and Townsend, 1981) and across the wall of the specimen (Symes, 1983; Hight et al., 1983; Gens and Potts, 1984; Sayao and Vaid, 1991; Wijewickreme and Vaid, 1991) have been proposed, from which there are recommendations for specimen sizes and the corresponding applicable stress paths such as the regions of no–go stress state (Figure 2.7). However, these studies considered the two effects separately rather than simultaneously. Furthermore, the specimen was not subjected to the complete boundary conditions anticipated in real tests. As a result the expected errors in terms of engineering properties (e.g. peak shear strength, angle of shearing resistance, strain components, inclination of stress path and shear stiffness) cannot be evaluated.

Such concerns bring forward the second question: to what extend the measured response would depart from the idealised behaviour. Here the idealised behaviour is of a single element for which the states of stress and strain are uniform because there is no curvature or end restraint effects. In comparison, the measured response is calculated from simulations that are modelled by realistic values of the specimen’s dimensions and the boundary loads. By comparing the stress-strain responses between these two cases the net errors of non-uniformity can therefore be quantitatively estimated.

A number of studies in this direction have been conducted at Imperial College using the in house programme ICFEP (Potts and Zdravkovic, 1999). These analyses employed the Fourier Se-
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

ries Aided Finite Element Method\(^1\) and isotropic elasto-plastic constitutive model\(^2\), either of Modified Cam Clay type with non-associated flow rule (e.g. \cite{Menkiti1995, Porovic1995, Rolo2003, Foundoukos2006}) or Lade’s double-hardening criteria (see \cite{Zdravkovic2005}). Each simulation was subjected to a stress history as in an experiment, i.e. including stages of consolidation, drained \(b\)-change, undrained unload and undrained shear with fixed \(\alpha\) angle. In addition, the stresses and strains were back calculated (from values at corresponding nodal points) as in the real case with local strain measurements. The effects of end restraint can be either accounted for or not by applying condition of perfect rough or smooth ends. An extensive analytical database has consequently been established for a wide ranges of specimen sizes \((H, D_o, D_i)\) and conditions of end restraint under different combinations of \(\alpha\) and \(b\) values in the final undrained shearing stage. This database is reported in details by \cite{Foundoukos2006}.

Table 2.1 summarizes the errors in the calculation of some engineering parameters at deviatoric shear strain of \(\varepsilon_d = 10\%\) found by \cite{Rolo2003} for three different specimen sizes and end conditions. The simulations reported here were for undrained shear conditions of \(\alpha = 0\) and \(30^\circ\) at \(b = 0.5\). The errors in \(\varepsilon_r\) and \(\gamma_{\theta\theta}\) were very small and therefore are not included in the summary, so as to the amount of errors in the predictions of strains for the smooth-end condition. The results illustrate three important points. First, his results confirm that a specimen of tall and thin wall with large outside diameter is an ideal configuration. Second, the disadvantages for specimen having small slenderness ratio, \(H/D_o\), can be compensated for if condition of perfectly smooth ends is presence. Third, the errors in predictions of strains are more significant than those of stresses. Consequently the measured undrained strength and mobilised shear resistance are likely less affected than the stress-strain relationship at large strains. Although the response at small strains was not directly discussed in his study, there were little deviations observed at strains \(\varepsilon_d < 3\%\) in terms of stress – strain and shear stiffness.

Recently \cite{Zdravkovic2005} conducted numerical simulations for four sets of specimen’s dimensions as seen in Table 2.2. Again the investigation highlights the similar advantages of smooth ends and the significant influences of the slenderness ratio. For example, for the undrained shear simulations at \(\alpha = 30^\circ\), \(b = 0.5\) the effective stress paths of the ideal test and specimens that have \(H/D_o = 2\) are almost identical, and those of smaller ratios are only slightly different under condition of smooth ends. In contrast, with rough ends the deviation magnifies; the largest is of the short and small specimen (RCTS 1:1) and the smallest is from the tall and large one (ICHCA 2H). Similar responses are noticed for the shear stress – shear strain relationships shown in Figure 2.9. However, the condition of smooth ends is rather optimistic for torsional shear test because the friction forces between the specimen and the loading platens are essential for torque transmission. It is interesting that the errors in shear stiffness \(G_{oct} = J/\varepsilon_d\) in rough ends cases are only significant at small strains, here at \(\varepsilon_d < 0.1\%\) (Figure 2.10).

Overall, the most practical solution for limiting the non-uniformity effects is to select a suitable specimen with due considerations for both aspect ratio and outside diameter. The dimensions chosen for the HCA Mark II specimens are in favor of these suggestions, as well as of the Japanese standard for hollow cylinder torsional shear tests \cite{JGS-0551, JGS-0543}. It is therefore expected that the levels non-uniformities have been effectively minimised and hence test results could be interpreted reliably in terms of the average stress and strain components, providing the imposed stress paths avoid the no-go regions.

---

\(^1\)This method can apply for problem that is axi-symmetric in geometry but whose boundary conditions and material properties can be non axi-symmetric. The gains in comparison with a full 3D analysis are less computational effort and simpler mesh.

\(^2\)Thereby avoiding the exaggerations in non-uniformities due to the use of a purely elastic model.
Table 2.1: Errors at $\varepsilon_d = 10\%$ in measured stresses and strains from HC simulations for undrained shear tests at $b = 0.5$ (after [Rolo2003])

<table>
<thead>
<tr>
<th>End condition</th>
<th>$\alpha$ [deg]</th>
<th>$p'$</th>
<th>$(\sigma_1 - \sigma_3)/2$</th>
<th>$\phi'$</th>
<th>$\varepsilon_z$</th>
<th>$\varepsilon_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough</td>
<td>0</td>
<td>+3.2</td>
<td>+6.1</td>
<td>+11.6</td>
<td>-22</td>
<td>+17</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>-1.9</td>
<td>-2.7</td>
<td>-0.8</td>
<td>-24</td>
<td>+9</td>
</tr>
<tr>
<td>Smooth</td>
<td>0</td>
<td>-2.8</td>
<td>+0.6</td>
<td>+4.2</td>
<td>not reported</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>-1.3</td>
<td>+0.1</td>
<td>+3.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) $H/D_o = 2$, $D_o = 250\text{mm}$, $D_i = 200\text{mm}$

<table>
<thead>
<tr>
<th>End condition</th>
<th>$\alpha$ [deg]</th>
<th>$p'$</th>
<th>$(\sigma_1 - \sigma_3)/2$</th>
<th>$\phi'$</th>
<th>$\varepsilon_z$</th>
<th>$\varepsilon_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough</td>
<td>0</td>
<td>+2.8</td>
<td>+0.6</td>
<td>-3.4</td>
<td>-5.6</td>
<td>+1.1</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>+1.4</td>
<td>+0.1</td>
<td>-1.2</td>
<td>-1.1</td>
<td>+1.6</td>
</tr>
</tbody>
</table>

C) $H/D_o = 2$, $D_o = 100\text{mm}$, $D_i = 70\text{mm}$

<table>
<thead>
<tr>
<th>End condition</th>
<th>$\alpha$ [deg]</th>
<th>$p'$</th>
<th>$(\sigma_1 - \sigma_3)/2$</th>
<th>$\phi'$</th>
<th>$\varepsilon_z$</th>
<th>$\varepsilon_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough</td>
<td>0</td>
<td>+0.3</td>
<td>+0.9</td>
<td>+1.9</td>
<td>-9.2</td>
<td>+5.3</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>-0.7</td>
<td>+0.3</td>
<td>+2.9</td>
<td>-14.1</td>
<td>+13.5</td>
</tr>
<tr>
<td>Smooth</td>
<td>0</td>
<td>-0.9</td>
<td>0</td>
<td>+2.1</td>
<td>not reported</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>-1.1</td>
<td>+0.7</td>
<td>+2.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Specimen dimensions used in the FE simulations by [Zdravkovic and Potts2005]

<table>
<thead>
<tr>
<th>Notation</th>
<th>$H/D_o$ [-]</th>
<th>$D_o$ [mm]</th>
<th>$D_i$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICHCA</td>
<td>1</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>RCTS</td>
<td>2.68</td>
<td>71</td>
<td>38</td>
</tr>
<tr>
<td>ICHCA-2H</td>
<td>2</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>RCTS-1:1</td>
<td>1</td>
<td>71</td>
<td>38</td>
</tr>
</tbody>
</table>
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

Figure 2.7: Regions of 'no-go' stress paths in the generalized stress space $t \sim b \sim \alpha$ (after Symes, 1983)

Figure 2.8: Effective stress paths for simulations of (a) smooth and (b) rough ends at $\alpha = 30^\circ$, $b = 0.5$ (after Zdravkovic and Potts, 2005)
2.3. STRESS AND STRAIN NON-UNIFORMITIES IN HC TESTING

Figure 2.9: Stress-strain relationship during shear for simulations at $\alpha = 30^\circ$, $b = 0.5$: a) Smooth and b) Rough ends (after Zdravkovic and Potts, 2005)

Figure 2.10: Shear stiffness $G_{\text{oct}}$ degradation curves for simulations at $\alpha = 30^\circ$, $b = 0.5$: a) Smooth and b) Rough ends (after Zdravkovic and Potts, 2005)
2.4 Review of HC tests on cohesive soils

Most of the available data of hollow cylinder tests on soils have focused on reconstituted samples of granular material, particularly sands and non-plastic silts. Examples are many, such as those described by Lade (1981), Saada and Bianchini (1987), and Vaid et al. (1990). In Japan, the application of hollow cylinder apparatus is also popular (e.g. Tatsuoka et al., 1986; Miura et al., 1986; Pradhan et al., 1988a,b; Ampadu and Tatsuoka, 1993; Ishihara, 1996; Habib and Towhata, 2002; Fukuda et al., 2003; Chaudhary, 2001; Nishimura and Towhata, 2004). Several systems of HCAs used by research groups in UK and Europe have been extensively reviewed by Rolo (2003).

On the contrary, fewer tests have been performed on clays, perhaps due to the complexity and expense of running long duration hollow cylinder tests. Other difficulties may include preparing tall and thin wall specimens from natural clays, or even making reproducible reconstituted specimens of suitable dimensions. To avoid being exhaustive, the following discussions only concentrate on HC tests on reconstituted and natural clays.

2.4.1 Tests on silt and reconstituted clays

The anisotropy of reconstituted $K_0$-consolidation Edgar Plastic Kaolinite (EPK) was investigated by Hong and Lade (1989a,b). The specimens were followed a $K_0 = 0.55$ consolidation path in the HCA to $p' = 196$ kPa (with back pressure of 196 kPa), and then subjected to undrained shear, except for two tests under drained shear. All tests were under the condition of constant and equal confining pressures ($p_o = p_i = 392$ kPa). It was observed that the strengths at the points of maximum effective stress ratio ($\sigma_1^e/\sigma_3^e$) were different. However, the authors argued that the specimens should possess insignificant initial anisotropy due to the high water content and short creep period from the reconstituted stage. Therefore the differences in shear strength were likely from the effects of $b$-change rather than principal stress rotation. They applied a three dimensional isotropic failure criterion\textsuperscript{3} (Figure 2.11) and found that it could predict reasonably well their experiment data.

In addition, strain cross-coupling was reported in these tests and this feature had also been observed from tests on other natural clays such as Mediterranean Sea soft clay (Frydman et al., 1995) and San Francisco Bay Mud clay (Lade and Kirkgard, 2000). Data obtained at Imperial College, which will be subsequently discussed, shows similar behaviour in tests on sands, silts and reconstituted clays.

Removing the condition of equal inner to outer cell pressure provides the flexibility to impose different values of $\alpha$ at the same $b$ value, thereby avoids any confusion between them. Table 2.3 summarizes the testing programme using this approach at Imperial College for reconstituted clayey soils and silts. Figure 2.12 illustrates a general decreasing trend at $b = 0.5$ with increasing $\alpha$ of the undrained shear strength after normalizing to the effective consolidation pressure, $S_u/p_0^0$, of some $K_0$ normally consolidated reconstituted soils.

\textsuperscript{3}Lade’s failure criterion for isotropic material: $(I_1^m/I_3^m - 27) I_3^m = \eta_1$, where $m$ and $\eta_1$ are material parameters; $I_1 = \sigma_1^e + \sigma_2^e + \sigma_3^e$; $I_3 = \sigma_1^e \cdot \sigma_2^e \cdot \sigma_3^e$ are the first and third invariants of the stress tensor. 

17
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

Figure 2.11: Applicability of Lade’s isotropic failure criterion to HCA tests on EPK specimens (Hong and Lade [1989a]): a) Determination of parameter $m$ and $\eta_1$; b) The fitting failure curve in the plane of $\sigma_0^r = 98\text{kPa}$; c) Predicted and observed angles of shearing resistance $\phi' = \arcsin\left(\frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}\right)$ with respect to $b$. 

18
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

Figure 2.12: Anisotropy of undrained shear strength at \( b = 0.5 \) of normally \( K_0 \)-consolidated soils \((OCR = 1)\) from the Imperial College database \((\text{Jardine et al., 1997})\). Values of \( S_u \) taken at peak shear stress, and at phase transformation point (PT) for HPF4 specimens.

Table 2.3: HCA tests on reconstituted \( K_0 \) recompression clayey soils and non-plastic silt at Imperial College London

<table>
<thead>
<tr>
<th>Soils</th>
<th>Author</th>
<th>( CF ) [%]</th>
<th>( p'_0 ) [kPa]</th>
<th>( K_0 )</th>
<th>( b )-ratios considered</th>
<th>Inclinations at shear ( \alpha ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HK</td>
<td>Menkiti (1995)</td>
<td>7</td>
<td>400</td>
<td>0.49</td>
<td>0, 0.5, 1</td>
<td>0, 22.5, 45, 90</td>
</tr>
<tr>
<td>KSS</td>
<td>Menkiti (1995)</td>
<td>44</td>
<td>300</td>
<td>0.585</td>
<td>0, 0.5, 1</td>
<td>0, 22.5, 45, 90</td>
</tr>
<tr>
<td>HPF4</td>
<td>Zdravkovic (1996)</td>
<td>0</td>
<td>200</td>
<td>0.5</td>
<td>0, 0.3, 0.5, 1</td>
<td>0, 15, 30, 45, 60, 90</td>
</tr>
<tr>
<td>HK15</td>
<td>Rolo (2003)</td>
<td>15</td>
<td>200</td>
<td>0.5</td>
<td>0, 0.3, 0.5, 1</td>
<td>0, 45, 90</td>
</tr>
</tbody>
</table>

Notes:
HK = a mixture of kaolin and sand (Ham River Sand)
KSS = a mixture of kaolin, sand and silt
HPF4 = non-plastic quart-based silt
HK15 = a mixture of kaolin and HPF4 silt
\( Zdravkovic (1996) \) also performed a series inclined-consolidation tests, which are not reported here.

\( CF \): Clay fraction

\( p'_0 \): Mean effective pressure at the end of anisotropic \( K_0 \) consolidation
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

2.4.2 Tests on natural clays

As mentioned previously, HCA tests on natural clay specimens have been rare. Torsional shear experiments on both reconstituted and intact specimens of Vallettica clay and Bisaccia clay (d’Onofrio and Magistris, 1998) indicated that the soil structure had notable effects on initial $G_0^\theta$ (Figure 2.13a) but negligible in damping ratio at small strain (Figure 2.13b). Using an HCA in which the inner and outer cell pressures were kept equal during shear, Kirkgard and Lade (1993) and Lade and Kirkgard (2000) conducted a series of tests on soft San Francisco Bay Mud. Figure 2.14, showing values of the angles of shearing resistance from torsion shear tests on $K_0$-consolidated samples and true triaxial tests on isotropically consolidated specimens, illustrates the degrees of initial anisotropy of the mud apart from the effects of $b$.

Hight et al. (1997) discussed the failure envelope and stiffness anisotropy of hard London Clay obtaining from depths greater than 50 m at Sizewell site. Three undrained and one drained torsional shear tests with resonant column measurements, as well as a number of triaxial compression tests, were conducted by Porovic (1995). Figures 2.15 and 2.16 show the torsional shear stress versus strain responses and the effective stress paths, respectively. It was found that at peak shear strength $\alpha \approx 40^\circ$ (associated with $b=0.41$) and the effective strength envelope in triaxial compression could be the upper bound to torsional shear strengths. In terms of shear modulus $G_{zb}$ at small strains ($\gamma_{zb} = 0.001\%$), the measured values by means of both resonant column and monotonic shear on hollow cylinder London clay specimens were roughly similar as evident in Figure 2.17. However, these were smaller than those measured by another laboratory from resonant column tests on solid specimens and from in-situ shear-wave measurements (see Figure 2.18). The authors argued that lower small strain shear stiffness from HCA tests was likely due to the effects of destructuring, shorter creep periods and imperfect compliance at specimen/end platen interfaces.

The same HCA system had been employed to investigate the stress-strain anisotropy of the soft Pentre clay-silt (Porovic, 1995) and soft Bothkennar clay (Albert et al., 2003). It is reported that the torsional peak shear strength of Bothkennar clay, where at failure $\alpha \approx 33^\circ$, was close to the triaxial compression strength envelope and higher than the triaxial extension envelope (Figure 2.19).

Leroueil et al. (2003) reported an investigation to the strength anisotropy of the soft Louiseville clay was carried out at the Universite Laval, Quebec, Canada. Two groups of strength envelopes were observed (Figure 2.20), one corresponding to $\alpha < 30^\circ$ and the other applying for $\alpha = 37 \sim 75^\circ$. Figure 2.21 shows the correlation between $\alpha$ values and the inclination angles $\theta$ of the observed failure surfaces, including the relationship predicted by Mohr’s failure criterion. It was recognised that the lowest strength associated with horizontal failure planes which are parallel to the bedding of Louiseville clay samples.

The tests on natural clays that are reviewed here share one common testing condition – during shearing the pressures in the inner and outer cells were kept equal. As a result, the mapping of the degree of anisotropy in $b - \alpha$ plane had been rather limited ($b = \sin^2 \alpha$), leaving yet investigations with more flexibility to control independently these two parameters. The HCAs employed by the Author and Nishimura (2006) to test natural London Clay specimens had the capability to avoid this restriction. It can also be seen that experiment data for stiff clays are limited, particularly for London Clay (see §3.2).
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

(a) Initial shear stiffness

(b) Hysteric damping ratio

Figure 2.13: The influence of soil structure on small strain stiffness parameters of two Italian stiff clays: A) Vallericca clay; and B) Bisaccaia clay ([d'Onofrio and Magistris](1998)).
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

(a) Stress paths

(b) Initial strength anisotropy

Figure 2.14: Strength anisotropy of San Francisco Bay Mud from torsional shear tests (after Lade and Kirkgard, 2000)
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

Figure 2.15: Torsional $\tau_\theta - \gamma_\theta$ in tests on London and Thanet clay specimens from Sizewell site (Hight et al., 1997).

Figure 2.16: Effective stress paths from torsional shear tests on clay specimens from Sizewell site (Hight et al., 1997). Also shown are failure envelopes obtained from triaxial compression tests.
Figure 2.17: Torsional shear modulus obtained from undrained torsional shear (both monotonic and resonant column) tests on hollow cylinder London clay samples (Hight et al. [1997]). a) Sample at 54.35m; and b) 57.50m.
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

Figure 2.18: Normalized modulus decay curves from resonant column and monotonic torsional shear tests [Hight et al. 1997]. RC tests on solid samples are shown with solid marks. Data from monotonic undrained torsional shear tests on hollow cylinder specimens (see Figure 2.17) are shown within the envelopes.

Figure 2.19: Profiles of peak $S_u$ from tests on samples at Bothkennar soft clay site [Albert et al. 2003]. All specimens are $K_0$ recompressed; and at failure the torsional shear tests associated roughly with $\alpha = 33^\circ$ ($b = 0.3$).
2.4. REVIEW OF HC TESTS ON COHESIVE SOILS

Figure 2.20: Strength envelopes of Louseveil clay (Quebec, Canada) from torsional shear tests \cite{Leroueil2003}

Figure 2.21: Relations between failure surface inclinations to $\alpha$ in torsional shear tests on Louseveil clay \cite{Leroueil2003}
2.5 Cross-anisotropic elasticity theory

2.5.1 Theoretical background

By definition, elasticity refers to a material response that involves no energy dissipation for any closed stress cycle irrespective of whether the stress-strain relationship is linear or non-linear. It should also be time-independent and not be influenced by the rate of stressing or straining. Soil behaviour at very small strain has been termed as quasi-elastic because it shows small hysteric loops of negligible energy losses and the stiffness within this range appears to be relatively insensitive to strain rate (Tatsuoka and Shibuya, 1991). However, the elastic compliance matrix, $[D_{ijkl}]$ or in short $[D]$, is (effective) stress-state dependent. In general the strain increment tensor can be related to a stress increment tensor by:

$$\Delta \varepsilon_{ij} = [D^{-1}] \cdot \{\Delta \sigma_{kl}\}$$  \hspace{1cm} (2.4)

One important feature of quasi-elastic behaviour is anisotropy. In a fully anisotropic material the directional dependence may develop in any of the $x$, $y$ or $z$ directions, giving a total of 36 terms in the compliance matrix. However, horizontally sedimented or reconstituted soils are often assumed to be isotropic in horizontal planes. Such soils possess cross-anisotropic properties and hence the number of elastic parameters are much less. Within the elastic limit and assuming the cross-coupling terms between normal strains and shear stresses are nil the compliance matrix can be expressed by:

$$[D^{-1}] = \begin{bmatrix}
\frac{1}{E_h} & -\nu_{hv}/E_v & -\nu_{vh}/E_v & 0 & 0 & 0 \\
-\nu_{hv}/E_v & \frac{1}{E_h} & -\nu_{vh}/E_v & 0 & 0 & 0 \\
-\nu_{vh}/E_v & -\nu_{hv}/E_v & \frac{1}{E_v} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{vh}} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{hv} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{hh}
\end{bmatrix}$$  \hspace{1cm} (2.5)

in which the Young’s moduli $(E_h, E_v)$ and Poisson’s ratios $(\nu_{vh}, \nu_{hv}, \nu_{hh})$ are defined in terms of effective stress for saturated soils, whereas the shear moduli $(G_{vh}, G_{hv}, G_{hh})$ are the same in drained or undrained conditions. Here $v$, $h$ indicate the vertical and horizontal direction, and the Poisson’s ratio $\nu_{ij}$ reflects the effect of normal strain in $i$-direction on $j$-direction strain.

The horizontal isotropic plane is the plane of symmetry:

$$G_{hh} = \frac{E_h}{2(1 + \nu_{hh})}$$  \hspace{1cm} (2.6)

and the symmetric condition about the diagonal of the compliance matrix gives (Love, 1927):

$$\nu_{vh}/E_v = \nu_{hv}/E_h, \hspace{1cm} G_{vh} = G_{hv}$$  \hspace{1cm} (2.7)

Equations 2.5, 2.6 and 2.7 indicate that five independent parameters are needed to describe a cross-anisotropic elastic material, normally chosen as $E_v$, $E_h$, $\nu_{vh}$, $\nu_{hv}$ and $G_{vh}$. Thermodynamic
2.5. CROSS-ANISOTROPIC ELASTICITY THEORY

law requires that stored energy must be positive for an elastic material and hence not only the Young's and shear moduli must be positive but the following conditions must also be satisfied (Pickering, 1970):

\[
-1 < \nu'_{hh} < 1
\]

\[
2E'_h \nu'_{ch}^2 \leq E'_v (1 - \nu'_{hh})
\]

\[
2(1 - \nu'_{hh}) \sqrt{E'_v / E'_h - \nu'_{ch}^2} + 2\nu'_{ch} (1 + \nu'_{hh}) \leq E'_v / G_{hv}
\] (2.8)

It is important to recognise that cross anisotropy is only applicable for horizontally-bedded soils in which the horizontal stresses have always been similar in all directions and the major principal stress has always acted vertically or horizontally. Natural or man-imposed stress changes can disrupt the horizontal anisotropy, as can sedimentation on slopping ground or due to other factors. Moreover, the compliance matrix is not unique but associated with a fixed stress point and soil state, and only applicable to very small imposed strain from that point (Hardin and Blandford, 1989; Tatsuoka and Shibuya, 1991; Porovic, 1995; Zdravkovic and Jardine, 1997; Kuwano and Jardine, 2002).

2.5.2 Determination of elastic parameters from laboratory shear tests

If the soil vertical axis of cross-anisotropy is aligned with the axis of the triaxial apparatus Equation 2.4 is simplified to:

\[
\begin{bmatrix}
\Delta \varepsilon_v \\
\Delta \varepsilon_h
\end{bmatrix} =
\begin{bmatrix}
1 / E'_v & -2\nu'_{ch} / E'_v \\
-\nu'_{ch} / E'_v & (1 - \nu'_{hh}) / E'_h
\end{bmatrix}
\begin{bmatrix}
\Delta \sigma'_v \\
\Delta \sigma'_h
\end{bmatrix}
\] (2.9)

It can be seen that only three of the five elastic properties can be directly defined in triaxial static probing tests: \(E'_v, \nu'_{ch}\), and the composite stiffness \(E'_h / (1 - \nu'_{hh})\). The compliance matrix is not symmetric because in this case the strain and stress increments are not work conjugate. To determine the full set of elastic parameters additional assumptions are needed (e.g. Graham and Houlsby, 1983; Hoque et al., 1996). To avoid this approach, static triaxial probing tests can be used together with bender element tests in vertical and horizontal directions (e.g. Pennington et al., 1997; Kuwano and Jardine, 1998; Lings, 2001; Kuwano and Jardine, 2002; Gasparre, 2005). However, such determination relies both on strain-rate independence and on the condition that bender element tests have unambiguous results.

One advanced method employing only static testing is to apply uniaxial stress probes with a hollow cylinder apparatus Equation 2.4 when expressed in terms of cylindrical coordinates \((z - r - \theta)\) space becomes:

\[
\begin{bmatrix}
\Delta \varepsilon_z \\
\Delta \varepsilon_\theta \\
\Delta \varepsilon_r \\
\Delta \gamma_{z\theta}
\end{bmatrix} =
\begin{bmatrix}
1 / E'_{zz} & -\nu'_{zz} / E'_z & -\nu'_{r\theta} / E'_z & -\nu'_{rz} / E'_r & 0 \\
-\nu'_{r\theta} / E'_z & 1 / E'_{\theta\theta} & -\nu'_{r\theta} / E'_r & 1 / E'_r & 0 \\
-\nu'_{rz} / E'_r & -\nu'_{r\theta} / E'_r & 1 / E'_r & 1 / G_{z\theta} & 0 \\
0 & 0 & 0 & 1 / G_{z\theta}
\end{bmatrix}
\begin{bmatrix}
\Delta \sigma'_z \\
\Delta \sigma'_\theta \\
\Delta \sigma'_r \\
\Delta \tau_{z\theta}
\end{bmatrix}
\] (2.10)

All the stress components in Equation 2.10 can be applied independently and corresponding strains measured in hollow cylinder (e.g. Zdravkovic, 1996; Zdravkovic and Jardine, 1997). As a consequence, it is possible to independently determine the five elastic parameters and checking the symmetry of the compliance matrix. The main difficulties in such tests are ensuring the necessary
2.5. CROSS-ANISOTROPIC ELASTICITY THEORY

stress-strain resolution and stress state uniformity.

2.5.3 Application of the cross-anisotropy elasticity theory in undrained multi-axial shear tests

It is generally acknowledged that for many soils there is only a small stress increment region within which the theory of elasticity can be reasonably assumed to describe the soil behaviour. Several researchers employed the cross-anisotropy elastic theory to predict the stress-strain responses of soils within this region and subjected to different shearing modes (e.g. [Zdravkovic 1996] [Pennington et al. 1997] [Kuwano 1999] [Lings 2001] [Nishimura 2006]) and compare that with the observed behaviour. [Nishimura 2005] showed how to predict the shear stiffness in undrained hollow cylinder shear tests using cross-anisotropic elastic parameters. His generalised formulations were employed in this study. Under undrained conditions the predicted elastic $G_{oct} = J/\epsilon_d$ (see also List of Symbols) can be determined from:

$$G_{oct} = \sqrt{\frac{1 - b + b^2}{(5B_1 + 2B_2) \cos^2 2\alpha + 2(B_1 + B_2)(1 - 2b)\cos 2\alpha + B_1(1 - 2b)^2 + 3B_3 \sin^2 2\alpha/4}}$$

The constants in this equation are:

$$B_1 = A^2_{22} - A_{22}A_{23} + A_{23}^2$$
$$B_2 = -A^2_{22} + 4A_{22}A_{23} - A^2_{23}$$
$$B_3 = 1/G_{ch}^2$$

(2.12)

where:

$$A_{22} = C_{22} - C_{12}^2/C_{11}$$
$$A_{23} = C_{23} - C_{12}^2/C_{11}$$

(2.13)

which link to the cross-anistropic elastic parameters

$$C_{11} = (1 - 4\nu_{ch}^{'})/3E_{v}^{'}, \quad (2 - 2\nu_{hh}^{'})/3E_{h}^{'}$$
$$C_{12} = (1 - \nu_{ch}^{'})/3E_{v}^{'}, \quad (1 - \nu_{hh}^{'})/3E_{h}^{'}$$
$$C_{22} = (1 + 2\nu_{ch}^{'})/3E_{v}^{'}, \quad (2 + \nu_{hh}^{'})/3E_{h}^{'}$$
$$C_{23} = (1 + 2\nu_{ch}^{'})/3E_{v}^{'}, \quad (1 + 2\nu_{hh}^{'})/3E_{h}^{'}$$

(2.14)

By this definition the function of octahedral shear stiffness with $\alpha$ is symmetric about $\alpha = 45^\circ$ axis for $b = 0.5$. The maximum limit is $G_{vch}$ at $\alpha = 45^\circ$, $b = 0.5$ and the minimum is corresponding to the triaxial compression condition, at which the above expression simplifies to:

$$G_{oct} = E_{u}/3 = \frac{E_{u}^{'}}{6} \times \frac{2E_{v}^{'}(1 - 2\nu_{hh}^{'}) + E_{h}^{'}(1 - 4\nu_{ch}^{'})}{E_{v}^{'}(1 - \nu_{hh}^{'}) - 2E_{h}^{'}(\nu_{ch}^{'})^2}$$

(2.15)

When undrained shear takes place from an isotropic stress state with fixed values of $\alpha$ and $b$, the inclination of effective stress-path as predicted by cross-anisotropy elastic parameter is:

$$\frac{\Delta(\sigma_{z}' - \sigma_{h}')}{2\Delta p'} = -\frac{3(1 - \nu_{hh}^{'}) - 2\nu_{ch}^{'})\nu_{hh}^{'}}{2E_{v}^{'2}E_{h}^{'}C_{12}(A_{22} + A_{23})} \times \frac{1 - 2b + \cos 2\alpha}{1 - 2b + 3\cos 2\alpha}$$

(2.16)

29
Taking derivative to $\alpha$ the slope is of its maximum at $\alpha = 0^\circ$ or $90^\circ$ and minimum at $\alpha = 45^\circ$. In addition for $b = 0.5$ condition all ESPs from isotropic axis have the same slope irrespective of changes in $\alpha$. For the particular condition of isotropic behaviour, the predicted slope would be -1:3.

2.6 Summary

Remarking on the study of the directional dependency of soils, [Saada and Bianchini (1975)] and [Arthur et al. (1980)] pointed out that anisotropy had been traditionally a missing parameter in geotechnical engineering, whereas they argued that the soil initial anisotropic fabrics could have as much influence as the water content for a clay or the void ratio with coarser soil. By now, recent developments (e.g. [Leroueil and Hight, 2003]; [Jardine et al., 2004]) show that soil anisotropy has become better understood. Indeed there have been successful examples of applying anisotropic soil models in some case histories and using available anisotropic strength data to select design parameters. In the long term, as more of such case studies are published, engineers will have greater confidence to account for this aspect in their designs if necessary.

The overview on laboratory testing devices in this chapter shows that HCA is well suited for the study of soil anisotropy. For the full exploration of stress-strain response in the generalized stress space, any carefully designed HCA that allows flexibility in its loading arrangement and has suitable sample dimensions is ideal. In addition the stress–strain response at small strains will be more accurately defined if high resolution local strain sensors are incorporated. As subsequently illustrated in Chapter 4 these requirements had been dealt with in the development of the Imperial College HCA Mark II system that was used in this study.

Over the past twenty years several HCA systems have been employed in a number of universities and some commercial laboratories. On the one hand, an extensive database of hollow cylinder tests on sands, silts and a few reconstituted clays is now available. One important observation is that the influences of principal stress rotation on both strength and stiffness are found to be appreciable. It has also been recognised that the initial anisotropy can be modified, sometimes significantly, by the pre-shear history. On the other hand, less has been known about the directional dependency of strength and stiffness, as well as the degrees of stiffness anisotropy at small and moderate strains from tests on natural clays. Although there have been some investigations using HCA on natural clayey samples, a majority of them did not separate the effects of $\alpha$ and $b$. Furthermore, it should be noted that in natural clays the inherent fabrics and discontinuities may contribute additional scale-dependent anisotropy. The Imperial College study of London Clay mechanical properties utilising two different HCA systems by the author and [Nishimura (2006)] aimed to quantify these issues.
Chapter 3

Review on the geology and engineering properties of London Clay

London Clay is a stiff, fissured clay and is widely distributed across the London and Hampshire Basins in southern England. In the London Basin its outcrop (Figure 3.1) is widespread but in the Hampshire Basin it is a narrow strip, usually less than 3 km in width. The clay is a marine clay deposited in Eocene times, and the formation consists chiefly of slightly calcareous, silty clay to very silty clay of high plasticity. Although apparently homogeneous the London Clay exhibits vertical variability in which different lithological units are marked and separated by the presences of glauconite-rich horizons or thin pebble beds (King, 1981; 1991; Ellison et al., 2004). In the overview on its geology (3.1) this characteristic is described and its significant influences, among other factors, on the shear strength and stiffness are reviewed in (3.2). The site-specific details of the London Clay material used in this research are not provided here but in Chapter 5.

3.1 Geology of London Clay

3.1.1 Depositional environment and post depositional processes

The London Clay was deposited in Early Eocene times (over 52 million years ago) in an entirely marine environment. It comprises a sequence of silty clays, clayey and sandy silts, and subordinate sands. Silty and very silty clay are prominent, and together with sandy silt they account for about 90% of the formation’s facies. Within the succession, the sandy beds were possibly formed in subtidal environment, whereas glauconite-rich sediments and lenticular sideritic concretions were formed during temporary breaks in sedimentation between transgressive-regressive movements of the shoreline (King, 1981; Ellison et al., 2004). The mud marine sediment was overlain by strata from younger periods. These are, in successive order, Claygate Member (upper part of London Clay) and the Bagshot Formation, Bracklesham Beds and Barton Beds of the Bracklesham Formation, the latter marks the end of known deposition. The total depositional thickness, including the London Clay, was approximated up to 350 m (Chandler, 2000).

During the Miocene Epoch, the subsiding London Platform and its sediments were compressed by tectonic forces associated with the Alpine Orogeny, which changed the platform to an eastwardly plunging syncline. There have been two main depocentres in the Thames estuary and in the south-
3.1. GEOLOGY OF LONDON CLAY

east of the Hampshire Basin. Away from these, there is a progressive westward thinning of the succession toward the presumed position of the basin margin. The westerly thinning and an increase in grain size are accompanied by an increase in the number of sand beds, which also become thicker westward ([King, 1981].

For the remainder of the Cenozoic times erosion took place and a great amount of material had been removed. For example, in Central London all the younger sediments and a significant thickness of London Clay have been eroded. Nevertheless, much of the formation is preserved in Hampstead Heath and Essex due to the easterly plunged of the London Basin ([King, 1981; Ellison et al., 2004]). This is probably the most significant post-depositional process influencing the behaviour of the London Clay.

One method to estimate the thickness of the removed sediments is using the yield stress ($\sigma'_y$) interpreted from oedometer tests. However, it is well known that this value is dependent on changes in cementing and structure caused by tectonism, aging, chemical alteration and weathering. Due to these influences, oedometer data gives a rough, and often over-predicted, estimation. Several authors have therefore combined this method with geological evidence. For example, the removal thickness was estimated of about 120–180 m in Central London ([Skempton and Henkel, 1957]) and ~150m in Bradwell ([Skempton, 1961]). At present, it is generally agreed that the amount of overburden removed is likely to be of around 200m based on the geological evidence presented by Chandler (2000).

The clay exposed by the erosion has been subjected to physical and chemical weathering processes. Deep ground freezing over the last 2 million years (during the Ice Age) affected the clay to a considerable depth. Chemical weathering (dissication, oxidation and precipitation) has affected to a less vertical extent. The products of chemical weathering have different features: rough and sub-vertical discontinuities due to dessication; colour changes from grey-blue to brown as a result of oxygenation in the unsaturated zone above the water table; and selenite (gypsum) crystals as pyrite reacts in the presence of dissolved calcium carbonate cement components. Chandler and Apted (1988) showed that weathering leads to a loss of $c'$, an apparent reduction in OCR, an increase in water content and to more intense fissuring.

The deposition of the recent Quaternary sediments involved successive Thames Gravel terraces in the London area, or soft Holocene deposits or hard brick-earths near the west of London. The presence of this superficial stratum and the types of the clay's top lithological units (either the clayey facies or the more permeable sandy silts) have strong influence to the extent of the weathered zone. On the one hand, where the gravel is in place this zone is thin (often < 1 m thick). On the other hand, where it is absent the weathered zone is thicker, generally 3–6m or even more ([Ellison et al., 2004]). The clay is strongly weathered near the top 1.5 m, showing a granulated or fragmented texture (Figure 3.2). At greater depth, the structure of the clay becomes increasingly clear and, except for oxidation, there is usually limited evidence of weathering below 3–4 m.

3.1.2 Lithological units

[King, 1981] subdivided the London Clay in the London Basin after identifying major depositional sequences from a combination of biostratigraphy and lithology, and the recognition of marine flooding cycles. This classification scheme was later applied to the succession in Isle of Sheppey ([King, 1984]), in Hampshire Basin ([King, 1991]), in Central London ([Standing and Burland, 2000]).

[Bishop et al., 1965] suggested an eroded thickness of 300 m at Wraysbury, west of the London Basin.
3.1. GEOLOGY OF LONDON CLAY

and most recently in the study at Heathrow T5 (Hight et al. 2003, 2007). In King’s classification scheme, the top of the London Clay is defined at the base of either the Bagshot Sands in the London Basin or the Bracklesham Group in the Hampshire Basin. At its base, the London Clay lies above the Harwich Formation (if presence) or rests on the Reading Formation or Woolwich Formation. Within the formation, there are five recognisable major transgressive-regressive cycles which are characterised by thin pebble or gauconite-rich beds. Figure 3.3 shows the five units, named in ascending order from A to E in the original scheme (King 1981). In each unit, there is a coarsening-upward facies sequence, indicating the progradation of the shoreline. Further sub-units can be defined but are generally harder to identify (King 1991). It has also been noted that generally in Central London only the lower part of the sequence is preserved (units B and A).

Hight (2003) illustrated that if the differences in stress level and lithological unit are taken into account then remarkable consistency can be found on geotechnical data from different sites, many of which are in the Central London. It is therefore important to identify these units. However, the identification itself is not simple and dependent greatly on the availability of borehole cores and the interpreter’s own experience. In addition, the recognition of these lithological units becomes increasingly more difficult eastwards across the London Basin due to the relatively deeper-water depositional environment and the particularly intense bioturbation. Fortunately, the thickness of each unit has been found to be relatively uniform across Central London. Therefore provided that its boundaries have been defined at one site, such information can be used for others (Hight et al. 2003).

3.1.3 Discontinuities

The London Basin is an extremely gentle syncline and dips of more than 3° are rarely met although there are some local folds (Ward et al. 1959). Small faults are commonly encountered but major ones are relatively rare. The predominant forms of discontinuities in the London Clay are:

Tectonic shears These discontinuities are the results of the tectonic forces that associated with the Alpine Orogeny. If presence these zones have potential influence on the stability of the temporary slopes and excavations. In the west of the London Basin they have been observed at similar elevations: 9.5–12.5 mOD at Prospect Park; 11.5 mOD at Wraysbury; and from 6.5 to 11.5 mOD at T5 (Hight et al. 2003). The shear zone's thickness varies from 3 to 75mm and Chandler et al. (1998) reported that the water content in the shear zones at Prospect Park site was lower than that of the adjacent un-sheared clay, indicating they were likely formed before erosion had taken place.

Fissures and joints These are known to have an important effect on the mass behaviour of the clay, particularly at shallow depths. By definition, joints are of larger scale discontinuities than fissures. It is observed that fissures are planar or conchoidal fractures, rarely extend more than 150mm long, and are predominantly sub-horizontal and sub-vertical. The orientations and spacing of joints are described by Fookes and Parrish (1969) as more systematic. However, due to the similarities they present in laboratory samples, generally it is difficult to have a clear distinction between them except prior observations in the field are available. It is recognised that the number of fissures per unit volume increases, with their size and spacing correspondingly decrease toward the ground surface (Ward et al. 1965; Skempton et al. 1969; Chandler and Apted 1988). In addition, the features of matt texture and no change in
orientation of the clay particles in the vicinity of the fissures’ surfaces imply that there were no appreciable prior relative movements along them.

One important observation with regard to fissures and lithological units is that unit A2 is not fissured, possibly due to its larger sand content than those in shallower depth. Gasparre (2005) also mentioned that of the eight undrained triaxial tests on samples from this unit, only in one sample the failure surface was influenced by pre-existing fissures. This is a small proportion in comparison with the cases seen in triaxial tests on samples from overlying units (Hight et al., 2003).

### 3.1.4 Microfabric

There are two typical types of microfabric observed on London Clay specimens, one is clay-rich and the other is silt-rich sample – the silt-rich sample has a denser fabric, a smaller specific surface area, a much broader range of pore radii and a much greater mean pore radius (Hight et al., 2003). It is also noted that the clay skeleton has open and large pores in the silt-rich clay, whereas in the clay-rich sample the silt grains float in the clay matrix and are not in contact with each other.

The recent microfabric studies (Gasparre, 2005) have confirmed the expectation of more compact structure and increasing level of preferred alignments of the clay aggregates with depth and maximum overburden pressure; unit C being of the most open and least anisotropic fabric, and the deeper units A2 and A3 having the most compact and anisotropic structures. This is seen as the causes of the differences in the mechanical response between units.

### 3.1.5 Hydrogeology

There are two aquifers that affect the London Clay – the first is a deep aquifer that includes the lower granular units of the Lambeth Group, the Thanet Sands and the Chalk. The second, a perched water table in the Terrace Gravels, is recharged from surface precipitation and locally from the River Thames.

At the start of the 20th century, the water pressures of the deep aquifer in the centre of the London Basin were artesian, linking to the water levels in the Chalk in the surrounding hills. Heavy extraction from this aquifer over a period of 150 years resulted to significant drops in its piezometric levels, which were in the order of 50 m in central London and caused under-drainage of the clay. At present the extraction has mostly ceased and piezometric heads near the base of this stratum are rising at rate of 1–2 m/year. The engineering implications due to the rising water levels in the lower aquifer are described in the CIRIA report by Simpson et al. (1989).
3.1. GEOLOGY OF LONDON CLAY

(a) Rock strata distribution

Figure 3.1: London Clay outcrops and distributions of other sedimentary rocks in London Basin (reproduced from BGS Website)

(b) Key units
Figure 3.2: Typical weathering profile in the London Clay (after Ellison et al. 2004)
Figure 3.3: Informal stratigraphical divisions of London Clay (King, 1981).
3.2 Laboratory shear strength and stiffness

Being important to infrastructure developments in the region, the mechanical properties of London Clay has been subjected to extensive investigations since the late 1950s. The first comprehensive one, the Ashford Common geotechnical study by the BRE and Imperial College (Bishop et al., 1965; Ward et al., 1965), discussed several aspects that strongly influence the clay’s shear strength, namely anisotropy, sample size, brittleness, and structure. Additional studies have been soon followed in which a wide ranges of testing devices and sampling techniques were employed (see Nishimura, 2006 Table 2.1). In particular, significant improvements in the understanding of small-strain behaviour have been gained since the early 1980s (e.g. Jardine et al., 1984; Atkinson et al., 1986; Jardine et al., 1991; Atkinson et al., 1993; Hight and Jardine, 1993; Jovicic and Coop, 1998; Clayton and Heymann, 2001). Overall, a substantial amount of data has been collected and reported in various general reports and papers.

Nevertheless, as also discussed in Chapter 2, previous investigations on the anisotropic mechanical response of natural London Clay have been limited both in terms of the imposed loading direction and the intermediate principal stress ratio. Many of these studies are triaxial compression tests on vertically- and horizontally-cut specimens with the exception of few torsional shear tests at Sizewell site (Porovic, 1995) and a number of direct simple shear tests at Brent Cross (Lau, 1988). The HCA tests carried out recently by Nishimura (2006) and the Author aimed to fill this gap.

With the comprehensive review by Hight et al. (2003) in mind, this section concentrates on the anisotropic shearing behaviour of London Clay obtained by laboratory tests. In particular, the effects of both micro- and macro-structure that manifest on anisotropy are discussed. The soil structure is responsible for the difference in mechanical response of the natural specimen with respect to the intrinsic behaviour of reconstituted specimen (after Burland, 1990). This definition is more specific than the term used by Mitchell (1976), in which structure generally means a combination of fabric and bonding, and therefore the term could be equally applied to both natural and reconstituted soils. The departure in mechanical response of intact sample from the reconstituted one is attributed to the micro-structure and macro-structure. The micro-structure comprises interparticle bonding and arrangement whereas the macro-structure includes discontinuities, laminations and bioturbations. Detail discussions of the structure of natural London Clay by comparing the mechanical behaviour of the intact samples from the different lithological units with that of reconstituted samples are presented by Gasparre (2005).

3.2.1 Shear strength

It is well known that the stress-strain response of natural London Clay is remarkably different from reconstituted one, a dilemma early addressed by Bishop et al. (1965); Bishop (1971b). For example, Figure 3.4 shows the relationships between water content and shear strength at failure established from CIU triaxial tests on intact, reconstituted and remolded blue London Clay specimens. In this figure, the relation established for samples consolidated from a slurry represents the Critical State Line (see Roscoe et al., 1958; Schofield and Wroth, 1968). It can be seen that the destruction of the clay’s micro-structure by strain or remolding is irreversible and that the shear strength of remolded sample was only half of the reconstituted one. In addition, there is no unique relationship between water content and ultimate shear strength. The effective strength envelope from these same tests

\[ F^2 \text{Formed by mixing the trimmings from corresponding intact samples at water contents about 1.25 times the liquid limit} \]
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

had been normalized by the Hvorslev’s equivalent stress \( \sigma'_{ve} \) to take into account the differences in states of specimens by Burland [1990] and Figure 3.5 clearly rules out the existence of such a Critical State in natural London Clay.

When subjected to shearing, London Clay exhibits clear brittle behaviour with well-defined failure plane after peak, then approaches a plateau and even reduces to residual strength if very large shear strain is allowed. The mobilised shear strength can be accordingly defined in relation to several reference points: peak, post-rupture, or residual strength. Moreover, due to its fissured nature the clay strength may be mobilised along these weak planes and resulted to a lower shear strength. However, the fissure strength is higher than the residual strength because little or no shear displacements imposed on fissure’s surfaces. The strength at critical state is defined for the isotropically consolidated reconstituted samples and is similar to the post-rupture strength envelope (Burland, 1990) [Hight and Jardine, 1993] [Jardine et al., 2004].

In the drained ring shear test shown in Figure 3.6 the intact strength dropped markedly from peak value to around 12° within 20mm of displacement. At larger relative slips the sample failed to a residual strength with \( \phi' = 0 \) and \( \phi' = 9.4° \) (Garga, 1970). A complete state of particle re-orientations and destruction of bonding within the slip zone must have been reached due to the large displacement and similar strength of remolded sample. The apparent difference in residual strength found by Agarwal (1968) on horizontally and vertically cut samples, from which \( \phi'_H / \phi'_V = 14.5° / 16.8° \), was due to the fact that complete residual condition had not been attained on his direct shear box reversal tests. Overall, the residual strength is expected to be isotropic with \( \phi'_s = 9 \sim 10° \).

With regard to the clay’s post-rupture strength, the envelope corresponds to \( \phi' = 20° \) at low to medium stresses (Figure 3.7b), but for high pressures (> 2000kPa) it becomes lower as seen on Figure 3.7 (Burland, 1990). Within the common pressure range of interest to shallow foundation design (< 1000kPa) the strength envelope of samples which failed prematurely on pre-existing fissures (denoted by F) lies below that of the post-rupture envelope. The fissure strength of London Clay is estimated with \( \phi' = 0 \) and \( \phi' \sim 17° \) (Hight and Jardine, 1993).

The anisotropy in peak strength of London Clay has been evaluated in terms of both total stress (e.g. Ward et al., 1959; 1965; Agarwal, 1968) and effective stress (see Bishop et al., 1965; Agarwal, 1968; Yimsiri, 2002) using triaxial apparatus. On the one hand, the undrained strength in total stress of horizontal-cut samples are higher than vertically-cut ones. On the other hand, their effective strength failure envelopes are nearly identical, as shown in Figures 3.8 and 3.9. Such coincidence is likely the direct consequence of the combined anisotropy in pore water pressure response and drained shearing resistance (\( \phi' \)), which will be the topic for discussion in Chapter 8. It is generally expected that the pattern of \( S_u \) anisotropy to be different from that of peak \( q / \rho' \).

There are serious concerns on the use of triaxial compression tests on samples cut at other orientations because they are subject to serious stress non-uniformity related to bending moments developed at the ends (Saada, 1970). A more reliable testing method, as explained in §2.2 is using HCA. In this section some of the results recently obtained by Nishimura (2006) will be selected for illustration. From a series of HCA tests at fixed \( b = 0.5 \) on specimens anisotropically consolidated prior to undrained shear it can be seen that the clay exhibited strong strength anisotropy (Figure 3.10). In addition, specimens subjected to torsional shear mode (\( \alpha = 30 \sim 70° \)) failed at lower strength, either in terms of \( S_u \) or \( (q / \rho')_{peak} \) than compression and extension modes. The undrained strength \( S_u \) is dictated by the effective stress strength parameters applicable to the particular volume of clay, as well as the direction of shearing and the slope of the effective stress path, the latter is dependent on the shearing direction and the clay’s stiffness anisotropy. The noticeable
influence of $b$-value to strength anisotropy had also been explored by other series of tests and Figure 3.11 shows the peak strength ratio mapped in the generalized stress space. Further discussions can be found in his thesis (Nishimura 2006) and are presented in [84] as the Author’s study also consisted of a programme of hollow cylinder tests but on samples at about 5m shallower.

It is important to note that the peak shear strength depends on the micro- and macro-structure presented on the specimen. As mentioned previously the micro-structure increases the intact shear strength (see Figure 3.5) and any destruction caused by undrained shear strains or volumetric strains will reduce it. On the other hand the macro-structure, e.g. fissures or joints, leads to sample-size effect. That means the larger the specimen the more discontinuities are presented, for which a lower peak strength is expected when the rupture surfaces and those planes of weakness are aligned in favorable shearing mode. These influences are discussed hereafter.

**Influence of micro-structure** The London Clay intact strength envelopes have two particular features. First, as shown in Figure 3.12 they are clearly well above the line defined from tests on reconstituted soil due to the effects of inter-particles bonding and fabric. In addition, a true cohesion due to cementing can be observed as the Mohr-stress circles of drained tension tests intercepted the shear stress axis (Bishop and Garga, 1969). Secondly, the envelopes gradually expand for deeper samples, indicating a higher apparent cohesion with depth (Bishop et al., 1965). In general the envelopes are curved in the range of medium stress, then flattened at higher stress level (Burland, 1990). Assembling data from other sites in Central London, Hight and Jardine (1993) established a family of upper and lower bound effective stress failure envelopes for London Clay at each depth range, e.g. Figure 3.13. The dependency on lithology was later found by Hight et al. (2003) as a reasonable template for triaxial test data from Heathrow T5. This template reflects the increase in fissure spacing and cohesion with depth.

Reductions in both stiffness and yield stress have been observed due to shear strains imposed from sampling process. There are various examples of the significant lost of shear strength of borehole samples with respect to block samples (Leroueil and Vaughan, 1990; Vaughan et al., 1993; Hight, 1998). The other cause of destruction may come from swelling in soils of expansive clay minerals. Bishop et al. (1965) showed that for London Clay samples that were allowed to swell isotropically (path OG in Figure 3.14) before shear in triaxial compression, swelling yields occurred at effective stress of about 40% of the in-situ stress. Similar evidence was also provided by Hight et al. (2003). However, Takahashi et al. (2005) conducted direct shear tests on samples that were first allowed to swell at 26kPa then recompressed to the in situ effective stress and found little influences on shear strengths from this swelling-compressing cycle. This could well be due to robustness of the structure for its high plasticity.

