Behaviour of buildings due to tunnel induced subsidence

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Abstract

In urban areas tunnelling can induce ground movements which distort and, in severe cases, damage overlying buildings and services. The conventional design methods, used in engineering practice to assess these deformations, are based on empiricism gained from green field sites and consequently do not account for important characteristics of these structures. Furthermore the mechanisms which control this tunnel-soil-structure interaction problem are not well understood.

Recently, a new approach, based on the relative stiffness of a building and the underlying soil, has been proposed to account for the effect of a building stiffness when predicting its deformation and potential damage. The objective of this thesis is to assess this new approach and extend its applicability. This has been achieved by performing parametric studies using both two and three dimensional finite element analyses and by reviewing field data obtained during the construction of the Jubilee Line Extension. In particular the effect of building weight, the nature of the contact between the building foundations and the soil and the geometry of the building with respect to the direction of tunnelling on the behaviour of the building has been investigated.

The results of these investigations have lead to an improved understanding of the tunnel-soil-building interaction problem. They have also verified and extended the applicability of the relative stiffness approach providing greater confidence for its use in engineering practice.
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# Contents

Acknowledgements

List of Figures

List of Tables

1 Introduction
   1.1 Background ........................................ 21
   1.2 Scope of research .................................. 22
   1.3 Layout of thesis .................................... 23

2 Tunnel induced ground and building deformation
   2.1 Introduction ........................................ 26
   2.2 Ground movement in greenfield conditions .......... 27
      2.2.1 Surface movement ............................... 27
         2.2.1.1 Transverse behaviour ....................... 27
         2.2.1.2 Longitudinal behaviour ..................... 30
         2.2.1.3 Volume loss ................................ 32
         2.2.1.4 Trough width parameter .................... 35
      2.2.2 Subsurface movement ............................ 36
      2.2.3 Summary ......................................... 41
   2.3 Numerical analysis of tunnel construction .......... 42
      2.3.1 Two dimensional analysis ....................... 42
      2.3.2 Three dimensional analysis ..................... 47
      2.3.3