**Sample-size effect** As a result of the presence of fissures, the strength of London Clay will vary with the volume under test. This effect has been investigated by, among others, Ward et al. (1965); Agarwal (1968); Bishop and Little (1967); Bishop (1971b) and Marsland (1974). For example, Figure 3.13 shows the substantial reduction in undrained strength as sample size increases (from 18 to 305mm), obtained from triaxial compression tests on blue London Clay samples taken at Wraysbury. The undrained strength ratio of the $\phi$38mm to large ($\phi > 100$mm) samples is 2.2. The smallest 18mm dia. samples, having no fissures, gave even higher undrained strength, about 400% higher than that of the large sample. By comparison, the large samples obtained by either block or tube sampling produced similar failure strength, which was approximately one third of the strength of remoulded sample at $w_n = 28\%$.
As the spacing between fissure increases with depth, much larger sample is needed to represent the discontinuities feature. Accordingly, the deeper the samples are the more pronounce effect of sample size is encountered (Bishop and Little 1967; Marsland 1974). For very large volume the strength of soil mass can be estimated from field tests or back analysis of full-scale failures. However, it is important to be aware of the differences in failure mechanisms and time to failure in the evaluation of sample-size effect from triaxial test to direct shear or plate loading tests. The back-analysed strength from the plate test or slope failure is affected by progressive failure, which is inevitable given the brittle behaviour of the clay and its anisotropic strength. This operational strength has been estimated within the post-rupture and fissure strength envelopes (e.g. Burland 1990; Hight et al. 2003).

The above mentioned studies clearly illustrate that any interpretation of natural London Clay shear strength requires the assessments of its micro- and macro-structure. Although the clay’s strength anisotropy has been explored recently with a wider range of imposing principal stress rotations (Hight et al. 1997; Nishimura 2006), more work in this direction is needed.

### 3.2.2 Shear stiffness

The advent in the 1980s of laboratory local strain measuring techniques has allowed the reconciliation of shear stiffness measurements in-situ and laboratory (e.g. Costa Filho 1984; Jardine et al. 1983; Tatsuoka and Shibuya 1991). The degradation of stiffness moduli, which starts early at small strains, found in laboratory experiments has been well established. However, with the exception of a limited number of torsional shear and resonant column tests conducted by Porovic (1995) (see 2.4.2), the available laboratory stiffness database of London Clay prior to 2003 has been largely confined in the axi-symmetric stress condition imposed in the triaxial apparatus.

One of the common ways to obtain stiffness anisotropy results is testing vertically- and horizontally- cut samples in triaxial cell. For example, Atkinson (1975) found an average ratio of 2 for Young's modulus in horizontal direction to vertical one under both undrained and drained conditions. Similar degree of anisotropy of the undrained Young's modulus $E_u$ is shown in Figure 3.16, with data from triaxial compression CIU tests on London Clay samples at Kennington by Yimsiri (2002). Stiffnesses were more than 50% higher for horizontal compression and the anisotropy was observed up to medium shear strain levels. As mentioned earlier, the undrained response is in fact influenced by stiffness of the soil-water as a whole. Nishimura (2006) observed from HCA tests that the development of pore water pressure during undrained shear was anisotropic.

The stiffness anisotropy at small strain of London Clay has been studied by several authors from bender element tests. Under isotropic stress condition, Jovicic and Coop (1998) reported ratios of $G_{hh}/G_{hv} = 1.5$, indicating the initial anisotropy of the soil fabric. They also observed an apparent isotropic shear stiffness in the plane parallel to the bedding. Hight et al. (2003) reported from field wave measurements ratios of $G_{hh}/G_{hv} = 1 \sim 3$ and $G_{vh}/G_{hv} = 0.7 \sim 1.5$ over the depth range at Heathrow T5 site. These ratios, given as average values, at Sizewell B site are 1.45 and 0.8~0.9, and at Kent are 1.4 and 0.6~0.8, respectively.

As part of the present project at Imperial College for Heathrow T5, the clay’s deformation characteristics over a wide range of shear strain had been measured in triaxial apparatus (Gasparre 2005) and RC-HCA and triaxial apparatus (Nishimura 2006). Assuming cross-anisotropy behaviour for the clay, the elastic stiffness matrix had been determined from static drained probes and dynamic shear wave measurements (see Gasparre 2005, Table 8.14). In general, it is confirmed
that the clay is stiffer in horizontal direction than in vertical, which is associated with its peculiar micro-structure feature. Bender element data gave shear stiffness ratio $G_{hh}/G_{hv} = 1.8 \sim 2.1$, whereas static probe tests yielded Young's modulus ratio $E'_{h}/E'_{v} = 1.6 \sim 2.6$ over the depth investigated (5 ~ 30m BGL). It has also been noted that these ratios show a tendency of increasing trend with depth.

Applying a set of elastic parameters to compare between predicted and measured strains in his HCA tests on 11.0m BGL samples, [Nishimura(2006)] found that $E'_{h}/E'_{v} > 1.9$ was more reasonable for the analysis of undrained shear stage with $b$-fixed. For the rotary-cored samples at various depth, he reported $G_{hh}/G_{hv} = 1.8 \sim 2.2$ from both resonant column and bender element tests. With respect to the shear stiffness in horizontal direction, the degradations of the dynamic equivalent shear modulus $G_{eq}$ by resonant column tests were practically similar between tests subjected to different rotations of principal stress direction. The same response was observed for the static secant shear stiffness, $G_{zz}$. The measurements were considered to be reliable for $\gamma_{zz} > 0.01\%$ but became more scatter at smaller static strain. At shear strain of about 0.005% the static shear stiffness was 10–30% smaller than the dynamic one, indicating that the elastic threshold was well below that level. The limitation in static strain measurement can be improved by using local strain sensors as employed in the Author’s study.

The effect of micro-structure has been shown to be more prominent although the degrees of influence were varied (e.g. [Hight et al., 1997; Jovicic and Coop, 1998; Hight et al., 2003]. Figure 3.17, reproduced from [Gasparre, 2005], shows the degradation curves of shear stiffness $G = q/\varepsilon_s$ for different lithological units. It is interesting to note that both [Nishimura, 2006] and [Gasparre, 2005] reported only small loss of shear stiffness in resonant column tests (sample size of 71 mm o.d.) before and after the presence of rupture surface, or in bender element tests on triaxial samples (100mm o.d.) that failed along a fissure and those that did not.

3 Calculating as the secant stiffness of a load-unload cycle
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.4: Shear strength–water content relationships for intact and remoulded samples of London Clay at Ashford Common, Level E at 34.8m BGL (Bishop et al., 1965)

Figure 3.5: Normalised effective stress paths from CIU compression tests of undisturbed London Clay samples at Ashford Common in Figure 3.4 (after Burland, 1990)
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.6: Drained ring shear stress ratio displacement relationships for undisturbed and remoulded blue London Clay from Wraybury (Garga 1970).
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.7: Post-rupture strength envelopes for intact samples from Ashford Common, levels C (20.1m) and E (34.8m) (Burland [1990]): a) for high pressures and b) for low to medium pressures. Also shown are the peak strengths of samples that failed on pre-existing fissures (denoted by F).

Figure 3.8: Triaxial CID and CIU compression tests on intact samples from Ashford Common, level E (34.8m). Here V, H denote vertical and horizontal cut samples (after Burland [1990]).
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.9: Triaxial CIU compression tests on 50mm dia. vertically and horizontal cut samples of London Clay from Kennington (Yimsiri, 2002)

Figure 3.10: Anisotropic effective strength envelope at $b = 0.5$ of London Clay, samples at 7.2mOD and anisotropically consolidated (Nishimura, 2006)
Figure 3.11: Peak strength ratio of London Clay at different combinations of $\alpha - b$, samples at depth of 7.2mOD (Nishimura, 2006)
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.12: Effective strength envelopes of undisturbed and remoulded blue London Clay from triaxial tests (Bishop and Garga [1969]). Drained triaxial compression and extension tests on undisturbed samples, and undrained triaxial compression tests on remoulded samples from 6m BGL at Wraysbury
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.13: Triaxial compression strength envelopes for London Clay (Hight and Jardine, 1993)
3.2. LABORATORY SHEAR STRENGTH AND STIFFNESS

Figure 3.14: Loss of structure from swelling, samples of London Clay at Level E, Ashford Common (Bishop et al., 1965): a) Changes in water content due to swelling; b) Triaxial CIU and CID tests.

Figure 3.15: Effect of sample size on triaxial undrained strength of undisturbed blue London Clay from Wraysbury (Bishop, 1971b)
Figure 3.16: Stiffness anisotropy in terms of $E_u/p_0^'$. Triaxial compression CIU tests on vertically and horizontally cut samples of London Clay from Kennington (Yimsiri 2002)
Figure 3.17: Degradation of shear stiffness from triaxial tests on samples of London Clay at Heathrow T5 (Gasparre 2005)
3.3 Summary

London Clay, a marine clay deposited in Eocene times, is spatially distributed in the London and Hampshire Basins. In both basins the clay exhibits significant vertical variability, particularly in the west of the London Basin. Lateral variability is observed to be insignificant at sites in and around Central London, unless there has been faulting or other tectonic events. It is recognised that cycles of relative sea-level changes had a major influence on the development of the clay’s lithological units (see, e.g., [King 1981; Ellison et al. 2004]). The clay, chiefly of silty to very silty clay facies, is characteristically stiff, heavily overconsolidated and of high plasticity. Fissures and joints are commonly found, and their presence is less pronounced with increasing depth.

It is widely acknowledged that the mechanical response of the natural London Clay is markedly different from that of a remolded soil. The intact sample exhibits clear brittle behaviour with well-defined failure plane after peak, then approaches post-rupture strength. In fact, it can be as low as the fissure strength if samples failed prematurely along existing fissures. The residual strength is lower and is found to be similar to the strength of the clay within the tectonic shear zones. It is recognised that the intact peak strength envelope seems to expand with increasing depth due to the clay’s vertical non-uniformity. From this observation, a lithology-dependence strength envelope has been proposed. The clay’s structure, a direct consequence from its geological stress history, can be modified by undrained shear strains due to sampling disturbance or volumetric strains due to swelling. A small number of direct simple shear and torsional shear tests has been carried out on blue London Clay samples to quantify its anisotropic mechanical properties. However, these investigations are limited in terms of the imposed loading directions and intermediate principal stress ratios.

Recent developments in small strain measurement have brought toward a significant body of stiffness data, especially those obtained from triaxial tests. It is recognised that the clay stiffness is highly non-linear and directionally dependent. In particular, the clay shows stiffer response under loading on horizontal direction than that on vertical direction, with the anisotropy presents not only at small strain but also up to relatively large shear strain. Nevertheless, much as with shear strength, additional experiments in suitable devices (DSC or HCA) are needed to improve our understanding about the effect of anisotropy on the clay’s mechanical properties.
Chapter 4

Laboratory equipment

The HCA apparatus used in this research is described in 4.1, including the developments of a local transducer system and the techniques employed to improve the measured resolutions at small strains. The expressions used for the calculation of stress and strain components in HC testing are explained in 4.2. Next 4.3 explains the method of calculation for strain components based on local strain transducers. Finally, 4.4 highlights the importance of using a complete system for local strain measurements if the pre-failure stress-strain response is investigated.

4.1 The Imperial College HCA Mark II

4.1.1 System components

The HCA described in this thesis, termed the HCA Mark II, was developed and built at Imperial College ([Jardine] 1996). The apparatus design was based on experience from the Bishop-Wesley triaxial stress path cell ([Bishop and Wesley] 1975), the large IC-HCA ([Hight et al.] 1983) and the resonant column HCA ([Drnevich] 1985; [Porovic] 1995).

HC specimens of two nominal sizes could be tested, namely 100 × 60 × 200 mm or 200 × 160 × 300 mm (o.d., i.d. and height, respectively). Triaxial samples of 100 mm dia. could also be used by using solid platens. The system is capable of generating axial load and torque of 20 kN and 1.6 kN.m at confining pressure of up to 1.2 MPa. Higher loads can be obtained if pressure multipliers or CRSP (Constant Rate Strain Pump) of higher capacity are deployed. Figure 4.1 shows the general layout of the apparatus, for which the main components are:

1. A stationary, large 600 mm dia. stainless steel re-action frame (Figure 4.2), sealed by large O-rings and reinforced by four internal 50 mm dia. steel pillars and two stiffening rings. The removable cell chamber, having 45 mm thick perspex and 12 tie bars, was designed to be lifted by an overhead crane. The frame's base plate was bolted firmly to a horse-shoe shaped concrete block that provided the main structural support.

2. A loading shaft incorporated with bearing and seal was aligned along the frame's central axis. This shaft carried the base platen which connected to the inner cell pressure line, the suction line and the back pressure line via short, small diameter and stiff plastic tubes. The inner
4.1. HCA MARK II

cell was filled with air-free water, whereas another line from the main water supply was used
to fill the outer cell. Between the top platen and the fixed, internal pressure-compensated
dual-axis load cell was an extension rod. The reaction at the top of the sample was provided
by rigid connections between the top platen and the extension rod with hardening resin. The
resin was a mixture of styrene polyester resin A (Scotter Bader UN1866) cured with 1.5 % of
catalyst M, for which hardening time in air at room temperature (21°) was approximately 20
minutes.

3. A system of actuators, mounted below the cell frame, was connected to the loading shaft (see
Figure 4.4). A single-acting Bellofram super-cylinder extended-travel actuator (P1) was used
to impose axial force and vertical displacement. On the movable transfer plate two in-plane
actuators (P2 and P3), connected to the loading shaft by chain and sprocket drive, produced
the torque and rotation in the horizontal plane. Actuator P3, under constant air pressure,
acted as a compensating actuator. Thrust bearings and a special low-friction statinal seal
were incorporated to de-couple the axial and torsional motions. The transfer plate was free
to move vertically through roller bearings that were mounted co-axially on hardened steel
shafts. These two shafts were greased and protected from dirt by concertina-type gaiters.

4. A loading system that could be set to either stress- or strain-controlled mode. The system in-
cluded electro-pneumatic controllers (capacity 10–800kPa, 0.07kPa/pulse) that act through
air/water (A/W) interfaces or water-filled geared constant rate of strain pumps (CRSP, max-
imum pressure of 1.0 MPa, 1.5 liter, 0.25 × 10⁻³ cc/pulse) to control the inner cell pressure,
outer cell pressure and back pressure, as well as the axial actuator P1 and torque actuator
P2. All the pressure lines were connected to a panel (Figure 4.5) for any manual control or
checking operations. The electro-pneumatic controllers and CRSPs were driven by individ-
ual stepper-motor regulator units which were connected to a computer-controlled system.

5. A system of stress and strain measuring transducers and a purpose-built signal condition-
ing 32-channel unit. The unit was connected to an IBM-compatible computer that used
specialised software (Takahashi, 2003) for automatic data acquisition and closed-loop servo
control.

Modifications to the HCA Mark II during the initial stage of the research (2003-2004) included:

• The porous-stone rings initially allowed torsional failure to develop without any end slips
when testing Ham River sand samples. However, problems were encountered with tests on
clay specimens. The stone rings were replaced by coarse brass rings, each were embedded
radially with 12 thin brass blades (Figure 4.3).

• An additional single-acting actuator (P4, 6in. dia.) was added between the cell’s base and
the transfer plate. The air pressure inside its chamber could be adjusted manually by an air
pressure regulator. By maintaining a constant pressure difference between actuators P1 and
P4, it was possible to raise the pressure in P1 while keeping the overall axial load constant to
the specimen.
Figure 4.1: General layout of HCA Mark II
4.1. HCA MARK II

Figure 4.2: HCA Mark II in Bishop Laboratory

Figure 4.3: Top and bottom annular rings with radial brass vanes
4.1. HCA MARK II

Figure 4.4: Loading system in HCA Mark II

Figure 4.5: Control panel (1) with: volume gauges (2 and 3); A/W interfaces (4 and 6); (5) air pressure regulator for actuator P4; (7) pressure multiplier; (8) signal conditioning unit for proximity transducers
4.1.2 Instrumentation

The two terms, inner and outer wall, are used hereafter to distinguish between the measurements on the inside and outside wall of the hollow specimen. Measurements taken between two points within the sample's height or on its mid-plane will be referred to as local, whereas measurements on the top and bottom faces (or using these as reference points) and volume changes of the specimen will be called global measurements.

At the start of this study and after each 6-month interval, the performance of each of the transducers had been evaluated in terms of its calibration characteristics (working range, linearity, accuracy and percentage of maximum error) as well as its short-term properties (resolution and accuracy) and long-term drift. The resolution, corresponding to one bit output of the A/D converter, was reported in both voltage and engineering units. The accuracy was determined by comparing the errors between the measured values and a linear regression of the calibration results (95% confidence limits of the errors to which a Gaussian distribution was assumed) (see Appendix A). Detailed discussions for each of the transducers are found in the following section, in which only the transducers' resolution and accuracy are quoted.

**Measurement of axial load, torque, and pressures**

The internal dual-axis pressure-compensated load cell (Maywood Instruments Ltd.) provided readings for both axial deviator force and torque, for which axial capacity of -6~8 kN and torque capacity of \( \pm 400 \) N.m. It had high sensitivity with low cross-sensitivity and hysteresis characteristics. The accuracy for axial load and torque were \( \pm 0.6 \) N and \( \pm 0.06 \) N.m respectively.

Two pressure transducers (MSI Sensors PR27) measured the pressures at the outlets of the 50 cc volume gauge and the outer cell A/W interface, respectively. The transducer had fast response and accuracy of \( \pm 0.1 \) kPa for pressure range between 400 and 800 kPa. The same type of pressure transducer was used to measure pore water pressure (p.w.p) at the top and bottom of the sample. Pressure at the sample's bottom was selected as input for effective stress calculation, and if required the operator could isolate the top and bottom drainage lines to estimate the degree of p.w.p non-uniformity.

**Measurement of global deformations**

The global strain measuring system used two strain-gauge type displacement transducers (HS-25 model) and two Imperial College-type volume gauges. The accuracy of the displacement transducer was \( \pm 0.5 \mu m \). One displacement transducer measured the vertical travel distance of the transfer plate relatively with the fixed stainless steel base. The other transducer measured the backward or forward movement in the plane of rotation of a stainless steel cantilever, which was fixed to the loading shaft.

One 100 cc volume gauge monitored the volume of water leaving or entering the sample within \( \pm 0.002 \) ml accuracy. There was also one 50 cc volume gauge used to measure the inner cell volume change (accuracy of \( \pm 0.001 \) ml).
4.1. HCA MARK II

Measurement of local deformations

The HCA Mark II local strain measuring system included one LVDT (Linear Variable Differential Transformer), three proximity transducers, two double-axis electrolevels and one single-axis electrolevel (Figure 4.6). The inner LVDT was aligned in the same horizontal plane with the outside proximeters, which were placed evenly at 120° interval around the specimen. Two double-axis electrolevels were mounted diametrically opposed positions on the specimen's outer side to access the amount of tilting, whereas the single-axis electrolevel was attached to the sample's mid-height.

The LVDT provided measurement of the inner wall deformation. Generally LVDT has superior performance due to its large linear range and very high resolution – as small as \( 0.01 \text{ » } 0.02 \mu m \) (Cuccovillo and Coop, 1997) and proven reliability in triaxial system (e.g. Kuwano, 1999; Rolo, 2003; Gasparre, 2005). For HCA Mark II a RDP Electronics LVDT - D5/200W was used and an accuracy of \( \pm 0.03 \mu m \) was achieved. It came together with a signal conditioning unit (RDP S7AC) and a pre-amplifier, both of which had been integrated in the data logger.

A set of three proximity transducers was used to measure the outer wall movement by detecting the gap between a sensor and its corresponding conductive target. The resolution of a non-contact proximeter, however, is generally poorer than that of a LVDT (see, e.g. Scholey et al., 1995). For example, Hird and Yung (1987, 1989) reported a resolution of 1 \( \mu m \) and accuracy of \( \pm 2 \mu m \) over the measurement range of 5 mm in their triaxial system. Later, using proximeter type of shorter working range, the resolution had been improved to 0.5 \( \mu m \) (Hird and Hajji, 1995; Hird and Pierpoint, 1997). For the large IC-HCA, the proximeter resolution was 1 \( \mu m \) (Zdravkovic, 1996) although recently Rolo (2003) reported that it became less satisfactory (3 \( \mu m \)) for very small strain determination, possibly due to the effects of deterioration.

For this study, Kaman type proximeter model KDM8200/6U1 of better resolution and stability characteristics was employed. In addition, each conductive target was changed from thin aluminium foil to a copper curved plate (0.7 mm thick) to avoid the electro-magnetic field penetration effects. Moreover these proximeters were calibrated in bi-polar adjustment instead of the full scale to ensure the best gain in both linearity and sensitivity. These resulted to significant improvements in accuracy of the proximeter system to \( \pm 0.3 \mu m \) (Figure 4.7).

At Imperial College the performance of the single-axis electrolevel for measuring torsional shear strain holds a good record. For instance, Rolo (2003) reported that the rotation angle can be determined accurately to \( \pm 1 \times 10^{-4} \), which was similar to the one employed in HCA Mark II. In contrast, the performance of the double-axis electrolevel was less satisfactory because of the inherent cross-sensitivity effects. At present (Foundoukos, 2006) reported that the achievable axial strain resolution in the large IC-HCA is about \( 1 \times 10^{-3} \% \). Kuwano (1999) recognised that such limited resolution was mainly due to the short gauge length and therefore extended it to 100 mm long, for which the axial strain resolution in her triaxial system was improved to \( 5 \times 10^{-4} \% \).

Taking these rationales into consideration, a new electrolevel-type design for axial deformation measurement in HC testing was developed. This design, available in late 2003, had three distinct features. First, the two triaxial-type pivots were replaced by a spherical ball (full rotation) at the top pad and a pivot with 2 degree of freedom (restricted rotation in the horizontal plane) at the base pad (see Figure 4.6b). Second, double-axis electrolyte was used to provide correction factor for the axial deformation due to the out-of-plane angular rotation. Finally the gauge length was extended, with upper-arm and lower-arm lengths of 150 mm and 20 mm respectively. As a result the axial deformation determination could be determined accurately within \( \pm 0.46 \mu m \).
4.1. HCA MARK II

To minimise the effects of electrical noises on the sensors’ performance, the following countermeasures had also been applied:

- Set the data-logger to 16-bit resolution and noise-rejection filter option
- Used individual cables to connect the M/D card and Datascan module thereby constraining the cross-talk effects.
- Set electrolevel input voltage to 0.5V instead of 5V to prevent the warming-up effect.
- Used a purpose-built printed circuit card for the electrolevels.
- Configured the sensors to work within the optimal range of the data-logger (150mV).
- The laboratory temperature was kept constant at $21 \pm 0.8^\circ C$ throughout this study.
- Wrapped the cell with a protection cover layer consisted of AirCap barrier-sealed bubble and aluminium foil to imitate the effects of temperature fluctuation during stages of small strain testing (see also Gasparre, 2005).

Further improvements

In considering the expecting level of strain at elastic behaviour of London Clay (e.g. Jardine, 1992; Clayton and Heymann, 2001; Gasparre, 2005), clearly further improvement to the accuracy of the local strain transducers is necessary, particularly to the proximeters and double-axis electrolevels. Experience indicates that better stability of readings can be achieved by real time numerical averaging scheme (e.g. Zdravkovic, 1996) and the following steps had been conducted to find out its effectiveness:

- The local transducers were set to their expected working ranges (150mV of the data logger) and their outputs were recorded continuously for every scanning in 4 minutes. Different averaging schemes were then applied (2-point, 4-point and mixed averaging, see also Figure 4.8), and in this figure each recording represented the mean of individual measurements.
- Statistics evaluations of the recording data were conducted (Table 4.1) to yield the accuracy for each local strain transducer, which was defined by the deviation according to 95% possibility of the average value. These results were compiled into Figure 4.9a, in which the accuracy achieved for each transducer were plotted against the number of averaging point.
- The accuracy in terms of strain (Figure 4.9b) was estimated for a hollow cylinder sample of nominal dimensions ($r_o = 50$mm, $r_i = 30$mm) with nominal gauge lengths of 150mm and 60mm for axial and torsional electrolevels.

Taking into account both the gain in precision and the required rate of data scanning (not slower than 6 sec/reading) the mixed-scheme averaging was selected (Table 4.2, Table 4.3) shows the estimated degrees of resolution and precision of the transducers used for HCA Mark II. It should be noted here that these estimations considered only the performance of the transducers, and therefore it appeared that the global strain measuring system can perform reasonably well. The typical changes of the local strain sensors’ readings with room temperature are illustrated in Figure 4.10. The overall performance of the global and local strain measurement systems in HCA Mark II in actual testing condition is investigated in 4.4.
### Table 4.1: Influences of averaging scheme to local transducers' precision in HCA Mark II

<table>
<thead>
<tr>
<th>Transducer type</th>
<th>Unit</th>
<th>Precision w.r.t averaging scheme</th>
<th>Without</th>
<th>2-point</th>
<th>4-point</th>
<th>Mixed scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer wall displacement</td>
<td>µV</td>
<td></td>
<td>30</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>µm</td>
<td></td>
<td>0.30</td>
<td>0.21</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>Inner wall displacement</td>
<td>µV</td>
<td></td>
<td>55</td>
<td>42</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>µm</td>
<td></td>
<td>0.03</td>
<td>0.02</td>
<td>0.015</td>
<td>0.013</td>
</tr>
<tr>
<td>Axial displacement</td>
<td>µV</td>
<td></td>
<td>50</td>
<td>42</td>
<td>26</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>µm</td>
<td></td>
<td>0.46</td>
<td>0.380</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>Rotational angle</td>
<td>µV</td>
<td></td>
<td>1.85</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>×10⁻³°</td>
<td></td>
<td>0.12</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

#### Notes:
- All transducers were set to their optimal working range (150 mV).
- Mixed scheme: 6-reading point for proximity transducers, and 4-reading point for LVDT, axial and shear electrolevels.
- Strain calculation: assuming sample 100mm O.D and 60mm I.D; gauge lengths of 150mm and 60mm for the axial and rotational electrolevel, respectively.

### Table 4.2: Degrees of precision in strain measurements in HCA Mark II

<table>
<thead>
<tr>
<th>Strain component</th>
<th>Global measurement</th>
<th>Local measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range (%)</td>
<td>Precision (% 10⁻³)</td>
</tr>
<tr>
<td>Radial strain, εᵣ</td>
<td>see notes</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circumferential strain, εθ</td>
<td>see notes</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial strain, εₚ</td>
<td>see notes</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear strain, γ₁θ</td>
<td>see notes</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric strain, εᵥ</td>
<td>see notes</td>
<td>0.45</td>
</tr>
</tbody>
</table>

#### Notes:
- Considering the performance of transducers only (i.e. assuming no errors from bedding and system compliance in case of global strain measurements). In normal working conditions, similarities between the global and the local strain measurements were noticed after 0.1 %, and 0.03 % for the semi-local strain measurement.
- Accuracy for strain components using mixed-averaging scheme within the optimal sensor working range (150mV).
- Global strain transducers are adjustable and therefore large range can be measured.
- Local volumetric strain was indirectly calculated as the sum of normal strain components.
Table 4.3: Resolution and precision of transducers in HCA Mark II

<table>
<thead>
<tr>
<th>Transducer type</th>
<th>Measurement</th>
<th>Range A (mV)</th>
<th>Range B (unit)</th>
<th>Sensitivity (mV/unit)</th>
<th>Resolution A (µV)</th>
<th>Precision B (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load cell</td>
<td>Axial deviatoric force</td>
<td>20</td>
<td>-1–6 kN</td>
<td>0.002 mV/N</td>
<td>0.625</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>Torque</td>
<td>20</td>
<td>±100 Nm</td>
<td>0.02 mV/Nm</td>
<td>0.625</td>
<td>1.25</td>
</tr>
<tr>
<td>Pressure transducer</td>
<td>Outer cell</td>
<td>20</td>
<td>400 kPa</td>
<td>0.1 mV/kPa</td>
<td>0.625</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>400–850 kPa</td>
<td>0.1 mV/kPa</td>
<td>0.625</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Inner cell</td>
<td>20</td>
<td>200 kPa</td>
<td>0.2 mV/kPa</td>
<td>0.625</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>200–850 kPa</td>
<td>0.2 mV/kPa</td>
<td>0.625</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Base &amp; top pwp</td>
<td>20</td>
<td>-200–200 kPa</td>
<td>0.1 mV/kPa</td>
<td>0.625</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>200–850 kPa</td>
<td>0.1 mV/kPa</td>
<td>0.625</td>
<td>0.06</td>
</tr>
<tr>
<td>Volume gauge</td>
<td>Inner cell</td>
<td>20</td>
<td>±25 cc</td>
<td>0.72 mV/cc</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>Sample</td>
<td>20</td>
<td>±50 cc</td>
<td>0.32 mV/cc</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td>Displacement trans.</td>
<td>Global axial</td>
<td>20</td>
<td>±1.25 mm</td>
<td>1.32 mV/mm</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>1.25 mm</td>
<td>1.32 mV/mm</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>Global horizontal</td>
<td>20</td>
<td>±1.5 mm</td>
<td>1.32 mV/mm</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>Equivalent rotation</td>
<td></td>
<td>±10°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proximeters</td>
<td>Outer wall displacement</td>
<td>150 mV</td>
<td>±1.5 mm</td>
<td>97 mV/mm</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>(KDM 8200-6U1)</td>
<td></td>
<td>1.3 V</td>
<td>±1.5–3.0 mm</td>
<td>40 mV/mm</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Inner wall displacement</td>
<td>150 mV</td>
<td>±0.38 mm</td>
<td>2045 mV/mm</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>(RDP D5/200W)</td>
<td></td>
<td>1.3 V</td>
<td>±0.38–0.7 mm</td>
<td>40 mV/mm</td>
<td>40</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 V</td>
<td>±0.7–2.5 mm</td>
<td>320 mV/mm</td>
<td>320</td>
<td>1280</td>
</tr>
<tr>
<td></td>
<td>Axial displacement</td>
<td>150 mV</td>
<td>±1.5 mm</td>
<td>110 mV/mm</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>(150mm-arm)</td>
<td></td>
<td>1.3 V</td>
<td>±1.5–5 mm</td>
<td>40 mV/mm</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Shear electrolevel</td>
<td>150 mV</td>
<td>±14°</td>
<td>11 mV/°</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>(60mm-arm)</td>
<td>Rotation angle</td>
<td>1.3 V</td>
<td>±14–30°</td>
<td>5 mV/°</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Notes:
- Under the condition of mixed-averaging scheme and considering the performance of transducers only.
- Range-A (mV): Sensing range of the A/D data logger
- Range-B (unit): Corresponding engineering range from calibration relationship
- Precision-A (µV): defined as 95% confidence of transducer’s readings to the mean value
- Precision-B (unit): Conversion to engineering unit of precision-A value by the sensitivity
Figure 4.6: The local strain transducer system for HCA Mark II
4.1. HCA MARK II

(a) Original condition: Foil target and full-scale calibration

(b) After improvements: Copper-pad target and bi-polar adjustment calibration

Figure 4.7: Improvements to regression characteristics of Kaman KDM8200/6U1 proximeter
4.1 HCA MARK II

(a) No averaging scheme  
(b) 4-point averaging  
(c) Mixed-averaging scheme

Figure 4.8: Short-term readings in local strain transducers w.r.t scanning scheme
Figure 4.9: Degree of precision w.r.t. averaging scheme for local strain measurements

(a) Deformation components

(b) Strain components

4.1. HCA MARK II
Figure 4.10: Typical degrees of changes in local strain sensors' readings with time (started 17:45 on 09th Aug 2004), illustrating the importance of controlling temperature and testing period for small strain measurements.
4.1.3 Computer control and data acquisition system

Hardware

Figure 4.11 shows a schematic diagram of the hardware employed for automatic control and data logging in the HCA Mark II system. Communication between the computer and the data logger was transferred via an RS232 port, and the transducers’ outputs could be monitored in voltage and engineering units. A 8255 I/O card installed in the PC motherboard provided means for the addressing of all stepper-motor controllers. The computer hard disk was used to store the recorded data files and a 1.44in floppy disk drive was employed for data transfer.

The data logger, designed and built at Imperial College, had two network-linked A/D converters (Datascan 7220) that could accommodate 32 transducers. For readings of high resolution and low noise the two Datascan units were programmed to 16-bit resolution and auto-ranging mode, which gave a resolution of 5µV over the 150mV working range. The first module was connected with DC-output type transducers, while the second module was specifically for AC-type transducers. As the signals from proximeters were demodulated by their own signal conditioning units (Kaman Instruments KM205), the resulting DC-signals could feed directly into the first module. The second Datascan, which incorporated a conditioning unit and M/D card, received signals from the LVDT and the inclinometer electrolevels. Both Datascan units provided steady input for low voltage DC transducers without the need of amplification, whereas additional modulator-demodulator (M/D) signal conditioning units were introduced for AC transducers.

Software

At Imperial College specific programmes had been written in QuickBasic language for the large HCA (Shibuya 1988; Menkiti 1995; Zdravkovic 1996) and the resonant column HCA (Porovic 1995). For HCA Mark II the new control programme (Takahashi 2003) was a further development from these codes and was called the Imperial College HCA Control Data Acquisition programme, or the HCA programme in short.

During automatic control, each HC test could be divided into several individual stages (e.g. saturation, B-value check, consolidation, change of intermediate principal stress ratio, unloading, principal stress rotation, uniaxial and multi-axial stress paths). The operator could terminate any of them manually or could set an automatic trigger. The following options were available for triggering the end of a stage:

- Any of the global strain transducers was out of its working range;
- Maximum and/or minimum pre-defined limits of global axial strain were reached;
- A pre-defined aging period was completed.

At the end of each stage the operator could select two options: (a) holding the stresses and strains (if strain-controlled) at their final target values; or (b) continuing with another testing stage.

The following modifications were added by the author in 2004 to the HCA programme (Ver. 0.3B):

- A specific data averaging scheme for each type of local strain transducers to minimise un-
4.1. HCA MARK II

wished readings’ noise while maintaining a reasonably quick scanning rate, and therefore to
improve the stability of measurements over the small strain range;

• An option for a stress-controlled mode employing CRSPs in addition to electro-pneumatic
axial and torque controllers;

• An option for automatically halting a test when the axial strain reached its pre-defined values
during a test stage.

The data-scanning rate of a system depends on the number of transducers employed and their
performance characteristics, the data logger and the software’s data-logging algorithm. The sys-
tem scanning rate was about 4 second for all 17 employed transducers, of which seven were local
strain transducers. In the data-recording sub-routine the number of scans for each record could
be specified before or during a testing stage. The recording rates were chosen by the author to en-
sure at least 60 readings for the first two log-cycles of strain (\(10^{-5} \sim 10^{-3}\%\)), and no less than 20
readings/log-cycle above this limit.

Stage testing-control parameters

In selecting the stage control parameters, the operator decided whether the stage would be drained
or undrained, and in stress- or strain-controlled mode. An undrained stage would have four control
parameters, and there would be one additional parameter (back pressure) for a drained stage. When
using CRSPs, both stress- and strain-controlled modes could be adopted. In stress-controlled mode,
the two CRSPs would be driven by the target stress values. In strain-controlled mode, either an axial
strain or rotational shear strain rate was selected; the corresponding CRSP would work in constant
flow rate (and therefore constant strain rate) while the other would be adjusted to maintain the
desired stress parameters.

The stress-controlled parameters could be the four individual stress components (\(\sigma_z, \sigma_\theta, \sigma_r\) and
\(\tau_{z\theta}\)) or the generalized stress path parameters \((p, t, b\) and \(\alpha)\). Their relationships are shown in
Equation 4.1

\[
\begin{align*}
\sigma_z &= p - t(2b - 1)/3 + t/\cos 2\alpha \\
\sigma_\theta &= p - t(2b - 1)/3 - t/\cos 2\alpha \\
\sigma_r &= p + 2t(2b - 1)/3 \\
\tau_{z\theta} &= t\sin 2\alpha
\end{align*}
\]  

Equation 4.1

The required axial load, torque and pressures are calculated from Equation 4.2

\[
\begin{align*}
W &= \pi \left( r_o^2 - r_i^2 \right) \left( 2\sigma_z - \sigma_r - \sigma_\theta \right)/2 \\
M &= 2\pi \tau_{z\theta} \left( r_o^3 - r_i^3 \right)/3 \\
p_o &= \left[ \sigma_r (r_o + r_i) + \sigma_\theta (r_o - r_i) \right]/2r_o \\
p_i &= \left[ \sigma_r (r_o + r_i) - \sigma_\theta (r_o - r_i) \right]/2r_i
\end{align*}
\]  

Equation 4.2

In cases where strain-controlled mode was required the corresponding load, torque and pressures
would depend on the sample’s stress-strain response, which was non-linear and time-dependent.
The programme therefore would drive each controller simultaneously in small steps, up-date the
corresponding strain value, and then re-drive the controllers. The value of the required stress pa-
4.1. HCA MARK II

Figure 4.11: Schematic diagram of HCA Mark II automatic servo-controlled system

A stress tolerance value of 0.5kPa and strain tolerance of 0.002~0.01% were commonly employed in this study. It was possible to set lower limits if very small stress/strain increments were required. However, values less than 0.2kPa and 0.001% would be practically implausible given the typical resolutions of the load cell and pressure transducers and of the global measuring system, which were used by the programme to calculate the control strain parameter values. One possible option was to use readings from local strain measurement, which were of higher resolution. However this might cause inconvenient consequences arising from problems in setting optimal resolution ranges, non-linear calibration and limited working range. Any of these could severely hamper the control process. Considering these factors and the stress/strain increment rates used in this study (4kPa/hr and 0.1%/hr), it was decided not to use strain parameters calculated from the local strain transducers.

Parameter $t$ were determined from:

$$
t = \frac{p - \sigma_z}{(2b - 1)/3 - 1/\cos 2\alpha} \quad \text{for axial strain-controlled, or}
$$

$$
t = \frac{\tau_{z\theta}}{\sin 2\alpha} \quad \text{for torsional shear strain-controlled}
$$

(4.3)
4.2 Calculation of stresses and strains in HCA

4.2.1 Individual stress and strain components

In the interpretation of HC test results, individual stresses and strains cannot be used directly as they vary across the wall and along the height due to the effects of wall curvature, end restraint and application of torque. There have been several expressions for these average stresses (e.g. Hight et al. [1983]; Miura et al. [1986]; Sayao and Vaid [1991]); the differences between them come from the applications of different constitutive laws (e.g., linear or plastic behaviour) and averaging methods (over the volume or across the wall). Nevertheless, Wijewickreme and Vaid [1991] pointed out that the differences among various expressions are negligible given that the specimen geometry have been chosen carefully to minimise the effects of stress non-uniformity.

This study employed the expressions of average stresses and strains recommended by Hight et al. [1983]:

\[
\bar{\sigma}_z = \frac{\int_0^H \sigma_z dz}{\int_0^H dz} = \frac{W}{\pi (r_o^2 - r_i^2)} + \frac{p_o r_o^2 - p_i r_i^2}{r_o^2 - r_i^2} \quad (4.4a)
\]

\[
\bar{\tau}_{z\theta} = \frac{\int_0^{2\pi} \int_{r_i}^{r_o} \tau_{z\theta} r^2 dr d\theta}{\int_0^{2\pi} \int_{r_i}^{r_o} r^2 dr d\theta} = \frac{3M}{2\pi (r_o^2 - r_i^2)} \quad (4.4b)
\]

\[
\bar{\sigma}_r = \frac{\int_{r_i}^{r_o} \sigma_r dr}{\int_{r_i}^{r_o} dr} = \frac{p_o r_o + p_i r_i}{r_o + r_i} \quad (4.4c)
\]

\[
\bar{\sigma}_\theta = \frac{\int_{r_i}^{r_o} \sigma_\theta dr}{\int_{r_i}^{r_o} dr} = \frac{p_o r_o - p_i r_i}{r_o - r_i} \quad (4.4d)
\]

where \( W \) and \( M \) are the applied axial load and torque, \( p_o \) and \( p_i \) are the outer and inner cell pressures. The height, outer and inner radius of the sample are \( H \), \( r_o \) and \( r_i \), respectively.

Similarly the individual average strains were (Hight et al. [1983]):

\[
\bar{\varepsilon}_z = \frac{\int_0^H \varepsilon_z dz}{\int_0^H dz} = \frac{\Delta h}{H} \quad (4.5a)
\]

\[
\bar{\gamma}_{z\theta} = \frac{\int_0^{2\pi} \int_{r_i}^{r_o} \gamma_{z\theta} r^2 dr d\theta}{\int_0^{2\pi} \int_{r_i}^{r_o} r^2 dr d\theta} = \frac{-2\Delta \theta (r_o^2 - r_i^2)}{3H (r_o^2 - r_i^2)} \quad (4.5b)
\]

\[
\bar{\varepsilon}_r = \frac{\int_{r_i}^{r_o} \varepsilon_r dr}{\int_{r_i}^{r_o} dr} = -\frac{\Delta r_o - \Delta r_i}{r_o - r_i} \quad (4.5c)
\]

\[
\bar{\varepsilon}_\theta = \frac{\int_{r_i}^{r_o} \varepsilon_\theta dr}{\int_{r_i}^{r_o} dr} = -\frac{\Delta r_o + \Delta r_i}{r_o + r_i} \quad (4.5d)
\]

where \( \Delta h \), \( \Delta r_o \), and \( \Delta r_i \) are the changes in height, external and internal radii respectively and \( \Delta \theta \) is the angular deformation (in radian) of the base relative to the top of the specimen (see Figure 4.12).

In these expressions the axial stress (Equation 4.4a) is calculated from stress equilibrium condition and therefore is valid irrespective of the material’s behaviour. However the torsional shear stress (Equation 4.4b) is calculated from perfectly plastic stress assumption. Equations 4.4c and 4.4d are derived by taking average of stresses across the wall and assumed linear elastic
stress distributions. Note that Equation 4.5a and Equation 4.5b are based purely on strain compatibility, whereas an additional assumption of linear strain distribution across the wall is used in the derivations of Equation 4.5c and Equation 4.5d.

Hereafter the overline notation for average stresses and strains will be dropped for simplicity. Standard soil mechanics convention is applied for the signs of stresses and strains, i.e. positive sign for compressive stress and strain, and the shear stress $\tau_{ij}$ is positive if it acts on a plane facing the positive $i$ direction but is directed in the negative $j$ direction.

4.2.2 Principal stress and strain components

The magnitudes and inclination angles of the principal stresses and strains in the $z \sim \theta$ plane can be accordingly determined from the Mohr circles for stresses and strains. For stress components (Figure 2.5):

$$\sigma_1 = \frac{\sigma_z + \sigma_\theta}{2} + \sqrt{\frac{(\sigma_z - \sigma_\theta)^2}{4} + \tau_{z\theta}^2}$$

$$\sigma_3 = \frac{\sigma_z + \sigma_\theta}{2} - \sqrt{\frac{(\sigma_z - \sigma_\theta)^2}{4} + \tau_{z\theta}^2}$$

$$\alpha = \frac{1}{2} \arctan \left( \frac{2\tau_{z\theta}}{\sigma_z - \sigma_\theta} \right)$$

(4.6)

and for strains (Figure 4.13):

$$\epsilon_1 = \frac{\epsilon_z + \epsilon_\theta}{2} + \sqrt{\frac{(\epsilon_z - \epsilon_\theta)^2}{4} + \gamma_{z\theta}^2}$$

$$\epsilon_3 = \frac{\epsilon_z + \epsilon_\theta}{2} - \sqrt{\frac{(\epsilon_z - \epsilon_\theta)^2}{4} + \gamma_{z\theta}^2}$$

$$\alpha_\epsilon = \frac{1}{2} \arctan \left( \frac{\gamma_{z\theta}}{\epsilon_z - \epsilon_\theta} \right)$$

(4.7)

4.3 Analysis of data from HC tests

The analysis on data files from any HCA Mark II testing stage was calculated in several steps as shown in Table 4.4. Clearly this approach provided two advantages: (i) automating the calculation process which was repeatable and often tedious given the large amount of employed transducers and collected data, and (ii) providing consistent output format that was suitable for presentation and data exchange.

In light of previous testing experience for the 100mm dia. natural London Clay specimens (e.g. Gasparre [2005]) the restraints from membranes and filter papers under extension shearing mode (Leroueil et al., 1988; La Rochelle et al., 1988) were negligible and therefore were not accounted for in this study. As in most cases the hollow cylinder tested specimen had failure plane geometry that was complex, no correction had been made to the cross-sectional area after rupture such as the one used for triaxial specimens (Bishop and Henkel [1957]; Chandler [1968]).
4.3. ANALYSIS OF DATA FROM HC TESTS

Figure 4.12: Displacement and strain components

Figure 4.13: Mohr circle of strain
4.3. ANALYSIS OF DATA FROM HC TESTS

Table 4.4: Order of data analysis for a testing stage in HCA Mark II

<table>
<thead>
<tr>
<th>Order</th>
<th>Run files</th>
<th>Input files</th>
<th>Dimensions</th>
<th>Output files</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HCA-P1</td>
<td>test.p (*)</td>
<td>$H_0, R_i, R_o$</td>
<td>out-p1.csv</td>
<td>Deformation analysis</td>
</tr>
<tr>
<td>2</td>
<td>HCA-P2</td>
<td>out-p1.csv</td>
<td>$H_0, R_i, R_o$</td>
<td>out-str1.csv</td>
<td>Strain analysis</td>
</tr>
<tr>
<td>3</td>
<td>HC-S1</td>
<td>out-str1.csv</td>
<td>$La_1, La_2, Ls_3$</td>
<td>out-str2.csv</td>
<td>(see 4.3.2)</td>
</tr>
<tr>
<td>4</td>
<td>HC-S2</td>
<td>test.str (*)</td>
<td>$hc-s1.csv$</td>
<td>hc-s2.csv</td>
<td>Increments of stresses &amp; strains</td>
</tr>
<tr>
<td>5</td>
<td>HC-S3</td>
<td>hc-s1.csv</td>
<td>$hc-s2.csv$</td>
<td>hc-s3.csv</td>
<td>Other parameters</td>
</tr>
</tbody>
</table>

(*) data files provided (directly) by the HCA programme:

`test.p` - sensor readings in engineering unit
`test.str` - individual effective stress and global strain components, also included $p', t, q/p', \alpha, b$ (see List of Symbols for explanations).

4.3.1 Deformation analysis

For a hollow cylindrical specimen (initial dimensions of $H_0, R_i, R_o$), the global deformations are:

\[
\begin{align*}
\Delta H &= - (Ax DT - Ax DT 0) \\
\Delta \Theta &= - \arctan \left( \frac{(Tr DT - Tr DT 0)}{a} \right) \\
\Delta R_i &= \sqrt{\frac{\pi R_i^2 H_0 + \Delta V_i}{\pi (H_0 + \Delta H)}} - R_i \\
\Delta R_o &= \sqrt{\frac{\pi R_o^2 H_0 + \Delta V_i + \Delta V_s}{\pi (H_0 + \Delta H)}} - R_o
\end{align*}
\]

(4.8)

in which $Ax DT$ is reading of the global axial displacement transducer; $Tr DT$ is reading of the global torsional displacement transducer (in mm) and $a = 63.26$mm is the distance from its axis to the center of the loading piston. The inner and specimen's volume changes are $\Delta V_i = (In VG - In VG 0)$ and $\Delta V_s = -(VG - VG 0)$ where $In VG$ and $VG$ are readings of inner and soil volume gauges (in mm$^3$), respectively.

On the other hand, the (average) deformations using local transducers’ readings are:

\[
\begin{align*}
\Delta h &= - H_0/2 \times \sum_{k=1}^{2} (ELa_k - ELa_k 0)/La_k \\
\Delta \theta &= \pi/180 \times (ELa_3 - ELa_3 0) \\
\Delta r_i &= (LVDT - LVDT 0) \\
\Delta r_o &= - \frac{1}{3} \sum_{j=1}^{3} (OPx_j - OPx_j 0)
\end{align*}
\]

(4.9)

where $LVDT$ is reading of the inner LVDT (in mm); $OPx_j$ are readings of the three proximeters (in mm); $ELa_k$ are readings of the two double-axis electrolevels in vertical direction (in mm); and $ELa_3$ is from the single-axis electrolevel (in degrees). The corresponding gauge lengths are $La_1, La_2$ and $Ls_3$.  

75
4.3. ANÁLISIS DE DATOS DE PRUEBAS HC

4.3.1. Análisis de datos de pruebas HC

Página 76

4.3.2. Componentes de esfuerzos globales e locales

Componentes de esfuerzos globales:

\[ \varepsilon_z = -\frac{\Delta H}{H_0} \]

\[ \gamma_{z\theta} = -\frac{2\Delta \Theta}{3H_0} \left( \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right) \]

\[ \varepsilon_r = -\frac{\Delta R_o - \Delta R_i}{(R_o - R_i)} \]

\[ \varepsilon_\theta = -\frac{\Delta R_o + \Delta R_i}{(R_o + R_i)} \]

Componentes de esfuerzos locales:

\[ \varepsilon_z = -\frac{\Delta h}{H_0} \]

\[ \gamma_{z\theta} = -\frac{2\Delta \Theta}{3Ls_3} \left( \frac{R_o^3 + b^3 - R_i^3}{(R_o + b)^2 - R_i^3} \right) \]

\[ \varepsilon_r = -\frac{\Delta r_o - \Delta r_i}{(R_o - R_i)} \]

\[ \varepsilon_\theta = -\frac{\Delta r_o + \Delta r_i}{(R_o + R_i)} \]

Aquí, \( b = 8 \) mm fue la distancia entre la superficie del especímen externo y el eje del electronivel un eje. El esfuerzo torsional local había sido corregido a la radio promedio \((R_o + R_i)/2\) del especímen como se muestra en la Figura 4.14.
4.3.3 Other parameters

From the geometry of the Mohr’s strain circle:

\[ \gamma_{\text{max}} = \epsilon_1 - \epsilon_3 = \sqrt{(\epsilon_z - \epsilon_\theta)^2 + \gamma_{z\theta}^2} \]
\[ \epsilon_1, \epsilon_3 = (\epsilon_z + \epsilon_\theta)/2 \pm \gamma_{\text{max}}/2 \]  
(4.14)

Similarly the directions of the major (increment) of principal stress \( (\alpha_{d\sigma'}) \) and strain \( (\alpha_{d\epsilon}) \) are:

\[ \tan \alpha_{d\sigma'} = 2\Delta \tau_{z\theta}/(\Delta \sigma'_z - \Delta \sigma'_\theta) \]
\[ \tan \alpha_{d\epsilon} = \Delta \gamma_{z\theta}/(\Delta \epsilon_z - \Delta \epsilon_\theta) \]  
(4.15)

The deviatoric shear strain and shear stress are:

\[ \epsilon_d = \sqrt{3/2} \cdot \gamma_{\text{oct}} = 2/\sqrt{6} \cdot \sqrt{(\Delta \epsilon_1 - \Delta \epsilon_r)^2 + (\Delta \epsilon_3 - \Delta \epsilon_r)^2 + (\Delta \epsilon_1 - \Delta \epsilon_3)^2} \]
\[ \Delta J = \sqrt{3/2} \cdot \Delta \tau_{\text{oct}} = 1/\sqrt{6} \cdot \sqrt{(\Delta \sigma'_1 - \Delta \sigma'_2)^2 + (\Delta \sigma'_2 - \Delta \sigma'_3)^2 + (\Delta \sigma'_1 - \Delta \sigma'_3)^2} \]  
(4.16)

4.4 Comparisons between global and local strain measurements

The fact that the bedding errors and system compliance problems can significantly affect the measurements of global strains at small strain range in laboratory tests has been highlighted by many researchers (e.g. Jardine et al., 1984; Tatsuoka et al., 1991). This important recognition leads to the general appreciation of employing local strain measuring systems for studies that are concentrated in the pre-failure stress-strain behaviour of soils. The Author therefore carried out comparisons of the differences between strain components from global and local measuring systems to:

1. Access the degrees of coincidence of the global strain measuring system with the local system.
2. Identify the strain limits after which the use of values calculated by semi-local strain calculation method (see §4.3) can be justified for tests that did not have a complete set of local strain transducers (i.e. without the axial and the torsional electrolevels).

4.4.1 In the calculation of individual strain components

Comparisons between global and local strains were systematically performed for all tests that employed the (local) axial and torsional electrolevels. The full details of these tests are explained in Chapter 5, and the results reported here are from the undrained shearing stages that involved in most cases the application of both axial and torsional shear stresses. Due to the stiff construction characteristics of the HCA Mark II, it was expected that the global axial strain \( (\epsilon_z) \) would have been subjected to more severe errors due to bedding and system compliance than the global torsional shear strain \( (\gamma_{z\theta}) \). The results shown in Figure 4.15 and Figure 4.16 confirm this trend. The global axial strain overestimated the local strain at small strain, with as much as 25 % of difference at 0.01 % strain level, and they were only coincided after 0.02 % strain. On the other hand, the global and local torsional shear strains were similar from very small strain (0.001 %) although departures could arise if failure planes did not formed within the two contacts of the torsional shear electrolevel as shown in test AM45-10. In this situation the local \( \gamma_{z\theta} \) was underestimated because it
4.4. COMPARISONS BETWEEN GLOBAL AND LOCAL STRAIN MEASUREMENTS

was only represented for the rotation of the upper intact part, not the relative value between the upper and lower specimen bodies registered by the global displacement transducer in the horizontal plane.

The similarities between global and local measurements of the radial and circumferential strain are often hard to achieve. This is because the global values are dependent on the soil’s creep response, the global axial strain, and the system compliance (including the tubes and inner volume gauge). In contrast, the local strains are calculated at only one specimen's section, and therefore implicitly assuming right cylinder deformation shape. Figure 4.17 illustrates two different relationships observed in the calculated \( \varepsilon_\theta \), in which the similarities appeared at either small strain range (test AM00-03i) or not (test AM45-00). The difficulties involving in \( \varepsilon_\theta \) and \( \varepsilon_r \) determinations (and therefore for local \( \varepsilon_v \)) should therefore be expected.

4.4.2 In the calculation of the deviatoric shear strain

In this study, the axial and torsional electrolevels were not available when the five multi-axial shear tests of series AM\( \alpha \)-05 were conducted. The tests therefore would have underestimated the stiffness moduli at small strains. It is of interest to know in simple terms (using the generalised \( \varepsilon_d \)) the limit after which the global strain value could be expected to provide similar reference as if local strain measuring system was employed. The comparisons made by the author indicated that in most this limit ranged between 0.02 – 0.04 %, with the average value of 0.03 % as seen in Figure 4.18.

In conclusion, the comparisons between actual measurement values of global and local strain components using the HCA Mark II system indicated the importance of having a complete local strain measurement system. It also showed that global strain measurements were likely to be reliable only after strains became larger than 0.03 %, and therefore were not appropriate for the investigation of pre-failure yielding characteristics (see also Chapter 9).
Figure 4.15: Comparison between global and local $\varepsilon_z$, shearing stage in tests AM00-00 and AM30-03
Figure 4.16: Comparison between global and local $\gamma_{z\theta}$, shearing stage in tests AM30-00 and AM45-10
4.4. COMPARISONS BETWEEN GLOBAL AND LOCAL STRAIN MEASUREMENTS

Figure 4.17: Comparison between global and local $\epsilon_\theta$, shearing stage in tests AM45-00 and AM00-03
4.4. COMPARISONS BETWEEN GLOBAL AND LOCAL STRAIN MEASUREMENTS

Figure 4.18: Comparison between global and local $\varepsilon_d$ during shearing stage in tests AM30-03 and AM00-03
Chapter 5

Descriptions of site, material and testing procedures

The London Clay samples used in this research were prepared from nine block samples taken at the same depth (12.5mOD) at the site of Terminal 5 (T5), Heathrow Airport, London. The site's location and block sampling procedures are introduced in §5.1, followed by descriptions of the index properties and material's characteristics in §5.3. Next, §5.4 explains the problems encountered in the preparation of the hollow cylinder specimens and the methods of countermeasures. The procedure for saturation is described in §5.5 and finally §5.6 describes the procedures followed after the tests were completed. Detailed descriptions of the uniaxial and multi-axial stress path tests will be presented in §7.1 and §8.1, respectively.

5.1 Site condition and block sampling at T5

5.1.1 Site condition

Terminal 5, a major expansion project started in 2002 at Heathrow Airport, includes a new terminal building, aircraft stands, taxiway, rapid transit tunnels and a variety of associated satellite and service structures. The site, about 15 miles to the west of Central London (see Figure 1.1), is close to the London Basin's western margin and there are several nearby sites whose ground investigations have been reported in the literature, such as Prospect Park in the north, Ashford Common in the south, Wraysbury in the south west, Hatton Cross and T4 in the south east. A recent overview on geology and engineering properties of London Clay obtained from these investigations has been presented by Hight et al. (2003).

It is known that in this area the London Clay is overlain by the Terrace deposits (BGS record sheet 269, 1:50000 Series Geological Map, Solid and Drift Edition), which are mainly layers of gravel or brick earth (see BAA Ground Investigation Report, Heathrow T5, [1995] 2000). One exception is in the north-east of T5, where they had been almost entirely removed during construction work of the Perry Oaks Sewage Sludge Treatment in the 1930s. Contaminated made ground, lagoons and former borrow pits are also commonly noted ground features at this site.

The T5 site map is shown in Figure 5.1 including the locations of the block sample area and other
5.1. SITE CONDITION AND BLOCK SAMPLING

Table 5.1: Locations and ground levels of boreholes and block sampling area mentioned in Figure 5.1

<table>
<thead>
<tr>
<th>Location</th>
<th>Easting [m]</th>
<th>Northing [m]</th>
<th>GL</th>
<th>Top of London Clay</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH402</td>
<td>5354.78</td>
<td>5919.13</td>
<td>+17.87</td>
<td>+17.57</td>
<td>-12.13</td>
</tr>
<tr>
<td>BH402A</td>
<td>5350.63</td>
<td>5922.42</td>
<td>+17.77</td>
<td>+17.77</td>
<td>-47.13</td>
</tr>
<tr>
<td>BH404</td>
<td>5379.86</td>
<td>5880.95</td>
<td>+18.01</td>
<td>+18.01</td>
<td>-11.95</td>
</tr>
<tr>
<td>BH406</td>
<td>4976.06</td>
<td>5707.22</td>
<td>+21.40</td>
<td>+17.40</td>
<td>-29.60</td>
</tr>
<tr>
<td>BH407</td>
<td>4971.25</td>
<td>5706.42</td>
<td>+21.27</td>
<td>+17.47</td>
<td>-43.73</td>
</tr>
<tr>
<td>PTBH1</td>
<td>5172.25</td>
<td>5724.22</td>
<td>+23.50</td>
<td>+17.50</td>
<td>-26.50</td>
</tr>
<tr>
<td>PTBH2</td>
<td>5177.65</td>
<td>5742.30</td>
<td>+23.50</td>
<td>+17.50</td>
<td>+8.50</td>
</tr>
<tr>
<td>Blocks</td>
<td>5845.00</td>
<td>5165.00</td>
<td>+17.70</td>
<td>+17.70</td>
<td>+12.50</td>
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<tr>
<td>Blocks</td>
<td>5865.00</td>
<td>5185.00</td>
<td></td>
<td></td>
<td>+16.50</td>
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<tr>
<td>Blocks</td>
<td>5855.00</td>
<td>5205.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
End elevation indicates the elevation of end-of-borehole or at which blocks were taken.

nearby boreholes of interest the London Clay characterisation study at Imperial College (Gasparre, 2005; Nishimura, 2006; Mannion, 2007). Figure 5.2 presents the geotechnical profiles of boreholes BH404 and BH406. For each borehole, there are profiles of index properties as well as of suction, undrained triaxial compression shear strengths and shear stiffness measurements based on in situ seismic tests. The suctions are either from field suction probe measurements on thin-walled tube samples or values of initial effective stress from triaxial tests. There are, as anticipated, significant vertical variabilities in the index and mechanical properties.

At T5 the top surface of the London Clay is practically level (17.2–18.0 mOD) while the ground surface levels (GL) vary, depending on the overlying gravel or made ground thickness (see Table 5.1). For example, in the block sampling area (location A) and nearby (e.g. location D) the GL is about 17.7 mOD and the gravel layer is thin (up to 0.5 m) or absent. At other places, such as location B of the rotary-cored boreholes (PTBH1 and PTBH2), the GL is higher (21–24 mOD) and the gravel layer is 4–6 m thick. The groundwater level (GWL) is about 19 mOD, generally showing hydrostatic or mildly artesian conditions.

5.1.2 Block sampling

The block sampling process involved four site excursions between March and May 2003, and carried out by the London Clay research group at Imperial College (see also Gasparre, 2005; Nishimura, 2006). The block samples were taken at three levels: 7.2, 12.5 and 16.5 mOD, i.e. at corresponding depths of 10.5, 5.2 and 1.2 m below GL (BGL). In total 21 blocks were collected, of which nine blocks were from 12.5 mOD that were used in this study.

At each sampling level the following general procedures were carried out. First, a hydraulic excavator removed the gravel layer and prepared orthogonal trenches of approximately 0.8 m wide, 1.0 m deep at 1.2 m apart. Next, the soil columns within these trenches were cut out using a pneumatic spade tool with a clay cutter until each of them had been reduced to about 60 cm square in plan. To minimise excessive disturbance, hand trimming was then employed, forming smaller blocks of 30 cm each side. Each block was immediately encased in several layers of cling film and wax, then was
5.1. SITE CONDITION AND BLOCK SAMPLING

Put within an open wooden box (35 cm size). After that polyurethane expanding foam from aerosol cans was sprayed gradually from the base upwards to the box's top, filling the surrounding gaps. A wooden top cover was then put on and secured by screws. The same procedures were repeated for the other soil blocks prepared on the same day. Once completed, a surveying team was called in to determine the national grid coordinates and the ordnance levels (mOD) at the top centers of these boxes. The next morning each soil block was carefully separated from the ground by trimming its base with clay blades. This bottom surface was again flattened and coated with cling film, wax layer, and polyurethane foam before finally covered with a wooden base plate. All the boxes were loaded to a van which then came directly to Imperial College. In general each sampling excursion required two working days and a crew of five to six people.

When a series of tests was to be made on an original large block, it was cut into several rectangular prisms measuring approximately 15 cm × 15 cm × 25 cm by a large electric band saw. These new prisms were again quickly protected by layers of cling film and wax, then enclosed in smaller wooden boxes. The prisms, except one used for the next specimen preparation stage, were returned to the store.
5.1. SITE CONDITION AND BLOCK SAMPLING

Figure 5.1: Plan of the Terminal T5 site, Heathrow Airport, with sampling locations reported in the present study: A) Block sampling; B) Rotary-cored boreholes (PTBH1 and PTBH2); C) Boreholes BH402 (reported in [Hight et al., 2003], BH402A and BH404; and D) Boreholes BH406 and BH407 (sources: DigiMAP archive and BAA Ground Investigation Report, Heathrow T5 [2000]).
5.1. SITE CONDITION AND BLOCK SAMPLING

(a) Borehole BH404, Location C

(b) Borehole BH406, Location D

Figure 5.2: Geotechnical profiles of two boreholes at T5
5.2 In situ stresses

High horizontal effective stresses are likely to exist in the London Clay due to its heavy overconsolidation state, and it is recognised that values of $K_0$ near the surface of the clay are reduced if it has been previously reloaded by old construction or the deposition of the Terrace Gravel (Burland et al., 1979). Hight et al. (2003) showed the $K_0$ profiles at T5, which were estimated using Imperial College suction probes (Ridley and Burland, 1993) on thin wall tube samples. Although higher $K_0$ values were noticed in the lagoon area where the gravel layer was removed but there was no evidence from the suction measurements of lower $K_0$ near to the surface in other areas overlain by the gravel layer. In this study, the filed $K_0$ profile where the gravel in place was selected as the basis for estimation of the in situ stresses.

It has been well recognised that London Clay was deposited 52 millions years ago and since then had been subjected to various cycles of loading, unloading and reloading for which their durations and degrees are largely ill-specified. Due to this complexity in this study only a simple geological model was used to estimate the clay’s stress history. This was done by simplifying the complex real events with a one-dimensional model, consisting of three distinct stages: (i) the deposition of the clay to its maximum thickness, i.e. virgin loading; then (ii) the erosion to the current level, an unloading stage of significant amount; and finally (iii) the reloading associated with the deposition of the Terrace Deposits, here represented by a gravel layer. The maximum thickness of deposition was determined from estimates of the maximum erosion thickness (see §3.1.2). To match the estimated current in-situ effective stresses, the model was calibrated to the field $K_0$ profile. An identical model had been employed by Gasparre (2005) and Nishimura (2006) in their studies on London Clay samples from the same site.

In this model, the coefficient of horizontal stress during deposition was calculated by Jaky’s formula:

$$K_{0c} = 1 - \sin \phi'$$

in which $\phi'$ is the effective angle of shearing resistance.

The changes of this coefficient due to a simple unloading ($K_{0u}$) or reloading ($K_{0r}$) process can be determined by the expressions (Mayne and Kulhawy, 1982):

$$K_{0u} = K_{0c} OCR_{\sin \phi'}$$

$$K_{0r} = K_{0c} \left[ OCR_{\sin \phi'} + 3 \left( 1 - \frac{OCR}{OCR_{\max}} \right) \right]$$

where $OCR = \sigma'_{\max} / \sigma'_{v0}$ is the over consolidation ratio and $OCR_{\max}$ is its maximum value which the soil experienced.

For the estimation of the in-situ stress state of the London Clay samples at T5, it was assumed that $\phi' = 26.5^\circ$ and there had been 175 m thick clay layer on top of the present elevation (at 17.5 mOD), which was later being eroded completely. During this period the ground water remained under hydrostatic condition with the GWL was alway at or above the ground level. In the final stage, a 6m-thick layer of gravel was deposited and the GWL was reduced to 4.5m BGL. The total unit weights of London Clay and gravel were 19.8 kN/m$^3$ and 19.1 kN/m$^3$, respectively.

The assumed distributions of the vertical effective stress is shown in Figure 5.4, whereas the...
5.3. SOIL DESCRIPTION

All samples tested in this study were from the blocks at 12.5 mOD (5.2 m BGL). In general the clay was stiff and of high plasticity but intensely fissured. The clay colour is light to dark olive grey, and when being fully exposed to atmosphere this turned to brown due to oxidation.
5.3. SOIL DESCRIPTION

5.3.1 Macrofabric, microfabric and mineralogy

Discontinuities found during block sampling and sample preparation include fissures (cracks) and foreign inclusions that were typically surrounded with a thin 1–3 mm band of brown oxidised silty clay. The presence of these oxidized layers, as also observed by Nishimura (2006) in the blocks taken at 7.2 mOD (10.5 m BGL), indicates that the discontinuities are natural and not due to artificial disturbance from the sampling process. The inclusions were either small sand and silt nodules or larger pyrite concretions (up to 4 cm wide). Whenever possible these large concretions were removed to ensure a successful trimming process and to minimise their potential influences on modifying the test results.

For each hollow cylinder sample, the pattern of major fissures on its outer surface was recorded after the cell pressure was applied, during which the membrane became practically transparent and the discontinuity pattern could be observed by eye. Generally the fissures were 3–6 cm apart and mostly sub-horizontal. Short vertical and sub-vertical discontinuities were also present but those extending fully along the specimen’s height were rare. In some extreme cases the fissures were open and their surfaces were marked with selenite crystals, disseminated mica plates and glauconite grains. Any soil block (or soil prism of smaller size) that had those features was very vulnerable to collapse during sample preparation.

Microfabric and mineralogical studies using SEM technique had been carried out on clay specimens from rotary cored samples at the same site (Gasparre 2005). For example, Figure 5.6 shows the SEM images (at magnification scales of 200X and 1500X) from a slightly deeper sample (at 7 m BGL). The images show a typical clay-rich soil configuration, in which the silt grains and other larger mineral particles float within the clay matrix. In addition, the fabric shows an open and disturbed structure, probably due to the influence of extensive bioturbation during depositional times. Gasparre also found that samples from shallow depths in units C and B2 had similar compositions that were characterized by a high proportion of smectite-based minerals. The mineralogical analysis for this sample indicated a significant content of clay mineral (60 % I-rich and S-rich illite-
5.3. SOIL DESCRIPTION

Smectite, 22% illite and 15% kaolinite) and high ratio of clay minerals to quartz (ratio of 34.0). The presences of calcite crystals in SEM images and chlorite (about 4%) in mineralogy analysis of the 16.5 mOD sample are the evidence of the limited influences of weathering process at this depth. However, the degree of fissuring in block samples was more severe because unlike the rotary cored samples they were taken from area where the gravel layers had been previously removed.

5.3.2 Index properties

The index properties reported herein for block samples from 12.5 mOD are $w_n$, $LL$, $PL$, $Gs$ and particle size distributions. These tests were carried out following British Standard (BS1377: Part 2) [1990] in which the cone penetration method was used for the determination of liquid limit. Figure 5.7 shows the results of hydrometer tests, where the clay fraction corresponds to percent finer than 2 $\mu$m and the sand content was determined from the proportion of soil mass remained on the 63 $\mu$m sieve after wet sieving. The clay's activity was determined by:

$$Activity\ A = \frac{PI}{CF}$$

in which $PI$ is the plasticity index ($= LL - PL$) and $CF$ is the clay fraction.

The specimens' index properties are summarised in Table 5.2, from which the following remarks can be made:

- All gradation curves were within a narrow range with small scatters in the values of clay fraction and sand content.
- The block samples were quite uniform in terms of natural water content, having value of 23.56 ± 0.5%. Similar degrees of consistency were noted for the Atterberg's limits (averages $LL = 69.1\%$, $PL = 25.4\%$, see Figure 5.8a).
- The clay was classified as silty clay of high plasticity

In reviewing the results from the advanced site characterisation programme, Hight et al. (2007) remarked that the lithological boundaries coincided more clearly with breaks in the trends in the water content data than in the plasticity data, and clay samples from unit B2 were generally of higher plasticity than those from units A2 and A3.

5.3.3 Initial effective stress from laboratory tests

The final stabilized effective stress of a specimen subjected to the first application of equal all around (isotropic) stresses under undrained condition is referred to as initial effective stress ($p'_i$). The values of initial effective stress for the specimens tested in this research were defined (see also §5.5) at pore water pressure close to +50 kPa. It was found that the average measured $p'_i$ for hollow cylinder specimens was 365 kPa. This was about 30% higher than the in-situ value of 280 kPa expected at this level, which was calculated based on assumptions of $K_0 = 2.3$ and 150 kPa vertical effective stress (see §5.2). One possible reason was the HCA specimen preparation process, which was time consuming (§5.4) and may have caused evaporation of the pore water. The major part of suction gain may have been related to the high current air flow used to blow soil from the specimen's interior during the boring of the inner cavity.
5.3. SOIL DESCRIPTION

Table 5.2: Index properties of 100mm hollow cylinder specimens

<table>
<thead>
<tr>
<th>Test code</th>
<th>$w_n$</th>
<th>LL</th>
<th>PL</th>
<th>PI</th>
<th>CF</th>
<th>A</th>
<th>SC</th>
<th>$\gamma_t$</th>
<th>$p'_i$</th>
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<td>%</td>
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<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>kN/m$^3$</td>
<td>kPa</td>
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<td>24.86</td>
<td>44.21</td>
<td>49.6</td>
<td>0.89</td>
<td>3.6</td>
<td>20.2</td>
<td>395</td>
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<td>25.74</td>
<td>42.98</td>
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<td>0.86</td>
<td>2.9</td>
<td>19.7</td>
<td>380</td>
</tr>
<tr>
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<td>43.83</td>
<td>52.2</td>
<td>0.84</td>
<td>2.3</td>
<td>20.3</td>
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<td>0.87</td>
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<td>49.4</td>
<td>0.87</td>
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<td>318</td>
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<td>52.1</td>
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<td>19.8</td>
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<td>43.74</td>
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<td>50.1</td>
<td>0.86</td>
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<td>AM60-10</td>
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<td>69.34</td>
<td>25.27</td>
<td>44.07</td>
<td>50.8</td>
<td>0.87</td>
<td>2.4</td>
<td>19.6</td>
<td>352</td>
</tr>
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<td>69.07</td>
<td>25.03</td>
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<td>49.4</td>
<td>0.89</td>
<td>2.5</td>
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</tr>
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<td>HC-DT</td>
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<td>43.66</td>
<td>50.9</td>
<td>0.86</td>
<td>3.1</td>
<td>20.2</td>
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</tr>
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<td>25.61</td>
<td>43.66</td>
<td>50.9</td>
<td>0.86</td>
<td>3.2</td>
<td>19.7</td>
<td>419</td>
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<td>25.61</td>
<td>43.66</td>
<td>50.9</td>
<td>0.86</td>
<td>3.9</td>
<td>19.4</td>
<td>350</td>
</tr>
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<td>69.16</td>
<td>25.30</td>
<td>43.86</td>
<td>51.0</td>
<td>0.86</td>
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<td>365</td>
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<tr>
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<td>0.33</td>
<td>0.36</td>
<td>0.49</td>
<td>1.3</td>
<td>0.02</td>
<td>1.0</td>
<td>0.2</td>
<td>42</td>
</tr>
</tbody>
</table>

Notes:

- AM or IM: Multi-axial shear tests from anisotropic ($q \neq 0$, CAU) or isotropic consolidation state ($q = 0$, CIU), followed by a 4-digit test code. For example, AM45-10 indicates a test on anisotropically consolidated sample with the final stress path followed $b = 1.0$, $\alpha = 45^\circ$ under undrained condition;
- HC-DT, HC-DQ, HC-DZ: Uniaxial drained shear tests.

Additional information on the measured $p'_i$ values on samples from the same site are also available in the studies by Nishimura (2006) and Gasparre (2005). These values are compiled in Figure 5.8b, together with the in-situ suction measurements on thin-wall samples reported by Hight et al. (2003). Most of the values found on samples from rotary cores were within the boundaries defined by in-situ suctions. In contrast, $p'_i$ of block samples (at 7.2 mOD and 12.5 mOD) were consistently higher the upper boundary, again possibly an indication of the effects of air flow during inner boring. For samples from rotary cores, 71 mm hollow cylinder specimens had higher $p'_i$ than triaxial 100 mm samples, confirming that the boring generated a net gain in initial effective stress measurement. The block samples are of higher overconsolidation ratio than the rotary cores from the same horizon as higher $K_0$ values were recorded for area not having gravel layer where the blocks were taken (Hight et al., 2003).
5.3. SOIL DESCRIPTION

(a) Magnification at 200X, indicating open structure

(b) At 1500X showing (1) calcite crystal, (2) silt grain within the clay matrix

Figure 5.6: SEM images of London Clay sample from 7m BGL. Images courtesy of Dr. J. Huggett, Natural History Museum, London (after Gasparre, 2005).
5.3. SOIL DESCRIPTION

Figure 5.7: Particle size distributions from hydrometer tests for samples at 12.5mOD

Figure 5.8: Profiles of (a) Atterberg's limits and (b) Initial effective stress measured in HCA and triaxial tests. Elevation 23.5mOD is GL for rotary-cored boreholes PTBH1 and PTBH2; whereas 18.0mOD is the average level for top of London Clay.
5.4 Specimen preparation and set-up

5.4.1 Hollow cylinder specimen preparation

The block samples used in this study came from a relatively shallow depth and presented several challenges for sample preparation. Indeed, the first two attempts that used common preparation technique for tube samples were unsuccessful. It was recognized that the features of the clay at this level, including fissures with thin silty/sandy partings and variable sizes of irregular embedded claystones, were responsible for such failure. Subsequently different methods and cutting tools were tried to overcome these problems and the following countermeasures were found to produce solid cylindrical specimens with satisfactory results:

- Immediately after removal from its wooden box, the integrity of the soil prism was maintained by applying several restraining membrane strips, each about 30 mm wide.
- The prism was cut to about 12 cm in diameter, proceeding with thin (5 mm) vertical soil parings with a sharp knife, cutting gradually from its top to bottom.
- A similar technique was employed during the trimming of the excess top and bottom, aiming to create a prism of 19 – 22 cm in height.
- The prism, with the strips in place, was placed into a manual soil trimming lathe, which had been modified so that an adjustable compression force could be applied to the sample through its top platen. This modification helped to restrain any tendency of fissure opening within the soil prism.
- The prism was gradually trimmed to a cylinder of 100 ± 0.3 mm with a wire saw.
- The cylinder was then encased within a three-part mould (inside diameter of 100.5 mm) and was tied by two outside Jubilee clips, ready for the next step of forming the inner cylinder cavity.

There are several available methods (e.g. [JGS-0551] 1998, Saada and Bianchini, 1975, Talesnick and Frydman, 1990, Lade and Kirkgard, 2000) for the preparation of the inner cavity; the choice of a particular method depends greatly on the soil type and past experience. The technique used by Porovic (1995) at Imperial College employed a metal working lathe to apply a series of successively increased drill sizes at slow advance rate. This was reported to cause relatively small degrees of disturbance on samples of soft Pentre clayey silt, hard London Clay and Thanet Clay. Hollow specimens of soft Bothkennar Clay were also prepared by the same technique (Albert et al., 2003).

For specimens of London Clay from the T5 site, the Author and Nishimura (2006) modified this technique slightly. The soil cylinder, encased securely in the steel mould, was set in a metal working lathe, and an inner hole was produced and enlarged by using five successive drill (nominal diameter of $\frac{7}{16}$, $\frac{29}{32}$, $\frac{3}{8}$, $\frac{3}{4}$, 2 inches) as shown in Figure 5.9. The largest drill was then replaced by a boring bar and the reaming was performed in two steps, first to $57 \sim 58$ mm diameter before completing at 60 mm. During this process compressed air was blown from the other end through the inner hole to clear the trimmed soils. Figure 5.10 illustrate the process of forming an HCA specimen, starting from a rectangular soil prism.

For each specimen, six moisture content measurements were made to check any change experienced before and after sample preparation. The specimen’s dimensions ($H$, $D_i$, $D_o$) were measured
at three positions. The values were recorded and average values were input into the controlling programme as the initial conditions. The specimen's weight was also measured for the calculation of the total unit weight.

5.4.2 Specimen set-up in the HCA Mark II

The set-up followed the main steps below:

**Preparation of specimen ends** Radial cuts (12 in total) at each of the specimen ends were prepared with a sharp knife to accommodate the fins protruding from the annular coarse-bronze porous ends.

**Side drain** The mould was removed and 12 damp strips of 1cm-wide Whatmann No.54 paper (four and eight strips uniformly distributed on inner and outer walls, respectively) were attached to the specimen walls to facilitate the consolidation or the equalization of the pore water pressure within the specimen.

**Outer membrane enclosure** Using a membrane stretcher a 100 mm dia. outer rubber membrane (made by Farrance Wykeham) was placed over the specimen with care to expel locally trapped air. After that four O-rings, two at each specimen end, were applied to seal the specimen from the outer cell.

**Inner membrane enclosure** The bottom and top drainage lines were securely connected and thoroughly flushed. Water left on the base bronze porous end was dried with tissue paper, and with the inner membrane located in position, the specimen was placed on the base pedestal. The top pedestal was set in place and sealed with inner and outer O-rings. A suction of 50~70 kPa was applied in the top suction line to extract the trapped air within the system, mainly presented in the beginning in the space between the inner membrane and the specimen.

**Installation of the suite of local strain sensors** The usual order of installation was: the inner LVDT, the three copper plates used as proximeters’ targets and finally the three electrolevels. During this stage care had to be taken to ensure: (i) the two double-axis electrolevels were symmetrically aligned; (ii) the axes of the inner LVDT and the external proximeters were in the same horizontal plane; (iii) the local strain sensors were located as centrally as possible on the specimen; and (iv) the sensors were adjusted to their optimal resolution ranges.

**Connection between load cell and the top cap** This connection used quick curing resin (see §4.1). It was important that the four bolts immersed firmly in the resin.

**Filling of cell water** The inner cell was filled with de-aired water with air being displaced through the inner air–bleed valve. Once completed, this valve was sealed. Next, the outer cell chamber was lifted into position by the overhead crane and then secured. With the outer air–bleed valves opened, the main water line was connected to fill the outer cell and the rate of inflow was reduced once the cell was nearly full to prevent excessive pressures being applied to the specimen. Finally, these air–bleed valves were closed.

The attachments made to the inner and outer membrane when placing the sensor’s pads were secured by super glue. Overall a typical setting-up process took 5~6 hours, of which the time-consuming stages were: trimming and boring of the HCA specimen, resin hardening time, installation of local strain sensors and filling the outer cell.
5.4. SPECIMEN PREPARATION AND SET-UP

(a) Outside trimming: 1) soil lathe; 2) wire saw; 3) knives

(b) Inside drilling: 1) Lathe drill bits; 2) boring bar

Figure 5.9: Tools used for the preparation of the London Clay hollow cylinder specimen
5.4. SPECIMEN PREPARATION AND SET-UP

(a) A soil prism  
(b) Cutting with electrical bandsaw

(c) Outside trimming in the soil lathe  
(d) Assembling of the three-part steel mould

(e) Drilling inner hole with boring bar  
(f) Preparing vane’s grips at specimen ends

Figure 5.10: Stages of preparation for 100mm OD hollow cylinder specimen
5.5 Saturation

After completing the specimen set-up in the cell, the top and bottom suction lines were closed and an equal all around pressure was applied to the specimen. The cell pressure was aimed to generate a positive pore water pressure (p.w.p) to avoid cavitation in the pressure measuring system. After about 4 hours, the air remained within the drainage lines was flushed with de-aired water sent from the 100cc volume gauge until no bubbles of air were seen in the air-trap unit (can be seen in Figure 4.5). The sample was then left in this state for about 24 hours so that the p.w.p readings reached equilibrium, at which the measured effective stress was assigned as the initial effective stress $p_i^0$.

However, initial saturation checks made by applying cell pressure increments (20~30 kPa) and calculating the corresponding B-values were usually gave low values ($B \sim 0.5$), indicating the whole system was still under-saturated. The saturation of the system was improved by using back pressure method, in which the cell pressure and the back pressure were simultaneously increased in several steps while keeping their difference constant and was equal to the initial effective stress. Typically the applied back pressure was between 250–300 kPa, sometimes up to 350 kPa, depending on the results of further B-tests. Generally a period of 3 to 5 days was required to achieve $B \geq 0.90$. After the saturation and B-check stages, a pause period of 24 hours was allowed to stabilise all sensor readings before carrying out subsequent testing stages.

5.6 Procedure after test completion

The final undrained shearing stage was terminated when either excessive deformation of the membrane or an apparently stable post-peak behaviour was observed. The natural London Clay specimens tested in this study showed clear shear banding with post-peak rupture. A residual state was not possible to reach due to the limited possible amount of imposed deformation in both the HCA and stress path cell.

Upon the termination of each test, photographs of the specimen were taken at four orthogonal views while keeping the stress state. Next the confining cell pressures were reduced to 100kPa, also under undrained conditions, then the outlet drainage valve was opened and the water in the outer cell was drained out. Once completed, the apparatus was subsequently disassembled and the local strain transducers were removed. The specimen was then wrapped with a transparent film on which the failure plane(s) and discontinuities were recorded. The specimens were wrapped with cling film and stored for any further inspection.
Chapter 6

Behaviour of London Clay during re-consolidation stages

This Chapter discusses the behaviour of the natural London Clay hollow cylinder samples during reconsolidation stages that brought the effective stresses of the samples from an initial isotropic state to the estimated in-situ state. This reconsolidation scheme consisted of two stages, first an isotropic consolidation or swelling to the estimated in-situ $p'$, and then drained triaxial extension to the anisotropic $K_0$, keeping the mean effective stress unchanged. Between them, pause periods were allowed for creep and ageing so that the rate of axial strain was less than 0.002 %/hour.

It is important in any drained test to ensure that full drainage condition is achieved. A variety of simplified theoretical approaches exist (e.g. Bishop and Henkel, 1957) to assess suitable loading rates. Alternatively, experimentally-based criteria can be applied based on mid-height pore water pressure (p.w.p) measurements (Hight, 1982; Baldi et al., 1988). Aiming to limit to 5% the difference between base and mid-height probe p.w.p with respect to the mean current effective stress ($\Delta u/p' = 0.05$), Gasparre (2005) found that a loading rate for axial stress of 1–2kPa/hour was appropriate for her 100mm diameter, 200mm high London Clay triaxial samples. In an ideal drainage condition the equivalent applicable loading rate for a HC specimen should be quicker because it has much a shorter horizontal drainage path due to its thin wall thickness. However, experiments have also indicated that the full effectiveness of side drains using filter paper in a stiff clay is not always perfect (see Leroueil et al., 1988). Therefore in this study stress changes during reconsolidation were applied at rates of 4–6 kPa/hour to be conservative.

The re-consolidation stages are introduced in §6.1, followed by §6.2 and §6.3 which describe the behaviour during the drained isotropic and anisotropic stages, respectively. The detailed testing data for each test is summarised in Appendix B.

Footnote 1: For a 100mm diameter cylinder sample, the theoretical horizontal drainage path is 18mm in HC specimen (12 strips of side drains, each 1cm wide, see §5.4) compared with the 50mm length in triaxial specimen. Hence the end-of-primary consolidation period can be $(50/18)^2 = 7.7$ times faster.
6.1 Re-consolidation stress path

It has been well recognised that the stress history prior to shear influences the subsequent soil response, particularly with regard to stiffness characteristics (e.g. Costa Filho 1984; Atkinson et al. 1986; Jardine 1992; Smith et al. 1992; Atkinson et al. 1993; Clayton and Heymann 2001). The effects depend not only on the induced strains but also on the relative direction between the incoming and outgoing stress paths, as well as the relative position of the current stress state to the soil’s bounding surfaces.

In addition, the duration of any creep periods imposed is a vital part of this recent stress history. The longer the period of creep or aging is, the less dependent the shear stiffness on the direction of previous loading path is (e.g. Jardine 1985; Jardine et al. 1991; Tatsuoka et al. 1997). It is therefore important to separate the effects of creep from those of the stress path by implementing acceptance criteria for strain rate.

The reconsolidation stress paths did not attempt to mimic the stress history model described in 5.2 because retracing such a complete path would cause considerable strains and would probably damage the soil’s natural structure severely. For example, Gasparre (2005) reported that volumetric strains as large as 2% were recorded when probing out towards the intact boundary surface under isotropic conditions. Moreover the block samples were highly overconsolidated and fissured, therefore they were prone to failure if the unloading path within extension zone was applied. Similar difficulties had been observed by Gasparre (2005) and Nishimura (2006) on rotary-cored and block samples at depths between 7~10m. As a result, a shorter reconsolidation path was applied for the majority of tests, except for tests IM-0005 and IM90-05 that started from isotropic stress conditions. The scheme consisted of two stages, as shown in Figure 6.1:

1. Isotropic re-consolidation from \( p' \) to \( p_0 \)

2. Anisotropic re-consolidation with constant \( p_0 \) and reduction of \( (\sigma_v - \sigma_h) \) toward the in-situ stress point

To limit the amount of pre-straining during re-consolidation stages, the path was set to terminate once the axial strain (extension) exceeded 0.5%. Since it was impossible to reach the estimated in-situ stress without risking extension failure, eventually the reconsolidation paths terminated at different points rather than A. Given that the Author’s tests were all from the same depth it was desirable to aim for a common set of stresses (e.g. point A1) although this was not always possible.

During reconsolidation, the cell pressure and axial load was automatically controlled following a constant stress rate (4~6 kPa/hr) while keeping the back pressure constant. At each end-of-consolidation point the specimen was allowed to creep until the strain rates of all strain components were smaller than 0.002%/hr. Depending on the relative difference between the initial effective stress and the imposed consolidation stress, the re-consolidation stages required from 8 to 12 days.

\(^2\)Two specimens were failed during anisotropic consolidation in the beginning of this study.
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

![Diagram showing stress points](image)

Initial $p'_0$, Final $p'_f$, Estimated in-situ $p_0$

A: Estimated in-situ stress point
A1: Final consolidation stress point

Figure 6.1: Scheme of reconsolidation

6.2 Isotropically re-consolidation stages

6.2.1 Changes in void ratio and normal strains

The changes in volumetric strain and void ratio of London Clay samples during isotropic consolidation are summarized in Table 6.1. As a result of the high initial effective stress, most of the samples were subjected to isotropic swelling except in two tests AM00-05 and AM90-05. Figure 6.2 and Figure 6.3 (from tests AM45-00 and AM50-05, respectively) show the typical development of normal strains during isotropic swelling. Overall it can be seen that the radial and circumferential strains were similar but not equal. If it is accepted that London Clay is a cross-anisotropic elastic material, then theoretically these two strains must be the same because $\Delta \sigma'_r = \Delta \sigma'_\theta$ in axi-symmetric stress conditions. Therefore the observed difference, although small, might possibly reflect the strain non-uniformity and the violation of right cylinder deformed shape of the hollow cylinder specimens. Similar responses have also been observed by Nishimura (2006) in his hollow cylinder tests on London Clay from another horizon, although it should be noted that only global strain measurements were available in his tests.

In addition, significant amount of strains generally developed after loading was completed. For example, an additional 0.12% of volumetric strain continued after completion of loading to $p' = 280$ kPa (see Figure 6.2), i.e. about 28% of the final strain. This indicated that the chosen loading rates were still relatively quick for a full drainage condition and may well be contributed residual straining to the secondary consolidation of the clay. Subsequently the applied loading rates during both isotropic and anisotropic consolidation were adjusted to 3–4 kPa/hour, and a lower bound of 2 kPa/hour stress increment was employed for all the drained uniaxial shear tests in Chapter 7.

Figure 6.4 plots the states of the clay specimens before and after the isotropic consolidation or swelling stage in the $v (= 1 + e)$ ~ $p'$ plane, whereas Figure 6.5 shows the relationships between the incremental final volumetric strains and mean effective stresses. Here the values of the initial void ratio were calculated from the specimen’s initial water content and assuming $G_s = 2.70$ (see Chapter 5), while the void ratios after isotropic consolidations were determined from the resulted volumetric strain. With respect to the isotropic intrinsic normal compression line (NCL) of reconstituted clay from lithological unit B (Gasparre, 2005) and given that the yield stress of the natural London Clay generally lies beyond the NCL line (Burland, 1990; Chandler, 2000), the stress level involved in this study was much smaller than the clay’s in situ yield stress.
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

Table 6.1: Values of mean pressure and void ratio of samples tested in HCA Mark II

<table>
<thead>
<tr>
<th>Test code</th>
<th>$p'_i$ [kPa]</th>
<th>$1 + e_i$ [-]</th>
<th>$p'_o$ [kPa]</th>
<th>$1 + e_o$ [-]</th>
<th>$\Delta p' = p'_o - p'_i$ [kPa]</th>
<th>$\epsilon_v$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM00-00</td>
<td>395</td>
<td>1.624</td>
<td>280</td>
<td>1.632</td>
<td>-115</td>
<td>-0.51</td>
</tr>
<tr>
<td>AM45-00</td>
<td>380</td>
<td>1.634</td>
<td>280</td>
<td>1.641</td>
<td>-100</td>
<td>-0.41</td>
</tr>
<tr>
<td>AM00-03</td>
<td>375</td>
<td>1.644</td>
<td>280</td>
<td>1.647</td>
<td>-95</td>
<td>-0.21</td>
</tr>
<tr>
<td>AM00-03b</td>
<td>320</td>
<td>1.649</td>
<td>280</td>
<td>1.656</td>
<td>-40</td>
<td>-0.40</td>
</tr>
<tr>
<td>AM30-03</td>
<td>318</td>
<td>1.642</td>
<td>280</td>
<td>1.644</td>
<td>-38</td>
<td>-0.15</td>
</tr>
<tr>
<td>AM00-05</td>
<td>270</td>
<td>1.641</td>
<td>312</td>
<td>1.639</td>
<td>42</td>
<td>0.11</td>
</tr>
<tr>
<td>AM30-05</td>
<td>375</td>
<td>1.642</td>
<td>312</td>
<td>1.644</td>
<td>-63</td>
<td>-0.13</td>
</tr>
<tr>
<td>AM50-05</td>
<td>380</td>
<td>1.642</td>
<td>312</td>
<td>1.648</td>
<td>-68</td>
<td>-0.35</td>
</tr>
<tr>
<td>AM90-05</td>
<td>304</td>
<td>1.642</td>
<td>312</td>
<td>1.643</td>
<td>8</td>
<td>0.05</td>
</tr>
<tr>
<td>IM00-05</td>
<td>387</td>
<td>1.630</td>
<td>280</td>
<td>1.637</td>
<td>-107</td>
<td>-0.43</td>
</tr>
<tr>
<td>IM90-05</td>
<td>415</td>
<td>1.628</td>
<td>280</td>
<td>1.640</td>
<td>-135</td>
<td>-0.74</td>
</tr>
<tr>
<td>AM45-10</td>
<td>356</td>
<td>1.630</td>
<td>280</td>
<td>1.637</td>
<td>-76</td>
<td>-0.44</td>
</tr>
<tr>
<td>AM60-10</td>
<td>352</td>
<td>1.632</td>
<td>280</td>
<td>1.637</td>
<td>-72</td>
<td>-0.33</td>
</tr>
<tr>
<td>AM90-10</td>
<td>382</td>
<td>1.640</td>
<td>280</td>
<td>1.646</td>
<td>-102</td>
<td>-0.38</td>
</tr>
<tr>
<td>HC-DT</td>
<td>405</td>
<td>1.607</td>
<td>280</td>
<td>1.617</td>
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<td>-0.62</td>
</tr>
<tr>
<td>HC-DQ</td>
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<td>280</td>
<td>1.616</td>
<td>-139</td>
<td>-0.78</td>
</tr>
<tr>
<td>HC-DZ</td>
<td>350</td>
<td>1.642</td>
<td>280</td>
<td>1.650</td>
<td>-70</td>
<td>-0.47</td>
</tr>
</tbody>
</table>

Notes:

$p'_i$ and $e_i$ Initial effective stress and void ratio, respectively

$p'_o$ and $e_o$ The effective stress and void ratio at end of isotropic re-consolidation stage

$\epsilon_v$ Resulted final volumetric strain due to isotropic re-consolidation

6.2.2 Bulk modulus

It has been well recognised that the relationship between mean effective stress and volumetric strain is highly non-linear. Hence the volumetric behaviour of the natural London Clay at 5.2mBGL have been discussed in term of the secant bulk modulus $K' = \Delta p' / \Delta \epsilon_v$ against volumetric strain. The individual testing results are listed in Appendix B and Figure 6.6 summarizes their ranges. Generally the majority of $K'$ values at 0.001% strain were within 100–300 MPa, which itself was a wide range given that the clay samples came from the same soil horizon and had similar physical properties (see Chapter 5). However, it is understood that measurement of bulk stiffness is notoriously difficult, being affected by creep and inaccuracies in volume strain measurements – external measurements are unreliable at small strains and internal assessments, though better, are usually based on the measurement at only one section of the sample.

Figure 6.7 plots the normalised bulk modulus ratio, $K'/p'$ which is of useful information for non-linear constitutive models (e.g. Jardine et al. [1991]; Hight and Higgins [1995]).
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

(a) Evolution with elapsed time

(b) With respect to mean effective stress

Figure 6.2: Typical development of strains during isotropic re-consolidation to $p' = 280$ kPa, test AM45-00
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

Figure 6.3: Typical development of strains during isotropic re-consolidation to $p' = 312\text{kPa}$, test AM50-05

(a) Evolution with elapsed time

(b) With respect to mean effective stress
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

Figure 6.4: Volume and effective stress state for 12.5mOD (5.2mBGL) natural London Clay samples

Figure 6.5: Incremental final volumetric strain and mean effective stress for 12.5mOD (5.2mBGL) natural London Clay samples
6.2. ISOTROPICALLY RE-CONSOLIDATION STAGES

Figure 6.6: Ranges of secant bulk modulus during isotropic re-consolidation

Figure 6.7: Ranges of normalised $K' / \rho'$ during isotropic re-consolidation
6.3 ANISOTROPIC RE-CONSOLIDATION STAGES

6.3 Anisotropic re-consolidation stages

During this stage, the clay samples were consolidated with constant mean effective stress $\Delta p' = 0$ and $\Delta (\sigma_0' - \sigma_t') < 0$ to reach to the estimated in-situ stress state or its vicinity as explained previously in Chapter 5. In Table 6.2, the final induced axial and volumetric strains are summarised together with the effective stresses at the end of the anisotropic consolidation stage. In this table the sample dimensions and the pad-to-pad lengths used in local axial and torsional shear strain calculations are also reported.

6.3.1 Developments of strains

The evolutions with time of resulting strains from test AM00-00 are plotted in Figure 6.8 as typical results of anisotropic consolidation stages. The anisotropy in stiffness of the London Clay is also noted due to the development of volumetric strain at early stages because for isotropic material this should be zero. Moreover, the radial and circumferential strains were seen as practically similar, a behaviour expected for a cross-anisotropic material.

Figure 6.9 shows the small creep rates of volumetric and axial strains at the end of the anisotropic consolidation stage, typically range between 0.002–0.003%/hour. In general the pause period was kept for a minimum of two days or until a small creep rate of this magnitude was achieved.

6.3.2 Relationships for stress ratio with induced strain

Figure 6.10 illustrates the developments of strain components with $\varepsilon_{13}(=\varepsilon_\theta - \varepsilon_\phi)$ of test AM00-00 during anisotropic consolidation. Again this highlights the lower stiffness in the vertical direction with regard to the horizontal direction of the London Clay, as well as the orthotropic response of the clay with similar magnitude of radial and circumferential strains.

The stress-strain relationships during anisotropic consolidation were normalised and plotted in terms of stress ratio $q/p'$ and strain $\varepsilon_{13}$ as shown in Figure 6.11. Generally tests that were consolidated to a similar anisotropic stress state showed small degree of scatter in their stress-strain response, reflecting the good repeatability in test data. Nevertheless, there are two tests that were of markedly softer response (AM50-05 and AM00-00).

Based on Figure 6.12, it can be seen that the ratio of $\varepsilon_v/\varepsilon_{13}$ at reference strain of $\varepsilon_{13} = 0.2\%$ are mainly within 0.3–0.8 for tests with local strain sensors (i.e. except tests AM00-05, AM30-05, AM50-05 and AM60-05), indicating the degrees of stiffness anisotropy of the clay. Interestingly Nishimura (2006) reported a range of 0.3–0.9 for this ratio in his tests on clay specimens from about 5m deeper horizon (10.5mBGL). To reduce the scatter, it is important to use a slower rate of loading and to apply some stabilisation techniques for local strain sensors such as thermal insulation recommended by Gasparre (2005).

\footnote{In which strains were measured by global strain transducers.}
### 6.3. ANISOTROPIC RE-CONSOLIDATION STAGES

#### Table 6.2: State of samples at the end of re-consolidation path

<table>
<thead>
<tr>
<th>Test code</th>
<th>( p' )</th>
<th>( \sigma_v - \sigma_h )</th>
<th>( \epsilon_v )</th>
<th>( \epsilon_s )</th>
<th>( H )</th>
<th>( R_o )</th>
<th>( R_i )</th>
<th>( L_o ) (_1 )</th>
<th>( L_o ) (_2 )</th>
<th>( L_s ) (_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM00-00</td>
<td>280</td>
<td>-110</td>
<td>-0.95</td>
<td>-1.28</td>
<td>212.84</td>
<td>49.93</td>
<td>30.10</td>
<td>151.15</td>
<td>151.05</td>
<td>67.48</td>
</tr>
<tr>
<td>AM45-00</td>
<td>280</td>
<td>-110</td>
<td>-0.47</td>
<td>-0.81</td>
<td>216.02</td>
<td>49.97</td>
<td>29.95</td>
<td>157.75</td>
<td>158.40</td>
<td>77.93</td>
</tr>
<tr>
<td>AM00-03</td>
<td>280</td>
<td>-152</td>
<td>-0.81</td>
<td>-1.36</td>
<td>199.17</td>
<td>50.46</td>
<td>30.03</td>
<td>150.24</td>
<td>151.29</td>
<td>66.90</td>
</tr>
<tr>
<td>AM00-03b</td>
<td>280</td>
<td>-160</td>
<td>-0.46</td>
<td>-0.70</td>
<td>209.14</td>
<td>50.30</td>
<td>29.62</td>
<td>153.79</td>
<td>151.89</td>
<td>78.39</td>
</tr>
<tr>
<td>AM30-03</td>
<td>280</td>
<td>-165</td>
<td>-0.63</td>
<td>-0.91</td>
<td>204.30</td>
<td>50.28</td>
<td>29.89</td>
<td>155.12</td>
<td>152.53</td>
<td>68.23</td>
</tr>
<tr>
<td>AM00-05</td>
<td>312</td>
<td>-160</td>
<td>-0.41</td>
<td>-1.36</td>
<td>199.81</td>
<td>50.49</td>
<td>30.05</td>
<td>see notes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM30-05</td>
<td>312</td>
<td>-160</td>
<td>-0.56</td>
<td>-0.80</td>
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<td>50.19</td>
<td>29.92</td>
<td>as above</td>
<td></td>
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<td>AM50-05</td>
<td>312</td>
<td>-160</td>
<td>-0.45</td>
<td>-0.78</td>
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<td>49.96</td>
<td>29.94</td>
<td>as above</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM60-05</td>
<td>312</td>
<td>-216</td>
<td>-0.47</td>
<td>-0.63</td>
<td>200.27</td>
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<td>30.01</td>
<td>as above</td>
<td></td>
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</tr>
<tr>
<td>AM90-05</td>
<td>312</td>
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<td>-0.44</td>
<td>-0.20</td>
<td>201.25</td>
<td>50.40</td>
<td>30.19</td>
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<td>143.03</td>
<td>142.05</td>
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<td>-0.89</td>
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<td>29.96</td>
<td>163.61</td>
<td>162.20</td>
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<td>146.20</td>
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<td>-1.10</td>
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<td>211.78</td>
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<td>147.18</td>
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<td>HC-DT</td>
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<td>-140</td>
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<td>50.49</td>
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<td>HC-DZ</td>
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<td>-110</td>
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<td>50.08</td>
<td>29.97</td>
<td>149.67</td>
<td>149.27</td>
<td>65.83</td>
</tr>
</tbody>
</table>

Notes:
- \( H, R_o, R_i \) were sample dimensions at the end of re-consolidation stages
- \( L_o \)\(_1\), \( L_o \)\(_2\) and \( L_s \)\(_3\) were lengths (pad-to-pad center) of the electrolevels (ELa1, ELa2 and ELS3, respectively) at the end of re-consolidation stages
- The values of axial and volumetric strains were calculated from local strain analysis
- The double-axis electrolevels (ELa1 and ELa2) were not available in some early tests (tests from series AM\( \alpha \)-05, except AM90-05)

#### Figure 6.8: Typical evolution of strains during anisotropic re-consolidation, test AM00-00

![Typical evolution of strains during anisotropic re-consolidation, test AM00-00](image-url)
6.3. ANISOTROPIC RE-CONSOLIDATION STAGES

Figure 6.9: Rates of creep at the end of anisotropic re-consolidation stage in test AM00-00

Figure 6.10: Typical development of strains during anisotropic re-consolidation path in test AM00-00
6.3. ANISOTROPIC RE-CONSOLIDATION STAGES

(a) For $p' = 280\text{kPa}$

(b) For $p' = 312\text{kPa}$

Figure 6.11: Stress ratio $q/p'$ and strain $(\varepsilon_1 - \varepsilon_3 = \varepsilon_\theta - \varepsilon_z)$ relationship during anisotropic re-consolidation stage
6.3. ANISOTROPIC RE-CONSOLIDATION STAGES

Figure 6.12: Observed $\varepsilon_{13} \sim \varepsilon_v$ relationships at early stages (shear strain < 0.2 %) during anisotropic re-consolidation stage
Chapter 7

Stiffness anisotropy of London Clay from uniaxial drained shear tests

Since the 1980s an improved understanding of pre-failure deformation characteristics of geomaterials has greatly advanced our capability to predict the ground movements (Jardine et al., 1986; Burland, 1989; Al-Tabbaa and Muir Wood, 1989; Simpson, 1992; Stallebrass et al., 1994; Potts, 2003). In design practice it has become possible to employ numerical analyses that capture the stress-strain non-linearity response of soils interpreted from locally instrumented laboratory tests (e.g. Jardine and Potts, 1988; Jardine et al., 1991; Hight and Higgins, 1995; Hird and Pierpoint, 1997; Potts and Zdravkovic, 2001). Several researchers (Lee and Rowe, 1989a,b; Gunn, 1993; Simpson et al., 1996; Hird and Pierpoint, 1997) have demonstrated that stiffness anisotropy might be an important factor to improve the predictions of ground deformations due to tunnelling or excavation in clays. Such analyses require experimental data considering the soils anisotropic deformation properties. However, limited experimental data of this type is currently available for the natural London Clay (Hight et al., 2003).

The work described in the present chapter concentrates on the anisotropic deformation characteristics of one horizon (at 12.5 mOD; 5.2m BGL) of the natural London Clay, which forms part of a much wider laboratory investigation undertaken at Imperial College using high quality soil samples from the T5 site at Heathrow Airport (Hight et al., 2007). The anisotropy in stiffness properties was studied by employing uniaxial static HCA tests, in which only one stress component was changed in each shearing stage. From these tests it was possible to explore aspects of the clay’s kinematic sub-yield surfaces and to evaluate its cross-anisotropic elastic parameters.

This chapter is organised into five sections. First (7.1) describes the detailed scheme of the uniaxial probing tests. It is followed by (7.2) which presents the results of a series of drained uniaxial tests with small stress load-unload cycles and details the measured values of the cross-anisotropic elastic parameters. Next, the characteristics of stiffness anisotropy at larger strains are reported in (7.3) in which three drained uniaxial tests to failure were carried out from anisotropic consolidation stress states and the evolution of secant stiffness and strain ratios as shear strains increased were established. The anisotropy in drained shear strength and stress-strain response of the clay are then discussed in (7.4). Finally (7.5) summarises the main findings. Overall the uniaxial test series described in this chapter forms the first set of measurements of the cross-anisotropic stiffness properties of London Clay under drained conditions using HCA.
7.1 Shearing programme and methods of measurement

The series of drained uniaxial tests was performed in which only one of the stress components \((\sigma_0^z, \sigma_0^\theta \text{ or } \tau_{z\theta})\) was changed while all other stresses, including \(\sigma_i^z\) in all cases, remained constant. The tests could be broadly divided into small and large stress probings as follows:

7.1.1 Small stress probing uniaxial tests

This series aimed to identify the elastic stiffness parameters of the London Clay, assuming the material was cross-anisotropic, and thereby to establish the parameters for, as well as the extent of, the inner most elastic sub-yield surface \(Y_1\) as suggested by Jardine (1992) (see also Chapter 9). The small probing tests were conducted at the three following effective stress points (see Figure 7.1):

- Point C: the isotropic initial set-up state (at \(p_i^0\));
- Point B: the estimated isotropic in-situ effective stress (at \(p_o^0 = 280\) kPa);
- Point A: the estimated anisotropic in-situ stress (at \(p_o^0 = 280\) kPa and \(K = \sigma_h^0/\sigma_v^0 > 1\)).

It was intended to investigate the influences of mean effective stress \(p'\) by comparing results from tests at Points B and C, and to study the effects of deviatoric stress by analysing data from Point A. Table 7.1 tabulates the test conditions conducted at each point. Because individual samples were subjected to a number of small probes, the applied stress changes were limited to 2 kPa in each probe to avoid disturbance to the sample. This condition resulted to very small strains, mostly in the order of \(\varepsilon_z < 0.0025\ \%\), \(\varepsilon_r < 0.0015\ \%\) and \(\gamma_{z\theta} < 0.004\ \%\). In total 33 probing tests were carried out on five different samples, in which 26 probes were performed at the estimated in-situ \(p_o^0 = 280\) kPa.

Analysing results from the consolidation stages of 100 mm London Clay triaxial samples Gasparre (2005) reported that a slow rate of stress change (about 1–2 kPa/hour) was required to satisfy the condition of full drainage. Applying this rate led to stage’s duration of up to 4–5 hours for each probe, and hence exposing the test to potential errors caused by secondary effects such as temperature changes or drift in the transducers. Due to the small magnitudes of the changes in stress and strains such effects can be very significant. As an example, in her triaxial stress path tests on London Clay samples Gasparre (2005) reported that typical daily cyclic changes of 0.7°C in temperature in a day led to cyclic variations of about 3.5 kPa and 0.003 % for the mid-height pore water pressure and strain measurements (using LVDTs), which in turn could cause serious errors in the measurements. Gasparre found that wrapping the cells in several layers of bubble wrap and aluminium foil helped to reduce the temperature change inside the cell to less than 0.1°C. This reduced the pore pressure changes to less than 0.5 kPa and the strain variations to around 0.001 %. She also conducted the small probes with rates of 2 kPa/hour in loading and 4 kPa/hour in unloading, reducing the total duration for each probed to 2 hours including the pause period. Not having a mid-height pore pressure transducer mounted in the HCA Mark II, a similar investigation of the cyclic variation in p.w.p measurement due to temperature change was not possible and the Author decided to apply the same technique as Gasparre in this study.

Jardine (1992) pointed out that the effects due to creep strains generated by the previous stress path could be effectively minimised by maintaining the stresses unchanged for long periods before starting the next stage. Accordingly he suggested that the creep rate should be less than a hundredth
7.1. SHEARING PROGRAMME AND METHODS OF MEASUREMENT

7.1.1 Small stress probe series

The strains in this small stress probe series were so small and owing to the abovementioned influences of temperature changes, the rest period between each probe was simply kept until the creep rate less than 0.0005 %/hour. This duration was typically between 4–6 hours. A longer pause period of no less than 24 hours was maintained after moving to each new stress point (B or A).

7.1.2 Uniaxial tests with large stress increments

In order to investigate the drained yielding characteristics of the clay over a wider range of strains, a series of five uniaxial drained shear tests with larger stress excursion were also carried out (see Table 7.2). Three tests started from anisotropic consolidation state (CAD tests, \( K = \sigma_h' / \sigma_v' = 1.55 \sim 1.7 \)) and continued to failure, while two others commenced from the isotropic condition (CID tests, \( K = 1 \)) and stopped with shear strains of about 0.1 %. All five tests were carried out in a stress-controlled mode, with a loading rate of 4 kPa/hour at a constant back pressure of 300 kPa.

The two CAD uniaxial tests (HC-DQ and HC-DZ) had \( \alpha_{d\sigma} = 90^\circ \) and projected towards the extension section of the clay’s local bounding surface; accordingly shear failure occurred early at relatively small strains. Another set of tests involving compressive failure (\( \alpha_{d\sigma} = 0^\circ \)) would have been more instructive. However, the quantity of suitable block samples was limited and it was not possible to allocate two specimens to this purpose.

To overcome this limitation, it was intended to carry out two uniaxial drained tests with large stress increment on the same sample. The applied stress increment was aimed to produce plastic strains that are small enough (about 0.1 %) to limit the potential modification to the initial fabric of the specimen. These two compressive uniaxial tests were carried out from similar isotropic stress states (CID tests) on the same specimen (IM90-05) and were named as IM90-DQ and IM90-DZ. An ageing period of one day was kept between these two tests so that the creep strain rate became less than 0.0002 %.

\[ \Delta \sigma' = 0 \]

1This sample (IM90-05) was then finally sheared under undrained condition with \( \alpha = 90^\circ, b = 0.5 \) (see Chapter 8).
### 7.1. Shearing Programme and Methods of Measurement

Table 7.1: Testing conditions of uniaxial drained shear probes on 5.2mBGL London Clay samples

<table>
<thead>
<tr>
<th>Test sample</th>
<th>Consolidation state</th>
<th>$p'$ [kPa]</th>
<th>$\sigma'_0 - \sigma'_h$ [kPa]</th>
<th>Uniaxial probing tests</th>
<th>Test results summary in</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC-DQ</td>
<td>Isotropic</td>
<td>420</td>
<td>0</td>
<td>$\Delta \sigma'_0$</td>
<td>Table 7.4</td>
</tr>
<tr>
<td>HC-DT $(K = 1)$</td>
<td>405</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM45-00</td>
<td>380</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC-DQ</td>
<td>Isotropic</td>
<td>280</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td>Table 7.5</td>
</tr>
<tr>
<td>HC-DT $(K = 1)$</td>
<td>280</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC-DZ</td>
<td>280</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM45-00</td>
<td>280</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IM90-05</td>
<td>280</td>
<td>0</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC-DQ</td>
<td>Anisotropic</td>
<td>280</td>
<td>-140</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td>Table 7.6</td>
</tr>
<tr>
<td>HC-DT $(K = 1.7)$</td>
<td>280</td>
<td>-140</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC-DZ $(K = 1.55)$</td>
<td>280</td>
<td>-110</td>
<td>$\Delta \sigma'<em>z$, $\Delta \tau</em>{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM45-00</td>
<td>280</td>
<td>-110</td>
<td>$\Delta \tau_{z\theta}$, $\Delta \sigma'_0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2: Testing conditions of uniaxial drained shear with large stress increments

<table>
<thead>
<tr>
<th>Test code</th>
<th>Void ratio [-]</th>
<th>$p'$ [kPa]</th>
<th>$\sigma'_0 - \sigma'_h$ [kPa]</th>
<th>Testing conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC-DQ</td>
<td>0.622</td>
<td>280</td>
<td>-140</td>
<td>CAD to failure with $\Delta \sigma'<em>0 &gt; 0$ ($\alpha</em>{d\sigma} = 90^\circ$)</td>
</tr>
<tr>
<td>HC-DT</td>
<td>0.624</td>
<td>280</td>
<td>-140</td>
<td>CAD to failure with $\Delta \tau_{z\theta} &gt; 0$ ($\alpha_{d\sigma} = 90^\circ$)</td>
</tr>
<tr>
<td>HC-DZ</td>
<td>0.656</td>
<td>280</td>
<td>-110</td>
<td>CAD to failure with $\Delta \sigma'<em>z &lt; 0$ ($\alpha</em>{d\sigma} = 45^\circ$)</td>
</tr>
<tr>
<td>IM90-DQ</td>
<td>0.639</td>
<td>280</td>
<td>0</td>
<td>CID load-unload cycle with $\Delta \sigma'_0 = 40$ kPa</td>
</tr>
<tr>
<td>IM90-DZ</td>
<td>0.641</td>
<td>280</td>
<td>0</td>
<td>CID load-unload cycle with $\Delta \sigma'_z = 30$ kPa</td>
</tr>
</tbody>
</table>
7.1.3 Methods of measurement and evaluations of errors

Assuming cross-anisotropic elasticity for the clay at very small strains and recalling Equation 2.5:

Under torsional shear stress change only ($\Delta \tau_{z\theta}$):

$$G_{vh} = G_{z\theta} = \frac{\Delta \tau_{z\theta}}{\Delta \gamma_{z\theta}}$$  \hspace{1cm} (7.1)

Under axial normal stress change only ($\Delta \sigma_{0z}$):

$$E_{0v}' = E_{0z}' = \frac{\Delta \sigma_{0z}}{\Delta \varepsilon_{z}}$$
$$\nu_{vh}' = \nu_{z\theta}' = -\frac{\Delta \theta/\Delta \varepsilon_{z}}$$  \hspace{1cm} (7.2)

and circumferential normal stress change only ($\Delta \sigma_{0\theta}$):

$$E_{0h}' = E_{0\theta}' = \frac{\Delta \sigma_{0\theta}}{\Delta \varepsilon_{\theta}}$$
$$\nu_{vh}' = \nu_{h\theta}' = -\frac{\Delta \varepsilon_{\theta}}{\Delta \gamma_{\theta}}$$  \hspace{1cm} (7.3)

By definition, elasticity refers to a material response that involves no energy dissipation for any closed stress cycle whether the stress-strain relationship is linear or non-linear. It should also be time-independent and not be influenced by the rate of stressing or straining. Soil behaviour at very small strain has been termed as quasi-elastic because it shows small hysteric loops of negligible energy losses and the stiffness within this range appears to be relatively insensitive to strain rate (Tatsuoka and Shibuya, 1991).

The strain ranges over which soils show quasi-elastic behaviour are generally very small, with non-linearity becoming noticeable after just 0.001% in many cases (e.g. Kuwano, 1999; Gasparre, 2005). It is therefore essential to achieve very good resolution and high accuracy in both stress and strain measurements. In this study, the scatter of stiffness measurements was evaluated by examining the variations in the measurement of the applied stress increment and the induced strain. Both the initial origin and the standard deviation values for stress and strain were obtained by statistical analysis from multiple data point recorded prior to the onset of shearing. For given scatters of axial strain and axial stress increments ($\delta\varepsilon_{z}$ and $\delta\sigma_{0z}$) the variation in vertical Young’s modulus can be determined from Equation 7.4:

$$\frac{\delta E_{0v}'}{E_{0v}} = \frac{\delta\varepsilon_{z}}{\Delta\varepsilon_{z}} + \frac{\delta\sigma_{0z}'}{\Delta\sigma_{0z}'}$$  \hspace{1cm} (7.4)

Similar analyses were carried out for the values of $E_{0h}'$ and $G_{vh}$. Table 7.3 indicates the potential scatter in the determination of stiffness moduli at two nominal small strain levels of 0.001 and 0.01%, and using data from actual tests. The scatter in strain measurement is clearly the dominant influential factor, which emphasizes the necessity of accuracy and high resolution in the measurements. At very small strain of 0.001%, the expected errors in $E_{0v}'$, $E_{0h}'$ and $G_{vh}$ are estimated at 20.5, 12.5 and 15%, respectively using the current HCA system. As shear strain increases, the degree of scatter in the measurement reduces significantly, becomes roughly 2% at strain level of 0.01%. However, these scatters corresponded to individual data points and could be further reduced by adopting regression analyses using multiple data values. It will be shown later that the potential scatters from similar tests were smaller, generally between 4–6% and at largest (for $E_{0h}'$) not more than 13%. 

117
Table 7.3: Degrees of accuracy in the determination of stiffness moduli at small strains

<table>
<thead>
<tr>
<th>Strain [%]</th>
<th>Stress levels [kPa]</th>
<th>% scatters in the measurements of $E'_z$ (= $E'_v$)</th>
<th>$E'_\theta$ (= $E'_h$)</th>
<th>$G_{z\theta}$ (= $G_{vh}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1.2 2 0.8</td>
<td>(0.00013 + 0.09) 100 = 13 + 7.5 = 20.5</td>
<td>(0.00001 + 0.05) 100 = 10 + 2.5 = 12.5</td>
<td>(0.0001 + 0.04) 100 = 10 + 5 = 15</td>
</tr>
<tr>
<td>0.01</td>
<td>11 14 6</td>
<td>(0.00013 + 0.09) 100 = 1.3 + 0.8 = 2.1</td>
<td>(0.0001 + 0.05) 100 = 1.0 + 0.4 = 1.4</td>
<td>(0.0001 + 0.04) 100 = 1.0 + 0.7 = 1.7</td>
</tr>
</tbody>
</table>

Notes
The scatters $\delta \varepsilon_z$, $\delta \varepsilon_\theta$ and $\delta \gamma_{z\theta}$ were 0.00013, 0.0001 and 0.0001%, respectively (Table 3.1)
The scatters $\delta \sigma'_z$, $\delta \sigma'_\theta$ and $\delta \tau_{z\theta}$ were 0.09, 0.05 and 0.04kPa, respectively (Table 3.2)
In the calculations, the first and second numbers link to errors due to strain and stress measurement.
Stress levels were based on actual uniaxial drained tests (HC-DT, HC-DQ and HC-DZ)

The Author therefore calculated the values of secant Young's modulus or shear stiffness as the inclination of the stress-strain curve with respect to a fixed origin. Each of these values was determined from a linear regression established through stress-strain origin and over the loading path region where the strains were 0.001 % or less, with regression coefficient $R^2 > 0.90$ as shown in Figure 7.3 The elasticity limit was identified at the point where the stress-strain response diverged from linearity, typically at a shear strain of 0.001 %. The same linear fitting method was applied to calculate the Poisson's ratios. The results from these measurements are presented in detail in the next section.

7.2 The clay’s cross-anisotropy elasticity at small strain

Results of the 33 uniaxial probing tests are shown in Figures 7.4 to 7.15, each figure plotting results from a same specimen and at a same effective stress state. The degree of confidence for each regression, represented by the regression coefficient $R^2$, was also included. There were some differences in accuracy of the readings between individual tests because both the testing technique and the resolution of the local strain transducers were continuously improved during the research. Tables 7.4 to 7.6 summarise the values of Young's moduli, shear stiffness and Poisson's ratios, which were calculated from the linear fitting outlined in the previous section.

7.2.1 Anisotropy in Poisson's ratios and stiffness moduli

Poisson's ratios

The Poisson's ratios have been determined directly from the measurements of the local normal strain components ($\varepsilon_z$, $\varepsilon_\theta$, $\varepsilon_r$). Because the induced strains were very small, particularly in uniaxial tests with $\Delta \sigma'_\theta$ changes, the values of Poisson's ratios were subjected to relatively large scatter, giving a broad range of values in different tests. On the one hand, in some tests conducted at isotropic stress states $\nu_{vh} \approx \nu'_{hw}$ was observed (see Figures 7.5, 7.8 and 7.11). This was in contradiction to the stiffer
response in the horizontal direction with that in the vertical direction and the assumption of cross-anisotropy property. On the other hand, at anisotropic stress states the average \( \nu_{vh}, \nu_{hv} \) and \( \nu_{hh} \) were 0.25, 0.44 and -0.17, respectively. As will be shown later, this data was more compatible with the cross-anisotropic assumption for London Clay.

Figure 7.16 and Figure 7.17 indicate no well-defined relationships between the Poisson’s ratios with stress levels. Similar observations have been obtained by other researchers, for example by Chaudhary (2001) from static uniaxial probing tests on Toyoura sand. In discussing her triaxial probing tests on natural London Clay, Gasparre (2005) suggested that the differences in Poisson’s ratios at different depths were more closely related with the initial fabric of the soil than of the applied effective stresses.

### Young’s and shear stiffness moduli

It can be seen (see Tables 7.4 to 7.6) that the drained Young's modulus in the horizontal direction \( E_0^h \) is significantly larger than that in the vertical direction \( E_0^v \). Under an isotropic stress state of \( p' = 280 \) kPa, the average horizontal and vertical Young’s moduli are 219MPa and 129MPa, respectively, with marginally lower magnitudes of 218MPa and 120MPa defined at the same \( p' \) with \( 1.55 < K = \sigma'_h/\sigma'_v < 1.7 \). The average ratios of \( E'_h/E'_v \) at isotropic and anisotropic stress states are 1.70 and 1.86, respectively. The results confirm the natural London Clay’s considerable degree of initial stiffness anisotropy, which has also been observed in the field and laboratory by shear wave velocity measurements (e.g. Hight et al., 2003).

The values of \( G_{vh} \) interpreted from tests under isotropic states at \( p' = 280 \) kPa fell between 73–85MPa and slightly lower (68–82MPa) under anisotropic stress conditions. On average, \( G_{vh} \) was 78MPa. Applying Equation 2.6, the shear stiffness in the horizontal plane \( G_{hh} \) was estimated between 126–160MPa under anisotropic stress conditions. The corresponding ratio \( G_{hh}/G_{vh} \) was 1.86–2.13, with the exception in test AM4500-ani where it was only 1.3 due to smaller observed values of both \( E'_h \) and \( \nu'_{hh} \). Such insensitivity of \( G_{hh}/G_{vh} \) to \( K \) indicated that the high \( G_{hh}/G_{vh} \) ratios were mainly due to the clay’s initial anisotropy and the stress-induced effects might be minor under the conditions imposed.

### 7.2.2 Effective stress dependency of stiffness moduli

#### Background

The quasi-elastic stiffness moduli has been shown empirically to be dependent on selected components of the effective stress tensor, following power law relationships (e.g. Hardin 1966, 1978; Hardin and Blandford, 1989) with additional terms relating to the density and stress history of soils. The variation in moduli due to change of density is often taken into account by normalizing to a void ratio function \( f(e) \), for which there are several proposals for sands and clays (e.g. Hardin and Richard Jr., 1963; Jamiolkowski and Lo, 1991; Jamiolkowski et al., 1994; Porovic, 1995; Ishihara, 1996; Rampello et al., 1997; Shibuya et al., 1997). Figure 7.2 plots two functions commonly used for clay soils and it can be seen that the difference can be significant for overconsolidated stiff clays \( (e < 0.7) \). In this study, the expression \( f(e) = e^{-1.3} \) by Jamiolkowski and Lo (1991) was used following experience reported with six different stiff Italian clays by Jamiolkowski et al. (1994).
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

The power law expressions in a general form are:

\[
\begin{align*}
E'_{v}/p_r &= C_v f(e)(\sigma'_{v}/p_r)^{nv}OCR^{kv} \\
E'_{h}/p_r &= C_h f(e)(\sigma'_{h}/p_r)^{nh}OCR^{kh} \\
G_{vh}/p_r &= C_{vh} f(e)(\sigma'_{v}/p_r)^{nv1}(\sigma'_{h}/p_r)^{nh1}OCR^{kvh1} \\
G_{hh}/p_r &= C_{hh} f(e)(\sigma'_{h}/p_r \cdot \sigma'_{h}/p_r)^{nh2}OCR^{kvh2}
\end{align*}
\] (7.5)

in which \(C_v, C_h, C_{vh}\) are material constants and \(p_r\) is the reference pressure. The exponent values \(nv, nh, nv1, nh1, nh2\) reflect the influence of effective stress level, and factors \(kv, kh, kvh1, kvh2\) indicate the effect of stress history.

As indicated by the above equations, the drained Young’s modulus in any particular direction depends on the normal effective stress acting in that direction, whereas the shear modulus is a function of the two normal effective stresses acting on the plane of shear but is independent of the normal effective stress acting normal to this plane. The effect of over consolidation ratio is often found to be negligible when the changes in void ratio and stress state have been taken into account (e.g., see Houlsby and Wroth, 1991; Rampello et al., 1997), giving \(kv = kh = kvh1 = kvh2 = 0\).

It should be noted that the above models were found applicable to non-cemented soils. Cemented soils may be almost insensitive to effective stress until a certain effective stress threshold is reached, above which the cementation starts to break down (e.g. Fernandez and Santamarina, 2001).

Assuming negligible influences from OCR, the power law expressions for Young’s moduli are:

\[
\begin{align*}
E'_{v}/p_r &= C_v f(e)(\sigma'_{v}/p_r)^{nv} \\
E'_{h}/p_r &= C_h f(e)(\sigma'_{h}/p_r)^{nh}
\end{align*}
\] (7.6)

In the special cases of tests starting from isotropic stress states, the shear stiffness moduli \(G_{vh}\) and \(G_{hh}\) in Equation 7.5 can be expressed by:

\[
\begin{align*}
G_{vh}/p_r &= C_{vh} f(e)(p'/p_r)^{nv1+nh1} \\
G_{hh}/p_r &= C_{hh} f(e)(p'/p_r)^{nh2}
\end{align*}
\] (7.7)
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

And if \( \sigma'_v \) and \( \sigma'_h \) cause equal effects to shear stiffness \( G_{vh} \) and \( G_{hh} \) then:

\[
\begin{align*}
G_{vh}/p_r &= C_{vh}f(e)(\sigma'_v/p_r \cdot \sigma'_h/p_r)^{m1} \\
G_{hh}/p_r &= C_{hh}f(e)(\sigma'_h/p_r \cdot \sigma'_h/p_r)^{m2}
\end{align*}
\] (7.8)

Void ratio function \( f(e) = e^{-1.3} \) and reference pressure \( p_r = 100 \text{ kPa} \) were used in the following analyses.

Analysis of experimental data

Figure 7.18 and Figure 7.19 plot the measured values of normalised Young's moduli following the expressions presented in Equation 7.5. Note that because only a limited number of tests have been carried out and only small changes in stress level (less than 150kPa) were considered, the interpreted trend lines shown in these figures are only approximate. Although there could be unlimited possibilities of fitting equations, it was recognised that the exponents were lay between 0.55–0.72.

Applying Equation 7.6, the interpreted trend lines for the Young's moduli obtained from the uniaxial tests are:

\[
\begin{align*}
E'_v/p_r &= 450e^{-1.3}(\sigma'_v/p_r)^{0.55} \\
E'_h/p_r &= 580e^{-1.3}(\sigma'_h/p_r)^{0.72} \text{ (upper bound)} \\
E'_h/p_r &= 500e^{-1.3}(\sigma'_h/p_r)^{0.70} \text{ (lower bound)}
\end{align*}
\] (7.9)

Considering only the isotropic stress state (Equation 7.7), the relationship for the normalised shear modulus \( G_{vh}/f(e) \) gives an exponent of 0.48, which is slightly smaller than the value of 0.50 quoted by Viggiani and Atkinson (1995) but larger than that found by Nishimura (2006) and Yimsiri (2002) (0.40 and 0.38, respectively).

\[
\begin{align*}
G_{vh}/p_r &= 280e^{-1.3}(p'/p_r)^{0.48} \\
G_{hh}/p_r &= 600e^{-1.3}(p'/p_r)^{0.48} \text{ (upper bound)} \\
G_{hh}/p_r &= 420e^{-1.3}(p'/p_r)^{0.50} \text{ (lower bound)}
\end{align*}
\] (7.10)

Using Equation 7.8, the measured values of \( G_{vh}/f(e) \) before and after two reconsolidation stages (Points B and A in Figure 7.1) are plotted against \( \sigma'_v \cdot \sigma'_h \) in Figure 7.20. On average, the constant \( m1 \) was about 0.22 which falls within the range (0.20–0.29) for natural Italian clays quoted by Jamiołkowski et al. (1994).

Similarly Figure 7.21 shows the changes in \( G_{hh}/f(e) \) with respect to \( \sigma'_h \cdot \sigma'_h \). Scatter in \( E'_h/f(e) \) and \( \nu'_vh \) caused significant difficulties in identifying the best fitting trend for \( G_{hh}/f(e) \). Under isotropic stress states the constant \( m2 \) could be estimated between 0.23–0.26. Both \( m1 \) and \( m2 \) values, however, are higher than that reported by Nishimura (2006) under isotropic stress conditions (0.09 and 0.12, respectively). The reason for this difference was not clear and further study is necessary to evaluate the exponents more accurately.

If the influences of the normal stresses \( \sigma'_v \) and \( \sigma'_h \) are assumed to be similar under isotropic and anisotropic stress conditions \( (n = n1 = n2) \), Equation 7.5 gives \( E'_h/E'_v = C_h/C_v \cdot (\sigma'_h/\sigma'_v)^n \). This expression describes the stress-induced stiffness anisotropy, in which the ratio \( C_h/C_v \) indicates the initial stiffness anisotropy. The data of \( E'_h/E'_v \) versus \( \sigma'_h/\sigma'_v \) from the uniaxial probing tests are summarised in Figure 7.22. Two possible interpretations are shown, one is independent
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

of stress ratio \( \frac{E'_{th}}{E'_{tv}} = 1.80 \) and the other is correlated with an exponent of \( n = 0.45 \) for anisotropic stress ratio. Again, further tests are needed to establish any reliable trend.

7.2.3 Symmetry of the compliance matrix

In applying cross-anisotropy elasticity, it is implicitly required that the compliance matrix is symmetric following equation 2.7. This assumption can be checked by comparing the ratio of \( \frac{E'_{th}}{E'_{tv}} \) to \( \nu'_{th}/\nu'_{tv} \), as shown in Figure 7.23. By definition, symmetry of the matrix requires that the data points lie on the 1:1 slope. The tests conducted at anisotropically consolidated states (filled symbols) showed a good correlation with this slope. On the other hand the average ratio found from tests conducted under isotropic stress states (open symbols) was 1:1.5, indicating significant departure from the symmetric condition.

7.2.4 Synthesizing with data from resonant column and triaxial tests

Gasparre et al. (2007a) present recent findings of deformation characteristics of natural London Clay obtained from the same site and tested with a suite of laboratory devices and testing techniques. In particular, the cross-anisotropic elastic parameters of the clay established from resonant-column, triaxial probing, bender element and hollow cylinder uniaxial tests could be well correlated with field geophysical data (Hight et al., 2007).

Figure 7.24 shows the measured profiles of \( G_{vh}, G_{hv}, E'_{hv}, E'_{vh}, \nu'_{vh}, \nu'_{hv}, \) and \( \nu'_{hh}, \) in which the vertical axis represents the elevation (in mOD) and the laboratory tests were performed on samples consolidated to their estimated in-situ anisotropic stress states. Note that for a cross-anisotropic elastic material under the same applied stress condition, values of \( G_{vh} \) from resonant column (Nishimura, 2006) and hollow cylinder (HCA) tests should be equal to \( G_{hv} \) from bender elements installed in the horizontal direction of the triaxial specimens (Gasparre, 2005).

The block samples tested in the present study were from 12.5mOD (denoted by solid symbols) and the corresponding data determined under anisotropic stress states are also plotted as open symbols. Overall there is good agreement between various laboratory tests regarding the values of Young's moduli and shear stiffness under similar anisotropic stress state conditions. However, larger degrees of discrepancies are seen between Poisson's ratios, reflecting their higher susceptibility to errors at very small strain.

Hight et al. (2007) report that the shear moduli \( (G_{hv} \) and \( G_{vh} \)) increase with depth but are practically equal. Both the \( E'_{th}/E'_{tv} \) ratio and \( \nu'_{tv} \) appeared to increase with depth, reflecting a greater increase in \( E'_{tv}. \) Gasparre et al. (2007a) and Hight et al. (2007) stress that the stiffness properties of a London Clay sample are not only dependent on the applied stresses but also on its lithological characteristics and perhaps also on the degrees of cementation. In particular, it is noted that the \( E'_{tv} \) values are larger and more dependent on the lithological unit than \( E'_{tv}. \) The differences of the stiffness ratio \( E'_{th}/E'_{tv} \) with depth are considered to reflect the changes of fabric imposed by depositional and post-depositional processes. These led to a more packed and sub-horizontally oriented clay structure, and possibly more significant cementation, for deeper units.
### 7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Table 7.4: Elastic parameters from uniaxial drained shear probes at initial effective stress

<table>
<thead>
<tr>
<th>Test code</th>
<th>Void ratio e</th>
<th>$G_{vh}$ [MPa]</th>
<th>$E'_v$ [-]</th>
<th>$\nu_{vh}$</th>
<th>$E'_h$ [MPa]</th>
<th>$\nu_{hw}$</th>
<th>$\nu_{hh}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCDQ-p420</td>
<td>0.603</td>
<td>340</td>
<td>0.45</td>
<td>-0.25</td>
<td></td>
<td></td>
<td></td>
<td>Figure 7.4</td>
</tr>
<tr>
<td>HCDT-p405</td>
<td>0.606</td>
<td>88</td>
<td>130</td>
<td>0.31</td>
<td>235</td>
<td>0.37</td>
<td>-0.12</td>
<td>Figure 7.5</td>
</tr>
<tr>
<td>AM4500-p380</td>
<td>0.634</td>
<td>73</td>
<td>169</td>
<td>0.37</td>
<td>250</td>
<td>0.24</td>
<td>-0.12</td>
<td>Figure 7.6</td>
</tr>
</tbody>
</table>

Table 7.5: Elastic parameters from uniaxial drained shear probes at isotropic $p' = 280$ kPa

<table>
<thead>
<tr>
<th>Test code</th>
<th>Void ratio e</th>
<th>$G_{vh}$ [MPa]</th>
<th>$E'_v$ [-]</th>
<th>$\nu_{vh}$</th>
<th>$E'_h$ [MPa]</th>
<th>$\nu_{hw}$</th>
<th>$\nu_{hh}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCDQ-p280</td>
<td>0.614</td>
<td>85</td>
<td>134</td>
<td>0.38</td>
<td>238</td>
<td>0.26</td>
<td>-0.34</td>
<td>Figure 7.7</td>
</tr>
<tr>
<td>HCDT-p280</td>
<td>0.617</td>
<td>79</td>
<td>129</td>
<td>0.36</td>
<td>200</td>
<td>0.33</td>
<td>-0.24</td>
<td>Figure 7.8</td>
</tr>
<tr>
<td>HCDZ-p280</td>
<td>0.650</td>
<td>78</td>
<td>124</td>
<td>0.36</td>
<td>247</td>
<td>0.44</td>
<td>-0.27</td>
<td>Figure 7.9</td>
</tr>
<tr>
<td>AM4500-p280</td>
<td>0.641</td>
<td>73</td>
<td>122</td>
<td>0.21</td>
<td>197</td>
<td>0.37</td>
<td>-0.15</td>
<td>Figure 7.10</td>
</tr>
<tr>
<td>IM9005-p280</td>
<td>0.641</td>
<td>80</td>
<td>135</td>
<td>0.38</td>
<td>215</td>
<td>0.34</td>
<td>-0.18</td>
<td>Figure 7.11</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>79</td>
<td>129</td>
<td>0.34</td>
<td>219</td>
<td>0.35</td>
<td>-0.24</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td>4</td>
<td>5</td>
<td>0.06</td>
<td>20</td>
<td>0.06</td>
<td>0.07</td>
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</tbody>
</table>

Table 7.6: Elastic parameters from uniaxial drained shear probes at anisotropic stress

<table>
<thead>
<tr>
<th>Test code</th>
<th>Void ratio e</th>
<th>$G_{vh}$ [MPa]</th>
<th>$E'_v$ [-]</th>
<th>$\nu_{vh}$</th>
<th>$E'_h$ [MPa]</th>
<th>$\nu_{hw}$</th>
<th>$\nu_{hh}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCDQ-ani</td>
<td>0.622</td>
<td>76</td>
<td>127</td>
<td>0.3</td>
<td>230</td>
<td>0.49</td>
<td>-0.29</td>
<td>Figure 7.12</td>
</tr>
<tr>
<td>HCDT-ani</td>
<td>0.624</td>
<td>72</td>
<td>112</td>
<td>0.19</td>
<td>226</td>
<td>0.36</td>
<td>-0.17</td>
<td>Figure 7.13</td>
</tr>
<tr>
<td>HCDZ-ani</td>
<td>0.656</td>
<td>68</td>
<td>123</td>
<td>0.27</td>
<td>217</td>
<td>0.47</td>
<td>-0.14</td>
<td>Figure 7.14</td>
</tr>
<tr>
<td>AM4500-ani</td>
<td>0.647</td>
<td>82</td>
<td>197</td>
<td></td>
<td></td>
<td>0.42</td>
<td>-0.07</td>
<td>Figure 7.15</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>75</td>
<td>120</td>
<td>0.25</td>
<td>218</td>
<td>0.44</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td>5</td>
<td>6</td>
<td>0.05</td>
<td>13</td>
<td>0.05</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.3: Fitting method for small uniaxial stress probes

Figure 7.4: Drained small stress probes at initial stress state, test HCDQ-p420
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.5: Drained small stress probes at initial stress state, test HCDT-p405

Stress conditions:
\( p' = 405 \text{kPa} \)
\( K = 1 \) (Isotropic axis)

**Graphs:**
- **Top Left:** Graph showing stress-strain relationship with annotations:
  - \( G_{vh} = 88 \text{MPa} \)
  - \( R^2 = 0.82 \)

- **Top Right:** Graph showing stress-strain relationship with annotations:
  - \( E'_{v} = 130 \text{MPa} \)
  - \( R^2 = 0.89 \)

- **Bottom Left:** Graph showing stress-strain relationship with annotations:
  - \( v'_{vh} = -0.12 \)
  - \( R^2 = 0.89 \)

- **Bottom Right:** Graph showing stress-strain relationship with annotations:
  - \( v'_{hv} = 0.37 \)
  - \( R^2 = 0.92 \)
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.6: Drained small stress probes at initial stress state, test AM4500-p380

Stress conditions:
\( p' = 380 \text{kPa} \)
\( K = 1 \) (Isotropic axis)

\( G_v = 73 \text{MPa} \)
\( R^2 = 0.89 \)

\( E'_v = 169 \text{MPa} \)
\( R^2 = 0.93 \)

\( E'_h = 250 \text{MPa} \)
\( R^2 = 0.97 \)

\( \nu'_{vh} = 0.37 \)
\( R^2 = 0.90 \)

\( \nu'_{hv} = -0.12 \)
\( R^2 = 0.84 \)

\( \nu'_{hv} = 0.24 \)
\( R^2 = 0.96 \)
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.7: Drained small stress probes at $p' = 280\text{kPa}$, test HCDQ-p280

Stress conditions:

- $p' = 280\text{kPa}$
- $K = 1$ (Isotropic axis)

HCDQ-p280

$G_m = 85\text{MPa}$
$R^2 = 0.93$

HCDQ-p280

$E'_v = 134\text{MPa}$
$R^2 = 0.81$

HCDQ-p280

$E'_h = 238\text{MPa}$
$R^2 = 0.97$

HCDQ-p280

$\nu'_vh = 0.38$
$R^2 = 0.82$

HCDQ-p280

$\nu'_vh = -0.34$
$R^2 = 0.97$

$\nu'_h = 0.26$
$R^2 = 0.94$
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.8: Drained small stress probes at $p' = 280$ kPa, test HCDT-p280

Stress conditions:
$p' = 280$ kPa
$K = 1$ (Isotropic axis)
Figure 7.9: Drained small stress probes at $p' = 280$ kPa, test HCDZ-p280
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.10: Drained small stress probes at $p' = 280$ kPa, test AM4500-p280.
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.11: Drained small stress probes at $p' = 280$ kPa, test IM9005-p280

Stress conditions:
$p' = 280$ kPa
$K = 1$ (Isotropic axis)

$G_v = 80$ MPa
$R^2 = 0.94$

$E'_v = 135$ MPa
$R^2 = 0.96$

$E'_h = 215$ MPa
$R^2 = 0.89$

$\nu'_{vh} = 0.38
R^2 = 0.96$

$\nu'_{hv} = 0.34
R^2 = 0.84$

131
Figure 7.12: Drained small stress probes at anisotropic stress state, test HCDQ-ani

Stress conditions:
- $p' = 280\text{kPa}$
- $q = 140\text{kPa}$
- $K = 1.7$
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.13: Drained small stress probes at anisotropic stress state, test HCDT-ani

- Stress conditions:
  - \( p' = 280 \text{kPa} \)
  - \( q = 140 \text{kPa} \)
  - \( K = 1.7 \)

- HCDT-ani
  - \( G_{nh} = 72 \text{MPa} \)
  - \( R^2 = 0.98 \)

- \( E'_v = 112 \text{MPa} \)
  - \( R^2 = 0.82 \)

- \( E'_h = 226 \text{MPa} \)
  - \( R^2 = 0.97 \)

- \( \nu'_{vh} = 0.19 \)
  - \( R^2 = 0.99 \)

- \( \nu'_{hv} = -0.17 \)
  - \( R^2 = 0.99 \)

- \( \nu'_{hv} = 0.36 \)
  - \( R^2 = 0.95 \)
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.14: Drained small stress probes at anisotropic stress state, test HCDZ-ani

Stress conditions:

- $p' = 280$ kPa
- $q = 110$ kPa
- $K = 1.5$
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.15: Drained small stress probes at anisotropic stress state, test AM4500-ani
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.16: Poisson’s ratios in the vertical plane with respect to stress levels and stress states

Figure 7.17: Poisson’s ratio $\nu'_{hh}$ in the horizontal plane with respect to stress levels and stress states
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.18: Dependency of $E'_{v}$ and $E'_{h}$ on normal effective stress levels

Figure 7.19: Dependency of $G_{vh}$ and $G_{hh}$ on mean stress $p'$ levels under isotropic stress conditions
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

(a) Isotropic stress states

(b) Anisotropic stress states

Figure 7.20: Dependency of $G_{vh}/p_r$ on $\sigma'_v/p_r \cdot \sigma'_h/p_r$

(a) Isotropic stress states

(b) Anisotropic stress states

Figure 7.21: Dependency of $G_{hh}/p_r$ with $\sigma'_h/p_r \cdot \sigma'_h/p_r$
7.2. THE CLAY’S CROSS-ANISOTROPY ELASTICITY AT SMALL STRAIN

Figure 7.22: Stress-induced stiffness anisotropy from uniaxial probing tests

Figure 7.23: Checking condition for the symmetry of the compliance matrix
Figure 7.24: London Clay cross-anisotropic elastic parameters from laboratory (triaxial, resonant column, hollow cylinder, BE) tests. Open symbols are results from tests conducted by Gasparre (2005) and Nishimura (2006), whereas solid symbols are test results performed by the author (see also Hight et al., 2007).
7.3 Stress-strain response in pre-failure condition of the drained uniaxial tests

In the previous section (7.2), interest was concentrated on the quasi-elastic response and the components of the cross-anisotropy stiffness matrix of natural London Clay at very small strains (0.001 %). The pre-failure deformation characteristics of the clay beyond such a small strain level and under drained conditions are the topic of this section, concentrating on the regions where the shear strains were between 0.001-0.1% or the applied stress increment less than 40kPa. The behaviour at larger strains and at failure is discussed in 7.4.

It is important to note here that the strain levels considered here were well above the elastic limits. The calculated parameters from the stress-strain curves of the HCA uniaxial tests are secant values, and therefore, except at very small strains, do not have the same physical meaning as the elastic parameters (Young’s moduli, shear modulus and Poisson’s ratios) mentioned in the previous section (see Tables 7.5 and 7.6).

7.3.1 Results of CAD uniaxial shear tests

Figure 7.25 to Figure 7.27 show the evolutions of the individual (local) strain components ($\varepsilon_z, \varepsilon_r, \gamma_{z\theta}$) under the imposed stress increment ($\Delta \sigma'_z, \Delta \sigma'_\theta, \Delta \tau_{z\theta}$) or with the deviatoric shear strain $\varepsilon_d$. The volumetric strain $\varepsilon_v = \varepsilon_z + \varepsilon_r + \varepsilon_\theta$ was calculated from local normal strain measurements. As expected, it was compressive in HC-DQ (loading) and dilative in HC-DZ (unloading). Prior to failure the torsional shear strains in tests HC-DQ and HC-DZ were small and therefore were excluded for clarity of presentation. This behaviour was also expected because the cross-coupling of the $\gamma_{z\theta}$ to normal stress components was not significant. In contrast the cross-coupling of normal strains to $\tau_{z\theta}$, although small, was larger. In fact, similar responses had also been observed in the uniaxial shear tests on HPF4 samples (Zdravkovic and Jardine, 1997).

The axial extension test (HC-DZ) produced similar strains in the horizontal plane ($\varepsilon_\theta \approx \varepsilon_r$, less than 3 % difference) up to $\varepsilon_d = 0.01 \%$, after which these two normal strains departed, with a difference of 12 % at $\varepsilon_d = 0.1 \%$. The similarity reflects the cross-anisotropic response of the clay at small strains, and the difference indicates the effects of strain non-uniformity under larger induced strains. For the same stress increment the magnitude of $\varepsilon_z$ in HC-DZ test was larger than that of $\varepsilon_\theta$ in HC-DQ test, indicating the stiffer in compressibility of the London Clay in horizontal plane to vertical one.

A very attractive feature of the uniaxial tests with large stress increments is that they provide stress-strain relationship over a much wider strain range than the small-stress probing tests. Such a plot is shown in Figure 7.28 presenting stress-strain curves and the calculated secant stiffness moduli with the corresponding individual strain components (i.e. $E'_z \sim \Delta \varepsilon_z, E'_\theta \sim \Delta \varepsilon_\theta, G_{z\theta} \sim \Delta \gamma_{z\theta}$). The average values of Young’s (elastic) moduli defined at strain level of 0.001 % from the small probing tests were also included in this figure. The degree of agreement was good except for the $E'_z (= E'_h)$ curve which was about 20 % stiffer than that seen in HC-DZ test. There was also significant scatter in the $E'_\theta (= E'_h)$ curve before $\varepsilon_d = 0.002 \%$ but a clearer trend became evident later.

With regard to the evolution of anisotropy in stiffness from very small to moderate strains, the
7.3. PRE-Failure Stress-Strain Response in Uniaxial Tests

Horizontal secant stiffness declined more rapidly with strain than the vertical one, remaining about 34% stiffer than vertical stiffness at strain of 0.01% strain. The degree of stiffness anisotropy became less noticeable after shear strain of 0.1%. The torsional shear mode showed a slow rate of shear stiffness degradation.

One way of comparing results from these three different shearing modes is to use stress and strain invariants such as octahedral shear stiffness and deviatoric shear strain \(G_{\text{oct}}\) and \(\varepsilon_{d}\). The calculated \(G_{\text{oct}}\) values are shown in Figure 7.29. At strain levels of 0.001%, all three tests had similar values of around 70 MPa, expectingly close to the (elastic) value \(G_{vh} = G_{\text{oct}}^{\text{max}} = 78\) MPa. Significant stiffness decay was evident over the strain range of 0.003-0.005%, with HC-DT and HC-DZ giving the fastest and lowest rate, respectively.

The evolutions of strain ratios with deviatoric shear strain \(\varepsilon_{d}\) are illustrated in Figure 7.29. Again, the Poisson’s (elastic) ratios as determined from small probing tests at anisotropic consolidation state were included for comparison. There were some discrepancies in values of \(\nu_{vh} = \nu_{\theta z}\) and \(\nu_{hh} = \nu_{r\theta}\) but the trends at small strains were broadly similar and in particular very good agreement was observed for \(\nu_{vh} = \nu_{\theta z}\).

7.3.2 Results of CID uniaxial shear tests

The changes during the loading stages of the CID tests in the individual strain components \((\varepsilon_{z}, \varepsilon_{r}, \varepsilon_{\theta}, \varepsilon_{v})\) with \(\varepsilon_{d}\) or under the applied stress increments are shown in Figure 7.30 and Figure 7.31 for IM90-DQ and IM90-DZ. As with the HC-DQ and HC-DZ tests, the values of \(\gamma_{z\theta}\) were small which indicated negligible torsional cross-coupling. The axial compression test (IM90-DZ) generated smaller strains in the horizontal plane than the axial strain (i.e. \(|\varepsilon_{\theta}| < \varepsilon_{z}\)).

Under the same stress increment of \(\Delta\sigma''_{\theta} = 30\) kPa, the lateral compression test (IM90-DQ) caused slightly smaller circumferential strain (0.26%) compared to that in HC-DQ (0.29%), whereas the axial strain remained similar and small. This response reflects the broadly similar horizontal stiffness modulus of the London Clay under isotropic and anisotropic stress states, although value of \(\nu_{rr}\) appeared to be higher under isotropic stress state. Similarly under the same \(\Delta\sigma''_{z} = 20\) kPa the \(E_{z}'\) was slightly smaller and \(\nu_{z\theta}\) was larger when shearing was initiated from isotropic state than that from anisotropic state.

Figure 7.32 plots the variations of the secant stiffness moduli \((E'_{z} \sim \Delta\varepsilon_{z}, E'_{\theta} \sim \Delta\varepsilon_{\theta})\) with strain up to 0.1%. Again the average drained stiffness moduli (true Young's moduli) as determined from small uniaxial probing tests are included for comparison. It can be seen that the anisotropic stiffness hierarchy \((E_{h}' > E_{v}'\)) persisted well beyond small strains. In addition, marked reductions in the stiffness curves started immediately after 0.001%, with the horizontal Young's modulus declining more rapidly.

The curves of \(G_{\text{oct}}\) with \(\varepsilon_{d}\) are shown as Figure 7.33. Similarity in its value from small strain up to \(\varepsilon_{d} = 0.01\%\) was found in both tests. At small strain of 0.002%, the initial \(G_{\text{oct}}\) was about 90 MPa, marginally higher (about 11%) than the value \(G_{vh} = 82\) MPa which was the average obtained from small-stress probing measurements from isotropic stress states.

In Figure 7.33, the strain ratios obtained from the CID tests are compared with the average (elastic) Poisson's ratios measured at isotropic stress states. At small strains (< 0.002%) the similarities could be recognised, particularly for \(\nu_{vh}\) and \(\nu_{hh}'\).
7.3. PRE-Failure Stress-Strain Response in Uniaxial Tests

Figure 7.25: Evolutions of strain components in test HC-DT
Figure 7.26: Evolutions of strain components in test HC-DQ
7.3. PRE-Failure STRESS-STRAIN RESPONSE IN UNIAXIAL TESTS

Figure 7.27: Evolutions of strain components in test HC-DZ
7.3. PRE-FAILURE STRESS-STRAIN RESPONSE IN UNIAXIAL TESTS

(a) At small strain up to 0.1%

(b) Stiffness moduli

Figure 7.28: Individual stress-strain relationships in drained CAD-uniaxial tests
7.3. PRE-FAILURE STRESS-STRAIN RESPONSE IN UNIAXIAL TESTS

Figure 7.29: Stiffness $G_{oct}$ and strain ratios in drained CAD-uniaxial tests
Figure 7.30: Evolutions of strain components in test IM90-DQ
Figure 7.31: Evolutions of strain components in test IM90-DZ
7.3. PRE-FAILURE STRESS-STRAIN RESPONSE IN UNIAXIAL TESTS

Figure 7.32: Individual stress-strain relationships in drained CID-uniaxial tests
7.3. PRE-FAILURE STRESS-STRAIN RESPONSE IN UNIAXIAL TESTS

Figure 7.33: Stiffness $G_{\text{oct}}$ and strain ratios in drained CID-uniaxial tests

(a) Stiffness $G_{\text{oct}}$

(b) Strain ratios
7.4 Observations of shear strength from the drained uniaxial tests

7.4.1 Effective stress paths and drained shear strength

Figure 7.34 shows the effective stress paths (ESP) of CAD uniaxial tests in two stress planes, \( t \sim p' \) and \( (\sigma_z - \sigma_0')/2 \sim p' \). The ESPs were straight lines and their slopes were indicated in this figure. Because test HC-DT was a loading test \( (\Delta \tau_{z\theta} > 0) \) its ESP travelled upward until hitting the failure surface, at which an anomalously strong drained peak strength of \( t = 180 \) kPa was recorded. After which the ESP returned backward and gradually departed from the vertical direction since although the control program tried to maintain the uniaxial testing condition but the newly formed shear surfaces facilitated the development of axial normal stress, creating \( \Delta \sigma_0 > 0 \). The two extension tests (HC-DQ and HC-DZ, \( \alpha = 90^\circ \pm \)) showed weaker drained shear strengths \( (t = 76 \) and 98kPa, respectively) in comparison with HC-DT (Figure 7.34b). On reaching the failure surface, failure occurred abruptly under the applied stress-controlled conditions.

Given that \( \sigma_0' \) was kept unchanged (at 280kPa) in these uniaxial tests, the mean stress parameter could be easily calculated as \( s = (3p' - \sigma_0')/2 \). The stress ratio \( t/s' \) and the mobilised shear resistance angle \( \phi' \) at peak (assuming \( c' = 0 \)) under drained conditions are shown in Table 7.7. It can be seen that the two extension tests HC-DQ and HC-DZ had similar behaviour at peak, with low stress ratios and small drained resistance angles \( (\phi' = 17 \sim 18^\circ) \). In contrast, test HC-DT showed a much stronger response.

Table 7.7: Drained shear strength parameters at peak of CAD uniaxial tests

<table>
<thead>
<tr>
<th>Test code</th>
<th>( t )</th>
<th>( s' )</th>
<th>( t/s' )</th>
<th>( \phi' = \sin^{-1}(t/s') )</th>
<th>( \alpha_f )</th>
<th>( b_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC-DT (loading, torsional shear)</td>
<td>180</td>
<td>280</td>
<td>0.64</td>
<td>40</td>
<td>45</td>
<td>0.82</td>
</tr>
<tr>
<td>HC-DQ (loading, extension)</td>
<td>98</td>
<td>310</td>
<td>0.32</td>
<td>18</td>
<td>90</td>
<td>0.72</td>
</tr>
<tr>
<td>HC-DZ (unloading, extension)</td>
<td>76</td>
<td>262</td>
<td>0.29</td>
<td>17</td>
<td>84</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The ESPs of the CAD tests are illustrated in three dimensional stress space (see Figure 7.35) which provides a picture of the changes in the angle of the major principal stress direction. As these tests were started from anisotropic states (with \( 1.53 < K < 1.7 \)), values of \( \alpha \) and \( \alpha_{d\sigma} \) were not the same. In the \( t \sim p' \sim \alpha \) space the ESP of HC-DT followed an upward bending curve in the plane of \( \alpha = 280 \) kPa, with \( \alpha \) changing from its initial \( 90^\circ \) to reach \( \alpha \approx 65^\circ \) at peak. On the other hand, the ESPs of HC-DQ and HC-DZ plotted as straight lines in the \( \alpha = 90^\circ \) plane. In terms of \( \alpha_{d\sigma} \) the expected ESPs should be straight, which is well identified in Figure 7.35b.

7.4.2 Stress-strain relationship up to failure condition

Figure 7.36 shows the individual stress-strain response of three CAD uniaxial tests, plotted up to 0.5 % to highlight the early failure points of HC-DQ (at \( \epsilon_\theta = 0.11\% \)) and HC-DZ (\( \epsilon_z = 0.22\% \)). Test HC-DT failed at a much larger strain \( (\gamma_{z\theta} = 3.1\%) \) and is therefore shown in a separate plot.

The relationships between stress ratio and shear strain \( q/p' \sim \gamma_1 \) (see Figure 7.37) again indicate the strong drained strength achieved in HC-DT test. The strain at peak \( (\gamma_1) \) was about 3.2 % in the HC-DT test and was of similar magnitude (0.3 %) in both HC-DQ and HC-DZ tests. However, additional tests are required to achieve a conclusive evaluation of drained shear strengths of
7.4. OBSERVATIONS OF SHEAR STRENGTH FROM THE DRAINED UNIAXIAL TESTS

Figure 7.34: Effective stress paths of CAD tests in $t \sim p'$ and $(\sigma_z - \sigma_0)/2 \sim p'$ planes

London Clay in HCA tests.
7.4. OBSERVATIONS OF SHEAR STRENGTH FROM THE DRAINED UNIAXIAL TESTS

Figure 7.35: Effective stress paths of CAD uniaxial tests in three dimensional stress space
Figure 7.36: Individual stress-strain relationships in CAD uniaxial tests
Figure 7.37: Stress ratio $q/p' \sim \gamma_{13}$ relationships in CAD uniaxial tests
7.5 Summary

The drained uniaxial tests carried out on samples of natural London Clay from the same depth (12.5 mOD; 5.2 mBGL) showed significant degrees of anisotropy of the clay’s stiffness. Overall, the yielding and deformation characteristics had been investigated extensively at very small strains up to failure. The investigation utilised an unique testing feature of the hollow cylinder apparatus that provides the independent control of any of the four stress components and the individual measurement of the corresponding axial, radial, circumferential and torsional strains. In addition, the determination of quasi-elastic parameters from uniaxial static test was greatly assisted by the presence of a high resolution and local stress-strain transducer system in the HCA Mark II.

The main findings are summarised below:

The elastic stiffness anisotropy of London Clay

- At very small strains (up to 0.001%) the clay’s stiffness matrix could be reasonably assumed as of a cross-anisotropic elasticity material with an isotropic horizontal plane. In particular the clay is less compressible in the horizontal direction than in the vertical one, and the weakest shearing mode is pure torsional shear.

- The components of this cross-anisotropic elasticity stiffness matrix have been established at the estimated in situ stress conditions. Although an identification criterion for the elastic strain limit based on a closed loop stress-strain response was not always satisfied due to the very small strains involved, generally there was high degree of confidence obtained in the calculation of the elastic stiffness moduli (see Table 7.5 and Table 7.6).

- The Poisson’s ratios were also strongly anisotropic ($\nu_{hv}^t > \nu_{vh}^t > \nu_{hh}^t$). However, there were large scatters in their measurements.

- The average elastic parameters of this cross-anisotropic compliance matrix at the in-situ effective stress ($p' = 280$ kPa and $K = \sigma_h^t/\sigma_v^t = 1.55 \sim 1.7$) were:

\[
\begin{align*}
E_v^t &= 120 \text{ MPa} & \nu_{vh}^t &= 0.25 \\
E_h^t &= 218 \text{ MPa} & \nu_{hv}^t &= 0.44 \\
G_{vh} &= 75 \text{ MPa} & \nu_{hh}^t &= -0.17
\end{align*}
\] (7.11)

- On average the inherent (at isotropic stress state) small strain stiffness anisotropy of London Clay, expressed in term of $E_h^t/E_v^t$, was 1.70. At anisotropic stress state, this ratio was about 1.82. The average small strain shear stiffness ratios of $G_{hh}/G_{vh}$ were 1.8 and 2.1 at the isotropic and anisotropic stress states, respectively. These values were also closely comparable with those found by Gasparre (2005) and Nishimura (2006).

- Under isotropic stress states the quasi-elastic Young’s and shear stiffness moduli showed dependency on effective stress level following a power law. The empirical relationships could be presented as follows, assuming the effect of stress history to be negligible:

\[
\begin{align*}
E_v^t/p_r &= 450e^{-1.3(\sigma_v^t/p_r)^{0.55}} \\
E_h^t/p_r &= 580e^{-1.3(\sigma_h^t/p_r)^{0.72}} \quad \text{(upper bound)} \\
E_h^t/p_r &= 500e^{-1.3(\sigma_h^t/p_r)^{0.70}} \quad \text{(lower bound)}
\end{align*}
\] (7.12)
in which $e$ is the current void ratio, the reference pressure $p_r = 100$ kPa and the void ratio function $f(e) = e^{-1.3}$.

- No well-defined trend of stress level dependency could be established for the Poisson’s ratios.
- Similar power law relationships were established for the elastic shear moduli $G_{vh}$ and $G_{hh}$. For tests initiated from isotropic stress states, the exponent in the expression $G_{vh}/f(e)/p_r \sim p'/p_r$ was 0.48, which is slightly smaller than the value of 0.50 quoted by Viggiani and Atkinson (1995) but larger than those found by Yimsiri (2002) and Nishimura (2006) (between 0.38–0.40). With regard to the $G_{vh}/f(e)/p_r \sim \sigma''_v/p_r \cdot \sigma'_h/p_r$ and $G_{hh}/f(e)/p_r \sim \sigma''_h/p_r \cdot \sigma'_h/p_r$ relationships, the average exponent was 0.22 and 0.24, respectively. These values are larger than the values of 0.09–0.12 reported by Nishimura (2006). The reason for these differences is not clear and further study is needed.
- Assuming the same degree of influences from normal stress applies to Young’s moduli (i.e. the same power law for the effective stress component), the suggested relationship for stress-induced stiffness anisotropy was $E'_h/E'_v = 1.6(\sigma'_h/\sigma'_v)^{0.45}$. However, this finding is inconclusive and more uniaxial tests at wider range of stress levels will be required to confirm the established trend.
- By and large the elastic stiffness parameters (Young’s and shear stiffness moduli) determined from hollow cylinder uniaxial probing tests showed very good agreements with results obtained from hollow cylinder resonant column, bender elements and triaxial probing tests on samples from similar depths and of the same site (Gasparre et al., 2007a).

The deformation characteristics from drained uniaxial shearing tests

- Similarities were observed between the elastic stiffness parameters defined from the small-stress probing tests and those calculated at very small strains (about 0.001 %) from the drained uniaxial tests with larger stress excursions.
- The large-stress drained uniaxial tests provided interesting information from small strains up to failure. It was found that the directional dependency of drained stiffness was clearly present up to intermediate strains (0.1 %), with persistent stiffer response in the horizontal direction compared to the vertical direction.
- Significant stress-strain non-linearity occurred early after the elastic strain limit (0.001 %) and the rate of degradation depended on the mode of shearing. Although there were differences in octahedral shear stiffness values for shear strain between 0.01-0.1 %, similar responses were not observed in bulk modulus. Accordingly the effects of the recent stress history on bulk modulus were not large providing sufficient creep periods had been allowed.
Chapter 8

Stress-strain-strength relationship from multi-axial shear tests

This Chapter describes the results of the multi-axial HCA tests, in which the final stages involved undrained shearing to failure with constant values of \( b \) and \( \alpha \). As described earlier, the specimens had all been taken by block sampling at the same location and depth (5.2 mBGL) at T5 to minimise the natural variations in the clay’s lithology, formation stress history and macro-fabric. In addition, they were also subjected to broadly similar reconsolidation stress paths (see Chapter 6). Any differences in stress-strain and strength responses of these tests should therefore reveal the influences of anisotropy (angle \( \alpha \)) and intermediate principal stress parameter (\( b \)).

The Author’s interpretation of multi-axis shear data was strengthened by the existence of an equivalent set of experiments performed on deeper samples (10.5 mBGL) by Nishimura (2006). Combining these two experimental programmes helped to confirm the influences of \( \alpha \) and \( b \), as well as identifying trends for the stress-strain relationships and pore water pressure response of London Clay under generalised stress conditions. The clay’s pre-failure yielding characteristics are presented later in Chapter 9.

The strains reported here are from local strain measurements, except in test series AM\( \alpha \)-05 in which only radial and circumferential strains were measured from local transducers but not axial and torsional shear strains. In §4.4 it has been illustrated that the well-known problems of global strain measurement system\(^1\) had a significant impact on the identification of stiffness and yielding characteristics at very small strains up to 0.1 %. Nevertheless, they did not affect significantly the strains associated with peak shear strength conditions.

The present Chapter is organised into seven sections. First, §8.1 explains the detailed scheme of the multi-axial shearing test series. The results from the drained \( b \)-change stages are described in §8.2. This is followed by §8.3 which presents the data from undrained shearing stages. Next, the undrained shear strength parameters of London Clay expressed both in total (\( S_u \)) and effective stress (\( t/s', q/p' \)) terms are shown in §8.4. The strength data is then investigated in the four dimensional stress space (\( q \sim \alpha \sim b \sim p' \)), as well as in the deviatoric stress plane (\( \pi \)-plane). After that, §8.5 reports the undrained deformation characteristics, and the observed shear stiffness \( G_{oc} \), at small strains were compared with those predicted by assuming cross-anisotropic elasticity theory (see §2.5) and applying the elastic parameters determined in the previous chapter (§7.2). This is fol-

\(^1\)As a results of bedding errors, compliance, etc, see Jardine et al. (1984)
8.1 TESTING SCHEME

which explores the possible influences of the failure mode and natural discontinuities on the measured shear strength. Finally, [8.7] summarises the key findings.

8.1 Testing scheme for the multi-axial shear series

The scheme set out for the multi-axial shearing tests aimed to achieve a range of \( \alpha \) and \( b \) combinations. By conducting the tests at different \( \alpha \) values under fixed \( b \) ratios and constant mean stress \( p \) values, it was possible to explore the anisotropy of the clay's shear strength and shear stiffness under undrained conditions. The multi-axial tests were performed at four fixed \( b \)–ratios (0, 0.3, 0.5 and 1.0) and the applied stress paths were intended to avoid the two no-go areas (in the vicinities of \([\alpha, b] = [0^\circ, 1]\) and \([90^\circ, 0]\)) for the reasons discussed previously in §2.2.

Figure 8.1 illustrates the stress path directions that were followed in the multiaxial tests, and Figure 8.2 illustrates the stress changes associated with the \( b \) reduction stages from 1 to 0.5 and 0.3, respectively. Due to the uncertainty in the early stage of this study in determining the estimated in situ stress condition (see §5.3), specimens in the AM\( \alpha \)-05 series [Figure 9.13] were re-consolidated to a different stress state \((p'_0 = 312 \text{ kPa})\) in comparison to those from the other test series \((p'_0 = 280 \text{ kPa})\). The results of the AM\( \alpha \)-05 series were therefore presented separately and in the discussions on shear strength the Author made allowance to take into account the differences in the samples’ effective stress states.

Drained \( b \)–change stage

After completing the anisotropic re-consolidation stages with \( K_0 > 1 \) (i.e. \([\alpha, b] = [90^\circ, 1]\)), the \( b \) values were reduced to either 0.3 or 0.5 under drained conditions while keeping \( \alpha \), \( p'_0 \) and \((\sigma'_{vo} - \sigma'_{ho})\) constant (stress parameters at the end-of-reconsolidation stages). Note that there was no need to conduct this stage in tests that were later sheared under \( b = 0 \) or \( b = 1 \) condition. The \( b \)-change stages represented true triaxial tests in which the maximum shear stress in the \( z \sim \theta \) plane was fixed but the shear stresses in the \( r \sim \theta \) and \( z \sim r \) planes were varied reciprocally. The required changes in the principal effective stress components can be calculated from Equation 8.1:

\[
\Delta \sigma'_z = \Delta \sigma'_\theta = -\frac{(\sigma'_{vo} - \sigma'_{ho})}{3 \cdot \Delta b} \quad \Delta \sigma'_r = \frac{2(\sigma'_{vo} - \sigma'_{ho})}{3 \cdot \Delta b}
\]

leading to \( \Delta \sigma'_r : \Delta \sigma'_z : \Delta \sigma'_\theta = -2 : 1 : 1 \). The stage was performed under stress-control, with the rate of \( \Delta b \) as specified in Table 8.1.
8.1. TESTING SCHEME

Undrained \( \alpha \)-rotation or undrained unloading stage

Rotations of the major principal stress axis direction to angles of \( 0^\circ < \alpha < 90^\circ \) with respect to the vertical axis involve the application of torsional \( \tau_{z\theta} \) to the specimens. The only exceptions are tests conducted precisely with \( \alpha = 0 \) and \( 90^\circ \). It has been known (Shibuya, 1985; Menkiti, 1995; Zdravkovic, 1996) from experiments on anisotropically normally consolidated sands, silts and clays (performed with \( K_o \approx 0.5 \) that unless the initial shear stress applied to the specimen is reduced, there are certain limitations to the range of undrained principal stress rotation (up to \( \alpha \approx 20^\circ \)) without causing failure. If this stage is performed drained, the associated yielding process leads to large volumetric strains and potential instability in low OCR sediments. There are therefore two strategies to overcome any analogous problem that might arise with high OCR samples tested with \( K_o > 1 \):

1. Unloading undrained to an isotropic stress state before applying the shear stress (Shibuya, 1985; Rolo, 2003; Foundoukos, 2006);

2. Unloading undrained to different values of \( t \), varying the degree of reduction depending on the desired value of \( \alpha \) while keeping \( p \) and \( b \) unchanged. This was followed by an \( \alpha \)-rotation stage (Menkiti, 1995; Zdravkovic, 1996).

The disadvantage of the first strategy is the potentially significant change in \( p' \), possible change in peak \( S_u \), and certain modification of the stress–strain behaviour from that expected in situ. With low OCR sediments complete unloading may involve tracing along the lower limits of the local bounding surface such as illustrated by Shibuya et al. (2003a). Tests that follow the second strategy start from different initial stress states—the stress state is dictated by the amount of unloading and excess pore water pressure generation during rotation of the principal stress axis direction and creep at the end of the stage.

In this study both strategies were used to examine their effects on shear strength and deformation properties of the London Clay samples. For undrained unloading stages the rate of stress change employed was \( \Delta q = 6 \text{kPa/hr} \), while in the AM\( \alpha \)-05 series continuous changes of \( \tau_{z\theta} \) from 0 to the final specified value in the undrained \( \alpha \)-rotation stages were applied to keep \( \Delta \alpha/\Delta t = 5^\circ/\text{hr} \). These stages were conducted under stress-control conditions.

Undrained shearing stage at fixed stress rotation (final stage)

The final undrained shearing stages to failure were performed under strain-control conditions, with the dominant strain being advanced by 0.1%/hr. This strain was chosen to be either \( \epsilon_{z\theta}(=1/2 \gamma_{z\theta}) \) when \( 45^\circ \leq \alpha \leq 60^\circ \) or \( \epsilon_z \) in all other cases. The program used feedback from the global deformation measurements to calculate the control strain, and the individual stress components were adjusted accordingly to maintain the constant condition of \( p, \alpha \) and \( b \).

Table 8.2 summarises the detailed conditions applying to the undrained shearing stages carried out in this study. The shearing directions of the stress paths from these multi-axial shear tests is best viewed in the \((\sigma'_z - \sigma'_\theta)/2 \sim \tau_{z\theta}\) stress plane. As shown in Figure 8.3 and Figure 8.4 tests with \( b = 0, 0.3 \) and 1 involved an unloading stage to the isotropic stress axis. Subsequently the samples were sheared under constant \( \alpha \) conditions. Note that test series IM\( \alpha \)-05 started from isotropic stress conditions and therefore required no unloading stage. Prior to the final shearing stages, the samples in series AM\( \alpha \)-05 were subjected to an \( \alpha \)-rotation stage during which \( t \) and \( p \) were kept...
### 8.1. TESTING SCHEME

Table 8.1: Conditions for drained $b$-change stages

<table>
<thead>
<tr>
<th>Test code</th>
<th>$b$</th>
<th>Rate $\Delta b$</th>
<th>$p_0$</th>
<th>$t_\alpha = (\sigma_{ho} - \sigma_{vo})/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM00-05</td>
<td>0.5</td>
<td>0.1</td>
<td>312</td>
<td>-80</td>
</tr>
<tr>
<td>AM30-05</td>
<td>0.1</td>
<td>0.1</td>
<td>312</td>
<td>-80</td>
</tr>
<tr>
<td>AM50-05</td>
<td>0.1</td>
<td>0.1</td>
<td>312</td>
<td>-80</td>
</tr>
<tr>
<td>AM60-05</td>
<td>0.1</td>
<td>0.1</td>
<td>312</td>
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<tr>
<td>AM90-05</td>
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<td>0.1</td>
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</tr>
<tr>
<td>AM00-03</td>
<td>0.3</td>
<td>0.05</td>
<td>280</td>
<td>-80</td>
</tr>
<tr>
<td>AM00-03b</td>
<td>0.1</td>
<td></td>
<td>280</td>
<td>-76</td>
</tr>
<tr>
<td>AM30-03</td>
<td>0.05</td>
<td></td>
<td>280</td>
<td>-83</td>
</tr>
</tbody>
</table>

Notes:
- These stages were conducted under conditions of constant $p_0$, $t_\alpha$ and $\alpha_\alpha = 90^\circ$ unchanged.

The HCA testing scheme used in this study was different from that employed by Nishimura (2006) in two main aspects. First, Nishimura conducted all his stages that followed the completion of re-consolidation under undrained conditions. As a result, his $b$-change path was part of the subsequent undrained shear stage. Secondly, he performed tests with specific constant $\alpha_{ds}$ rather than fixing $\alpha$, which consequently changed continuously during the final shear stages of his tests, except in the axial compression ($\alpha_{ds} = \alpha = 0^\circ$) and axial extension ($\alpha_{ds} = \alpha = 90^\circ$) tests. The $\alpha_{ds}$ values in his tests had been chosen so that the specimens would fail near to the intended $\alpha_f$ values.
8.1. TESTING SCHEME

Figure 8.1: Test scheme for mapping of undrained anisotropy in multi-axial shear series: a) In the $\alpha \sim b$ plane; b) In the 3D stress space $\tau_{z\theta} \sim (\sigma_z - \sigma_\theta)/2 \sim b$
8.1. TESTING SCHEME

Figure 8.2: Stress changes during drained $b$-change stage
8.1. TESTING SCHEME

(a) Series AM\(\alpha\)-00 (constant \(b = 0\))

(b) Series AM\(\alpha\)-10 (constant \(b = 1\))

Figure 8.3: Effective stress paths of multi-axial shear tests in \((\sigma'_z - \sigma'_\theta)/2 \sim \tau_{z\theta}\) plane
8.1. TESTING SCHEME

(a) Series AM\(\alpha\)-03 (constant \(b = 0.3\))

(b) Series AM\(\alpha\)-05 and IM\(\alpha\)-05 (constant \(b = 0.5\))

Figure 8.4: Effective stress paths of multi-axial shear tests in \((\sigma'_z - \sigma'_\theta)/2 \sim \tau_{z\theta}\) plane (continued)
### Table 8.2: Conditions of tests under undrained conditions

<table>
<thead>
<tr>
<th>Test code</th>
<th>Type</th>
<th>$K_0$</th>
<th>$\alpha$</th>
<th>$b$</th>
<th>Shear mode</th>
<th>$p'_o$</th>
<th>$t_o$</th>
<th>Including stage of</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM00-00</td>
<td>CAU</td>
<td>1.53</td>
<td>0</td>
<td>0</td>
<td>TC</td>
<td>280</td>
<td>55</td>
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<tr>
<td>AM45-00</td>
<td></td>
<td>1.53</td>
<td>45</td>
<td></td>
<td>Torsional</td>
<td>280</td>
<td>55</td>
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<tr>
<td>AM00-03</td>
<td>CAU</td>
<td>1.85</td>
<td>0</td>
<td>0.3</td>
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<td>80</td>
<td>Unloading</td>
</tr>
<tr>
<td>AM00-03b</td>
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<td>1.92</td>
<td>0</td>
<td></td>
<td>TTC</td>
<td>280</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>AM30-03</td>
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<td>1.97</td>
<td>30</td>
<td></td>
<td>Torsional</td>
<td>280</td>
<td>83</td>
<td>Unloading</td>
</tr>
<tr>
<td>AM00-05</td>
<td>CAU</td>
<td>1.78</td>
<td>0</td>
<td>0.5</td>
<td>TTC</td>
<td>312</td>
<td>80</td>
<td>$\alpha$-rotation</td>
</tr>
<tr>
<td>AM30-05</td>
<td></td>
<td>1.78</td>
<td>30</td>
<td></td>
<td>Torsional</td>
<td>312</td>
<td>80</td>
<td>$\alpha$-rotation</td>
</tr>
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<td>1.78</td>
<td>50</td>
<td></td>
<td>Torsional</td>
<td>312</td>
<td>80</td>
<td>$\alpha$-rotation</td>
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<td>2.29</td>
<td>60</td>
<td></td>
<td>Torsional</td>
<td>312</td>
<td>108</td>
<td>$\alpha$-rotation</td>
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<td>1.78</td>
<td>90</td>
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<td>TTE</td>
<td>312</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>IM00-05</td>
<td>CIU</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>TTC</td>
<td>280</td>
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<td></td>
</tr>
<tr>
<td>IM90-05</td>
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<td>1</td>
<td>90</td>
<td></td>
<td>TTE</td>
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<td>CAU</td>
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<td>45</td>
<td>1</td>
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<td>280</td>
<td>77</td>
<td>Unloading</td>
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<td>AM60-10</td>
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<td>1.75</td>
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<td>90</td>
<td></td>
<td>TE</td>
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<td>75</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- $\alpha$, $b$: Fixed conditions applied during the final shearing stages (see also Figure 8.1)
- TC, TE, TTC, TTE: Triaxial and true triaxial testing conditions
- Torsional: Tests involved torsional shear stress $\tau_{z\theta}$
8.2 Stress-strain behaviour in b-change stages

Two multi-axial HCA tests series AMα-03 and AMα-05 involved stages where the values of $b$ were changed gradually from 1 to 0.3 or 0.5, respectively. During this stage the drainage valves were open with constant back pressure values, and the rate of stress change was kept lower than 6 kPa/hour. These conditions were similar to those applied during re-consolidation stages (stress rate of 4–6 kPa, see §6.1) and therefore the conditions were considered to be effectively drained. The remained, although small, excess pore pressures could be quantified if local measurement of porewater pressure was available. However, due to the long (typically 8 hours) constant stress holding period that followed each b-change stage it was expected that it became negligible prior to the undrained shearing stages.

8.2.1 Stress-strain response during $b$-change to 0.5

The development of the strain components observed as $b$ reduced from unity to 0.5 are shown in Figure 8.5 and Figure 8.6. Under this testing condition, $\epsilon_z$ and $\epsilon_\theta$ were compressive, whereas $\epsilon_r$ was extensive. The evolution of strains with time is plotted in Figure 8.7 and Figure 8.8. It was observed that the strain rates generally reduced markedly when the applied stress changes halted after 5 hours. The rates decreased further over the following 8 hours, as any remaining excess pore pressures dissipated and slow creep process started to govern the clay’s response.

It can be seen that results from all five tests were similar, with the end-of-stage and creep (plus any primary) strains are tabulated in Table 8.3. Differences were observed between the $\epsilon_z$ and $\epsilon_\theta$ strain components and finite volumetric strains were generated (although the mean effective stress $p'$ was kept constant). These observed features at small strains reflect the stiffness anisotropy of the London Clay.

During $b$-change stages the horizontal plane is the maximum shear plane, i.e. $\sigma_r' - \sigma_\theta' = \sigma_1' - \sigma_3'$. Figure 8.9a and 8.9b show the relationships of the shear stress and shear strain in this plane ($\sigma_r' - \sigma_\theta' \sim \epsilon_r - \epsilon_\theta$) and the generalised deviatoric stress and strain components ($\Delta J \sim \epsilon_d$), respectively. The responses were very similar between five tests, indicating good repeatability. In addition, and as expected, with the reduction of $b$ the response became progressively more non-linear.

The stress-strain non-linearity is also clearly illustrated in Figure 8.10 which magnifies the relationship in the small strain region (less than 0.02%) and plots the values of equivalent secant octahedral shear stiffness ($G_{oct}$). The observed ($G_{oct}$) developed at deviatoric shear strains around 0.001% was about 80MPa, significantly lower (30%) than the bender element test $G_{bb} = 115$MPa found by Gasparre [2005]. This underestimation was expected because of the overestimation of axial strains due to global axial strain measurement.

8.2.2 Stress-strain response during $b$-change to 0.3

A full set of local strain transducers was available in this test series. The developments of strain components as $b$ reduced from 1 to 0.3 are illustrated in Figure 8.11 to Figure 8.13 while the values of total and creep strains are detailed in Table 8.4. It was observed that results from these three stages
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

tests were similar, with compressive strains in radial and circumferential directions. In all tests, the volumetric strains were not zero at early stages of shearing, indicating as before the clay’s anisotropic stiffness properties.

As seen in Figure 8.14, test AM00-03b which was subjected to faster shearing rate ($\Delta b = 0.1$) showed a stiffer response both in the plane of maximum shear and in $\Delta J \sim \epsilon_d$ relationship than those in AM00-03 and AM30-03 tests (where $\Delta b = 0.05$). There was a large scatter in the small strain region (Figure 8.15) of test AM30-03, which evidently affected the calculated values of $G_{oct}$ between 0.002 – 0.006 % in this test. The average upper limit of $G_{oct}$ at $\epsilon_d = 0.001$ % was 100 MPa (see Figure 8.15) which is close to the value of 115 MPa identified from bender element tests (Gasparre, 2005). This better match was the result of using a complete set of local strain measurements in series AM$\alpha$-03.

8.2.3 Comparison with predictions from elastic theory

The pattern of straining observed at small strains can be compared with that predicted by the cross-anisotropic elastic theory set out in 2.5. With the changes of radial stress were double those of axial and circumferential stresses, and adopting the cross-anisotropic model the elastic parameters established for London Clay at 5.2mBGL (see 7.2), it can be shown that the ratios of $\Delta \epsilon_r : \Delta \epsilon_z : \Delta \epsilon_\theta$ are expected to be -2 : 1 : 1 and -1.2 : 1 : 0.2 for isotropic and cross-anisotropic elastic material, respectively. Note, again, that these patterns are only expected to apply within the elastic region.

The relationships between radial and axial strain ($\Delta \epsilon_r : \Delta \epsilon_z$) in series AM$\alpha$-05 are plotted in Figure 8.16a. The observations made in test AM50-05 were very close to the cross-anisotropic elastic prediction. For the other tests to $b = 0.5$, it can be seen that the general response up to strain of 0.05 % was closer to the prediction made from the isotropic model than that from cross-anisotropic elasticity. However, it should be noted that the strains were semi-local strain measurements and therefore were not reliable for strains less than 0.03 %.

Figure 8.16b show the data obtained from series AM$\alpha$-03 and the relationship predicted from elastic theory. Overall the slopes of the observed data were again similar to that of isotropic material at strains larger than 0.05 %. In both cases the clear departure of measured data from the expected cross-anisotropic elastic behaviour after small strains (about 0.005 %) indicated the limiting space in which elastic theory can be assumed to be valid.
8.2. STRESS-STRAIN BEHAVIOUR IN $b$-CHANGE STAGES

Table 8.3: Strains developed during $b$-change stages in AM$\alpha$-05 series

<table>
<thead>
<tr>
<th>Test code</th>
<th>Total strains at end-of-stage</th>
<th>Amount of ‘creep’ strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_z$</td>
<td>$\epsilon_\theta$</td>
</tr>
<tr>
<td></td>
<td>[%]</td>
<td>[%]</td>
</tr>
<tr>
<td>AM00-05</td>
<td>0.061</td>
<td>0.037</td>
</tr>
<tr>
<td>AM30-05</td>
<td>0.043</td>
<td>0.036</td>
</tr>
<tr>
<td>AM60-05</td>
<td>0.073</td>
<td>0.071</td>
</tr>
<tr>
<td>AM90-05</td>
<td>0.047</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Notes:
- Strains were from semi-local strain measurements.
- End-of-stage included the constant stress-holding period after reaching the intended $b = 0.5$.
- Creep strains were those developed within the constant stress holding periods.

Table 8.4: Strains developed during $b$-change stages in AM$\alpha$-03 series

<table>
<thead>
<tr>
<th>Test code</th>
<th>Total strains at end-of-stage</th>
<th>Amount of creep strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_z$</td>
<td>$\epsilon_\theta$</td>
</tr>
<tr>
<td></td>
<td>[%]</td>
<td>[%]</td>
</tr>
<tr>
<td>AM00-03</td>
<td>0.104</td>
<td>0.089</td>
</tr>
<tr>
<td>AM00-03b</td>
<td>0.063</td>
<td>0.056</td>
</tr>
<tr>
<td>AM30-03</td>
<td>0.074</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Notes:
- Strains were from local strain measurements.
- End-of-stage included the constant stress-holding period after reaching the intended $b = 0.3$.
- Creep strains were those developed within the constant stress holding periods.
Figure 8.5: Development of normal strains during drained $b = 1 \rightarrow 0.5$ (semi-local strain measurements)
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

Figure 8.6: Development of normal strains during drained $b = 1 \to 0.5$ (contd.) (semi-local strain measurements)
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

Figure 8.7: Evolution of normal strains during drained $b = 1 \rightarrow 0.5$ (semi-local strain measurements)
Figure 8.8: Evolution of normal strains during drained $b = 1 \rightarrow 0.5$ (contd.) (semi-local strain measurements)
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Shear stress - shear strain in the horizontal plane $\tau \sim \theta$

(b) Deviatoric stress-strain $\Delta J \sim \epsilon_d$

Figure 8.9: Stress-strain relationships during drained $b = 1 \rightarrow 0.5$ (semi-local strain measurements)
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Deviatoric stress-strain $\Delta J \sim \varepsilon_d$ in small strain region

(b) Octahedral shear stiffness

Figure 8.10: Shear stiffness during drained $b = 1 \rightarrow 0.5$ (semi-local strain measurements)
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

Figure 8.11: Development of normal strains during drained $b = 1 \rightarrow 0.3$
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Test AM00-03

(b) Test AM30-03

Figure 8.12: Evolution of normal strains during drained $b = 1 \rightarrow 0.3$
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Development of normal strains

(b) Time evolution

Figure 8.13: Test AM00-03b during drained $b = 1 \rightarrow 0.3$ stage, rate $\Delta b = 0.05$ per hour
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Shear stress - shear strain in the horizontal plane $\tau \sim \theta$

(b) Deviatoric stress-strain $\Delta J \sim \varepsilon_d$

Figure 8.14: Stress-strain relationships during drained $b = 1 \rightarrow 0.3$
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

(a) Deviatoric stress-strain $\Delta J \sim \epsilon_d$ in small strain region

(b) Octahedral shear stiffness

Figure 8.15: Shear stiffness during drained $b = 1 \rightarrow 0.3$
8.2. STRESS-STRAIN BEHAVIOUR IN B-CHANGE STAGES

Figure 8.16: Observed strains and predicted values by cross-anisotropic elasticity theory (see also Chapter 7)
8.3 Overall stress-strain response in undrained shear stages

8.3.1 Overview

This section presents the results of the undrained shear stages from the four multi-axial HCA test series outlined in Table 8.5. Due to the complexity of the testing scheme, the present section concentrates on the overall stress-strain response of the London Clay under conditions of constant $b$. Subsequent sections, §8.4 and §8.5, discuss the shear strength and stiffness characteristics of the tested samples.

Throughout the testing programme, sufficient pause periods were allowed after the end of each stage for undrained creep to stabilise (to strain rates below 0.002 %/hour, following the recommendations by Jardine (1992)), which typically required pauses extending for several hours. The main parts of the final shearing stages to failure were carried out with a nominal shear strain rate of $d(\varepsilon_1 - \varepsilon_3)/dt = 3.5$ %/day, while maintaining fixed values of $p$, $\alpha$ and $b$ via automatic servo-control (§4.2). To help define the small strain response at shear strain of less than 0.2 %, the earlier stages of the tests were conducted with slower shear strain rates (about 0.7 %/day) over the first 10–12 hours, after which the nominal rate was applied for the rest of the test. It was recognised that this arrangement was likely to create some influences to the stiffness characteristics at very small strains, which are strain rate or time dependent, of the London Clay.

Because of the extra degrees of stress and strain freedom provided by HCA testing in comparison with the triaxial testing, a variety of 2-D plots is necessary to present the effective stress paths (ESPs) and stress-strain relationships. The testing results are presented in three types of plot: 1) the ESPs in the $p_0 - t$ and the $(\sigma_z - \sigma_\theta)/2 - \tau_{z\theta}$ planes; 2) the development of individual strains ($\varepsilon_z, \varepsilon_r, \varepsilon_\theta, \gamma_{z\theta}$) and the principal strains ($\varepsilon_1, \varepsilon_2, \varepsilon_3$) during shear with respect to the deviatoric shear strain ($\varepsilon_d$); and 3) the stress-strain response in the vertical plane $z \sim \theta$ in which the maximum shear stresses $t$ were applied.

The undrained shear strengths $S_u$ and the peak stress ratios $q/p'$ from an undrained test were identified as in Figure 8.17. However, the generalisation of the shear strength of London Clay in terms of effective stress is difficult because samples tend to fail at different $p'$, depending systematically on $b$, $\alpha$ as well as $p'$ level and the consolidation $K$ ratio. Previous experience with triaxial tests (see, e.g., Bishop et al., 1965; Hight and Jardine, 1993; Hight et al., 2003) shows curves of peak $q \sim p'$ envelopes, indicating a dependency on $p'$ of the $q/p'$ at peak. In this study it was not possible to define the potential envelope curvature but recognising that the range of $p'$ values at which the HCA tests failed was relatively narrow (less than 30% of the average values, see Table 8.6), hence the Author defined shear strengths in terms of $q/p'$ or $S_u$. These two approaches, as noted by Nishimura.

### Table 8.5: Direction of undrained multi-axial shear tests to failure

<table>
<thead>
<tr>
<th>Test series</th>
<th>$b$-value</th>
<th>$\alpha$-value</th>
<th>Including stages of</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM$\alpha$-00</td>
<td>0</td>
<td>0 and 45</td>
<td>Unloading to isotropic axis</td>
</tr>
<tr>
<td>AM$\alpha$-10</td>
<td>1</td>
<td>45, 60 and 90</td>
<td>Unloading to isotropic axis</td>
</tr>
<tr>
<td>AM$\alpha$-03</td>
<td>0.3</td>
<td>0 and 30</td>
<td>Unloading to isotropic axis</td>
</tr>
<tr>
<td>AM$\alpha$-05</td>
<td>0.5</td>
<td>0, 30, 50, 60 and 90</td>
<td>$\alpha$-rotation</td>
</tr>
<tr>
<td>IM$\alpha$-05</td>
<td>0.5</td>
<td>0 and 90</td>
<td></td>
</tr>
</tbody>
</table>

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183
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

\[ t = q/2 \]

Line of peak stress ratio

Maximum undrained strength

Figure 8.17: Identification of undrained shear strength \( S_u \) and peak stress ratio

\textit{et al.} (2007), bracket the available possibilities when assessing the anisotropic shear strength trends - one for a purely frictional material with constant \( \phi' \) and the other for a purely cohesive material where strength is independent of \( p' \). The actual effective stress-dependency of anisotropy remains to be established.

8.3.2 Stress-strain response in the intermediate stages prior to shear to failure

Stages of unloading to isotropic stress axis

Undrained unloading stages (reducing shear stress while keeping \( p \) and \( b \) constant) were carried out on a sub-set of tests from series AM\( \alpha \)-03 and AM\( \alpha \)-10. Figure 8.18 to Figure 8.21 show the development of strain components with respect to \( \epsilon_r \) in AM60-10, AM30-03, AM00-03 and AM00-03b. With no torsional shear stress applied to the specimens, the values of \( \gamma_z \theta \) were very small and there were close similarities between \( \epsilon_r \) and \( \epsilon_\theta \) up to a shear strain level of 0.1–0.2 %, indicating a predominantly axi-symmetric mode of deformation of the specimens. Although the induced \( \epsilon_z \) were nearly the same (about 0.2 %), tests under \( b = 0.3 \) condition generated larger strains (50-75 % larger) in the horizontal plane than test AM60-10 under \( b = 1.0 \) condition.

The stress-strain relationships recorded during the unloading stages (Figure 8.22) confirm the stiffer response of test AM60-10 at strains level below 0.1 % in comparison with AM\( \alpha \)-03 series, reflecting the shear strain dependency on \( b \). It can also be seen in Figure 8.23 that during unloading all samples showed contractant behaviour, with \( p' \) falling by 22–35 kPa (8–12 %) from the typical initial reconsolidation value \( (p'_o = 280 \text{ kPa}) \).

Stages of \( \alpha \)-rotation

The semi-local strain measurements during \( \alpha \)-rotation stages (keeping \( q \) and \( p \) constant) in tests AM50-05, AM30-05 and AM00-05 are shown in Figure 8.24, Figure 8.25 and Figure 8.26 respectively. Because these tests started from \( \alpha_0 = 90^\circ \), the induced shear strains increased as \( \alpha \) reduced (Figure 8.27). With the introduction of \( \gamma_z \theta \), significant \( \gamma_z \theta \) developed and it eventually governed the strain responses. The shear strains \( \epsilon_{13} \) were significantly larger than those developed in the unloading stages. Note also that \( \epsilon_r \) were small and close to zero, indicating the near plane-strain testing condition in the \( b = 0.5 \) series.
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.28 illustrates the contractant behaviour ($\Delta p' < 0$) observed in London Clay samples during the undrained $\alpha$-rotation stages, with $p'$ reducing by of 6–17 % of $p'_0 = 312$ kPa. As expected, test AM00-05 showed the largest strains and change in $p'$. Over their common parts, the responses were similar although AM00-05 appeared to be both stiffer and more contractant than the other two tests (Figure 8.29). The changes in $p'$ led to $q/p'$ rising by modest amounts during these stages.

8.3.3 Shearing to failure with constant $\alpha$ and $b$ (final stages)

Once the unloading or $\alpha$-rotation stage was completed, each sample was sheared to failure with a fixed combination of $\alpha$ and $b$, following the scheme described earlier in Table 8.2. The stress-strain responses observed in these stages are described in this section.

In four tests, of which three in axial compression ($\alpha = 0^\circ$, AM00-03, AM00-05 and IM00-05) and one axial extension ($\alpha = 90^\circ$, AM90-10), noticeable periods of interruption can be seen in the ESPs and stress-strain responses. These interruptions (identified with $a$ and $b$ for the beginning and ending points in each figure, e.g. Figure 8.33) were the results of the unintentional halts of the computer control program that unfortunately happened when the apparatus was unattended. The program had been set up to stop when any of the global transducers reached the end of its working range in order to prevent damage and unfortunately the settings adopted led to unanticipated problems. During these periods, the total applied stresses were unchanged but the effective stress points moved to the left ($\Delta p' > 0$) at nearly constant $t$. This response could be due to a time or strain-rate dependency of the clay behaviour, or equally because a possible time lag in pore water pressure measurement due to the compliance of the leads between the transducers and the samples. As a result, creep strains developed over the same constant $t$ and that led to the plateaux seen in the stress-strain curves (e.g. Figure 8.35). These unwanted stops could have been avoided if the Author had been present to re-set the external transducers, or to select more suitable starting times for shearing, taking note of the experience gained during testing of the London Clay.

The volumetric strains reported here are the sum of the measured normal strains (i.e. $\varepsilon_v = \varepsilon_z + \varepsilon_r + \varepsilon_\theta$), not the global values calculated from volume gauge ($\varepsilon_v = -\Delta V/V$), which by definition were nil during undrained ($\Delta V = 0$) stages. This local $\varepsilon_v$ was used as an indicator of the degree of strain non-uniformity within the specimen, which was expected to increase significantly as rupture planes formed and failure stages were approached.

Stress-strain response under $b = 0$ and 1 conditions

The ESPs recorded during shear from the AM$\alpha$-00 test series are shown in Figure 8.30. The upper plot presents the ESP trajectories in the $t \sim p'$ plane, whereas the lower one views them in the $(\sigma'_z - \sigma'_\theta)/2 \sim p'$ plane. In these plots lines of constant $q/p' (= 2t/p')$ are drawn through the peak points as if no apparent effective cohesion intercept existed ($c' = 0$). Test AM45-00 showed an anomalous result, giving significantly higher strength than test AM00-00 (i.e. triaxial compression). This result is contradict to the behaviour expected from all other studies on London Clay (e.g. Bishop, 1966; Porovic, 1995; Nishimura, 2006) where shear strength at $\alpha = 0^\circ$ has been found to be higher. No particular differences have been found in the initial or index properties of this sample and the Author has not been able to identify any cause for its abnormal response.

*A much stiffer system has now been installed in this HCA.*
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.31 shows the development of strains in AM00-00 test series. For each test, values of \( \varepsilon_z, \varepsilon_r, \varepsilon_{\theta} \) and \( \gamma_{z\theta} \) are shown in the left hand figure, where as \( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) are presented in the right hand plot. Failure was reached in triaxial compression (AM00-00) with axial strain being the major principal strain (\( \varepsilon_1 \)) and torsional \( \gamma_{z\theta} \) was negligibly small (see Figure 8.31b). Test AM45-00 was dominated by torsional \( \gamma_{z\theta} \) but the normal strains were also significant, particularly the radial \( \varepsilon_r \).

Table 8.6 summarises the peak values of \( S_u \) and \( q/p_0 \), as well as the shear strains \( \gamma_{13} = \varepsilon_1 - \varepsilon_3 \) found at these values. It can be seen that the shear strains associated with peak \( q/p_0 \) and \( S_u \) were not necessarily the same although the two points were reached at similar stages (e.g. Figure 8.32).

Table 8.6 summarises the peak values of \( S_u \) and \( q/p_0 \), as well as the shear strains \( \gamma_{13} = \varepsilon_1 - \varepsilon_3 \) found at these values. It can be seen that the shear strains associated with peak \( q/p_0 \) and \( S_u \) were not necessarily the same although the two points were reached at similar stages (e.g. Figure 8.32).

The trajectories of the ESPs for the AM0-10 test series are shown in Figure 8.33. The tests conducted in the triaxial extension mode (AM90-10) had the lowest shear strength, reaching just \( q/p_0 = 0.6 \) at peak. In Figure 8.34 large \( \gamma_{z\theta} \) values can be observed in the AM45-10 and AM60-10 tests. As expected, negligible \( \gamma_{z\theta} \) appeared in the AM90-10 test (Figure 8.34c) in which normal strains were predominant. In the AM00-00 (TC mode) and AM90-10 (TE mode) tests, \( \varepsilon_2 (= \varepsilon_r) \) and \( \varepsilon_3 (= \varepsilon_\theta) \) were similar but not exactly equal, suggesting some divergence from axi-symmetric strain conditions and tranverse isotropy.

Figure 8.35 demonstrates that the peak shear strength for the AM90-10 test (developed at \( \gamma_{13} = 0.35\% \)) was much smaller than those seen in the AM45-10 and AM60-10 tests (developed at 1.48% and 1.72%, respectively). The interuptions due to the unexpected stop of the control programme, as described previously, are indicated in Figure 8.33 and Figure 8.35. During these periods, the samples were contractive, with \( \Delta p' = 5 \text{kPa} \) in AM45-10 and \( \Delta p' = 40 \text{kPa} \) in AM90-10, and plateaux could be observed in the corresponding stress-strain curves.

One common observed response under undrained multi-axial testing conditions is that the clay was contractive (\( \Delta u > 0 \)) when \( (\Delta \sigma_z - \Delta \sigma_\theta) = (\Delta \sigma_r - \Delta \sigma_\theta) > 0 \) and vice versa. Similar response was also noted by [Nishimura (2006)] in his multi-axial HCA tests. This response is common to all the undrained tests and contributes to the anisotropy in shear strength of the London Clay [Nishimura et al. (2007)]. The slopes \( (\Delta \sigma_z - \Delta \sigma_\theta)/2\Delta p' \) of the initial parts of the ESPs were relatively similar (between -1:2.9 and -1:3.1), which can be compared to the theoretical slopes of -1:3 for an isotropic elastic material and of -1:2.6 to -1:4.2 predicted for a cross-anisotropic elastic material (see §2.5 and [Nishimura 2006]). The reasons for this mismatch between the theoretical and observed values can be:

1. Uncertainty in the average elastic parameters, in particular the values of Poisson’s ratios

2. Potential stress and strain non-uniformities in the HC specimens

In addition, as will be shown later for tests under \( b = 0.3 \) and 0.5 conditions, the non-orthotropic stress state after the drained \( b \)-change could change and distort the initial cross-anisotropy of the clay. [Nishimura (2006)] also argued that the early yielding (especially in his HCA CAU test series as a result of the foregoing undrained \( b \)-change stage) may also be a reason for the above disagreement.

Because the post-peak behaviour was not followed to large strains in the HCA tests in the present study, the softening behaviour in stress-strain response, and hence any post-rupture strength (Burland and [1990]) or residual strength development, was not defined. In general, as noticed in the uniaxial drained shear tests, ruptures were formed prior to peak strength and tests that were dominated by

\[ \text{Note that } \varepsilon_r \text{ can become any of the principal strains depending on the mode of shearing.} \]
torsional shear stresses (30° < α < 60°) facilitated the formation of ruptures (shear planes) at smaller shear strains than in compression. Their patterns will be described in Section 8.6.

**Stress-strain response under b = 0.3 condition**

The ESPs from the AM0-03 test series are plotted in Figure 8.36. In the AM00-03 and AM00-03b tests, considerable reductions in p′ developed once γ13 = 0.4%, which caused the ESPs to bend significantly to the left well ahead of the development of the peak strengths. Reductions in p′ were evident in all tests, showing an average slope of -1:3.4 for the initial parts of ESPs. This can be compared with the observed values reported by Nishimura (2006) from HCA tests on samples at 10.2 mBGL (between -1:2.2 and -1:2.8), while the cross-anisotropic elastic theory predictions ranged from -1:3.13 to -1:4.17.

The two tests (AM00-03 and AM00-03b) in axial compression (α = 0° ± 90°) showed higher shear strength than the AM30-03 test which involved torsional shear (α = 30°). The upper bound of stress ratio at peak can be taken as q/p′ = 1.37 from test AM00-03b. This strength was higher than that in triaxial compression mode (AM00-00), indicating a positive effect of b on shear strength. Figure 8.37 shows the developments of strain components during undrained shearing stages in the AM0-03 test series. Because values of ᵉᵣ were not close to zero, these tests were not representative of the plane strain condition.

Considering the two compression tests with α = 0, the AM00-03b test showed a stiffer response, and was affected by a longer period of unintentional halt, than the AM00-03 test (Figure 8.38). The higher shear strength and stiffer response in the AM00-03b test may reflect the individual macro-fabric of the test specimen, as the AM00-03 sample had more fissures than the AM00-03b sample. It is noted that their index properties were similar but the AM00-03 sample started from a higher initial effective stress on set-up stage (at 375kPa) than the AM00-03b sample (at 320kPa). When reconsolidated to the same effective stress point, larger strains were therefore induced in the AM00-03 test than those in the AM00-03b test. As a result, the AM-00-03 sample was subjected to more plastic strains prior to shear.

**Stress-strain response under b = 0.5 condition**

Multi-axial HCA tests can be performed over the whole range of principal stress rotation angle (α = 0° ~ 90°) under b = 0.5 condition without entering the no-go zones (Symes, 1983). The Author performed two series of b = 0.5 tests, one from isotropic consolidation conditions (CIU, known as IMα-05) and the other from anisotropic consolidation points (CAU, termed AMα-05). However, due to the limited number of available samples the IM series was limited to only two tests, one with α = 0° and the other with α = 90°.

It can be seen from Figure 8.39 that the CIU shear strength of samples from 5.2 mBGL were significantly higher in the ‘plane-strain’ compression test (IM00-05) than in the extension test (IM90-05). Observing that ᴇᵣ developed at b = 0.5 was nearly zero in his hollow cylinder test series, Nishimura (2006) suggested that under this particular condition his tests were close to the plane strain shearing mode. It is noted that in the Author’s study ᴇᵣ were small but not zero in the IMα-05 test series (Figure 8.40).

The early parts of the ESPs (see Figure 8.39b) indicated a slope of -1:3.3 in IM90-05 and -1:5.7
in IM00-05. However, if the part after halt point $b$ was accounted for in IM00-05 then this slope was -1:3.3, which was similar to that of IM90-05 and close to the ratio of -1:3.13 predicted by cross-anisotropic elasticity theory.

Figure 8.41 shows that the shear strain $\gamma_{13}$ at peak strength in extension condition (at 0.86% in IM90-05) was less than a third of that in compression (at 2.21% in IM00-05). Due to yet another unintentional halt, the ESP in test IM00-05 showed a staggered section (marked by points $a$ to $b$) at around $q/p_{0} = 0.75$ where $\gamma_{13}$ was around 0.36%. The observed low extension shear strength at $b = 0.5$ was in contrast with the CIU multi-axial HCA tests on London Clay samples at 10.5 mBGL reported by Nishimura (2006). This difference will be discussed in §8.4.

As mentioned earlier, the AM$\alpha$-05 series involved undrained principal stress rotation stages after the drained $b$-change stages. The corresponding ESPs are shown in Figure 8.42. Two interesting observations can be made here. First, the slopes of the ESPs were again relatively similar in all tests. Secondly, the bounds for shear strength were of the same order to those found from the CIU tests (see again Table 8.6), with an upper bound of $q/p' = 1.44$ for AM00-05 (axial compression) and $q/p' = 0.75$ for AM90-05 (axial extension). Note that test AM50-05 ($\alpha = 50^\circ$) also showed low strength ($q/p' = 0.85$).

Figure 8.43 and Figure 8.44 indicate that $\epsilon_r$ in the CAU HCA $b = 0.5$ test series were nearly zero in the AM00-05, AM50-05, AM60-05 tests and very small (less than 0.5%) in the AM30-05 and AM90-05 test. It is therefore reasonable to conclude that these tests involved near plane strain conditions, which appeared to generate a higher shear strength in axial compression than the equivalent triaxial compression test. The gain of shear strength under plane strain extension condition over to that from triaxial extension condition was not significant.

Figure 8.45 shows the stress-strain response to shearing stages where both $\alpha$ and $b$ were kept constant of the AM$\alpha$-05 series. As with the IM90-05 test, peak was reached after only a small shear strain (0.27%) in the AM90-05 test. The AM60-05 test indicated a marked strain-softening response post peak while the AM30-05 test showed strain-hardening behaviour.

Overall the behaviour observed in tests at $b = 0.5$ was compatible with the trend seen at other $b$ values. The samples (5.2 mBGL) tested in this study generally had lower shear strength under extensional and torsional loading conditions than those under compressional conditions. In addition, the general trend of anisotropic pore pressure response, giving an average ESP slope of -1:3.3 (see Table 8.7) and independent of $\alpha$ and $b$, was one of the causes for the anisotropy of the clay shear strength. The next section, §8.4, discusses the anisotropy in the observed shear strength and the influences of the intermediate principal stress and sample’s macro fabric in more detail.

---

Data of the $\alpha$-rotation stage for AM60-05 was unfortunately not recorded.
### Table 8.6: Shear strength from multi-axial undrained shear tests on London Clay at 5.2mBGL

<table>
<thead>
<tr>
<th>Test code</th>
<th>b</th>
<th>α</th>
<th>t/s'</th>
<th>φ'</th>
<th>q/p'</th>
<th>γ₁₁₂</th>
<th>S_u</th>
<th>γ₁₁²</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM0005</td>
<td>0.5</td>
<td>0</td>
<td>0.70</td>
<td>44.4</td>
<td>1.40</td>
<td>2.21</td>
<td>170</td>
<td>2.21</td>
</tr>
<tr>
<td>IM9005</td>
<td>90</td>
<td>0.28</td>
<td>16.3</td>
<td>0.560</td>
<td>0.86</td>
<td>93</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>AM0005</td>
<td>0.5</td>
<td>0</td>
<td>0.72</td>
<td>46.1</td>
<td>1.44</td>
<td>1.57</td>
<td>180</td>
<td>1.35</td>
</tr>
<tr>
<td>AM3005</td>
<td>30</td>
<td>0.56</td>
<td>34.1</td>
<td>1.12</td>
<td>3.14</td>
<td>165</td>
<td>16</td>
<td>3.16</td>
</tr>
<tr>
<td>AM5005</td>
<td>50</td>
<td>0.43</td>
<td>25.2</td>
<td>0.85</td>
<td>1.18</td>
<td>135</td>
<td>15</td>
<td>1.67</td>
</tr>
<tr>
<td>AM6005</td>
<td>60</td>
<td>0.55</td>
<td>33.4</td>
<td>1.10</td>
<td>1.75</td>
<td>195</td>
<td>17</td>
<td>2.17</td>
</tr>
<tr>
<td>AM9005</td>
<td>90</td>
<td>0.38</td>
<td>22.0</td>
<td>0.75</td>
<td>0.33</td>
<td>120</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>AM0003</td>
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<td>0</td>
<td>0.65</td>
<td>40.4</td>
<td>1.42</td>
<td>2.95</td>
<td>144</td>
<td>3.21</td>
</tr>
<tr>
<td>AM0003b</td>
<td>0</td>
<td>0.63</td>
<td>38.9</td>
<td>1.37</td>
<td>3.48</td>
<td>138</td>
<td>3.64</td>
<td></td>
</tr>
<tr>
<td>AM3003</td>
<td>30</td>
<td>0.46</td>
<td>27.1</td>
<td>0.97</td>
<td>1.78</td>
<td>113</td>
<td>1.78</td>
<td></td>
</tr>
<tr>
<td>AM4510</td>
<td>1</td>
<td>45</td>
<td>0.40</td>
<td>23.7</td>
<td>0.71</td>
<td>1.72</td>
<td>100</td>
<td>1.75</td>
</tr>
<tr>
<td>AM6010</td>
<td>60</td>
<td>0.40</td>
<td>23.3</td>
<td>0.70</td>
<td>1.48</td>
<td>102</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>AM9010</td>
<td>90</td>
<td>0.33</td>
<td>19.5</td>
<td>0.60</td>
<td>0.35</td>
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<td>0.35</td>
<td></td>
</tr>
<tr>
<td>AM4500</td>
<td>0</td>
<td>45</td>
<td>0.58</td>
<td>35.7</td>
<td>1.45</td>
<td>2.15</td>
<td>192</td>
<td>2.15</td>
</tr>
<tr>
<td>AM0000</td>
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<td>0.54</td>
<td>32.5</td>
<td>1.31</td>
<td>3.72</td>
<td>136</td>
<td>3.72</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8.7: Observed and predicted slopes of ESPs in multi-axial shear tests

<table>
<thead>
<tr>
<th>Test code</th>
<th>b</th>
<th>α</th>
<th>Theoretical</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[-]</td>
<td>[deg]</td>
<td>1: slope</td>
<td>1: slope</td>
</tr>
<tr>
<td>IM0005</td>
<td>0.5</td>
<td>0</td>
<td>-4.17</td>
<td>-5.7 or -3.3 (corrected)</td>
</tr>
<tr>
<td>IM9005</td>
<td>90</td>
<td>0</td>
<td>-4.17</td>
<td>-3.3</td>
</tr>
<tr>
<td>AM0005</td>
<td>0.5</td>
<td>0</td>
<td>-4.17</td>
<td>-3.1</td>
</tr>
<tr>
<td>AM3005</td>
<td>30</td>
<td>0</td>
<td>-4.17</td>
<td>-3.3</td>
</tr>
<tr>
<td>AM5005</td>
<td>50</td>
<td>0</td>
<td>-4.17</td>
<td>-3.3</td>
</tr>
<tr>
<td>AM6005</td>
<td>60</td>
<td>0</td>
<td>-4.17</td>
<td>-3.1</td>
</tr>
<tr>
<td>AM9005</td>
<td>90</td>
<td>0</td>
<td>-4.17</td>
<td>-3.3</td>
</tr>
<tr>
<td>AM0003</td>
<td>0.3</td>
<td>0</td>
<td>-3.13</td>
<td>-3.4</td>
</tr>
<tr>
<td>AM3003</td>
<td>30</td>
<td>0</td>
<td>-4.05</td>
<td>-3.4</td>
</tr>
<tr>
<td>AM6010</td>
<td>1</td>
<td>60</td>
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<td>-3.0</td>
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<td>AM9010</td>
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<td>0</td>
<td>-3.13</td>
<td>-2.9</td>
</tr>
<tr>
<td>AM0000</td>
<td>0</td>
<td>0</td>
<td>-3.13</td>
<td>-3.0</td>
</tr>
</tbody>
</table>

Notes: Theoretical values predicted from cross-anisotropic elastic theory, with average elastic parameters (at anisotropic stress state) as defined in Chapter 7.
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.18: Strains developed during unloading stage, test AM60-10

Figure 8.19: Strains developed during unloading stage, test AM30-03
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.20: Strains developed during unloading stage, test AM00-03

Figure 8.21: Strains developed during $d\alpha = 45^\circ$, test AM00-03i
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.22: Normalised $q/p' \sim \gamma_{13}$ relationship during unloading stages

Figure 8.23: Unloading stages $p' \sim \gamma_{13}$ relationship
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.24: Strains developed during $\alpha$—rotation stage, test AM50-05

Figure 8.25: Strains developed during $\alpha$—rotation stage, test AM30-05
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.26: Strains developed during $\alpha$-rotation stage, test AM00-05

Figure 8.27: Development of shear strain $\gamma_{13}$ w.r.t $\alpha$
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.28: Changes in $p'$ during $\alpha$—rotation stages

Figure 8.29: Normalised stress-strain relationship $q/p' \sim \gamma_{13}$
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.30: Effective stress paths in test series AM\(\alpha\)-00 with constant \(b = 0.0\)
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

(a) Test AM00-00

(b) Test AM45-00

Figure 8.31: Developments of strains during undrained shear in series AM$\alpha$-00
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.32: Stress-strain response in test series AMα-00

(a) Shear stress - shear strain in $z - \theta$ plane $t \sim (\varepsilon_1 - \varepsilon_3)$

(b) Stress ratio - shear strain $q/p' \sim (\varepsilon_1 - \varepsilon_3)$
Figure 8.33: Effective stress paths in test series AM$\alpha$-10 with constant $b = 1.0$
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.34: Developments of strains during undrained shear in series AMα-10
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

(a) Shear stress - shear strain in $z - \theta$ plane $t \sim (\epsilon_1 - \epsilon_3)$

(b) Stress ratio - shear strain $q/p' \sim (\epsilon_1 - \epsilon_3)$

Figure 8.35: Stress-strain response in test series $\text{AM}_{\alpha-10}$
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.36: Effective stress paths in test series AMα-03 with constant $b = 0.3$
Figure 8.37: Developments of strains during undrained shear in series AMα-03

203
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

(a) Shear stress - shear strain in $z - \theta$ plane $t \sim (\epsilon_1 - \epsilon_3)$

(b) Stress ratio - shear strain $q/p' \sim (\epsilon_1 - \epsilon_3)$

Figure 8.38: Stress-strain response in test series AMα-03
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.39: Effective stress paths in test series IM\(\alpha\)-05 with constant \(b = 0.5\)
Figure 8.40: Developments of strains during undrained shear in series IM$\alpha$-05
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

(a) Shear stress - shear strain in $z - \theta$ plane $t \sim (\epsilon_1 - \epsilon_3)$

(b) Stress ratio - shear strain $q/p' \sim (\epsilon_1 - \epsilon_3)$

Figure 8.41: Stress-strain response in test series IM$\alpha$-05
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.42: Effective stress paths in test series AM$\alpha$-05 with constant $b = 0.5$
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

Figure 8.43: Developments of strains during undrained shear in series AMα-05

(a) Test AM00-05

(b) Test AM30-05

(c) Test AM50-05
Figure 8.44: Developments of strains during undrained shear in series AM$\omega$-05
8.3. OVERALL STRESS-STRAIN RESPONSE IN UNDRAINED SHEAR STAGES

(a) Shear stress - shear strain in \( z - \theta \) plane \( t \sim (\varepsilon_1 - \varepsilon_3) \)

(b) Stress ratio - shear strain \( q/p \sim (\varepsilon_1 - \varepsilon_3) \)

Figure 8.45: Stress-strain response in test series AM\( \alpha \)-05
8.4 Shear strength anisotropy of London Clay

This section discusses the anisotropy in peak shear strength in terms of both undrained strength \( S_u \) and effective stress ratios \( (q/p' \text{ and } t/s') \) for the London Clay samples at 5.2 mBGL. The pattern of anisotropy in shear strength is also compared with that established by Nishimura (2006) from his multi-axial HCA tests on clay samples from a deeper horizon (10.5 mBGL). Further discussion concerning the overall anisotropy and the influence of shearing mode for the T5 profile (up to 50 mBGL) are discussed later, drawing on the summaries offered by Nishimura et al. (2007) and Hight et al. (2007).

8.4.1 Experimental-fitting expression for shear strength anisotropy from triaxial tests

The strong anisotropy in shear strength and stiffness of the natural stiff London Clay has been well recognised (see §3.2 and Ward et al., 1959, 1965; Bishop et al., 1965; Agarwal, 1968; Atkinson, 1975). Generally the techniques of investigation employed in these studies were similar and often limited to triaxial compression tests on samples that were cut with different inclinations to the vertical (depositional) direction of the clay. Therefore the testing mode is equivalent to a variation of \( \alpha \) with fixed \( b = 0 \) condition in HCA test. However, the application of this method on diagonally cut samples is questionable due to the associated problems of stress non-uniformity and bending moments (see §2.2), as well as the impossibility of applying effective \( K_o \) stress conditions prior to shear. Nevertheless, even until very recently these studies provided the only available database of shear strength anisotropy of natural London Clay (Hight et al., 2003).

Based on triaxial experiments on inclined samples, Bishop (1966) proposed the following expression for the dependency of undrained shear strength to loading direction:

\[
S_u(\beta) = S_u(\beta=0)(1 - m \sin^2 \beta)(1 - n \sin^2 2\beta) \tag{8.2}
\]

where \( \beta \) is the direction of the sample's axis with respect to the vertical direction (Figure 8.46), and \( m, n \) are soil parameters defined by fitting to the experiment values.

The Bishop's fitting curves that followed Equation 8.2 for (i) heavily OC London Clay and (ii) lightly OC Welland Clay are shown in Figure 8.47. The undrained shear strengths are normalized by \( S_u(\beta=0) \), which corresponds to the strength obtained on a vertically cut sample. Note also that this approach implicitly neglects all \( b \) effects.

For low OCR soils such as Welland Clay, the inclined samples only show a slight reduction in
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

Figure 8.47: Experimental-based fitting curve for shear strength anisotropy by Bishop (1966) from triaxial tests on diagonally-cut samples. Also presented are database of HCA tests under $b = 0.5$ condition on low OCR, $K_0$-consolidated soils at Imperial College.

Undrained shear strength with $\beta$, reaching a minimum value at $\beta = 90^\circ$. In contrast, for the heavily OC London Clay undrained shear strength ratio reduced with $\beta$ from $0^\circ$ to $45^\circ$ but then it gained an additional of 46% at $\beta = 90^\circ$. Depending on the fitting parameters selected, Equation 8.2 can reproduce either type of trend reported here for tests involving the $b = 0$ triaxial condition.

For the more general conditions, this figure also includes results of the HCA tests on $K_0$-consolidated and low OCR soils under $b = 0.5$ conditions in the large Imperial College HCA (see §2.3). It is important to note that in such HCA tests, values of $b$ can be controlled independently, with $K_0$ consolidation stress state prior to shear can be imposed on the vertically cut sample, and $\alpha$ acting as $\beta$ for principle stress loading direction. The IC-HCA data of low OCR reconstituted soils at $b = 0.5$ condition shows a reducing trend of undrained shear strength with $\alpha$ but with a more remarkable anisotropic dependency than the response of triaxial tests on Welland Clay.

The disadvantages involving in inclined triaxial testing can be avoided by conducting hollow cylinder shear tests, and it is also possible to investigate the influences of $b$. Equation 8.3 can be extrapolated to:

$$S_u(\alpha, b) / S_u(\alpha = 0, b) = (1 - m \sin^2 \alpha)(1 - n \sin^2 2\alpha)$$

The normalized $S_u(\alpha = 0, b)$ can be taken at the undrained shear strength in compression at specific $b$ ratio in case $0 \leq b \leq 0.5$, or $S_u(\alpha = 0, b = 0)$ (triaxial compression condition) in other cases as a HCA test can't be conducted at $[\alpha, b] = 0, 1$ due to problems of stress non-uniformity. The empirical parameters $(m, n)$ have similar meanings as in Bishop (1966) expression but are dependent on $b$.

In addition to the effects of $b$, the shear strength anisotropy of low OCR soils has been found to be sensitive to the consolidation history, with patterns that depend on the ratio $K_c = \sigma'_3 / \sigma'_1$ and the principal stress direction applied during consolidation ($\alpha_c$) (Jardine et al., 1997; Zdravkovic and Jardine, 2001; Shibuya et al., 2003b). The comparison between CIU and CAU HCA test data on samples from 10.5 mBGL (Nishimura, 2006) indicated that consolidation state may have a modest
influence on the peak strength under some conditions, particularly with $\alpha = 90^\circ$. It was noted that this study also involved simple triaxial re-consolidation stress paths. Overall, a modest dependency of shear strength on consolidation state could be expected in the present study.

### 8.4.2 Anisotropy in shear resistance

The observed effective stress ratios $q/p'$ developed at peak resistance are plotted in Figure 8.48, Figure 8.49 and Figure 8.50 for conditions of $b = 0, 0.3, 0.5$ and $1$, respectively. Similarly, Figure 8.51, Figure 8.52, Figure 8.53 and Figure 8.54 show the peak undrained shear strengths $S_u$ and the normalised ratios $S_u/p_0'$ that take into account the difference in the mean effective consolidation stress prior to shear ($p_0' = 312$ kPa in series $AM\alpha-05$ versus $280$ kPa in all other tests).

As previously mentioned, the $AM45-00$ test gave abnormally high shear strength although no special difference was noticed in this specimen in terms of its physical properties, initial fissuring pattern and failure mode. The limited number of tests in the $AM\alpha-00$ series, and the abnormality of $AM45-00$ result, meant that insufficient information was available to confirm the anisotropy trend under the $b = 0$ condition. While test $AM45-00$ is reported, the strength data were not included in the summary plots that synthesize the overall undrained shear strength behaviour of the London Clay.

The shear stress anisotropy (i.e. dependence on $\alpha$) was clearly evident in the test series under $b = 0.3$ and $0.5$ conditions but was less recognisable in those under $b = 1.0$ condition. In general, there was a trend of shear strength reduction as $\alpha = 0 \rightarrow 45^\circ$ in which $q/p'$ reduced to approximately $35\%$ and $42\%$ from $\alpha = 0^\circ$ to $\alpha = 30^\circ$ and $\alpha = 45^\circ$, respectively. The maximum effective stress ratios developed with $\alpha = 0^\circ$ and $0 \leq b \leq 0.5$ ($q/p' = 1.37 \sim 1.44$) with undrained shear strengths in the range of $140$–$170$ kPa.

If the differences in $p_0'$ are taken into account, the tests conducted under $b = 0.5$ and $\alpha = 0^\circ$ conditions (Figure 8.54) gave the highest shear strength ($\max S_u/p_0' = 0.6$), which was about $15\%$ higher than those found under $b = 0$ and $0.3$ conditions. Triaxial extension test ($\alpha = 90^\circ$, $b = 1$) gave the lowest bound of shear strength ($q/p' = 0.55$, $S_u/p_0' = 0.3$). All in all, the variations in $q/p'$ and $S_u/p_0'$ were relatively minor (between $0.6$–$0.7$) in the tests conducted with $45^\circ \leq \alpha \leq 90^\circ$ and $b = 1.0$.

It can be seen from Figure 8.50 and Figure 8.53 that the tests in near plane strain conditions ($b = 0.5$ and $\epsilon_r \approx 0$) showed two possible interpretive trends with $\alpha > 45^\circ$. The shear strength either increased with $\alpha$ (test $AM60-05$) or decreased (AM90-05 and IM90-05 tests). Additional tests in this region, such as $\alpha = 75^\circ$ (or between $80^\circ > \alpha > 50^\circ$), would be helpful to establish the trend more clearly. It is also interesting to note that the rising trend between $\alpha = 45^\circ$ or $90^\circ$ was found in tests on London Clay at $10.5$ mBGL (Nishimura, 2006). This significant difference in behaviour might be attributed to an effect of sample size as suggested by Nishimura et al. (2007).

Figure 8.55 summarises the patterns of anisotropy in undrained shear strength observed from hollow cylinder tests on natural London Clay samples at $5.2$ and $10.5$ mBGL. The $b = 0.5$ condition is of practical interest as it represents the near plane strain condition and encompasses the entire range of variation in the major principal stress loading direction. Overall there was striking agreement for values of $\alpha < 45^\circ$ but a marked difference at $\alpha = 90^\circ$, with the heavily OC samples showing a different pattern after $\alpha > 45^\circ$ to that of the low OCR soils mentioned earlier (see Figure 8.56). It is likely that gravitational compaction and heavy overconsolidation alter the mi-
crostructural anisotropy more significantly in London Clay than the low OCR reconstituted soils and that its overall behaviour is also far more affected by the discontinuities formed as part of the natural geological processes.Apparently for the tested London Clay samples, the sub-horizontal fissures, which are more abundant at shallower depth, facilitated failure mechanisms that prefer torsional, or possibly extensional loading ($\alpha > 60^\circ$).

### 8.4.3 Comparison with the Bishop’s (1966) empirical curved fitting expression

The variations in anisotropy of undrained shear strength of London Clay at 5.2 mBGL were compared with the empirical curve fitting expression proposed by Bishop (1966). Figures 8.57, 8.58 and 8.59 plot the testing data and Equation 8.3 at $b = 0.5$, $0.3$ and $0$ and $1$, respectively. The $m$ and $n$ parameters of the curves plotted are those suggested by Bishop (see Figure 8.47), as Nishimura (2006) showed that these values offered reasonable matches in certain conditions in his study.

At first sight, the expression can describe reasonably the decrease in shear strength from $\alpha = 0 \rightarrow 45^\circ$ for $b = 0.5$ and $0.3$. However, it cannot match the low values of $S_u$ seen with $b = 1$ when $\alpha > 45^\circ$. If the data for samples at 10.5 mBGL (after Nishimura, 2006) is included (Figure 8.60), good agreement is found for $b = 0.5$ and $\alpha > 45^\circ$. Although this approach seemed offer a reasonable fit for compressional loading conditions ($\alpha < 45^\circ$, $b < 0.5$), further terms are required in Equation 8.3 to take account of the effects of $b$ on shear strength.

### 8.4.4 Influence of $b$ on shear strength

It is widely recognised that one of the key factors that influence the shear strength of soil is the intermediate principal stress ($\sigma_2'$). The commonly applied Mohr-Coulomb failure condition implicitly assumes that shear strength is independent of $\sigma_2'$. However, real soil behaviour shows noticeable departure from this assumption (e.g. Bishop, 1971b; Symes et al., 1982; Shibuya, 1983; Tatsuoka, 1988; Tatsuoka et al., 1988; Lade and Kirkgard, 2000; Leroueil and Hight, 2003), indicating the important effects of $\sigma_2'$, or of the $b$-ratio on the soil’s shear strength. Potts (2003) highlighted that this is an important but often overlooked factor in numerical analyses, although part of the difficulties come from the fact that only limited experimental data is available.

In this aspect, HCA tests can offers useful experimental data of shear strength in the generalized $\alpha \sim b$ plane and the deviatoric $\pi$-plane. The influence of $b$ can be investigated by establishing the shear parameters in constant-$\alpha$ sections. The discussions within this section use this approach, which has also been employed by Nishimura (2006). In addition to the Mohr-Coulomb criterion, the alternative failure criteria proposed by Matsuoka and Nakai (1974) and (Lade and Duncan, 1975) were also chosen for comparison purposes.

In the normalised $\pi$-plane the stress point can be determined as follows:

$$
X = (\sigma'_2 - \sigma'_3)/\sqrt{2}\sigma' = b/\sqrt{2} \cdot q/p' \\
Y = (2\sigma'_1 - \sigma'_2 - \sigma'_3)/\sqrt{6}\sigma' = (2 - b)/\sqrt{6} \cdot q/p'
$$

(8.4)

It follows that if a failure criterion of constant $q/p'$ is adopted, its projection (in one section of the $\pi$-plane) is a line and perpendicular to the $b = 0.5$ line. The other failure criteria are illustrated in Figure 8.61 considering the $b = 0$ to $1$ range. It is further assumed that all criteria pass through a common point representing a constant mobilised angle of shearing resistance ($\phi' = 30^\circ$) with
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

$b = 0$ on the left hand plot (compression tests, $\alpha < 45^\circ$), and $b = 1$ on the right (extension tests, $\alpha > 45^\circ$). It can be seen that the $b$ value at which the functions are fixed affects the position of the constant $q/p'$ and Lade-Duncan criteria but not those associated with the Mohr-Coulomb or Matsuoka-Nakai criteria.

**Analysis of test results**

The normalised stress states at peak strength plots from undrained HCA tests on samples from 5.2 mBGL in HCA are grouped together in the $\pi$-plane for conditions of $\alpha = 0, 30, 45, 60, 90^\circ$ in Figure 8.62 and Figure 8.63. It was further assumed that the differences in consolidation stresses (between $p' = 312$ and 280 kPa) caused negligible changes to the peak shear strength parameters. Such an assumption, as discussed previously (see also Nishimura, 2006), has been found to be acceptable for the shallow London Clay at Heathrow T5.

Figure 8.62 shows the data under $\alpha = 45^\circ$ condition which covers the full testing range of $b = 0 \sim 1$. Except the abnormally high strength observed in test AM45-00, a Mohr-Coulomb criterion with $\phi' = 24^\circ$ appeared to be a reasonably good fit. However, the results reported by Nishimura (2006) on London Clay samples from a deeper horizon (10.5 mBGL) indicated that $\phi'$ at peak increased with $b$ and his data was found to fit better with the constant $q/p$ criterion in this condition.

For other $\alpha$ sections (Figure 8.63), it is obvious that there was no single failure criterion that can be used to fit all the experimental data. The observed deviations from any single Mohr-Coulomb failure line, particularly in the $\alpha = 0, 30$ and $60^\circ$-sections, indicated significant degrees of influence from $b$. The best fitting $\phi'$ values tend to reduce with increasing value of $\alpha$, dropping from $\phi' = 32^\circ$ at $\alpha = 0^\circ$ to $\phi' = 20^\circ$ at $\alpha = 90^\circ$. Practically speaking, for any fixed value of $\alpha$ a Mohr-Coulomb failure calibrated to data obtained at $b = 0$ (in case $\alpha \leq 45^\circ$) or $b = 1$ (in case $\alpha > 45^\circ$) seemed to be a safe option although not a best fit.

In general, both Matsuoka-Nakai and Lade-Duncan criteria showed poorer agreement, being either underestimated or overestimated the experimental results. In addition, a constant $q/p'$ criterion fitted to $b = 0$ was found to overestimate the shear strength in $\alpha \leq 45^\circ$ condition in most cases and to underestimate that value in $\alpha < 45^\circ$ condition if fitted to $b = 1$.

To have a broader view, the results obtained under $\alpha = 45^\circ$ condition for samples at both 5.2 and 10.5 mBGL are plotted together in Figure 8.64. The majority of data fell within two bounds that pass through the $b = 0$ point established by Nishimura (2006). The lower bound represented a Mohr-Coulomb line with $\phi' = 23.5^\circ$, while the upper limit can be the $q/p' = 0.92$ line, or perhaps the Lade-Duncan failure criterion. For other $\alpha$ sections, it was generally found that $b$ had a strong influence on peak $\phi'$. Nishimura et al. (2007) report that no specific theoretical failure criterion could be fitted to the full set of observed strengths for the London Clay samples obtained at shallow depths at T5 and tested in the two HCA studies.

The effective stress ratios mobilised at peak by the London Clay samples from 5.2 mBGL are shown as functions of $b$ in Figures 8.65 to 8.67. In general, it is not reasonable to assign fixed values of stress ratio to the generalised stress conditions. This observation reconfirms the previous analysis of the use of constant $q/p'$ failure criterion in the deviatoric plane.

216
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

8.4.5 Combined effects of \( \alpha \) and \( b \) on peak shear strength parameters

Figure 8.68 plots the measured stress ratios (\( q/p' \) or \( t/s' \)) at peak shear strength against \( b \) and \( \alpha \) in comparable three-dimensional stress spaces. As mentioned earlier, test AM45-00 had untypical high shear strength and this data point is excluded from the current interpretation. All in all, the data formed irregular surfaces which were depressed near \( \alpha > 45^\circ \) and \( b > 0.5 \) but elevated in the \( \alpha < 45^\circ \) and \( b < 0.5 \) zones. The lowest strengths are associated with the \([\alpha, b] = [45^\circ, 0.5]\) and \([90^\circ, 1]\) spots.

Nishimura et al. (2007) review the combined hollow cylinder shear testing database from the two main horizons explored at T5 (5.2 and 10.5 mBGL) and show that despite the differences between the clay specimens in terms of soil fabric, consolidation state and applied stress path, the overall three dimensional shapes are generally similar, with the main exception being the response with \([\alpha, b] = [90^\circ, 0.5]\).

The high peak stress ratios in compressional loading modes were compatible with the triaxial compression failure envelope curvatures of London Clay established for the Heathrow T5 site (see Figure 8.69). The shear strength envelopes tend to expand with depth, but flatten for any given depth with higher \( p' \) or \( s' \) values. The curved en-echelon pattern reflects the dependency of shear strength with \( OCR \) and lithology (Gasparre, 2005; Hight et al., 2007), as well as stress level. Nishimura et al. (2007) also report that in all cases the mobilised shear strengths were between the indicated boundaries and above the lower limit of fissure strength (\( \phi' = 17^\circ, c' = 0 \) kPa), and there is a tendency of shear strength anisotropy to grow with depth.

Nishimura (2006) and Nishimura et al. (2007) presented the undrained shear strength profiles (Figure 8.70) obtained with three shear modes (triaxial compression, triaxial extension and simple shear) on London Clay from the same site. In simple shear tests the peak shear stress was reached at \( 50^\circ < \alpha < 65^\circ \) and \( 0.5 < b < 0.7 \) and therefore the observed shear strengths subjected to mixed conditions of \( \alpha \) and \( b \). Nevertheless, comparison of such test data provides useful bench marks for the possible variations in anisotropy throughout the stratum.
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

Figure 8.48: Anisotropy of stress ratio at peak $q/p'$ at $b = 0$

Figure 8.49: Anisotropy of stress ratio at peak $q/p'$ at $b = 0.3$
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

Figure 8.50: Anisotropy of stress ratio at peak $q/p'$ at $b = 0.5$

Figure 8.51: Anisotropy of stress ratio at peak $q/p'$ at $b = 1$
Figure 8.52: Anisotropy of undrained shear strength $S_u$ at $b < 0.5$
Figure 8.53: Anisotropy of undrained shear strength $S_u$ at $b = 0.5$
Figure 8.54: Anisotropy of undrained shear strength $S_u$ at $b = 1$
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

Figure 8.55: Anisotropy of undrained strength ratio $S_u/S_u(\alpha=0)$ under near plane strain condition ($b = 0.5$) of natural London Clay from hollow cylinder tests.

Figure 8.56: Shear strength anisotropy of natural London Clay and other $K_o$-consolidated low OCR soils under $b = 0.5$ (near plane strain) condition tested in HCA at Imperial College.
Figure 8.57: Comparison of \( S_{u,b=0.5}/S_{u,\{\alpha=0,b=0.5\}} \) ratio at \( b = 0.5 \) with Bishop (1966) expression.

Figure 8.58: Comparison of \( S_{u,b=0.3}/S_{u,\{\alpha=0,b=0.3\}} \) ratio at \( b = 0.3 \) with Bishop (1966) expression.
Figure 8.59: Comparison of $S_u/S_u(\alpha=0)$ ratio at $b = 0$ and 1 with Bishop (1966) expression.

Figure 8.60: Experimental-based expression (Bishop, 1966) and shear strength anisotropy of London Clay. Hollow cylinder tests at two depths 5.2 and 10.5 mBGL.
Figure 8.61: Failure criteria in \( \pi \)-plane, fitting to \( \phi' = 30^\circ \) at \( b = 0 \) or 1. Failure criteria: Mohr-Coulomb (M-C), Matsuoka-Nakai (M-N) and Lade-Duncan (L-D).

Figure 8.62: Peak shear strength on \( \pi \)-plane of \( \alpha = 45^\circ \) for London Clay samples at 5.2 mBGL
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

(a) $\alpha = 0^\circ$

At $\alpha = 0^\circ$

L-D

$\phi' = 32^\circ$

M-N

$q/p' = 1.05$

(b) $\alpha = 30^\circ$

At $\alpha = 30^\circ$

L-D

$\phi' = 26^\circ$

M-N

$q/p' = 0.84$

(c) $\alpha = 60^\circ$

At $\alpha = 60^\circ$

L-D

$\phi' = 23^\circ$

M-N

(d) $\alpha = 90^\circ$

At $\alpha = 90^\circ$

L-D

$\phi' = 20^\circ$

M-N

Figure 8.63: Peak shear strength on $\pi$-planes for London Clay samples at 5.2 mBGL
Figure 8.64: Deviatoric plane at $\alpha = 45^\circ$ for London Clay at 5.2 and 10.5 mBGL. Failure criteria: Mohr-Coulomb (M-C), Matsuoka-Nakai (M-N) and Lade-Duncan (L-D).

Figure 8.65: Peak $q/p'$ at section of $\alpha = 45^\circ$ for London Clay samples at 5.2mBGL
8.4. SHEAR STRENGTH ANISOTROPY OF LONDON CLAY

Figure 8.66: Peak $q/p'$ at sections of $\alpha < 45^\circ$

Figure 8.67: Peak $q/p'$ at sections of $\alpha > 45^\circ$
Figure 8.68: Changes in stress ratio $q/p'$ at peak with $\alpha$ and $b$ London Clay samples at 5.2mBGL
Figure 8.69: Comparison of TE and SS shear strengths with Hight and Jardine (1993) TC envelopes (after Nishimura et al., 2007)

Figure 8.70: Undrained shear strength profiles of three different shear modes (after Nishimura et al., 2007)
8.5. Deformation characteristics in the final shearing stages to failure

This section presents the non-linear stress-strain relationships recorded during the final undrained shearing stages conducted with constant $\alpha$ and $b$ conditions, concentrating on $G_{\text{z}\theta}$ and $G_{\text{oct}}$. These are secant stiffness values determined by the methods expressed earlier in §4.3.

It has been mentioned that $\varepsilon_z$, $\varepsilon_r$, $\varepsilon_\theta$ and $\gamma_{\text{z}\theta}$ were local strain measurements in all but except in the AM$\alpha$-05 series. In this series, the local axial and torsional shear strains were measured from global strain transducers. From the comparison shown in §4.4, the semi-local strain measurements did not coincide with the local strain measurements at deviatoric shear strains below 0.03%. Therefore this section only present the stiffness data for tests that had full set of local strain measurements, while Appendix C reports the results of the AM$\alpha$-05 series for completeness.

With regard to the deformation characteristics under undrained conditions, the main points of interest are:

1. Comparison between the observed stiffness moduli at small strains ($< 0.001 \%$) from the undrained multi-axial HCA tests with those predicted from the cross-anisotropic elastic theory using elastic parameters established through the uni-axial drained HCA tests in Chapter 7;
2. Evaluation of the rate of stiffness degradation under undrained conditions as shear strain accumulates;
3. Comparison between the measured stiffness data with the bounds of stiffness $E_u/p_0$ established in triaxial tests for London Clay (e.g. Hight and Jardine, 1993; Hight and Higgins, 1995; Hight et al., 2003).

For the first point, the changes in the elastic $G_{\text{oct}}$ with variations in $\alpha$ and $b$ can be calculated from cross-anisotropic elastic theory as shown in §2.5 (Nishimura, 2006). The chosen drained stiffness parameters (small stress probes at in situ anisotropic stresses, see Table 7.6) were: $E_h' = 120\text{MPa}$, $E_v' = 218\text{MPa}$, $G_{vh} = 75\text{MPa}$, $\nu_v' = 0.25$ and $\nu_h' = -0.17$.

The second and third points have often been evaluated using triaxial data. Limited information is available for the more generalised stress condition, notably with the torsional $G_{\text{z}\theta}$ results from tests on deep London Clay samples at Sizewell site reported by Hight et al. (1997). Only recently that the Author’s study and that of Nishimura (2006) have explored both $G_{\text{z}\theta}$ and $G_{\text{oct}}$ under wider combinations of constant $\alpha$ and $b$. In Chapter 7 the anisotropy in the drained secant stiffness moduli appeared to persist until moderate strains ($< 0.2 \%$), and it is of interest to investigate if such response was also observed under undrained conditions.

8.5.1 Shear stiffness under constant $\alpha$ and $b$ conditions

Figure 8.71 shows the predicted trends for shear stiffness at different $b$ values (0 to 1) between $\alpha = 0 \sim 90^\circ$. The upper bound, for which max $G_{\text{oct}} = G_{\text{vh}}$, is identified at $b = 0.5$ and $\alpha = 45^\circ$. The minimum values at $\alpha = 0^\circ$ or $\alpha = 90^\circ$ are known to be related to the Young’s modulus in triaxial tests ($E_u/3$ or $E_u/3$). With a maximum variation of predicted elastic $G_{\text{oct}}$ less than 6 %, it can be seen that the influences from variations in $\alpha$ and $b$ are not expected to be significant.

Figure 8.72 plots $G_{\text{z}\theta}$ for tests that involved $\tau_{\text{z}\theta}$ components. The $G_{\text{oct}} \sim \varepsilon_d$ relationships found
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.71: Variation of elastic $G_{\text{oct}}$ values with $\alpha$ and $b$ based on cross-anisotropic elastic theory (analysis after Nishimura [2005]). Elastic parameters were $E'_{v} = 120$ MPa, $E'_{h} = 218$ MPa, $G_{vh} = 75$ MPa, $\nu'_{vh} = 0.25$ and $\nu'_{hh} = -0.17$.

in the AM$\alpha$-00 and AM$\alpha$-10 test series ($b = 0$ and 1) are grouped together in Figure 8.73, while the AM$\alpha$-03 test series (under $b = 0.3$ conditions) is shown in Figure 8.74. Results from the IM$\alpha$-05 test series (CIU, $b = 0.5$ condition) are plotted separately in Figure 8.75.

The data indicate noticeable early degradations of shear stiffness moduli from shear strains as low as 0.001%. However, the AM45-00 test showed an anomalously stiff response, as well as the unusual high shear strength reported in §8.4. While the causes for this deviation from normal behaviour have not been identified, and unfortunately it was unable to repeat this test due to the shortage of specimens near the end of the present experimental programme, the Author regards this experiment with suspicion. Nevertheless, as seen in Figure 8.74 it is encouraging that the results of the two similar tests in approximate plane strain compression (AM00-03 and AM00-03b) are in good agreement. Figure 8.75 also shows similar trends in stiffness response between two CIU tests (IM00-05 and IM90-05), which indicate comparable trends under the (near) plane strain compression and extension conditions.

The drained uniaxial shear test series in Chapter 7 reported average $G_{z\theta} = G_{vh}$ value of $75 \pm 5$ MPa. The undrained experimental results at small shear strains (0.001%) showed $G_{\text{oct}}$ between 78–97 MPa, where the upper bound (97 MPa, test AM45-00) probably reflected the abnormally strong sample as noted earlier. The lower limit (78 MPa) was close to the range established from the drained uniaxial tests.

The differences between the experimental data at small strains (< 0.001%) and the predicted values using cross-anisotropic elastic theory for $G_{\text{oct}}$ are reported in Table 8.8. With regard to the observed data in the CAU test series, the maximum $G_{\text{oct}} = 61 \sim 81$ MPa. This became even higher (100–110 MPa) in the CIU tests. Considering the maximum values established from the CAD and CID test series, the CAU tests fell 20% below the drained $G_{z\theta}$ whereas the CIU tests indicated 25% stiffer secant moduli. These differences might be explained by the following differences in test procedures:

1. Tests in the CIU series were subjected to a longer creep/pause period (1 day longer) than those
in CAU series.

2. The multi-axial undrained tests were performed at significantly faster strain rate (0.7 \% per day for the first 10–12 hours) in comparison with the slow rates (0.02 to 0.05 \% per day) employed in uniaxial drained tests. The effect, however, was expected to be less significant than the first reason.

With the fast shear rates employed in the multi-axial undrained shear stages, the limits to the stiffness plateau expected within the elastic limit \( Y_1 \) surface were not well-defined, although marked stiffness degradation could be confirmed at shear strains exceeding 0.002 \%. The rates of stiffness degradation with shear strain shown in the \( G_{\alpha \theta} \) and \( G_{\text{oct}} \) plots are similar, giving \( \frac{G_{\text{oct}}}{G_{\text{oct}}^{0.01}} = 0.27 \sim 0.42 \) and \( \frac{G_{\text{oct}}}{G_{\text{oct}}^{0.001}} = 0.16 \sim 0.35 \).

It is difficult in these experiments to resolve clear effects on undrained stiffness of both \( \alpha \) and \( b \) over the pre-failure strain ranges (\( \epsilon_d < 0.1 \% \)). The only remarkable difference appeared to be that \( b = 0 \) tests gave a stiffer response than those at \( b = 1 \) (see Figure 8.73). However, this behaviour should not be treated as representative of true soil response as it was not repeated under other \( b \) conditions. In addition, as discussed earlier, cross-anisotropic elastic theory predicts relatively minor effects of \( \alpha \) and \( b \) on \( G_{\text{oct}} \). The expected trend for \( G_{\text{oct}} \) to be at a maximum with \( \alpha = 45^\circ \) was observed with \( b = 1 \), but not under other \( b \) conditions.

### 8.5.2 Normalisation of the undrained stiffness ratios

[Hight and Jardine (1993)] reported bounds of shear stiffness decay curves obtained in locally instrumented undrained triaxial tests. These are plotted in terms of secant \( E_u/p_0^0 \) versus the triaxial shear strain invariant \( \epsilon = 2/3(\epsilon_1 - \epsilon_3) = \epsilon_a \) where \( \epsilon_a \) is the undrained axial strain. Since then their template for stiffness degradation curve has been used extensively for checking the quality of site investigation data and for predicting deformations in numerical modelling (e.g. [Jardine et al. 1991], [Hight and Higgins 1995], [Jardine et al. 2004]).

Most recently the normalised stiffness template has been updated for the Heathrow T5 investigation [Hight et al. 2003]. It has been widely acknowledged that the normalised curves are influenced by the direction of loading (compression or extension), the direction of the re-consolidation stress path and the creep strain prior to shear. In general, the bounds show maxima for \( E_u/p_0^0 \) ranging between 700 and 1400 [Hight et al. 2007].

The stress and strain parameters applying to axi-symmetric conditions are related to those in the generalised stress space by \( E_u/p_0^0 = 3G_{\text{oct}}/p_0^0 \) and \( \epsilon_a = \epsilon_d/\sqrt{3} \), and this allows the results of multi-axial undrained HCA shear tests to be checked against the abovementioned template. As mentioned already, cross-anisotropic elastic theory predicts that the triaxial data should provide the lower bound limit for undrained stiffness within the \( Y_1 \) surface.

The normalised stiffness ratios for the CAU tests under \( b = 0 \) and 1 conditions are shown in Figure 8.76 and Figure 8.77, while data for those tested under \( b = 0.3 \) conditions are plotted in Figure 8.78. At very small shear strains (0.001 \%), the observed experimental results fall within 600–1000, and the trends suggested a greater dependence on \( b \) than on \( \alpha \). For the CIU tests under \( b = 0.5 \) condition, the corresponding curves are plotted in Figure 8.79. This indicated a ratio of about 1200 for both compressional and extensional loading. Tests that were performed under near plane strain conditions (with \( b = 0.3 \) and 0.5) also indicated the largest values of \( 3G_{\text{oct}}^\text{max}/p_0^0 \), showing a range of 800–1200. Overall, the observed data was within the range reported by [Hight et al. 2007].
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Table 8.8: Comparisons with predictions by cross-anisotropic elastic theory

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>$b$</th>
<th>$\alpha$</th>
<th>Elastic $G_{oct}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAU</td>
<td>0</td>
<td>0</td>
<td>81.0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>30</td>
<td>62.0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>60</td>
<td>61.0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>45</td>
<td>78.0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>45</td>
<td>62.0</td>
</tr>
<tr>
<td>CIU</td>
<td>0.5</td>
<td>0 or 90</td>
<td>105.0</td>
</tr>
</tbody>
</table>

Notes:

Elastic stiffness parameters are as determined in §7.2, having $G_{vh} = 75 \pm 5$MPa for the CAD tests and $G_{vh} = 79 \pm 4$MPa for the CID tests.

On the other hand, Figures 8.80 and 8.81 show the bounds established from triaxial CAU a) compression tests (Gasparre, 2005) and b) extension tests (Nishimura, 2006), respectively. These normalised stiffnesses were significantly lower (ratios between 400–600) than the database and commercial values. (Hight et al., 2007) suggested that the apparent anomaly was due to the slower rates of stress change along the approach path to the in situ stresses and the extended drained pause periods (normally 7 days) prior to the undrained shearing stages. The Author’s HCA tests, conducted with shorter pause periods (1 day) and faster stress change rates prior to final shearing stages, were therefore expected to conform with the Hight and Jardine (1993) database.
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.72: Torsional shear stiffness from CAU tests at $b = 0, 0.3$ and $1$

Figure 8.73: Octahedral shear stiffness in series AM$\alpha$-00 and AM$\alpha$-10

236
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.74: Octahedral shear stiffness in series AMα-03

Figure 8.75: Octahedral shear stiffness from CIU tests (IMα-05)
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.76: Normalised $3G_{\text{oct}}/p'$ under $b = 0$ condition

Figure 8.77: Normalised $3G_{\text{oct}}/p'$ under $b = 1$ condition
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.78: Normalised $3G_{oct}/p'$ under $b = 0.3$ condition

Figure 8.79: Normalised $3G_{oct}/p'$ under $b = 0.5$, CIU tests
8.5. DEFORMATION CHARACTERISTICS IN THE FINAL SHEARING STAGES TO FAILURE

Figure 8.80: Normalised stiffness data of triaxial CAU compression tests, Heathrow T5 site, after Hight et al. (2007)

Figure 8.81: Normalised stiffness data of triaxial CAU extension tests, Heathrow T5 site, after Hight et al. (2007)
8.6 Modes of failure

This section describes the observed failure modes of the hollow cylinder specimens subjected to undrained multi-axial stress paths. For a stiff fissured soil like natural London Clay, it is well known that the discontinuities presented within the specimen can influence the peak and post-rupture strengths (e.g. Bishop, 1966; Burland, 1990; Hight et al., 2003; Jardine et al., 2004) thereby contributing to the pattern of shear strength anisotropy.

The degree of progressive failure in laboratory shear tests is known to vary considerably, depending significantly on the initial consolidation stresses, the kinematic restraint offered by the equipment, the degree of stress and strain non-uniformities within the specimen and the soil's stress-strain behaviour—in particular its stiffness and brittleness. With high plasticity soils such as London Clay, the discontinuity patterns also play a crucial role.

8.6.1 Previous studies

Nishimura (2006) discussed the kinematic post-rupture responses of his torsional HCA shear tests on London Clay specimens and linked the relationships between failure planes (planes of weakness) with the abovementioned factors. Assuming the soil is in a perfectly plastic state at failure, he identified the following four possible kinematic failure modes:

**Mode 1**  Formation of primary diagonal followed by a secondary near horizontal shear plane (Figure 8.82a) facilitated by a plane of weakness, with stress concentration at the points where the plane has unfavourable conditions and bifurcation of the intact parts under a modified stress field;

**Mode 2**  Bending of a specimen (Figure 8.82b) facilitated by the non-uniformity in deformation of the specimen and possibly by apparatus compliance under torsion that can be accentuated by rigid body rotation of the specimen. A small diameter but tall specimen may provide favourable conditions for this mode;

**Mode 3**  A spiral shear plane, specifically associated in a hollow cylinder specimen when shear band develops for an extensive length along the specimen's curvature wall (see Figure 8.82c);

**Mode 4**  Formation of two shear bands that lead to the formation of a wedge combined with rigid body movement. This mode is a combination of Mode 1 with Mode 2 (Figure 8.82d).

Depending on the degree to which pre-formed planes of weakness can contribute to the final failure mechanism, the peak strength inferred from the boundary measurements can fall between the extremes of intact peak strength available to unfissured clay and the fissured strength.

By detailed observation of the specimen states when first ruptures initiated, Nishimura (2006) found that the ruptures frequently developed along natural discontinuities regardless of the loading direction (i.e. independent to $\alpha$). Nevertheless, the values of $q$ and $q/p'$ ratio at first rupture exhibited significant anisotropy. He argued that one factor that affected this anisotropy was the complex interaction between the strength of the discontinuity, the modification in the local stress field caused by slippage on the latter, and the deformation characteristics of the sample's intact parts. Moreover, he observed that strain hardening continued after rupture until a peak state was reached, typically between 84-90% of the strain at peak. Similar observations have been reported.
8.6. MODES OF FAILURE

earlier, for example, Bishop (1971a) noted that the clay natural discontinuities can promote strain localisation and a degree of hardening after initial rupture.

The abovementioned points show that detailed observation of samples’s shear modes is necessary in understanding the response of the soils at stages of failure and post-peak. It is important to note that the exact states of first ruptures in test specimens were not identified in the present study. The Author’s test results did not indicate marked post-peak strain softening except in the AM60-05 test.

8.6.2 Observed failure patterns in multi-axial shear tests

The natural discontinuities and failure planes on the specimen’s outer surface were recorded separately at two stages of each test. Once the set-up stage was completed and the cell was filled with water, any visible natural discontinuities were recorded by eye, matching their positions with the square grids marked on the outer latex membrane. At the end of the shearing stage, the rupture lines on the sample’s outside surface were recorded by tracing their form on transparent film that wrapped around the specimen. These two traces are shown together in the same 2-D sketches in Figures D.1 to D.13 where the natural discontinuities are dashed lines and continuous lines represent failure surfaces. The three-dimensional forms of the failed specimens are plotted to the left of the 2-D sketches to aid the visualisation. In Table 8.9 the observed dip angles of the shear planes are compared with those of the maximum stress ratio planes predicted by the Coulomb failure criterion.

To assist the comparisons between HCA failure modes and those developed in other types of apparatus, the results here are arranged into three groups: triaxial, true triaxial and torsional shear stress conditions. The subsequent discussions are followed this order.

7The expecting accuracy was about 5% of the grid size, i.e. ±1 mm
Tests in triaxial stress conditions

The AM00-00 and AM90-10 tests were in triaxial compression and extension stress conditions. It was observed that the specimens failed along single or multiple diagonal shear planes (Mode 4, see Figure D.1 and Figure D.2), with dip angles matching closely (within $5^\circ$) with the inclinations of the planes at maximum stress ratio (see Table 8.9). This observation was similar with Nishimura’s finding and indicating Coulomb type bifurcation of intact parts.

Tests in true triaxial conditions

Tests in this group are that conducted under $0 < b < 1$ and with no torsional shear stress, i.e. pure compressional or extensional loading ($\alpha = 0^\circ$ or $90^\circ$). Due to the difference in boundary constraint with regard to the axi-symmetric triaxial stress condition, there are only two possible directions for planes of maximum stress ratio–both of them must also be perpendicular to the $z \sim \theta$ plane. Accordingly the shear displacement fields are confined in the plane of the specimen wall. The shear planes are either of wedge form (Mode 4) or if they are extended for a long length, would form spiral shape (Mode 3).

For this group, Coulomb-type of bifurcation was commonly observed because the shear planes were found to develop close to the planes of maximum stress ratio. Overall, the dip angles of the observed shear planes were within 2–8 % of those predicted by Coulomb criterion. One exception was in test IM90-05, where the observed dip angles were significantly lower than the predicted one. One reason could be that in cases where the final failure mechanism coincided with the initial natural discontinuities, the dip of the shear plane became less steep.

On the one hand, the multiple shear planes with limited rigid body movements in the radial direction were regularly associated with compressional loading conditions ($\alpha = 0^\circ$). On the other hand, the extensional loading seemed favour the spiral shear as seen in Figure D.5 and Figure D.7.

Tests involved torsional shear stress conditions

Figure D.9 to Figure D.15 illustrate the modes of failure developed in tests that subjected to torsional shear stress conditions ($0^\circ < \alpha < 90^\circ$). The three Modes 1, 3 and 4 were all recognised in these tests, in which Mode 4 was a combination of Mode 1 and Mode 2. Tests that subjected to $\alpha = 30^\circ$ or $\alpha = 60^\circ$ showed multiple shear plane pattern but those carried out at $\alpha = 45 - 50^\circ$ (see Figure D.11, Figure D.12 and Figure D.13) had single plane shear.

Generally the recorded dips of rupture planes were better fitted with predictions under loading conditions with $\alpha = 30^\circ$ than with $\alpha = 60^\circ$. The differences increased substantially in tests with $\alpha = 45 - 50^\circ$, where the shear planes became near horizontal.

In summary, Mode 4 failures were seen as the most frequent, followed closely by Mode 3. In particular, Mode 1 was often associated with pure torsional loading conditons where $\alpha = 45 \sim 50^\circ$. It should be noted that Mode 2 was recognised as part of Mode 4. The predicted angle of failure planes by Coulomb-type bifurcation seemed to best match for loading condition of $\alpha \leq 30^\circ$ but not with $\alpha = 40^\circ \sim 60^\circ$ when near horizontal failure planes were dominant.
### 8.6. Modes of Failure

#### Table 8.9: Patterns of shear planes in multi-axial undrained shear tests

<table>
<thead>
<tr>
<th>Stress condition</th>
<th>Test code</th>
<th>Mode</th>
<th>Shear plane pattern Type</th>
<th>Dip of plane(s) of shear</th>
<th>Dip of plane(s) of max $\sigma'_1/\sigma'_3$ ratio</th>
<th>Passing fissures?</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>AM00-00</td>
<td>4</td>
<td>Mutiple</td>
<td>54–58</td>
<td>61.3</td>
<td>No</td>
<td>Figure D.1</td>
</tr>
<tr>
<td>TE</td>
<td>AM90-10</td>
<td>4</td>
<td>Single</td>
<td>31</td>
<td>35.3</td>
<td>No</td>
<td>Figure D.2</td>
</tr>
<tr>
<td>TTC</td>
<td>AM00-03</td>
<td>3</td>
<td>Mutiple</td>
<td>52–64</td>
<td>65.2</td>
<td>No</td>
<td>Figure D.3</td>
</tr>
<tr>
<td>$\alpha = 0^\circ$</td>
<td>AM00-03b</td>
<td>3</td>
<td>Mutiple</td>
<td>54–60</td>
<td>64.4</td>
<td>No</td>
<td>Figure D.4</td>
</tr>
<tr>
<td></td>
<td>AM00-05</td>
<td>3</td>
<td>Mutiple</td>
<td>52–65</td>
<td>68.0</td>
<td>No</td>
<td>Figure D.5</td>
</tr>
<tr>
<td></td>
<td>IM00-05</td>
<td>3</td>
<td>Mutiple</td>
<td>62–64</td>
<td>67.2</td>
<td>No</td>
<td>Figure D.6</td>
</tr>
<tr>
<td>TTE</td>
<td>AM90-05</td>
<td>4</td>
<td>Single</td>
<td>32</td>
<td>34.0</td>
<td>No</td>
<td>Figure D.7</td>
</tr>
<tr>
<td>$\alpha = 90^\circ$</td>
<td>IM90-05</td>
<td>4</td>
<td>Multiple</td>
<td>17–27</td>
<td>36.9</td>
<td>Partly</td>
<td>Figure D.8</td>
</tr>
<tr>
<td>TS</td>
<td>AM30-03</td>
<td>4</td>
<td>Mutiple</td>
<td>24–30</td>
<td>31.5</td>
<td>No</td>
<td>Figure D.9</td>
</tr>
<tr>
<td>$\alpha = 30^\circ$</td>
<td>AM30-05</td>
<td>4</td>
<td>Mutiple</td>
<td>12–30</td>
<td>28.0</td>
<td>No</td>
<td>Figure D.10</td>
</tr>
<tr>
<td>$\alpha = 45 - 50^\circ$</td>
<td>AM45-00</td>
<td>1</td>
<td>Single</td>
<td>10–21</td>
<td>27.0</td>
<td>No</td>
<td>Figure D.11</td>
</tr>
<tr>
<td></td>
<td>AM50-05</td>
<td>2</td>
<td>Single</td>
<td>17</td>
<td>32.5</td>
<td>Partly</td>
<td>Figure D.12</td>
</tr>
<tr>
<td></td>
<td>AM45-10</td>
<td>2</td>
<td>Single</td>
<td>22</td>
<td>33.1</td>
<td>Partly</td>
<td>Figure D.13</td>
</tr>
<tr>
<td>$\alpha = 60^\circ$</td>
<td>AM60-05</td>
<td>3</td>
<td>Mutiple</td>
<td>20–26</td>
<td>28.3</td>
<td>No</td>
<td>Figure D.14</td>
</tr>
<tr>
<td></td>
<td>AM60-10</td>
<td>4</td>
<td>Mutiple</td>
<td>12–30</td>
<td>33.3</td>
<td>No</td>
<td>Figure D.15</td>
</tr>
</tbody>
</table>

#### Notes:

- The sketches showing patterns of discontinuity and failure surface are presented in Appendix [D](#).
- TC, TE: Triaxial compression and extension condition, respectively
- TTC, TTE: True triaxial compression and extension condition, respectively
- TS: Torsional shear condition
- Dip of plane(s) of shear: from measurement on each failed sample.
- Dip of plane(s) of max $\sigma'_1/\sigma'_3$ ratio: from Mohr-Coulomb stress circle, with $\phi'$ calculated directly from peak $q/p'$ value.
8.7 Summary

The multi-axial HCA shear tests carried out in the HCA Mark II extended the database of HCA tests on the natural London Clay sampled at the T5 site at relatively shallow depth (5.2 to 10.5 mBGL). In total 15 multi-axial shear tests (four test series) were performed. The multi-axial test series involved various combinations of the principal stress rotation angle ($\alpha = 0^\circ, 30^\circ, 45^\circ, 50^\circ, 60^\circ$ and $90^\circ$) and the intermediate principal stress ratio ($b = 0, 0.3, 0.5$ and $1$) during the shearing paths to failure under undrained conditions. Tests in one series were conducted from isotropic consolidation states (CIU, known as IM series), while all the other tests were started from anisotropic stress conditions (CAU, known as AM series). The main findings are summarised as follows:

8.7.1 Effective stress paths and shear strength parameters under undrained conditions

- Under undrained conditions, the clay's pre-failure response appeared to show contractive behaviour ($\Delta u > 0$) under compressional loading ($\Delta \sigma_z - \Delta \sigma_\theta = (\Delta \sigma_u - \Delta \sigma_h) > 0$) and dilative behaviour under extensional loading. The slopes $\left(\Delta \sigma_z - \Delta \sigma_\theta\right)/2\Delta p'$ measured over the initial parts of the effective stress paths had similar inclinations, ranging between $-1.2.9$ to $-1.3.4$. These observed values were within the theoretical slopes of $-1:3$ predicted for a linearly isotropic elastic material and those of $-1:2.6$ to $-1:4.2$ predicted for a linearly cross-anisotropic elastic material that had the (drained) deformation characteristics identified at very small strains (see Chapter 7). The pre-failure pore water pressure development seen in undrained tests contributed to the anisotropy seen in the clay's undrained shear strength ($\text{Nishimura, 2006; Nishimura et al., 2007}$).

- In the present study, the range of the pre-shear values of $p'$ was narrow, making it impossible to define the potential curvature of the peak $q - p'$ envelopes for London Clay (see, e.g. $\text{Bishop et al., 1965; Burland, 1990; Hight and Jardine, 1993; Hight et al., 2003}$). The author therefore defined peak shear strength in terms of $q/p'$ and $S_u$, which bracketed the available possibilities when assessing the anisotropic shear strength trends - one for a purely frictional material with constant $\phi^0$ and the other for a purely cohesive material where strength is independent of $p'$ ($\text{Nishimura, 2006}$).

- The clay shear strength characteristics were markedly anisotropic as reported in §8.5. The dependence on $\alpha$ was seen most clearly under conditions of $b = 0.5$ and $0.3$ where near plane strain conditions were presented ($\varepsilon_2 \approx 0$). Less significant variations in shear strengths with $\alpha$ were observed in the more limited testing conditions available under other $b$ conditions, particularly with $b = 1$. Regarding the effects of $b$, when subjected to the same loading direction (i.e. equal $\alpha$ value) the strengths in plane strain condition ($b = 0.3 \sim 0.5$) were generally higher than in other conditions.

- The maximum shear strength was found to develop under plane strain compression, with much lower strengths found in triaxial extension or maximum torsional shear ($\alpha = 45^\circ$) conditions. The higher $q/p'$ ratios developed for compressional loading modes were compatible with the general failure envelope curvature, shown by $\text{Nishimura et al., 2007}$ for the London Clay at T5 and reflecting variations with depth of OCR and possibly of lithology.

- There was a trend of shear strength reduction as $\alpha = 0^\circ \rightarrow 45^\circ$, which corresponded reasonably well with the trends reported by $\text{Nishimura, 2006}$. However, for tests with extensional
loading ($\alpha > 45^\circ$) the clay’s macrofabric seemed to become a controlling factor and a reducing strength pattern was noticed in the Author’s tests. This was different from Nishimura’s findings under $b = 0.5$ condition. The effects of anisotropy led to maximum and minimum $q/p'$ values that could differ significantly (as much as 40 %) at $b = 0.5$.

- The experimental curve-fitting expression for strength anisotropy of London Clay proposed by Bishop (1966) could only be applied reasonably for compressional loading ($\alpha < 45^\circ$). Due to the limitation in neglecting $b$ effects, it was not able to represent the clay’s undrained shear response outside this region in the generalised stress space.

- The relative value of $\sigma_2$, as represented by the $b$-ratio, also had a strong influence on the clay’s shear strength, with the peak shear strength stress states clearly deviating from the Mohr-Coulomb failure line in the deviatoric plane. The influence of $b$ on shear strength was not always well-defined. For the same loading direction, the strength in plane strain condition was generally higher than that under other modes. The Matsuoka-Nakai and Lade-Duncan failure criteria, which account for the effect of $\sigma_2$ explicitly, showed good agreement with the observed data in some conditions, but neither provided a secure upper bound or lower bound when fitted to the shear strength at $b = 0$ or 1. In the absence of further data, an anisotropic Mohr-Coulomb envelope calibrated to shear strength at $b = 0$ or 1 may be considered as a conservative assumption, although not the best fit.

- The generalised pattern of shear strength variation due to the combined effects of anisotropy and intermediate principal stress ratio observed by the author on samples from 5.2 mBGL were largely in good agreement with those reported by Nishimura (2006) for samples from 10.5 mBGL. Nevertheless, there were deviations for the tests under $[\alpha, b] = [90^\circ, 0.5]$ condition. Under extensional loading, two bounds of strength were noticed with the upper limit corresponding to the samples from deeper depths.

8.7.2 Deformation characteristics under undrained conditions

- In general the stress-strain response was highly non-linear as expected. The observed secant stiffness moduli at small strains of less than 0.005 % were subjected to a relatively wide scatter. Such scatter, which was more significant than that observed in the uniaxial drained test series, could be the result of the combined effects of faster shear rates and shorter periods of creep/pause employed in the multi-axial undrained shear stages.

- The limits to the stiffness plateau expected within the elastic limit $Y_1$ surface were less well defined in comparison with the drained uniaxial tests, although marked stiffness degradation could be seen at shear strain greater than 0.001 %. The dependency of octahedral shear stiffness $G_{\text{oct}}$ on $\alpha$ and $b$ within the elastic region was difficult to register. Both $G_{\text{vh}}$ and $G_{\text{oct}}$ showed similar rates of degradation, with $G_{\varepsilon_d}/G_{\varepsilon_d=0.001}$ ratio reduced to $0.27\text{–}0.42$ at $\varepsilon_d = 0.01$ % and further down to $0.19\text{–}0.35$ at $\varepsilon_d = 0.1$ %.

- The normalised undrained shear stiffness modulus degradation curves ($E_u/p'_0 \sim \log \varepsilon_d$) fitted reasonably well within the bounds set up for London Clay (Hight and Jardine 1993; Hight et al. 2003) from triaxial tests with local strain measurements. Data from tests performed under near plane strain conditions ($b = 0.3$ and 0.5) were located near the upper bound limit, with a ratio between 800–1200 at $\varepsilon_d = 0.001$ %. In comparison, the data from triaxial tests on London Clay specimens (Gasparre 2005; Nishimura 2006) showed lower values.
at small strains, and the apparent anomaly is attributed to the slower rates of stress change along the approach path to the in situ stresses and the extended drained pause periods prior to undrained shearing (Hight et al., 2007).

8.7.3 Modes of failure

- The observations made in this study confirmed the close link between the modes of failure and the applied stress conditions. The shear planes often developed in the specimens’ intact parts. The frequently observed failure modes were described in \(8.6\).

- In shear tests under \(\alpha \leq 30^\circ\) conditions, the dips of shear planes were consistent (within \(5^\circ\)) with those predicted using Coulomb failure criterion. The agreement was not achieved in tests with \(\alpha \geq 60^\circ\). Tests with \(\alpha = 45 - 50^\circ\) showed an eventual near-horizontal shear plane, which departed noticeably from the Coulomb failure criterion’s predictions.
Chapter 9

Pre-failure yielding characteristics of natural London Clay

It is well recognised that the quasi-elastic behaviour of natural London Clay, as presented in Chapter 7, is restricted to a very small strain range (of the order of 0.001%). With further straining the stress-strain response becomes quickly non-linear with the accumulation of plastic strains. In the traditional treatment of elasto-plastic constitutive models, a single transition phase from elastic to plastic behaviour is identified as a yielding limit, for example as in the original Cam Clay model (Schofield and Wroth, 1968). This is a simplification of the stress-strain response of soils where yielding generally occurs in a progressive manner, with possibly different distinct phases that might be defined by clear changes in response. Recognising the importance of progressive yielding for the prediction of the ground deformations, constitutive soil models that have multiple sub-yielding surfaces have become more common. However, there is an urgent need for experimental measurements that link observations of the detailed pre-failure yielding characteristics to the assumptions made by modellers.

This Chapter focuses on the identification of possible kinematic yield surfaces for London Clay samples from 5.2 m BGL. Results are presented from both the uniaxial and multi-axial stress path HCA tests. The yield surfaces considered are those defined in the framework of multiple kinematic yield-surface proposed by Jardine (1992), which probably represents the currently accepted view. This framework is re-summarised in 9.1. The limits of the sub-yield loci are identified in 9.2 and 9.3 based on drained and undrained HCA tests, respectively. In 9.4 the sub-yield loci are reconstructed in incremental effective stress planes and are compared with the equivalent loci found from the triaxial probing tests on samples from similar depths at T5 site reported by Gasparre (2005). Finally, 9.5 summarises the main findings.

As explained earlier in Chapter 8, the test series AM$\alpha$-05 was limited by its lack of local axial and torsional shear strain measurements; the data calculated up to $\gamma_d = 0.03$ % were not considered reliable. Noting that the $Y_2$ locus of London Clay (see 9.1 for definition) has been identified at strains of this magnitude (Jardine, 1992; Hight and Higgins, 1995; Gasparre, 2005), the results from the AM$\alpha$-05 series are not included in the discussions presented in this Chapter.

Note: Reviewing the extensive family of kinematic surfaces constitutive models is out of the scope of this section. For reference, see Potts and Zdravkovic (1999), Muir Wood (2004), Grammatikopoulou (2004) and Yamamuro and Kaliakin (2005).
9.1 Introduction

9.1.1 Multiple yield surfaces framework

In classical elastic-plastic theory (with strain rate or creep independency and no thermal effects), the stress-strain behaviour of soils is described by a large-strain yield locus; within it the soil response is assumed to be linear elastic and the plastic strains occur only when the current stress point engages the locus. This fixed yield locus is often defined at relatively large shear strains, associated with a point of large curvature on the stress-strain curve. Such theory, though having advantages in simplifying the numerical calculation and the selection of input parameters, is incapable of simulating the non-linear and inelastic pre-failure responses of real soils.

This shortcoming can be improved by using models that incorporate multiple kinematic yield surfaces (after Mroz [1967]). It is noteworthy that the term yielding used here merely indicates a recognisable and distinct change in the material’s mechanical behaviour that leads to an increasingly ‘plastic’ response, and not the classical boundary between perfectly elastic and plastic responses. In this study the framework postulated by Jardine (1992) was employed to describe the observed stiffness anisotropy and non-linearity in stress-strain response of London Clay. This experimentally-based framework has been found to be useful in describing the pre-failure yielding characteristics of soils (e.g. Smith [1992]; Zdravkovic [1996]; Kuwano [1999]; Lings [2000]; Chaudhary, 2001; Yimsiri [2002]; Rolo [2003]; Gasparre [2005]), as well as in indicating the aims that need to be addressed in constitutive modelling (Hight and Higgins [1995]; Leroueil and Hight [2003]).

Jardine’s model has three distinct sub-yield loci, each representing a yielding boundary for a specific feature of the soil’s mechanical behaviour:

- Around the current stress point and within the classical large-scale yield locus \(Y_3\) there are at least two mobile (kinematic) sub-yield surfaces \(Y_1\) and \(Y_2\) as shown in Figure 9.1. The yield loci are dependent on the initial effective stress level; in uncemented soils their sizes are expected to grow as the initial effective stress increases.

- It is assumed that inside the \(Y_1\) locus (Zone I) the soil behaviour is linear and quasi-elastic. Within this limit little energy is dissipated in static strain loops, although dynamic tests indicate finite (very low) damping ratios (Porovic [1995]). The elastic zone is very small in its size, and behaviour within it may be anisotropic.

- After engaging \(Y_1\) and before reaching \(Y_2\) (Zone II) the stress-strain response is hysteretic and non-linear. However, the total strain increments developed with fixed stress path directions keep their directions approximately unchanged. The intersection between the outgoing stress path and the \(Y_2\) locus is also thought to correspond to the strain at which shear-induced pore pressures start to build up under undrained conditions. It has been suggested that \(Y_2\) surface corresponds to the limit beyond which particle contacts fail and relative particle movements occur (Jardine, 1995).

- After engaging \(Y_2\) (Zone III) more significant plastic deformations occur, with the stress-strain relationship becoming highly non-linear and the behaviour markedly rate-dependent. In addition, the strain increment directions may rotate and deviate from their initial elastic pattern. As the outgoing stress path approaches \(Y_3\), while dragging the kinematic \(Y_1\) and

\[\text{This is not a rigorous definition because once the small scale elastic yield limit is passed, the sample has yielded and plastic strains occur.}\]
Y₂ loci behind it, the ratio of plastic to total strain progressively increases and the stiffness reduction rate increases.

- The outer (conventional) yield surface Y₃ is associated with a change in fabric and in the normalised stress space it corresponds to the local boundary surface (LBS), which cannot be crossed by undrained stress paths. Soils experiencing stress paths that reach this surface undergo very large plastic strains and beyond this locus plastic straining dominates the soil behaviour. The LBS exists within the more extensive state boundary surface (SBS) which provides the outermost limit between admissible and non-admissible normalised effective stress states (see Jardine et al., 2004).

Paths that continue after engaging the Y₁ surface may show elastic behaviour if there is a sharp change in the stress path direction, or there are straining halts that allow the soil to benefit from ageing. If a very long period of ageing is allowed, then Y₁ and Y₂ could possibly re-centre around the current stress state. Tatsuoka et al. (1997) and Clayton and Heymann (2001) suggested that with decreasing strain rate, Zones I and II may grow in size and may become less dependent on previous stress history. Outside Zone I the non-linearity in stress-strain response depends on several factors, notably the previous stress and strain history, the stress path and strain rate, as well as the soil type.

Stiffness anisotropy has also been gradually recognized as an important feature when describing the stress-strain response of many types of soil. Within Zone I, linear anisotropic elasticity has been found as an appropriate soil model in experimental studies on horizontally bedded samples (e.g. Zdravkovic and Jardine, 1997; Pennington et al., 1997; Kuwano, 1999; Lings, 2000; Gasparre, 2005). Cross-anisotropic elasticity has been applied in Zone II, but with less direct meaning because the stress-strain is non-linear. The parameters used in such non-linear elastic constitutive models are not elastic moduli but are the tangential slopes of the appropriate stress-strain relationships.

For stiff clays, it has also been found that employing both non-linearity and anistropic stiffness elasticity in numerical analyses helped to improve the predicted displacement fields near excavations or tunnels (e.g. Gunn, 1993; Simpson et al., 1996; Hird and Pierpoint, 1997; Addenbrooke et al., 1997; Franzius, 2003).

³See Leroueil and Hight (2003) for an extensive review.
9.1.2 Experimental methods to identify sub-yield loci

The relatively small strains and stress changes associated with $Y_1$, together with its mobility and the rate-dependent properties of $Y_2$ create many technical challenges to their determination in laboratory experiments. Accurate stress control and strain measuring systems are therefore essential. In addition, although plastic straining is relatively small until the $Y_2$ locus is engaged, creep and rate-dependency phenomena start to play a non-negligible role as this surface is approached. Potential delays in pore water pressure equalisation experienced in low permeability soils inevitably affect the behaviour at small strains, so the effects of loading rate need to be considered for both drained and undrained tests.

The elastic $Y_1$ locus

Within the $Y_1$ region the stress and strain increments are linearly related, and therefore the first deviation from linearity has been employed to identify the $Y_1$ limits. Experimental data (Jardine, 1992, 1995; Kuwano, 1999; Hird and Pierpoint, 1997; Cuccovillo and Coop, 1997; Leroueil and Hight, 2003; Rolo, 2003; Gasparre, 2005) indicate that the strain limits at which the $Y_1$ locus is engaged are very small, around 0.001 % for many reconsituted and natural soils, although higher values have been reported for well cemented materials.

The $Y_2$ locus

The $Y_2$ surface that bounds Zone II is identified within the non-linear and inelastic range of stress-strain behaviour. Using triaxial cells with very high resolution strain measurement systems, Kuwano (1999) and Gasparre (2005) showed that identification of the $Y_2$ yielding proposed earlier based on fully closed hysteretic loops (see Jardine et al., 1985; Smith et al., 1992; Jardine, 1995) may not always be applicable. Possibly this could be due to the strain measurement limitations of the earlier investigations. Smith (1992) and Kuwano (1999) recognised that both clays and sands undergoing fixed stress paths often show a sharp change in the direction of the strain increment vector ($\Delta \epsilon_\varphi / \Delta \epsilon_\sigma$) at relatively small strains and these points were associated with $Y_2$ yielding. Such features can be difficult to detect in undrained tests although attempts have been made by searching for changes in the directions of the effective stress paths. For example, Zdravkovic (1998) defined $Y_2$ yielding at points of sharp change in the slope of the $\Delta p'/\Delta \epsilon_\varphi$ curves found in her undrained HCA and triaxial tests, whereas Kuwano (1999) preferred to interpret shifts in the $\Delta u/\Delta \sigma'_z$ relationship or effective stress path direction $\Delta q/\Delta p'$ as the $Y_2$ limits in her undrained triaxial tests on sands.

The large scale $Y_3$ surface

The onset of $Y_3$ yielding can be defined as corresponding to sharp changes in the overall stress-strain response or in the undrained effective stress path direction. However, it has been found that such indicators were not always obvious in the triaxial tests on Ham River sand and Dunkerque sand reported by Kuwano (1999), particularly in paths involving $dq/dp'$ lower than 2.2. To improve the identification procedure, Kuwano and Jardine (2002) recommended locating $Y_3$ yield by applying a criterion involving a limiting incremental ratio of plastic strain to total strain.
9.1. INTRODUCTION

For lightly overconsolidated soils and cemented soils and weak rocks [Hight and Higgins (1995)] related the $Y_3$ limits to the points at peak seen in shear tests to failure. They reported a reference shear strain $\varepsilon_{13} = \varepsilon_1 - \varepsilon_3$ of 0.7% in triaxial compression for natural stiff plastic clays. For stiff brittle clays, clear ruptures indicating strain localisations often precede the peak conditions [Nishimura, 2006] and it can be argued that the $Y_3$ yield should be assigned to the point where a shear band first appeared.

**The application of energy-based criteria to identify yield limits**

Considering alternative ways of identifying yield points, some researchers have advocated the application of energy-based criteria, either as the total strain energy $U$ [Tavenas et al., 1979; Graham et al., 1983; Yasufuku et al., 1991] or its incremental value $\Delta U$ [Burland and Georgiannou, 1991; Hird and Pierpoint, 1997]. The postulation was that the total or incremental work done by stress increments to reach the yield locus along any stress path was constant. These two parameters are defined in Equation 9.1:

$$U = \int \{\sigma'_i\}^T \cdot d\{\varepsilon_{kl}\} \quad \text{Total strain energy}$$

$$\Delta U = \int \{\Delta \sigma'_{ij}\}^T \cdot d\{\varepsilon_{kl}\} \quad \text{Incremental strain energy}$$

(9.1)

where $\{\sigma'_{ij}\} = \{\sigma'_x, \sigma'_y, \sigma'_\theta, \gamma_{z\theta}\}$ and $\{\varepsilon_{ij}\} = \{\varepsilon_z, \varepsilon_r, \varepsilon_\theta, \gamma_{z\theta}\}$, which are the components of effective stresses and corresponding strains in a HCA test.

Figure 9.2 shows the schematic diagram for the calculation of $\Delta U$. It appears (see Figure 9.2b) that for the two special cases of stress-strain behaviour, in which the response is either linear elastic or perfectly plastic, the incremental strain energy is:

$$\Delta U = \frac{1}{2} [D_{ijkl}] \{\varepsilon_{kl}\} \cdot \{\varepsilon_{kl}\} \quad \text{for linear elastic material}$$

$$\Delta U = B \cdot \{\varepsilon_{kl}\} \quad \text{for perfectly plastic material}$$

(9.2)

Later in this chapter it will be shown that test data suggest an approximate power law relationship $\Delta U \sim \varepsilon_{kl}$ within Zone II with the form:

$$\log(\Delta U) = C + n \log(\varepsilon_{kl})$$

(9.3)
which indicates that the exponents should satisfy $1 < n \leq 2$, assuming that the stress-strain response shows a trend for tangent stiffness to fall with strain.

The definition of yielding through strain energy criteria has been considered attractive because it represents the soil’s stress-strain response in a convenient way and is applicable to general stress conditions. Traditionally the total strain energy $U$ has been used to determine the large ($Y_3$) yield, and Smith (1992) demonstrated that certain contours correlated with the $Y_2$ surfaces of the soft Bothkennar clay at in-situ stresses. However, because the sub-yield surfaces are kinematic it may be more appropriate to use the incremental strain energy ($\Delta U$) for $Y_1$ and $Y_2$.

Kuwano (1999) noted that both the values of $U$ and $\Delta U$ required to reach yielding points ($Y_1$ to $Y_3$) were dependent on the effective stress path direction for her triaxial tests on sands. This implied that the energy contours were not always representative for the yield surfaces. In the following sections, the applicability of the incremental strain energy to define $Y_1$ and $Y_2$ limits will be examined.

9.1.3 Descriptions of testing schemes and employed methods of interpretation

The pre-failure yielding characteristics of London Clay samples from 5.2 mBGL at T5 were investigated by both drained and undrained stress paths that started at or near to the estimated in situ mean effective stress (Table 9.1). The outgoing stress paths involved changes not only in $t$ and $p'$ but also in $\alpha$ and $b$ from different initial effective stress origins. The changes were therefore related to a 4-D stress space and several methods of presentation can be used. Figure 9.3 shows the stress space used in this study. The 3-D stress space $(\sigma_0^z - \sigma_0^\theta)/2 \sim \tau_{z\theta} \sim (\sigma_0^2 - p')$, as suggested by Muir Wood (2004), was selected because it can conveniently express the changes in loading direction and $b$ in terms of stress components (see Figure 9.3a). In addition, this stress space helps to illustrate the isotropic stress states clearly. Changes in $p'$ can be visualized by plotting results in Figure 9.3b.

It is also important to recall from Chapter 6 that the final effective anisotropic consolidation points were not the same in all tests, which caused differences (though small) in the locations of the points B, C and D in stress space. To generalise the results, the shapes of $Y_1$ and $Y_2$ loci were constructed in the incremental effective stress space (see 9.5).

In this study the $Y_1$ and $Y_2$ yielding were defined as follows:

$Y_1$ yielding The end of the linearity response in a stress-strain relationship

$Y_2$ yielding A change in the slope of two locally measured strains (e.g. $\Delta \epsilon_\theta \sim \Delta \epsilon_z$). The relationships between $\Delta u$ and shear stress increment ($\Delta t$) from undrained tests were also used.

Each of these limits was determined from a linear regression with regression coefficient $R^2 > 0.95$ within the range of strains involved. Once the stress and strain limits of $Y_1$ and $Y_2$ were identified by the above methods, all other stress and strain components at this particular state could be easily defined, as could the stress and strain invariants. The values of the incremental strain energy at these yielding points were then reported to assess the applicability of the energy-based criterion for identifying sub-yield loci. It is noted here that due to the reason of not having sufficiently detailed observation on the states in which shear bands first appeared, the Author did not investigate the large yield $Y_3$ in the present study.
9.1. INTRODUCTION

Legend

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Effective stress point after the end-of-stage of</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>A Isotropic re-consolidation</td>
</tr>
<tr>
<td>•</td>
<td>B Anisotropic re-consolidation</td>
</tr>
<tr>
<td>□</td>
<td>C Drained b-change</td>
</tr>
<tr>
<td>◻</td>
<td>D Undrained unloading</td>
</tr>
</tbody>
</table>

Figure 9.3: Schematic presentation of the initial effective stress origins and outgoing stress paths performed in this study

Table 9.1: Scheme of drained and undrained tests used in the identification of small strain yield loci

<table>
<thead>
<tr>
<th>Initial effective stress origin</th>
<th>Outgoing stress paths</th>
<th>Drainage condition</th>
<th>$\alpha_{d\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ($b = 0$, $p' = 280$ kPa)</td>
<td>Anisotropic re-consolidation</td>
<td>Drained</td>
<td>90°</td>
</tr>
<tr>
<td></td>
<td>IM90-DQ (uniaxial with $\Delta \sigma'_0$)</td>
<td>Drained</td>
<td>90°</td>
</tr>
<tr>
<td></td>
<td>IM90-DZ (uniaxial with $\Delta \sigma'_z$)</td>
<td>Drained</td>
<td>0°</td>
</tr>
<tr>
<td>B ($b = 1$, $p' = 280$ kPa)</td>
<td>Drained b-change</td>
<td>Drained</td>
<td>90°</td>
</tr>
<tr>
<td></td>
<td>HC-DQ (uniaxial with $\Delta \sigma'_0$)</td>
<td>Drained</td>
<td>90°</td>
</tr>
<tr>
<td></td>
<td>HC-DZ (uniaxial with $\Delta \sigma'_z$)</td>
<td>Drained</td>
<td>0°</td>
</tr>
<tr>
<td></td>
<td>HC-DT (uniaxial with $\Delta \tau_{z\theta}$)</td>
<td>Drained</td>
<td>45°</td>
</tr>
<tr>
<td></td>
<td>Unloading to isotropic axis</td>
<td>Undrained</td>
<td>0°</td>
</tr>
<tr>
<td>C ($b = 0.3$, $p' = 280$ kPa)</td>
<td>Unloading to isotropic axis</td>
<td>Undrained</td>
<td>0°</td>
</tr>
<tr>
<td>D ($b = 0$, $p' = 248 \sim 255$ kPa)</td>
<td>Multi-axial stress paths</td>
<td>Undrained</td>
<td>0° $\sim$ 90°</td>
</tr>
</tbody>
</table>

Notes:

$\alpha_{d\sigma}$ : the angle of major principal stress increment rotation between $d\sigma_1$ direction and the vertical axis.

254
9.2 Sub-yield loci observed in drained tests

This section discusses the sub-yield \(Y_1\) and \(Y_2\) loci as determined from anisotropic re-consolidation, uniaxial stress probing and \(b\)-change stages, all of which were conducted under drained conditions. The initial effective stress origins from which these stress paths originated are points A and B illustrated in Figure 9.3.

The author’s interpretations of the sub-yield loci followed the methods established in 9.1.3. In particular, the limits of the elastic surface \(Y_1\) were those identified from the small stress probing uniaxial tests that had been discussed in 7.2. These carefully monitored static probes usually went beyond the linear elastic range (see e.g. Figure 7.3) and therefore the limits could be well defined by reference to the linear regression analyses of the stress-strain data up to strain of 0.001 %.

To avoid repeating the discussion of the elastic limits, reference is made to Figures 7.7 to 7.11 and Figures 7.12 to 7.15 which indicate the elastic limits for tests originating at the isotropic stress point A and the anisotropic stress states point B. These elastic limits were used for the analysis of the incremental strain energy \(\Delta U\) and the shear stress increment \(\Delta t\) associated with \(Y_1\), and these values were reported hereafter.

In general, the values of \(\epsilon_d\) and \(\Delta t\) corresponding to the elastic limits were found to change only within a limited range, of about 0.001 – 0.002 % and 1.5–2 kPa, respectively. The incremental strain energy found at the \(Y_1\) limits was between \(1\times10^{-5}\) kJ/m\(^3\), and the average \(\Delta U\) was therefore slightly larger than the value of \(1\times10^{-5}\) kJ/m\(^3\) reported in static probing tests on London Clay specimens from a similar depth (12.5 mOD) using triaxial cells (Gasparre, 2005). Detailed comparison between these two studies is given later in 9.4.

The \(Y_2\) limits were defined at points of changes in the slopes of two strain (locally calculated) components (in linear scale). Plots using logarithmic scales were employed to highlight the behaviour in the small strain region, particularly in the assessment of the incremental strain energy. The following discussions concentrate on the \(Y_2\) limits found from the drained stress paths.

9.2.1 Tests from isotropic initial stress states

The outgoing drained stress paths conducted from point A (isotropic conditions with \(p' = 280\) kPa) are those involving the start of the anisotropic re-consolidation stages and two monotonic uniaxial shear probes (see Figure 9.3). Figure 9.4 shows the relationships between the axial and circumferential strains in the early parts of the anisotropic re-consolidation stress paths of the AM45-00 and HC-DT tests. These changes were tentatively identified as \(Y_2\) yielding, and appeared to be at similar points in both tests. In terms of the maximum shear strain, the \(Y_2\) yielding was at \(\epsilon_{13} = 0.03\) %. Note that the approximate limit of \(Y_1\), determined from the uniaxial small stress probing tests, was also plotted in this figure at \(\epsilon_{13} = 0.001\) %.

Figure 9.5 and Figure 9.6 show the magnitudes of the incremental shear stresses (\(\Delta t\)) and the incremental strain energy (\(\Delta U\)) at \(Y_1\) and \(Y_2\) yield points. The data indicated that similar values of \(\Delta t\) and \(\Delta U\) were associated with the \(Y_2\) limits for the re-consolidation stress paths on different specimens that had been subjected to a similar stress history. On average, values of \(\Delta t \simeq 11\) kPa and \(\Delta U \simeq 0.001\) kJ/m\(^3\) could be quoted for the \(Y_2\) yield point in these tests.

*The results from other tests were similar and therefore were omitted to ensure clarity in the presentation.
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Within these two limits, the $\Delta U \sim \epsilon_d$ relationships appeared to follow the same power law with an average exponent $n$ of 1.41. This value is expected because plastic strains start developing after the stress path engages $Y_1$ surface, causing a reduction (from 2) in the value of the exponent.

The flexibility of the HCA in applying stress paths that differ from triaxial stress conditions provided an opportunity to investigate the sub-yield loci in the 4-D stress space. This advantage has been illustrated earlier in the identification of the $Y_1$ locus from small-stress uniaxial probing tests. To search for $Y_2$ limits, the data from the two CID uniaxial shear tests (IM90-DQ and IM90-DZ) that originated from point A are shown in Figure 9.7. The departures from linear relationships between volumetric and deviatoric strains (both measured locally) were chosen here as giving the best indicators of the $Y_2$ limits.

From this figure, it can be seen that the strain limits were noticeably different, one at $\epsilon_d = 0.013\%$ (IM90-DQ) and the other at 0.022\% (IM90-DZ). This reflected the strong anisotropic deformation properties of the clay, with a markedly stiffer response being noted as a result of loading in the horizontal direction than the vertical one. By cross-referencing the values of the individual stress and strain components (Figure 9.8), it was found that the strain limits were again not equal ($\epsilon_\theta = 0.01\%$ and $\epsilon_z = 0.025\%$, respectively). However, at these limits the corresponding incremental individual effective stresses were roughly similar ($\Delta \sigma_\theta' = 16$ kPa and $\Delta \sigma_z' = 14$ kPa). As a result, average $\Delta t \approx$ 8 kPa at $Y_2$ limits was identified from uniaxial tests, which was only marginally smaller than those observed in the anisotropic re-consolidation results.

Figure 9.9 illustrates the $\Delta U \sim \epsilon_d$ relationships of the two CID uniaxial tests. Values of $\Delta U$ in the order of 0.001–0.002 kJ/m$^3$ were noted for $Y_2$ limit points. Within the two sub-yield limits, the relationships showed different exponents (1.52 and 1.27 for the IM90-DQ and IM90-DZ tests, respectively) but are not far from those noted earlier in the anisotropic re-consolidation tests.

9.2.2 Tests from anisotropic initial stress states

The outgoing drained stress paths from the anisotropic stress state (point B in Figure 9.3) involved sets of CAD uniaxial HCA tests and $b$-change stages.

Figure 9.10 shows the sub-yield limits identified using results of the three CAD uniaxial static tests HC-DT, HC-DQ and HC-DZ. The strain limits in terms of $\epsilon_d$ at $Y_2$ points were 0.025, 0.02 and 0.025 %, respectively. As shown in Figure 9.11, these points were associated with individual stress increments of 10, 16 and 14 kPa for $\Delta \tau_{\theta \theta}$, $\Delta \sigma_\theta'$ and $\Delta \sigma_z'$ (or $\Delta t = 10, 8$ and 7 kPa, respectively for HC-DT, HC-DQ and HC-DZ). The smaller values of stress increment for $Y_2$ in the uniaxial HC-DT test reflects the use of shear stress rather than axial stress in the pure torsional shear plot. Again $Y_2$ yielding corresponded in all cases to an increase in the rate of volume change development with contractant response.

The values of $\Delta U$ for the two sub-yield loci were calculated and shown in Figure 9.12 for the CAD uniaxial HCA test series. These ranged between $5–6 \times 10^{-5}$ kJ/m$^3$ for the $Y_1$ limits and 0.002–0.004 kJ/m$^3$ for the $Y_2$ limits. These limits were not as closely related as they apparently appeared to be in the CID uniaxial HCA test series or in the anisotropic re-consolidation stress paths that originated from the isotropic stress point A. The relevance of using an unique $\Delta U$ criterion to identify yielding states is further investigated with data from the undrained stress path tests (see §9.3). The relationships showed an average exponent of 1.21, indicating that a more non-linear response occurred in the CAD uniaxial tests (which commenced closer to the outer $Y_3$ yield...
boundary) than in the CID uniaxial tests.

Other points on the $Y_2$ locus surrounding the effective stress point B can be located by studying the results of the drained $b$-change tests. During the $b$-change tests, the maximum shear stress was developed in the horizontal $r \sim \theta$ plane. Figure 9.13 shows the limits for $Y_2$ as identified from relatively gentle changes in the slopes of the $\epsilon_r \sim \epsilon_\theta$ relationships of the AM00-03 and AM30-03 tests. In both cases, these limits were equivalent to a deviatoric shear strain $\varepsilon_d$ of about 0.015% (see Figure 9.14).

The (average) values of $\Delta U$ and $\Delta b$ at $Y_2$ yielding were $6 \times 10^{-4}$ kJ/m$^3$ and 0.05, respectively (see Figure 9.15 and Figure 9.16). The exponent of the linear power law relationship between the two small strain sub-yield limits was 1.45, confirming the presence of plastic strains after the stress path crossed $Y_1$ surface. The corresponding value of shear stress increment (in the horizontal plane) was then $(\Delta \sigma_r - \Delta \sigma_\theta)/2 = 80/3 \times 0.05 \approx 13$ kPa.

Table 9.2 summarizes the values of the $Y_1$ and $Y_2$ limits in terms of $\varepsilon_d$, $\Delta U$ and shear stress $\Delta t$ using data of the drained stress paths. The following points of interest can be made:

- The observed yielding response was progressive, with distinct changes in behaviour being noted in terms of stress-strain linearity (for $Y_1$) and strain increment directions ($Y_2$ yielding) in all tests.
- The $Y_1$ and $Y_2$ loci are kinematic; they moved with the current stress state from point A to point B as a result of anisotropic re-consolidation.
- Values of $\Delta U$ had been identified between $1-7 \times 10^{-5}$ kJ/m$^3$ for the $Y_1$ and 0.001-0.006 kJ/m$^3$ for the $Y_2$ boundaries. Between these two loci, the relationship $\Delta U \varepsilon_d$ seemed to follow a linear power law, with the exponents ranging between 1.21 and 1.52. The smaller this value was, the more important plastic strainings is thought to be within Zone II.
- Values of $\Delta U$ corresponding to the $Y_1$ and $Y_2$ limits fell within the same order of magnitude but significant variations existed. This implied that an incremental strain energy criterion may not be suitable for independent use without any other method of identification.

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$^5$Again it is noted that data from the AM$\alpha$-05 series was not appropriate to use in the identification of pre-failure yielding characteristics.
## 9.2. Sub-Yield Loci Observed in Drained Tests

### Table 9.2: Identification of small strain yield loci $Y_1$ and $Y_2$ from drained tests

<table>
<thead>
<tr>
<th>Test code</th>
<th>Initial stress state</th>
<th>Strain $\epsilon_d$ [%]</th>
<th>$\Delta U$ [kJ/m$^3$]</th>
<th>$\Delta t$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM90-DQ</td>
<td>Isotropic</td>
<td>0.001 0.013</td>
<td>$3 \times 10^{-5}$</td>
<td>0.002</td>
</tr>
<tr>
<td>IM90-DZ (Point A)</td>
<td></td>
<td>0.001 0.025</td>
<td>$3 \times 10^{-5}$</td>
<td>0.001</td>
</tr>
<tr>
<td>Anisotropic re-consolidation</td>
<td></td>
<td>0.001 0.035</td>
<td>$1 \times 10^{-5}$</td>
<td>0.001</td>
</tr>
<tr>
<td>Small stress probes (avg.)</td>
<td></td>
<td>0.001 n.a.</td>
<td>$3 \times 10^{-5}$</td>
<td>n.a.</td>
</tr>
<tr>
<td>HC-DQ</td>
<td>Anisotropic (Point B)</td>
<td>0.001 0.021</td>
<td>$7 \times 10^{-5}$</td>
<td>0.002</td>
</tr>
<tr>
<td>HC-DT</td>
<td></td>
<td>0.001 0.025</td>
<td>$5 \times 10^{-5}$</td>
<td>0.006</td>
</tr>
<tr>
<td>HC-DZ</td>
<td></td>
<td>0.001 0.025</td>
<td>$6 \times 10^{-5}$</td>
<td>0.004</td>
</tr>
<tr>
<td>$b$-change</td>
<td></td>
<td>0.001 0.015</td>
<td>$1 \times 10^{-5}$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Small stress probes (avg.)</td>
<td></td>
<td>0.001 n.a.</td>
<td>$4 \times 10^{-5}$</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Notes:
- Initial effective stress points A and B as shown in Figure 9.3
- Small stress probes (avg.): Average data from the HCA uniaxial small-stress probing tests (see §7.2)
- n.a. = not applicable
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.4: Identification of $Y_2$, anisotropic reconsolidation stage

Figure 9.5: Limits of shear stress $t$ for $Y_2$, anisotropic reconsolidation stage
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.6: Incremental strain energy, anisotropic reconsolidation stage

Figure 9.7: Identification of $Y_2$ from $\varepsilon_v \sim \varepsilon_d$, CID uniaxial tests
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.8: Stress limits of $Y_2$, CID uniaxial tests

Figure 9.9: Incremental strain energy $\Delta U$, CID uniaxial tests
Figure 9.10: Identification of $Y_2$ in $\epsilon_v \sim \epsilon_d$ or $\epsilon_z \sim \gamma_{z\theta}$, CAD uniaxial tests.
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.11: Stress limits of $Y_2$, CAD uniaxial tests

Figure 9.12: Incremental strain energy $\Delta U$, CAD uniaxial tests
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.13: Identification of $Y_2$ limits, drained $b = 1 \rightarrow 0.3$ stages.

Figure 9.14: Limits of $Y_2$ in terms of deviatoric shear strain as established in Figure 9.13 from drained $b = 1 \rightarrow 0.3$ stages.
9.2. SUB-YIELD LOCI OBSERVED IN DRAINED TESTS

Figure 9.15: Incremental strain energy, drained $b = 1 \rightarrow 0.3$ stages

Figure 9.16: Corresponding $Y_2$ limits as established in Figure 9.13 from drained $b = 1 \rightarrow 0.3$ stages
9.3 Sub-yield loci observed in undrained tests

The previous section illustrated the application of the multiple yield surfaces scheme proposed by Jardine [1992] to drained stress path tests. The undrained tests considered here presented an opportunity to check the scheme under conditions in which a coupled soil-water response was expected. It was also the first time that the framework had been applied for London Clay under general stress conditions rather than being restricted to axi-symmetric triaxial states.

It is important to note here that the instrument reading and test strain rates adopted by the Author led to only a limited amount of data being obtained at the early stages of the undrained tests when shear strains were less than 0.001 %. As a consequence, these tests did not allow a proper evaluation of the undrained \( Y_1 \) limits. Based on the available information, \( Y_1 \) yielding was assumed to take place at \( \varepsilon_d \) values of around 0.001 %, as indicated in the drained uniaxial probing test series described earlier in [9.2].

Table 9.3 lists the test conditions and the corresponding reference figures, which illustrate the identifications of the \( Y_2 \) limits in each test series. For convenience, the results were again discussed with respect to the initial effective stress points (A to D) of the outgoing undrained stress paths.

<table>
<thead>
<tr>
<th>Undrained stress paths</th>
<th>Initial stress point</th>
<th>Reference figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undrained unloading</td>
<td>Points B and C (Anisotropic state)</td>
<td>Figure 9.17 to Figure 9.20</td>
</tr>
<tr>
<td>Multi-axial at ( b = 0 ) AM( \alpha )-00</td>
<td>Point D (Isotropic state)</td>
<td>Figure 9.22 to Figure 9.25</td>
</tr>
<tr>
<td>Multi-axial at ( b = 0.3 ) AM( \alpha )-03</td>
<td>Point D (Isotropic state)</td>
<td>Figure 9.26 to Figure 9.29</td>
</tr>
<tr>
<td>Multi-axial at ( b = 1 ) AM( \alpha )-10</td>
<td>Point D (Isotropic state)</td>
<td>Figure 9.30 to Figure 9.33</td>
</tr>
<tr>
<td>Multi-axial at ( b = 0.5 ) IM( \alpha )-05</td>
<td>Point A (Isotropic state)</td>
<td>Figure 9.34 to Figure 9.37</td>
</tr>
</tbody>
</table>

Two methods of identification for \( Y_2 \) limits (see again [9.1.3]) were employed in the following analyses. The first method locates the departures from linearity in the relationships between relevant pairs of individual strain components (e.g. \( \varepsilon_\theta \sim \varepsilon_z \) or \( \gamma_{z\theta} \sim \varepsilon_z \)) within the small strain range (up to 0.1 %). The second method also finds possible shifts from linearity in measurements of excess pore water pressure (\( \Delta u \)) versus shear stress increment (\( \Delta t \)).

In comparing Figure 9.17 with Figure 9.18 it appears that to identify the \( Y_2 \) limits the strain-strain approach offered more recognisable points of marked changes than using p.w.p measurements. If the limits identified by these two methods are plotted together to locate the corresponding values of \( \varepsilon_d \) (see Figure 9.19), differences (within a strain range of 0.01 ~ 0.04 %) of as much as 0.01 % were found. As will be shown later, this observation holds true for other undrained test data. The local strain measurements are considered to be more indicative than the p.w.p measurements, which were measured externally and subjected to potential time lag errors. If a local p.w.p probe was available, the resolution in p.w.p measurements would had been improved and better well-defined \( Y_2 \) limits might have been established using the second approach. In the following discussions, both approaches are shown for completeness but the reported values of \( Y_2 \) limits were established by the first approach.
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

9.3.1 Tests from anisotropic stress points B and C

Undrained unloading stress paths originated from point C for the three tests in series AM\(\alpha\)-03, and from point B for the other five tests under \(b = 0\) and 1 conditions. We recall that local strain transducers for axial and torsional shear strain measurements were not available for the AM\(\alpha\)-05 test series; the pre-failure yielding data from this series therefore is not discussed here.

Two typical unloading results were selected for discussion, one from test AM00-03 (initial stress point C) and the other from test AM45-00 (initial stress point B). Other undrained unloading test data showed similar responses, and the \(Y_2\) limits identified from them were bracketed within the mentioned values.

The differences in the identification of \(Y_2\) limits by using strain or p.w.p measurements have been discussed earlier. Note again that the first approach was a more reliable method in the present study, and these limits are illustrated in Figure 9.17 to Figure 9.19. On average, the limits for \(Y_2\) were identified at \(\Delta t \approx 10\) kPa, \(\epsilon_d = 0.014\%\), with only small changes in excess p.w.p (\(\Delta u = 2\) kPa) being developed up to these points. The incremental strain energy was 0.004 to 0.007 kJ/m\(^3\). Between \(Y_1\) and \(Y_2\), its relationships with \(\epsilon_d\) followed similar power law, having an exponent of 1.98 (see Figure 9.20).

As mentioned previously, an exponent of 2.0 indicates the response of a linear elastic material, which could be applied within Zone I in Jardine’s framework. Theoretically it also is expected that the exponent \(n\) should reduce as the stress paths engage the \(Y_1\) surface. The high value of the observed exponent could be the result of the difference between the computed local and global volumetric strains. It was recognised that globally the volumetric strain was zero (constant volume) but locally it was not the case (see §8.2), particularly if the samples approached failure conditions. Figure 9.21 illustrates that if semi-local strain measurements were used (keeping the condition of \(\epsilon_z + \epsilon_\theta + \epsilon_r = 0\), see §4.3 for details) then the average exponent was indeed less than 2. However, such method of presentation is inconsistent to the objective of identifying small strain yield loci in the present study, which requires the use of local strain measurements. The Author therefore decided to keep using local strains in the calculation of \(\Delta U\) for the other undrained tests, and noted that the reported exponents were indicative values only.

The similarities of the \(Y_2\) strain limits and the shear stress increment limits at point B and C, also taking into account the equivalent limits established from drained stress path tests, indicated that the shape and size of \(Y_2\) surface were not dependent on the pre-probing values of \(t\) and \(b\) within the range considered in this study. However, their sizes were expected to increase with \(p’\) as shown by Gasparre (2005) and Gasparre et al. (2007b).

9.3.2 Tests from isotropic stress points D and A

Eight multi-axial undrained HCA tests that involved outgoing stress paths were conducted from point D in which fixed values of \(\alpha\) and \(b\) were controlled, while two undrained tests (IM90-05 and IM00-05) started from point A without experiencing earlier re-consolidation to point B. The tests were performed with \(b = 0, 0.3, 1\) and 0.5, and were accordingly grouped under the same \(b\) condition as shown in Figure 9.22 to Figure 9.37 Again, the same (two) identification methods for \(Y_2\) limits were applied.

As seen in the undrained unloading tests, the limits of \(Y_2\) defined from strain or p.w.p measure-
ments were generally not exactly the same. The differences in term of $\varepsilon_d$, though small in absolute value (between 0.01 and 0.03 %), were significant ($\pm 20\%$ different). This observation again highlights the importance of employing local p.w.p measurement if the limits of yield loci rely only on stress components (i.e. the second approach).

On average, the limits for $Y_2$ were identified at $\Delta t = 7 \sim 10$ kPa, with small changes in excess p.w.p ($\Delta u = 0.5 \sim 1.5$ kPa) up to this point. The incremental strain energy $\Delta U = 0.001 \sim 0.006$ kJ/m$^3$, as shown in Figure 9.25, Figure 9.29, Figure 9.33 and Figure 9.37.

It is also noted that between the two small strain sub-yield loci, the relationship of $\Delta U$ appeared to follow an approximately linear power law form. The observed average exponents were 1.78, 1.54 and 2.00 for the tests conducted at point D with $b = 0, 0.3, 1.0$. For the two tests conducted at point A under $b = 0.5$ condition, the average exponent was 1.54. Note again that the incremental strain energy values were calculated from local strain measurements (see earlier discussion).

The limits of $Y_2$, as identified and shown in Table 9.4, provided information allowing its shape to be constructed in the $\Delta(\sigma_z' - \sigma_\theta')/2 \sim \Delta \tau_z\theta$ plane, adding a dimension. Note also that the shear stress increment $\Delta \tau$ is the radius of the Mohr stress circle. The shapes and sizes of the sub-yield loci $Y_1$ (nominally identified as corresponding to $\varepsilon_d = 0.001\%$ in undrained tests) and $Y_2$ from the undrained stress paths will be synthesized and discussed with the findings from drained tests ($\S$9.2) in the following section.

A summary of the limits of $Y_2$ in terms of deviatoric shear strain, shear stress increment and incremental strain energy is given in Table 9.4. The following points of interest can be made:

- $Y_2$ yielding appeared to indicate a shift in the pattern of the strain increments that developed at relatively small strains in tests that followed stress paths of constant direction.

- The $Y_2$ loci are kinematic as they moved with the current stress state from points B and C to points A and D as a result of the undrained unloading tests.

- Values of $\Delta U$ had been identified between 0.001–0.007 kJ/m$^3$ for the $Y_2$ boundaries. This implied that this parameter was as sensitive to influences from $p'$, stress history and applied stress condition as other parameters (shear stress increment or deviatoric shear strain). While these values were within the same order of magnitude but noticeable variations were found, suggesting that a criterion of identifying $Y_2$ yielding in terms of constant $\Delta U$ values may not always be appropriate.

- For stress-strain behaviour within Zone II, the $\Delta U \sim \varepsilon_d$ relationships appeared to follow an approximately constant power law function with the exponents ranging between 1.54 and 2.00. The high exponent values observed in some tests could be explained as results of the use of local strain measurements that sometimes indicated a (local) reduced volume of the sample in contrast to the (global) constant volume during undrained testing condition.
### 9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Table 9.4: Yielding limits identified from undrained tests

<table>
<thead>
<tr>
<th>Stress paths &amp; origin point</th>
<th>Test</th>
<th>$\alpha_{\sigma\tau}$ [deg]</th>
<th>$b$ [-]</th>
<th>$\epsilon_d$ [%]</th>
<th>$\Delta U$ [kJ/m$^3$]</th>
<th>$\Delta t$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloading, point C</td>
<td>AM00-03</td>
<td>0</td>
<td>0.3</td>
<td>0.013</td>
<td>0.004</td>
<td>9</td>
</tr>
<tr>
<td>Unloading, point B</td>
<td>AM45-00</td>
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<td>0.0</td>
<td>0.015</td>
<td>0.007</td>
<td>11</td>
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<td>Average values</td>
<td></td>
<td></td>
<td></td>
<td>0.014</td>
<td>0.006</td>
<td>10</td>
</tr>
<tr>
<td>Multi-axial, point D</td>
<td>AM00-00</td>
<td>0</td>
<td>0.0</td>
<td>0.015</td>
<td>0.001</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>AM45-00</td>
<td>45</td>
<td>0.0</td>
<td>0.017</td>
<td>0.006</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>AM00-03</td>
<td>0</td>
<td>0.3</td>
<td>0.020</td>
<td>0.0009</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>AM45-10</td>
<td>45</td>
<td>1.0</td>
<td>0.018</td>
<td>0.006</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>AM60-10</td>
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<td>1.0</td>
<td>0.016</td>
<td>0.004</td>
<td>9</td>
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<tr>
<td>Average values</td>
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<td></td>
<td></td>
<td>0.017</td>
<td>0.003</td>
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<tr>
<td>Multi-axial, point A</td>
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<td>0.002</td>
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<tr>
<td></td>
<td>IM90-05</td>
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<td>0.5</td>
<td>0.020</td>
<td>0.001</td>
<td>5</td>
</tr>
<tr>
<td>Average values</td>
<td></td>
<td></td>
<td></td>
<td>0.015</td>
<td>0.0015</td>
<td>7</td>
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</tbody>
</table>

Figure 9.17: Identification of $Y_2$ limits in undrained unloading stages using strain data
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.18: Identification of $Y_2$ limit in undrained unloading stages using pwp measurement

Figure 9.19: Limits of $Y_2$ in $\epsilon_d$, undrained unloading stages
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.20: Incremental strain energy during undrained unloading stages

Figure 9.21: Comparison of $\Delta U$ calculated by local and semi-local strain measurements
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.22: Identification of \( Y_2 \) limit in series AM\( \alpha \)-00 (at \( b = 0.0 \)) using strain data, multi-axial probes from point D

Figure 9.23: Identification of \( Y_2 \) limit in series AM\( \alpha \)-00 (at \( b = 0.0 \)) using pwp measurement, multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.24: Limits of $Y_2$ in $\epsilon_d$, series AM$\alpha$-00 (at $b = 0.0$), multi-axial probes from point D

Figure 9.25: Incremental strain energy in series AM$\alpha$-00 (at $b = 0.0$), multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.26: Identification of $Y_2$ limit in series AM$\alpha$-03 (at $b = 0.3$) using strain data, multi-axial probes from point D

Figure 9.27: Identification of $Y_2$ limit in series AM$\alpha$-03 (at $b = 0.3$) using pwp measurement, multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.28: Limits of $Y_2$ in $\varepsilon_d$, series AM$\alpha$-03 (at $b = 0.3$), multi-axial probes from point D

Figure 9.29: Incremental strain energy in series AM$\alpha$-03 (at $b = 0.3$), multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.30: Identification of $Y_2$ limit in series $AM\alpha - 10$ (at $b = 1.0$) using strain data, multi-axial probes from point D

Figure 9.31: Identification of $Y_2$ limit in series $AM\alpha - 10$ (at $b = 1.0$) using pwp measurement, multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.32: Limits of $Y_2$ in $e_d$, series AM$\alpha$-10 (at $b = 1.0$), multi-axial probes from point D

Figure 9.33: Incremental strain energy in series AM$\alpha$-10 (at $b = 1.0$), multi-axial probes from point D
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.34: Identification of $Y_2$ limit in series IM$\alpha$-05 (at $b = 0.5$) using strain data, multi-axial probes from point A

Figure 9.35: Identification of $Y_2$ limit in series IM$\alpha$-05 (at $b = 0.5$) using pwp measurement, multi-axial probes from point A
9.3. SUB-YIELD LOCI OBSERVED IN UNDRAINED TESTS

Figure 9.36: Limits of $Y_2$ in $\varepsilon_d$, series IM$\alpha$-05 (at $b = 0.5$), multi-axial probes from point A

Figure 9.37: Incremental strain energy in series IM$\alpha$-05 (at $b = 0.5$), multi-axial probes from point A
9.4 The kinematic sub-yield loci in effective stress space

The previous two sections discussed the identification of the sub-yield $Y_1$ and $Y_2$ loci using data from drained and undrained stress paths originating from different initial effective stress points at mean effective stress ($p' = 280$ kPa) expected in situ at the T5 site at 5 mBGL. In this section, the sizes and shapes of these two loci are constructed and discussed. Due to the limited number of tests available in the present study, such a construction exercise should be considered as having an indicative nature. These results are also compared with the findings by Gasparre (2005) through her triaxial stress path tests which included experiments on samples from similar depths to the Author’s.

To account for the small differences of $p_0$ and $t$ values applying at the end of the anisotropic re-consolidation stress states, the sub-yield loci were presented in terms of the effective stress increments with respect to the initial stress point, i.e. plotted in the $\Delta p'$ and $\Delta \tau_{z\theta}$ planes. The first plot is similar to that in triaxial stress space, while the latter provides additional information for tests involving torsional loading. As a result, the trajectories of the two kinematic sub-yield loci can be visualised and the potential effect (if any) of $\alpha$ on their shapes and sizes can be estimated.

9.4.1 Previous research on the shapes and sizes of sub-yield loci

It is known that the size of the $Y_1$ region, apart from growing in size with $p'$, is also likely to be dependent on strain rate (Tatsuoka and Shibuya, 1991) and the ageing period imposed prior to shearing (Clayton and Heymann, 2001). In terms of its shape when plotted in $p'$ vs $q$ plane, Kuwano (1999) reported that the $Y_1$ surface of Ham River sand was markedly elongated in the direction of the previous effective stress path direction, while Rolo (2003) found it was positioned eccentrically with respect to the current effective stress from tests on Bothkennar clay samples. These observations reflected the influences of the recent effective stress history. In the incremental stress plane $\Delta p' \sim \Delta q$, Gasparre (2005) showed that the $Y_1$ surface of London Clay constructed using triaxial probing test results was approximately symmetric and circular around the effective stress origin, provided that long creep pause periods were employed. It could be argued that the difference was due mainly to the differences in the imposed ageing periods.

Figure 9.38 and Figure 9.39 present the interpretations made by Gasparre (2005) for the $Y_1$ and $Y_2$ loci, using data from triaxial tests on samples from similar depths and re-consolidated to the estimated in situ anisotropic stresses. Note that she did not present the yield limits identified from her anisotropic re-consolidation stages and accordingly information of the two sub-surface developed around isotropic stress points is not available. At the anisotropic stress point, the two loci were rounded, but only the $Y_1$ surface seemed to be located symmetrically around the initial effective stress point. On the other hand, the $Y_2$ locus appeared to extend further on the compression side than on the extension side, which was explained as a result of the proximity of the initial stress state to the failure line in extension.

Gasparre (2005) also reported that if the equivalent sub-yield loci for samples from different lithological units were plotted together in the plane of stress increments and the differences in the (initial) mean current effective stresses were accounted for by normalising to $p_{0}''$, each of the sub-locus would be located eccentrically with respect to the current effective stress origin. However, due to the limited number of tests available at point B with changes in $b$ and therefore its influences on the features of $Y_2$ could not be investigated.
yield surfaces tended to coincide to a unique rounded shape. The normalised $Y_1$ and $Y_2$ surfaces were well centred on the initial effective stress state (see *Figure 9.40* and *Figure 9.41*). This observation implied that the sizes of the kinematic surfaces were linearly dependent on the level of the applied mean effective stresses. Once the initial stresses were taken into account, no apparent influence of lithology could be found. Nevertheless, it is important to note that the lithology influenced the magnitudes of the elastic stiffness parameters as illustrated in Chapter 7 and by *Gasparre et al.* (2007b).

### 9.4.2 Analysis of the Author’s data

The previous review provides an useful template to compare the studies by the Author and *Gasparre* (2005). The following discussions concentrate on the values of the observed $\Delta U$ at yielding, as well as the shapes and sizes of $Y_1$ and $Y_2$ surfaces. It is important to note that the results were presented in different plot styles, and therefore a circle in the $\Delta p' \sim \Delta q$ plane (as commonly used for triaxial stress space) becomes an ellipse in the $\Delta p' \sim \Delta (\sigma_0 v - \sigma_0 h)/2$ plane with its longer axis parallel to the $\Delta p'$ axis.

There were also some differences in terms of shearing rate and ageing period between these two studies. Gasparre reported shearing rates between 0.0003–0.0007%/hr in her triaxial static probing tests and between 0.0005–0.01%/hr in the monotonic loading tests. In the Author’s study, the static uniaxial HCA probing tests had slightly faster shearing rates (0.0005–0.001%/hr, see §7.1); much faster rates (about 0.03%/hr in the first 12 hours, see §8.3) were applied in the monotonic multi-axial HCA shear tests. Another significant difference was the length of the ageing periods imposed prior to shear in the monotonic loading test series, which were significantly longer (5 to 7 days) in Gasparre’s triaxial tests, compared to the 1–3 days involved in the Author’s HCA test series. These differences will be discussed in this section.

**Incremental strain energy**

*Gasparre* (2005) quoted incremental strain energy values at her $Y_1$ and $Y_2$ limits of $1 \times 10^{-5}$ and $4.6 \times 10^{-4}$ kJ/m$^3$, respectively. By comparing these with the Author’s findings (see *Table 9.2* and *Table 9.4*), the main difference is in the average value for the $Y_2$ limits, which was around 0.002 kJ/m$^3$ on average, or four times larger. This follows from differences (though small) in the corresponding (average) deviatoric shear strain limits, which may well be related to strain rate and ageing phenomena. On the one hand, the reasonable agreement found at elastic limits highlighted the relative rate and ageing independence of the elastic stiffness parameters between the two studies. On the other hand, the more significant difference observed at $Y_2$ limits reflected the larger values of $\Delta t$ recorded at $Y_2$ in the present study (with faster tests) and the more gradual rates of stiffness decay from the elastic values (see also §8.6).

Considering the exponents in the $\Delta U \sim \epsilon$ power law relationships, it was found that $n = 1.37$ in the drained HCA tests and $n = 1.77$ in the undrained multi-axial HCA tests, while $n = 1.66$ in the triaxial drained tests (*Gasparre* 2005). Both studies indicated plastic strainings in the stress-strain response within Zone II as the stress paths crossed the elastic surface.
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

**Y\textsubscript{1} and Y\textsubscript{2} surfaces**

A common observation made in all the tests under both drained and undrained conditions (originating from points A, B, C and D, [Figure 9.3]) was that distinct Y\textsubscript{1} and Y\textsubscript{2} loci could be identified around each point, implies that the Y\textsubscript{1} and Y\textsubscript{2} loci are kinematic and can move with the current effective stress point.

The limited range of stress states (with potential variations in p\textsubscript{0}, t and b) imposed in this study made it impossible to identify their separate influences on the sizes, shapes and positions of the two kinematic yield loci. Further investigation is recommended to provide quantitative evaluations of these effects.

[Figure 9.42] shows the idealised shapes of Y\textsubscript{1} as constructed by the Author using data from drained HCA tests. The data points at elastic limits were plotted separately with respect to the initial stress points, either at point A (isotropic stress states) or at point B (anisotropic stress conditions). [Table 9.5] lists the (average) increments of stress components identified in [9.2].

It can be seen that the elastic yield locus, though still of indicative nature, was rounded and is shown with a nominal near circular shape, with a small radius (about 1 kPa) in both of the considered incremental stress planes (see [Figure 9.42]). The observed shape of the elastic surface was different from Gasparre (2005), being extended along the shear stress axis. However, it should be noted that the triaxial data were at anisotropic stress states (point B) only, and both studies showed noticable scatter (see again [9.33]) due to the very small stresses involved. The faster shearing rates employed in this study led both Y\textsubscript{1} and Y\textsubscript{2} expanding. On the other hand, shorter ageing periods would have affected the positions and sizes of these two yield loci. Although it was impossible to quantify these features in this study, the net effects were expected to be substantial.

When the effects of p\textsubscript{0}' are taken into account ([Figure 9.43]), the limits seen in this study are nearly equal in radius from the initial stress point (ratio of \(\Delta p' / p_0' = \Delta t / p_0' \sim 0.003\)). Further tests of different multi-axial shearing modes and at different mean stress levels will help to confirm the above observation of the Y\textsubscript{1} locus and examine the influences of loading direction to the shape of the elastic locus.

[Table 9.6] summarises the identified limits of Y\textsubscript{2} yielding from the drained and undrained tests. The observed kinematic feature of the sub-yield Y\textsubscript{2} is illustrated in [Figure 9.44], using data from the drained tests conducted at points A and B. When plotted in the considered incremental stress space, it appeared that the shape of Y\textsubscript{2} was not distorted as found by Gasparre (2005). It was also bigger than the Y\textsubscript{2} surface identified by her from triaxial tests (see again [9.33]). Note that both studies offered a very limited number of data points and the combined effects of shearing rates and ageing periods have not been quantitatively evaluated. Further investigation in this direction is therefore recommended.

[Figure 9.45], plotting test results from undrained stress paths, shows many similarities with those observed in the drained tests. Overall, the shape and size of the Y\textsubscript{2} locus could be confidently established in the \(\Delta \tau_{z\theta} \sim \Delta (\sigma'_{z} - \sigma'_{\theta})/2\) plane because a greater number of data points were available from tests that were sheared at different \(\alpha\). Again, it was noted that the loading directions did not significantly distort the Y\textsubscript{2} surface. This observation implied that the shape of Y\textsubscript{2} locus was independent of changes in \(\alpha\).

Finally, [Figure 9.46] shows the normalised Y\textsubscript{2} loci when the influences of p\textsubscript{0}' is taken into account. A near circular shape could have been interpreted, with a radius of approximately \(\Delta p' / p_0' = \cdots\).
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Table 9.5: $Y_1$ yielding limits in terms of (average) incremental stress components

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Test stage</th>
<th>$p_0^\prime$</th>
<th>$b$</th>
<th>$\alpha_{ds}$</th>
<th>$\Delta p^\prime$</th>
<th>$\Delta t$</th>
<th>$\Delta (\sigma_z^\prime - \sigma_\theta^\prime)/2$</th>
<th>$\Delta \tau_{z\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drained</td>
<td>Uniaxial tests $\Delta \sigma_{z}^\prime$</td>
<td>280</td>
<td>0</td>
<td>90</td>
<td>0.67</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Uniaxial tests $\Delta \sigma_{\theta}^\prime$</td>
<td>280</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.7</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Re-consolidation</td>
<td>280</td>
<td>1</td>
<td>45</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(point A)</td>
<td>Uniaxial tests $\Delta \sigma_{z}^\prime$</td>
<td>280</td>
<td>1</td>
<td>90</td>
<td>0.67</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Uniaxial tests $\Delta \tau_{z\theta}$</td>
<td>280</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
<td></td>
<td>Unitaxial tests $\Delta \sigma_z^\prime$</td>
<td>280</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>0.7</td>
<td>0.8</td>
<td>0</td>
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<td></td>
<td>$b$-change</td>
<td>280</td>
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<td>90</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$\Delta t/p_0^\prime \simeq 0.035$. On the other hand, [Gasparre (2005)] interpreted the normalised $Y_2$ surface as slightly skewed along the direction of undrained stress paths. As mentioned earlier, at present the data are inconclusive due to both the limited number of data and the combined effects of shearing rates and ageing periods.
### 9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Table 9.6: \( Y_2 \) yielding limits in terms of incremental stress components

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Test stage</th>
<th>( p'_0 )</th>
<th>( b )</th>
<th>( \alpha_{d\sigma} )</th>
<th>( \Delta p' )</th>
<th>( \Delta t )</th>
<th>( \Delta(\sigma'<em>z - \sigma'</em>\theta)/2 )</th>
<th>( \Delta\tau_{z\theta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drained</td>
<td>Uniaxial, IM90-DZ</td>
<td>280</td>
<td>0</td>
<td>90</td>
<td>-5.3</td>
<td>-8.0</td>
<td>-8.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(point A)</td>
<td>Uniaxial, IM90-DQ</td>
<td>280</td>
<td>0</td>
<td>0</td>
<td>4.7</td>
<td>7.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Re-consolidation</td>
<td>280</td>
<td>0</td>
<td>90</td>
<td>0.0</td>
<td>-11.0</td>
<td>-11.0</td>
<td>0.0</td>
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<tr>
<td>(point B)</td>
<td>Uniaxial HC-DQ</td>
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<td>1</td>
<td>90</td>
<td>-5.3</td>
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<td>Uniaxial HC-DT</td>
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<td>Unitaxial HC-DZ</td>
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<td>0</td>
<td>4.7</td>
<td>7.0</td>
<td>7.0</td>
<td>0.0</td>
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<td></td>
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<td>90</td>
<td>0.0</td>
<td>12.5</td>
<td>12.5</td>
<td>0.0</td>
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<tr>
<td>Undrained</td>
<td>Unloading</td>
<td>280</td>
<td>0</td>
<td>0</td>
<td>-2.7</td>
<td>9.0</td>
<td>9.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(points B, C)</td>
<td>Multi-axial shear</td>
<td>280</td>
<td>0</td>
<td>0</td>
<td>-2.6</td>
<td>8.0</td>
<td>8.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(point D)</td>
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<td>251</td>
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<td>8.0</td>
<td>0.0</td>
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<tr>
<td></td>
<td></td>
<td>252</td>
<td>0</td>
<td>45</td>
<td>-3.2</td>
<td>10.0</td>
<td>0.0</td>
<td>10.0</td>
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<td></td>
<td></td>
<td>248</td>
<td>0.3</td>
<td>0</td>
<td>-2.6</td>
<td>9.0</td>
<td>9.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
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<td>30</td>
<td>-3.2</td>
<td>11.0</td>
<td>5.5</td>
<td>9.5</td>
</tr>
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<td></td>
<td></td>
<td>256</td>
<td>1</td>
<td>45</td>
<td>3.4</td>
<td>-10.0</td>
<td>0.0</td>
<td>-10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250</td>
<td>1</td>
<td>60</td>
<td>3.4</td>
<td>-10.0</td>
<td>5.0</td>
<td>-8.7</td>
</tr>
<tr>
<td>(point A)</td>
<td>Multi-axial shear</td>
<td>280</td>
<td>0.5</td>
<td>90</td>
<td>1.8</td>
<td>-6.0</td>
<td>-6.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>280</td>
<td>0.5</td>
<td>0</td>
<td>-3.0</td>
<td>10.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

284
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Figure 9.38: Experimental $Y_1$ surface in $\Delta p' \sim \Delta q$ space, triaxial tests on London Clay samples from unit B2 \cite{Gasparre2005}. Sample from 11.0 and 12.5 mOD, $p'_0 = 260$, $q_o = 86$ kPa.

Figure 9.39: Experimental $Y_2$ surface in $\Delta p' \sim \Delta q$ space, triaxial tests on London Clay samples from unit B2 \cite{Gasparre2005}. Sample from 11.0 and 12.5 mOD, $p'_0 = 260$, $q_o = 86$ kPa.
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Figure 9.40: Normalised $Y_1$ locus in $\Delta p'/p'_0 \sim \Delta q'/p'_0$ space, triaxial testing results after Gasparre (2005)

Figure 9.41: Normalised $Y_2$ locus in $\Delta p'/p'_0 \sim \Delta q'/p'_0$ space, triaxial testing results after Gasparre (2005)
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Figure 9.42: Yield $Y_1$ in the incremental effective stress space, (uniaxial, b-change and anisotropic-reconsolidation) drained tests. The $Y_1$ surface plotted in continuous line is after Gasparre(2005).

Figure 9.43: $Y_1$ in the normalised incremental effective stress space. The $Y_1$ surface plotted in continuous line is after Gasparre(2005).
Figure 9.44: Yield $Y_2$ in the incremental effective stress space, (uniaxial, b-change and anisotropic-reconsolidation) drained tests. The $Y_2$ surface plotted in continuous line is after Gasparre(2005).
9.4. THE KINEMATIC SUB-YIELD LOCI IN EFFECTIVE STRESS SPACE

Figure 9.45: Yield $Y_2$ in the incremental effective stress space, (unloading and multi-axial stress path) undrained tests. The $Y_2$ surface plotted in continuous line is after Gasparre(2005).

Figure 9.46: $Y_2$ in the normalised incremental effective stress space. The $Y_2$ surface plotted in continuous line is after Gasparre(2005).
9.5 Summary

The yielding characteristics of the London Clay were investigated in this Chapter using the multiple yield surfaces framework suggested by Jardine (1992). These characteristics were studied at four initial stress points in the 4-D stress space, and the outgoing stress paths consisted of tests under both drained and undrained testing conditions as shown in Figure 9.3.

Using a consistent set of procedures, the author identified the limits of the two sub-yield $Y_1$ and $Y_2$ loci in terms of the deviatoric shear strain, shear stress increments and incremental strain energy (see Table 9.2 and Table 9.4). In addition, the sizes and shapes of these loci were constructed in the incremental stress space. Overall, the following important points could be made:

- The methods employed for the identification of the $Y_1$ and $Y_2$ limits worked reasonably well, particularly if individual strain measurements were used for both drained and undrained test data.
- Clear evidence of $Y_1$ yielding was found in the drained tests (and inferred for the undrained tests) in terms of the end of quasi linear elastic behaviour.
- The points where strain increment directions changed were interpreted as $Y_2$ yielding in both the drained and undrained tests. In general, these points corresponded to the onset of more contractant behaviour (either volume reduction or $p'$ reduction).
- Considering the data in the present study, the values of $\Delta U$ at sub-yield limits were not unique and varied significantly. This implied that this parameter was as sensitive to influences of $p'$, stress history and applied stress condition as other parameters (shear stress increment or deviatoric shear strain). At least in this study it was not suitable to be used independently without other methods.
- The kinematic nature of both the $Y_1$ and $Y_2$ loci, as suggested by the framework, was verified by the experiments.
- The elastic yield $Y_1$ locus was near circular in shape, with a radius of roughly 1 kPa ($\Delta t/p'_0 \approx 0.003$) in the incremental stress plane or at about 0.001 % deviatoric shear strain.
- The observed shape of $Y_2$ locus had a radius $\Delta t/p'_0 \approx 0.035$ in the normalised stress plane and the limits were reached at around 0.01–0.03 % deviatoric shear strain.
- The sizes and shapes of the sub-yield loci appeared to be independent of the $t$ and $b$ levels considered in the present study. As mentioned earlier, when normalising to the initial effective stress $p'_e$, each locus seemed to coincide with a unique rounded surface around the initial effective stress point. This observation supported the finding of Gasparre (2005).
- It appeared that the loading directions did not significantly distort the $Y_2$ surface. This observation implied that the shape of $Y_2$ locus was independent of changes in $\alpha$.
- Further investigation is recommended to quantify the effects of variations in shearing rates and ageing periods to the positions, shapes and sizes of the two kinematic yield loci.
Chapter 10

Conclusions and recommendations

The thesis has described a laboratory investigation into the anisotropy in mechanical behaviour of the natural London Clay, an Eocene clay that is widely distributed in both London and Hampshire Basins in the south of England. All samples were obtained by block sampling from one horizon (12.5mOD, i.e. 5.2mBGL) at the construction site of the new Terminal 5 at Heathrow Airport in London. The clay was stiff and of high plasticity (the average liquid and plastic limits are 69% and 25%, respectively).

The author’s work was a contribution to the comprehensive London Clay characterisation programme conducted by a team at Imperial College, which applied advanced laboratory testing, lithological study (Gasparre, 2005; Nishimura, 2006; Mannion, 2007) and in situ testing as summarised by Hight et al. (2007).

In the present study, the author focused on the following points of interest:

1. The application of the new HCA Mark II to study the mechanical anisotropy of the stiff clay from very small strains up to failure;
2. The degrees of anisotropy in shear strength and deformation properties in a particular key horizon representative of the natural clay at a relatively shallow depth (5.2 mBGL);
3. The influences of the intermediate principal stress $\sigma_2$ on the clay’s mechanical behaviour;
4. The clay’s deformation characteristics at very small strains and the possible application of cross-anisotropic elastic theory to predict the stress-strain response within this elastic zone;
5. The non-linear stress-strain relationships of the clay and the relevance of multiple sub-yield surface framework to interpret the pre-failure behaviour.

The findings from this research are summarised in §10.1 while §10.2 outlines the potential applications to design practice. Finally §10.3 provides suggestions for future work.
10.1 Conclusions

10.1.1 Hollow cylinder apparatus, material and testing techniques

The recently developed HCA Mark II ([Jardine] 1996) proved to be a versatile laboratory shearing device. It offers capabilities of computerised stress-path or strain-path control, automatic data acquisition coupled with an extensive local stress-strain measuring system and flexibility in imposing stress state in the four-dimensional space \( (q \sim p' \sim \alpha \sim b) \), which are far exceeding those of triaxial stress path testing. The review in §2.2 shows that HCA offers more advantages than the Direction Shear Cell ([Arthur et al.] 1977), a device of potential similar capability in changing the direction of loading. In addition, as shown in §2.3, the problems of stress and strain non-uniformities within the HC specimens could be effectively minimised in the HCA Mark II by selecting suitable specimen geometries and avoiding the pre-defined no-go stress regions.

In the HCA Mark II the specimen's deformation components were measured by two independent (local and global) strain measuring systems. The author improved the resolution and stability of the local strain measuring system by 1) using a revised geometry for the double-axis electrolves; 2) changing the material for the proximeter transducers's targets from foil to copper curved plates; and 3) optimising the data acquisition programme. By doing so the potential errors in the determination of small strain stiffness moduli were effectively reduced, allowing reasonable estimations of the clay's pre-failure yielding response and deformation characteristics over the very small strain range (about 0.001%). In the context of hollow cylinder testing devices at Imperial College, the HCA Mark II approached, although not exactly matched, the degrees of accuracy and resolution for strain measurement system that have previously been only available in specialised triaxial apparatus (e.g. [Cuccovillo and Coop] 1997; [Kuwano] 1999; [Gasparre] 2005).

Comparison between the global and local strain measurements in the present HCA Mark II system is reported in §4.4. It showed that the semi-local strain measurements were likely to be reliable only after strains became larger than 0.03 %, and therefore were not appropriate for the investigation of pre-failure yielding characteristics at small strains.

During the course of this study, considerable efforts were also made in the preparation of hollow cylinder specimens with minimal disturbance from block samples. These techniques and the specimen set-up procedure are described in Chapter 5.

Two series of HCA tests on London Clay specimens were carried out in this research (see §7.1 and §8.1). The first series consisted of drained uniaxial probing tests, in which only one stress component \( (\sigma_z', \sigma_\theta' \text{ or } \tau_{z\theta}) \) was allowed to change. Both small-stress excursions and shear to failure stages were carried out. The second series consisted of 13 CAU and two CIU multi-axial stress path tests. In all of these multi-axial tests the loading direction (\( \alpha \)) and the intermediate principal stress ratio (\( b \)) were controlled independently during the final undrained shear stage. Overall, the experimental programme required a wide range of testing scheme and involved long periods.

10.1.2 Anisotropy in drained stiffness moduli at very small strains and the assumption of cross-anisotropic elastic behaviour

- The series of static uniaxial shear tests comprised the first hollow cylinder investigation into the anisotropic drained deformation characteristics of the natural London Clay. Two CID
and three CAD uniaxial tests to failure were conducted from the estimated in situ effective stress state. In addition, 33 small static uniaxial probes were performed at several effective stress levels and stress states (isotropic and anisotropic conditions) to provide the measurements of stiffness moduli and Poisson's ratios.

- At very small strains (up to 0.001 %) the London Clay can be reasonably modelled as a cross-anisotropic elastic material with the vertical direction as the axis of symmetry. The experimental results indicated that its parameters were dependent on the stress level and the stress state.

- The elastic components of this cross-anisotropic stiffness matrix at in-situ effective stress were established in §7.2. The elastic stiffness moduli determined in this study showed broad agreement with results obtained from hollow cylinder resonant column, bender elements and triaxial probing tests on samples from similar depths and subjected to similar stress history (see [Gasparre et al., 2007a; Hight et al., 2007]). However, the Poisson's ratios were subjected to large scatters in measurements due to the very small magnitudes of strain components in the calculations.

- The calculated quasi-elastic (drained) stiffness parameters found for \( p' = 280 \text{ kPa} \) and \( K = \sigma'_h/\sigma'_v = 1.55 \sim 1.7 \):

\[
\begin{align*}
E'_v &= 120 \text{ MPa} \quad \nu'_{vh} = 0.25 \\
E'_h &= 218 \text{ MPa} \quad \nu'_{hv} = 0.44 \\
G_{vh} &= 75 \text{ MPa} \quad \nu'_{hh} = -0.17
\end{align*}
\]

- It was reconfirmed that the clay was significantly stiffer in the horizontal direction than in the vertical direction. On average, the inherent small strain stiffness anisotropy of London Clay established at isotropic stress state, expressed in term of \( E'_h/E'_v \), was 1.7. At the in situ anisotropic stress state (\( K = 1.55 \sim 1.70 \)), this ratio rose to about 1.82. In addition, the average small strain shear stiffness ratios of \( G_{hh}/G_{vh} \) were 1.8 and 2.1 at isotropic and anisotropic stress states, respectively.

- The Young's moduli appeared to be dependent on the normal effective stress level following a power law relationship as suggested by [Hardin and Blandford, 1989]. The relationships could be normalised to a void ratio function, \( f(e) = e^{-1.3} \), and assuming negligible effect of stress history with the following empirical parameters:

\[
\begin{align*}
E'_v/p_r &= 450e^{-1.3}(\sigma'_v/p_r)^{0.55} \\
E'_h/p_r &= 580e^{-1.3}(\sigma'_h/p_r)^{0.72} \quad (\text{for upper bound})
\end{align*}
\]

in which the reference pressure \( p_r = 100\text{ kPa} \). The effective stresses \( \sigma'_v \) and \( \sigma'_h \) were in the vertical and horizontal directions, respectively.

- Similar power law relationships were established for the elastic shear moduli \( G_{vh} \) and \( G_{hh} \). For tests initiated from isotropic stress states, the exponent in the expression \( G_{vh}/f(e)/p_r \sim p'/p_r \) was 0.48, which is slightly smaller than the value of 0.50 quoted by [Viggiani and Atkinson, 1995] but 20 % larger than those found by [Yimsiri, 2002] and [Nishimura, 2006]. With regard to the \( G_{vh}/f(e)/p_r \sim \sigma'_v/p_r \cdot \sigma'_h/p_r \) and \( G_{hh}/f(e)/p_r \sim \sigma'_h/p_r \cdot \sigma'_h/p_r \) relationships, the average exponent was 0.22 and 0.24, respectively. These values are larger than the values of 0.09–0.12 reported by [Nishimura, 2006]. Further study is needed to investigate these trends.
10.1. CONCLUSIONS

- No indicative trend of stress level dependency could be established for the Poisson’s ratios.

10.1.3 The deformation characteristics from drained uniaxial shearing tests

- Reasonable agreements were observed (see §7.3) between the elastic stiffness parameters defined from the small-stress probing tests and those calculated at very small strains (about 0.001 %) from the drained uniaxial tests with larger stress excursions.

- The large-stress drained uniaxial tests provided information over a wider range of strains (from small strains up to failure). It was found that the directional dependency of drained stiffness was clearly present up to intermediate strains (0.1 %) with persistently stiffer response in the horizontal direction compared to the vertical direction.

- Significant stress-strain non-linearity occurred early after the elastic strain limit (0.001 %) and the rate of degradation depended on the mode of shearing.

10.1.4 Stress-strain-strength response in multi-axial HCA tests

A total of 15 multi-axial undrained shear tests was performed with various combinations of the principal stress rotation angle $\alpha$ (at $0^\circ$, $30^\circ$, $45^\circ$, $50^\circ$, $60^\circ$ and $90^\circ$) and the intermediate principal stress ratio $b$ (0, 0.3, 0.5 and 1). This test series helped to establish an experimental database on the mechanical properties for the natural London Clay at shallow depth (5.2 mBGL) under undrained conditions.

Under undrained conditions, the clay’s pre-failure response appeared to show contractive behaviour ($\Delta u > 0$) under compressional loading $(\Delta \sigma_z - \Delta \sigma_\theta) = (\Delta \sigma_v - \Delta \sigma_h) > 0$ and dilative behaviour under extensional loading. The slopes $(\Delta \sigma_z - \Delta \sigma_\theta)/2\Delta \sigma'$ measured over the initial parts of the effective stress paths had similar inclinations, ranging between -1:2.9 to -1:3.4. These observed values were within the theoretical slopes of -1:3 predicted for a linearly isotropic elastic material and those of -1:2.6 to -1:4.2 predicted for a linearly cross-anisotropic elastic material that had the (drained) deformation characteristics identified at very small strains (see Chapter 7). The pre-failure pore water pressure development seen in undrained tests contributed to the anisotropy seen in the clay’s undrained shear strength ([Nishimura] 2006; [Nishimura et al.] 2007).

Influences of $\alpha$ and $b$ on the shear strength of natural London Clay

Previous experience involving triaxial tests on good quality samples (see, e.g [Bishop et al.] 1965; [Hight and Jardine] 1993; [Hight et al.] 2003) showed curved peak $q \sim \sigma'$ envelopes, indicating a dependency on $\sigma'$ of the $q/\sigma'$ at peak. In this study the pre-shear values of $\sigma'$ were not varied greatly, making it impossible to define the potential envelope curvature. The author therefore defined shear strength in terms of $q/\sigma'$ or $S_u$ which bracket the available possibilities when assessing the anisotropic shear strength trends - one for a purely frictional material and the other for a purely cohesive material where strength is independent of $\sigma'$.

- The clay’s shear strength characteristics were strongly anisotropic, particularly under conditions of $b = 0.5$ and 0.3, which were also close to plane strain condition ($\epsilon_2 = 0$). Compressional loading conditions ($\alpha < 45^\circ$) generally resulted in high shear strength values, and the
maximum observed shear strength was associated with plane strain compression ($\alpha = 0^\circ$).
In contrast, lower strengths were found for triaxial extension ($\alpha = 90^\circ$) and torsional shear ($\alpha = 45^\circ$) conditions. The higher $q/p'$ ratios developed by compressional loading modes are compatible with the general failure envelope curvature, shown by Nishimura et al. (2007) for the London Clay at T5, when consideration is taken of variations with depth of effective stress level, OCR and lithology.

- Anisotropy in shear strength led to a range of $q/p'$ values that could differ by as much as 40% at $b = 0.5$. The experimental fitting expression proposed by Bishop (1966) for strength anisotropy of London Clay could be applied reasonably to compressional loading ($\alpha < 45^\circ$). However, without an extension to consider $b$ effects, Bishop’s expression could not represent the clay’s undrained shear response under four dimensional stress conditions. No definitive trend of shear strengths could be asserted for tests under extensional loading ($\alpha > 45^\circ$).
- The relative value of $\sigma_2$, as represented by the $b$ ratio, also had a strong influence on the clay’s shear strength, with the peak shear strength stress states clearly deviating from the Mohr-Coulomb failure line in the deviatoric plane. However, the effect of $b$ was not always well defined. For the same loading direction (i.e. $\alpha$ value) the strength in plane strain condition was generally higher than that under other conditions. The Matsuoka-Nakai and Lade-Duncan failure criteria, which account for the effect of $\sigma_2$ explicitly, showed good agreement with observed data in some conditions, but neither provided a secure upper bound or lower bound when fitted to the shear strength at $b = 0$ or 1. In the absence of further data, an anisotropic Mohr-Coulomb envelope calibrated to shear strength at $b = 0$ or 1 may be considered as a conservative assumption, although not the best fit.
- The generalised pattern of shear strength variation due to the combined effects of anisotropy and intermediate principal stress ratio observed by the author on samples from 5.2 mBGL were largely in good agreement, except the result at $\alpha = 90^\circ$ and $b = 0.5$ condition, with those reported by Nishimura (2006) for samples from 10.5 mBGL.

Deformation characteristics in drained $b$-change and undrained multi-axial shear tests

- The multi-axial shear test series provided additional interesting data during the drained $b$-change stages, when $b$ was reduced from 1.0 to the final values of 0.5 (series AM$\alpha$-05) and 0.3 (series AM$\alpha$-03). Under this stress condition, the observed shear stiffness of $G_{oct}$ developed at deviatoric shear strains around 0.001% was 100 MPa which is close to the value of $G_{hh}$ = 115 MPa identified from bender element tests (Gasparre, 2005).
- The lower limit of experimental $G_{z\theta}$ was seen to fit within the value of $G_{vh}$ established from drained uniaxial torsional shear tests.
- In considering the undrained shear stiffness decay curves under conditions of constant $\alpha$ and $b$, the limits to the stiffness plateau expected within the elastic limit $Y_1$ surface were not well defined, although marked stiffness degradation could be located at shear strain of around 0.002%. The dependency of octahedral shear stiffness on $\alpha$ and $b$ were difficult to register over the observed strain range. Both $G_{z\theta}$ and $G_{oct}$ showed similar rates of degradation, with $G_{\epsilon_d}/G_{\epsilon_d=0.001}$ ratio reduced to 0.27–0.42 at $\epsilon_d = 0.01$ % and further down to 0.19–0.35 at $\epsilon_d = 0.1$ %.
- The normalised undrained shear stiffness modulus degradation curves ($E_u/p'_0 \sim \log \epsilon_d$)
fitted reasonably well within the bounds set up for London Clay \cite{Hight1993, Hight2003} from triaxial tests with local strain measurements. Data from tests performed under near plane strain conditions ($b = 0.3$ and 0.5) were located near the upper bound limit, with a ratio between 800–1200 at $\epsilon_d = 0.001\%$. In comparison, the data from triaxial tests on London Clay specimens \cite{Gasparre2005, Nishimura2006} showed lower values at small strains, and the apparent anomaly is attributed to the slower rates of stress change along the approach path to the in situ stresses and the extended drained pause periods prior to undrained shearing \cite{Hight2007}.

10.1.5 Pre-failure yielding characteristics

The yielding characteristics of the London Clay were investigated using the multiple yield surface framework suggested by \cite{Jardine1992}. The $Y_1$ and $Y_2$ kinematic yield loci were identified at different effective stress points using the results from uniaxial and multi-axial test series under both drained and undrained conditions. The following important points could be made:

- Behaviour within the relatively small kinematic $Y_1$ region was essentially cross-anisotropic with the compliance matrix terms varying with effective stress level. The second kinematic $Y_2$ surface that surrounded $Y_1$ reflected the states at which the soil micro-structure was first altered by continued straining. The limits to the two sub-yield $Y_1$ and $Y_2$ loci were identified in terms of the deviatoric shear strain, shear stress increments and incremental strain energy (see Chapter 9).

- The methods employed for the identification of the $Y_1$ and $Y_2$ limits worked reasonably well, particularly if using individual strain measurements for both drained and undrained test data. Clear evidence of $Y_1$ yielding was found in the drained tests (and inferred for the undrained tests) in terms of the end of quasi linear elastic behaviour. The points where strain increment directions changed were interpreted as $Y_2$ yielding.

- The application of a fixed incremental strain energy contour to identify the sub-yield limits was not always satisfactory. The values of $\Delta U$ at sub-yield limits were found to vary greatly. This implied that this parameter was as sensitive to influences from $p'$, stress history and applied stress condition as other parameters (shear stress increment or deviatoric shear strain).

- The kinematic nature of both the $Y_1$ and $Y_2$ loci, as suggested by the framework, was observed from the experimental data at different initial effective stress points.

- In this study it appeared that the kinematic sub-yield surfaces were rounded and placed relatively symmetrically on the initial stress point.

- The elastic yield $Y_1$ locus was near circular in shape, with a radius of roughly 1 kPa ($\Delta t/p'_o \approx 0.003$) in the incremental stress plane or at about 0.001 % deviatoric shear strain.

- The observed shape of $Y_2$ locus had a radius $\Delta t/p'_o \approx 0.035$ in the normalised stress plane and the limits were reached at around 0.01–0.03 % deviatoric shear strain.

- The sizes and shapes of the sub-yield loci appeared to be independent of the $t$ and $b$ levels considered in the present study. As mentioned earlier, when normalising to the initial effective stress $p'_o$, each locus seemed to coincide with a unique rounded curve around the initial effective stress point.
Further investigation is recommended to quantify the effects of variations in shearing rates and ageing periods to the positions, shapes and sizes of the two kinematic yield loci.

10.2 Practical applications

10.2.1 Soil characterisation using laboratory tests

To ensure the success of obtaining representative results, it is important to employ a sampling method that avoids significant damage to the clay’s structure, fabric and state. The block and rotary-coring techniques used in the T5 study by the Imperial College research team gave good quality samples, as demonstrated by generally good correlations between the mechanical and physical properties seen in laboratory and field tests (Hight et al., 2003, 2007).

The London Clay shows a strong degree of anisotropy in its mechanical behaviour, and generally the stress-strain-strength characteristics depend on the stress state prior to shear. It is therefore necessary to perform laboratory tests on samples that are subjected to representative recent stress histories and experienced stress paths relevant to those imposed in the field (see Jamiolkowski et al., 1983; Tatsuoka et al., 1997; Jardine et al., 2004). As the HCA is not always available in normal practice, the use of simple shear tests together with triaxial tests (TC and TE) can provide some indications of the influences of $\alpha$ and $b$ on peak shear strength. In addition, if residual shear strength is another important factor in the particular problem investigated then a programme of ring shear tests would be necessary.

It has been shown that similar ranges of shear stiffness moduli were found between in-situ shear wave measurements and laboratory (uniaxial HCA, resonant column HCA, bender element, static triaxial probing) tests (see also Gasparre et al., 2007b; Hight et al., 2007). This compatibility requires careful testing techniques and proper methods of interpretation, with due consideration given to creep and ageing.

10.2.2 Numerical analysis and constitutive modelling for practical projects

Nishimura (2006) reviewed the wide variety of practical problems that will benefit from improved knowledge regarding the London Clay’s anisotropy. The aim of this subsection is to discuss possible benefits from the author’s findings. These can be broadly divided into two main applications:

1. Predictions of ground movements and settlements that relate to the construction of underground structures in London Clay;
2. Analyses or predictions of short-term failures for large foundations, slopes, excavations or tunnels.

The first application requires the use of advanced constitutive soil models that can account for the pre-failure yielding characteristics, the anisotropy in stiffness and the effects of recent stress history. One commonly approach in practice is to use simple cross-anisotropic non-linear elastic models (e.g. Addenbrooke et al., 1997; Hird and Pierpoint, 1997; Franzius, 2003). These models, although clearly an idealization of soil behaviour, are attractive to engineers due to their relatively easy implementations in numerical analyses to solve boundary values problems. The observed
10.3. SUGGESTIONS FOR FURTHER STUDY

degrees of elastic stiffness anisotropy, expressed in terms \( E'_h / E'_v \) and(or) \( G_{hh} / G_{vh} \) and the values of the Poisson's ratios, are useful in this respect.

The present study provides a new experimental database for the relationships of \( G_{oct} \sim \epsilon_d \) and \( K'_0 \sim \epsilon_o \) for use with non-linear elastic approaches such as those described by [Jardine et al. (1986)] and [Hight and Higgins (1995)]. However, it should be noted that due to the influences of recent stress history and stress state, stiffness decay curves are not unique and no single set can be universally applicable for every stress conditions.

An alternative is to incorporate bounding surface plasticity into the general elastic-plastic models (e.g. [Al-Tabbaa and Muir Wood, 1989]; [Puzrin and Burland, 1998]; [Grammatikopoulou, 2004]). However, while the latter approach can describe some features of soil response more realistically than the simple nonlinear approach, difficulties are still remained on the selection of hardening parameters albeit with large number of parametric studies ([Jardine et al., 2001]). Moreover, the elastic-plastic models ignore the effects of creep and ageing on relocation of the kinematic surfaces.

Referring to the second application, it was shown that the common practice of using triaxial compression tests for shear strength characterisation involves considerable inaccuracies if the in situ stress field is subjected to changes in \( \alpha \) and \( b \). In cases where higher values of \( \alpha \) dominate, a triaxial compression approach may overestimate shear strength significantly. Equally, the shear strength interpreted from triaxial extension tests may not provide a safe lower bound for the London Clay as shown here and by [Nishimura, 2006].

To stimulate the influence of anisotropy on shear strength, there have been several models proposed to date (e.g. [Whittle and Kavvadas, 1994]; [Gajo and Muir Wood, 2001]; [Li and Dafalias, 2004]; among others). The shear strength data presented in Chapter 8 provides useful information for this aspect. The present study showed that the intermediate principal stress also strongly influences the shear strength characteristics. It was noted that a bounding surface of circular shape in the deviatoric plane generally overestimates the shear strength of soils in torsional shear modes, and the observed failure points fitted neither a Mohr-Coulomb hexagon nor Matsuoka-Nakai or Lade-Duncan curve. It is therefore desirable to allow more flexible formulation of the failure envelope (see [Rouainia and Muir Wood, 2000]; [Georgiadis et al., 2004]).

10.3 Suggestions for further study

10.3.1 Improvements for the HCA Mark II

This device can be further improved as follows:

- Increasing the accuracy and stability in axial strain measurement by replacing the double-axis electrolevel with higher resolution instruments such as the LVDT system described by [HongNam and Koseki, 2005].

- Using a (local) mid-height pore pressure probe to assist the proper selection of loading rates for drained and undrained tests.

- Introducing new algorithms for strain-controlled phases of uniaxial and simple shear tests.
10.3. SUGGESTIONS FOR FURTHER STUDY

10.3.2 Small strain deformation characteristics

In this study it was recognised that the extent of the linear elastic strain range of the clay was relatively independent of the values of deviatoric shear stress $t$ and intermediate principal stress ratio $b$ (within the imposed stress conditions). Therefore, it will be useful to investigate the possible influences of recent stress history and stress state by conducting other series of uniaxial probing tests from the same stress point but subjecting the samples to different recent stress histories (e.g. changes in the direction of the reconsolidation stress path). Such tests will also help to re-evaluate the suggested parameters in the power law relationships for small strain Young’s and shear moduli found in the present study.

In the identification of the shapes of the sub-yield surfaces $Y_1$ and $Y_2$ for London Clay, the present study has expanded the available stress condition from the (limited) triaxial stress condition to a four dimensional stress space (at one specific mean stress). To fully establish the shapes of these loci, it is desirable to perform other uniaxial tests at several other levels of mean effective stress.

10.3.3 Shear strength anisotropy

Additional tests would be useful to clarify the influence of the intermediate principal stress ratio on the shear strength parameters of the clay. This could be achieved by conducting multi-axial HCA shear tests with $\alpha = 45^\circ$ but under other $b$ conditions, such as 0.3 and 0.7.

The potential influences of the consolidation regime to shear strength anisotropy could be further investigated from CIU multi-axial tests under $b = 0.5$ for the range of $\alpha = 30^\circ \sim 60^\circ$ and then compared with the results from the corresponding CAU series. Alternatively, a different consolidation path (such as inclined consolidation) to that employed in this study could be considered.

In order to improve the understanding of the inherent anisotropy of the natural London Clay, tests on reconstituted samples will be required. Although a limited number of such tests have been performed by Nishimura (2006), the alternative consolidation, swelling and re-consolidation regimes could be considered, along with a broader spread of stress conditions at failure. The series that is representative for plane strain condition (possibly at $b = 0.3 \sim 0.5$) should be of main interest as for the case of natural clay.
References


REFERENCES


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REFERENCES


Lade, P. V., 1976. Interpretation of torsion shear tests on sand.


REFERENCES

cal characterisation and properties of a sensitive clay from Quebec. In: Tan, T. S., Phoon, K. K.,
Hight, D. W., Leroueil, S. (Eds.), Characterisation and engineering properties of natural soils.

T. S., Phoon, K. K., Hight, D. W., Leroueil, S. (Eds.), Characterisation and engineering properties

triaxial testing. In: Donaghe, R. T., Chaney, R. C., Silver, M. L. (Eds.), Advanced triaxial testing

Leroueil, S., Vaughan, P. R., 1990. The general and congruent effects of structure in natural soils
and weak rocks. Geotechnique 40 (3), 467–488.

Li, X., Dafalias, Y. F., 2004. A constitutive framework for anisotropic sand including non-

Geotechnique 50 (2), 109–125.

51 (6), 555–565.

York, USA, Ch. III and IV.


Marsland, A., 1974. Comparison of the results from static penetration tests and large in-situ plate
245–252.

Matsuoka, H., Nakai, T., 1974. Stress-deformation and strength characteristics of soil under three
JSCE, pp. 59–70.

ASCE 108 (GT6), 851–872.


Miura, K., Miura, S., Toki, S., 1986. Deformation behavior of anisotropic dense sand under principal

15, 163–175.

REFERENCES

Nishimura, S., April 2005. Calculation of undrained ESPs and octahedral shear stiffness in HCA testing based on cross-anisotropy elastic parameters. Internal report.


REFERENCES


thesis, Imperial College of Science, Technology and Medicine, London, UK.


REFERENCES


Appendix A

Calibration of transducers

An outline of the calibration methodology of the employed transducers is given in this section. Calibration procedures of the combined load cell and all local strain transducers, which required specific arrangement, are also discussed. A two-step calibration procedure has been applied to all local strain transducers to ensure the least error for each of the transducers over the small strain working ranges. Table A.1 summarises the typical calibration characteristics of the HCA Mark II transducers.

A.1 Calibration methodology

A consistent calibration procedure, having the following steps, was adopted:

- Calibration of the transducer, including the data logger, against a suitably accurate reference. No less than 20 calibration points were used over the calibration range.

- Calculation of a least square linear fit to describe the relationship between the voltage output and the reference value.

- Determination of the linear regression error (in engineering unit) for each calibration point as the difference between the reference and the converted values.

- Plotting of the reference and error at each calibration point with respect to the voltage output. Also conducting statistical assessment of the transducer calibration’s maximum error, non-linearity (regression) and standard deviation.

- Determination of the ratio (in %) of maximum error over the calibration full-scale-output (F.S.O) and the accuracy (engineering unit), defined as ±2 times the standard deviation of the errors. Assuming a Gaussian distribution, this corresponds to a confidence level of 95% that the difference between a measured value and the true value would be within the reported accuracy.
A.2 Calibration of the combined load cell

The expected working range for all tests was at axial load of 1 – 4 kN and torque of ±100 Nm. Consequently the load cell was calibrated over this range only. The calibration factors of the load cell to axial force and torque components, as well as their cross-effects, were established from three separate calibrations.

The compression deviator force was calibrated with a dead-weight tester in a purpose-built load frame. Another different purpose-built frame with dead-weight hanger was used to calibrate the deviator force in tension. In both cases the load cell was arranged to react with the load frames via steel ball bearings to ensure axi-symmetrical point load application. The linear fit calibration factors due to axial force only were determined (Figure A.1).

Next, the torque component of the load cell was calibrated using a purpose-built torque calibration frame. The transducer was rigidly fixed onto the base medal pedestal of the resonant column HCA. A 1.2 m-length steel rod was fixed to the bottom of the load cell via four screws. High strength cables were attached to both ends of the rod and extended in opposite directions, perpendicular to the rod, over low friction pulleys to the hanger on which the weights were placed. Linear fit calibrations of both clockwise and anti-clockwise torque moment directions were carried out (Figure A.2).

Finally, the cross-effects between axial load and torque were calibrated by applying several combinations of axial and torque, shown schematically in Figure A.3. Again using linear fit function, it was able to obtain the load cell calibration factors, which was a 2 by 2 matrix (four calibration factors).

A.3 Calibration of the local strain transducers

All the local strain transducers had limited linear working ranges compare to their full-scale outputs. Typically, the linear range was between a third to a half of the transducer's F.S.O, except for the LVDT. As such, the calibration for each of them was modified into two-calibration procedure:

- Calibration of the transducer over the F.S.O. Often, a third order polynomial fit was applied to achieve regression of 0.99999 or better. The first calibration was used to determine the linear portion of the transducer's working range.

- Next, a second calibration was carried out in the linear portion and the linear fit calibration's factors were defined.

Figures A.4, A.5 and A.6 are typical calibration graphs of the LVDT and electrolevels used in this study. The shear strain electrolevel was housed in a purpose-built holder that attached to a theodolite for calibration. By mounting the axial strain electrolevel between two pads of a purpose-built micrometer, direct relationship between the electrolyte's output and relative vertical movement could be obtained.
### Table A.1: HCA Mark II transducer calibration characteristics

<table>
<thead>
<tr>
<th>Transducer type</th>
<th>Measurement</th>
<th>Range</th>
<th>Calibration range</th>
<th>Fitting function</th>
<th>Accuracy (±)</th>
<th>Max error</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Load cell</strong></td>
<td>Axial deviatoric force</td>
<td>-8–8 kN</td>
<td>-1–0 kN</td>
<td>Linear</td>
<td>2.19 N</td>
<td>2.59 N</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0–1 kN</td>
<td>–</td>
<td>–</td>
<td>2.23 N</td>
<td>2.35 N</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1–6 kN</td>
<td>–</td>
<td>–</td>
<td>4.82 N</td>
<td>5.26 N</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Torque</strong></td>
<td>±400 Nm</td>
<td>±100 Nm</td>
<td>–</td>
<td>–</td>
<td>0.47 Nm</td>
<td>0.56 Nm</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Pressure transducer</strong></td>
<td>Outer cell pressure</td>
<td>1000 kPa</td>
<td>69–862 kPa</td>
<td>–</td>
<td>0.41 kPa</td>
<td>0.41 kPa</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Inner cell pressure</td>
<td>–</td>
<td>69–862 kPa</td>
<td>–</td>
<td>0.40 kPa</td>
<td>0.40 kPa</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Base pore water pressure</td>
<td>–</td>
<td>69–862 kPa</td>
<td>–</td>
<td>0.38 kPa</td>
<td>0.38 kPa</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Top pore water pressure</td>
<td>–</td>
<td>69–862 kPa</td>
<td>–</td>
<td>0.32 kPa</td>
<td>0.30 kPa</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Volume gauge</strong></td>
<td>Inner cell volume change</td>
<td>50 cc</td>
<td>-20–21 cc</td>
<td>–</td>
<td>0.16 ml</td>
<td>0.18 ml</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>Sample volume change</td>
<td>100 cc</td>
<td>-35–39 cc</td>
<td>–</td>
<td>0.10 ml</td>
<td>0.10 ml</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Displacement</strong></td>
<td>Global axial displacement</td>
<td>25 mm</td>
<td>0–22 mm</td>
<td>–</td>
<td>19 μm</td>
<td>18 μm</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Global horizontal disp.</td>
<td>25 mm</td>
<td>0–22 mm</td>
<td>–</td>
<td>19 μm</td>
<td>20 μm</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Equivalent rotation</td>
<td>±10°</td>
<td>±10°</td>
<td>Linear</td>
<td>0.02°</td>
<td>0.02°</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Proximeters</strong></td>
<td>Outer wall displacement 1</td>
<td>6 mm</td>
<td>±3.0 mm</td>
<td>3rd order</td>
<td>7.9 μm</td>
<td>5.9 μm</td>
<td>0.16</td>
</tr>
<tr>
<td>(KDM 8200-6U1)</td>
<td></td>
<td></td>
<td>±1.0 mm</td>
<td>Linear</td>
<td>4.0 μm</td>
<td>3.0 μm</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Outer wall displacement 2</td>
<td>6 mm</td>
<td>±3.0 mm</td>
<td>3rd order</td>
<td>7.3 μm</td>
<td>7.0 μm</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±1.0 mm</td>
<td>Linear</td>
<td>4.9 μm</td>
<td>3.9 μm</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Outer wall displacement 3</td>
<td>6 mm</td>
<td>±3.0 m</td>
<td>3rd order</td>
<td>8.9 μm</td>
<td>9.8 μm</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±1.0 mm</td>
<td>Linear</td>
<td>4.2 μm</td>
<td>4.4 μm</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>LVDT</strong> (RDP D5/200W)</td>
<td>Inner wall displacement</td>
<td>±2.5 mm</td>
<td>±2.5 mm</td>
<td>Linear</td>
<td>7.1 μm</td>
<td>6.3 μm</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.5 mm</td>
<td>Linear</td>
<td>1.5 μm</td>
<td>1.2 μm</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>Axial electrolevel</strong> (150mm-arm)</td>
<td>Local axial displacement 1</td>
<td>±7 mm</td>
<td>±5 mm</td>
<td>3rd order</td>
<td>13.2 μm</td>
<td>11.9 μm</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±1.5 mm</td>
<td>Linear</td>
<td>6.7 μm</td>
<td>5.1 μm</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Local axial displacement 2</td>
<td>±7 mm</td>
<td>±5 mm</td>
<td>3rd order</td>
<td>37.9 μm</td>
<td>26.6 μm</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±1.5 mm</td>
<td>Linear</td>
<td>3.1 μm</td>
<td>2.7 μm</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Shear electrolevel</strong> (60mm-arm)</td>
<td>Local rotation angle</td>
<td>±30°</td>
<td>±30°</td>
<td>3rd order</td>
<td>0.167°</td>
<td>0.283°</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±10°</td>
<td>Linear</td>
<td>0.033°</td>
<td>0.019°</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Notes:**
- Fitting functions, either linear or 3rd order polynomial, had regression of no less than 0.99999
- Accuracy, defined as 95% confidence of measured value within this range to the true value
- % error = Ratio of maximum error over a calibration range
A.3. LOCAL TRANSDUCERS

Figure A.1: Calibration for axial load component of load cell

Figure A.2: Calibration for torque component of load cell

Figure A.3: Calibration map for combined effects in load cell
A.3. LOCAL TRANSDUCERS

Figure A.4: Calibration of the inner LVDT.

Figure A.5: Calibration of the bi-axes electrolevel.

Figure A.6: Calibration of the single-axis shear electrolevel.
Appendix B

Test results of the re-consolidation stages

This Appendix presents the detailed testing results from re-consolidation stages introduced in Chapter 6. The data are shown for isotropic re-consolidation stages, which was mainly swelling condition, and anisotropic re-consolidation stages in which the mean effective stress was kept constant.
Figure B.1: Bulk modulus during isotropic consolidation, series AMω-00
Figure B.2: Bulk modulus during isotropic consolidation, series AMα-03
Figure B.3: Bulk modulus during isotropic consolidation, series IMα-05
Figure B.4: Bulk modulus during isotropic consolidation, series AM\(\alpha\)-10
Figure B.5: Bulk modulus during isotropic consolidation, series HC-uniaxial
Figure B.6: Development of strains during anisotropic re-consolidation, series AMα-00
Figure B.7: Development of strains during anisotropic re-consolidation, series AM\(\alpha\)-03
Figure B.8: Development of strains during anisotropic re-consolidation, series AMα-00
Figure B.9: Development of strains during anisotropic re-consolidation, series AMα-10
Figure B.10: Development of strains during anisotropic re-consolidation, series HC-uniaxial
Figure B.11: Stress-strain relationship $t \sim \epsilon_{13}$ during anisotropic re-consolidation stage, series AM$\alpha$-00

Figure B.12: Stress-strain relationship $t \sim \epsilon_{13}$ during anisotropic re-consolidation stage, series AM$\alpha$-03
Figure B.13: Stress-strain relationship $t \sim \epsilon_{13}$ during anisotropic re-consolidation stage, series AM$\alpha$-10

Figure B.14: Stress-strain relationship $t \sim \epsilon_{13}$ during anisotropic re-consolidation stage, series HC-uniaxial
Appendix C

Undrained stiffness moduli, tests with semi-local strain measurements

This Appendix presents the stiffness data calculated using the semi-local strain approach from the final undrained shear stages in series AMα-05 introduced in Chapter 8. It is noteworthy to recall here that strain measurements using global transducers were only considered to be representative after $\gamma_d = 0.2 \sim 0.3 \%$ (see [4.4]).

Figure C.1 and Figure C.2 show the stiffness data of the CAU tests under $b = 0.5$ condition, in which only radial (inner and outer) local strain sensors were available. In most of the cases the stiffness values are underestimated, particularly in tests AM30-05 and AM60-05 with $G_{oct} < 40$MPa. Less significant underestimations ($G_{oct} = 64$MPa) are found in pure compressional and extensional loading conditions. Interestingly test AM50-05 which involved mainly changes in torsional shear stress showed response as stiff ($G_{oct} = 105$MPa) as local strain measurement in IMα-05 series. It was possible that the effects from bedding and compliances errors on torsional shear strain was much less than on axial strain.
Figure C.1: Torsional shear stiffness in series AMα-05 (semi-local strain measurement)

Figure C.2: Octahedral shear stiffness in series AMα-05 (semi-local strain measurement)
Appendix D

Patterns of discontinuities and shear surfaces in HCA specimens

This Appendix presents sketches of the discontinuity and shear surface patterns observed on the HCA test specimens outer surfaces. Because it was difficult to capture the thin discontinuities on the specimens with a standard camera, sketches are preferred here to photographs.

The legends used for all the sketches in this Appendix are:

- Continuous and dotted lines: Shear surfaces and discontinuities, respectively.
- Arrows: Relative movements in the $z \sim r \sim \theta$ plane along shear surfaces
- Hatched zone: Shear zone consisting of a number of complex small shear surfaces
Figure D.1: Failure pattern of sample in axi-symmetric triaxial compression condition

Figure D.2: Failure pattern of sample in axi-symmetric triaxial extension condition
Figure D.3: Failure pattern of sample in true triaxial condition with $\alpha = 0^\circ, b = 0.3$

Figure D.4: Failure pattern of sample in true triaxial condition with $\alpha = 0^\circ, b = 0.3$
Figure D.5: Failure pattern of sample in true triaxial condition with $\alpha = 0^\circ, b = 0.5$

Figure D.6: Failure pattern of sample in true triaxial condition with $\alpha = 0^\circ, b = 0.5$
Figure D.7: Failure pattern of sample in true triaxial condition with $\alpha = 90^\circ, b = 0.5$.

Figure D.8: Failure pattern of sample in true triaxial condition with $\alpha = 90^\circ, b = 0.5$. 
Figure D.9: Failure pattern of sample in torsional shear condition with $\alpha = 30^\circ$, $b = 0.3$.

Figure D.10: Failure pattern of sample in torsional shear condition with $\alpha = 30^\circ$, $b = 0.5$. 
Figure D.11: Failure pattern of sample in torsional shear condition with $\alpha = 45^\circ, b = 0$

Figure D.12: Failure pattern of sample in torsional shear condition with $\alpha = 50^\circ, b = 0.5$
Figure D.13: Failure pattern of sample in torsional shear condition with $\alpha = 45^\circ, b = 1$

Figure D.14: Failure pattern of sample in torsional shear condition with $\alpha = 60^\circ, b = 0.5$
Figure D.15: Failure pattern of sample in torsional shear condition with $\alpha = 60^\circ, b = 1$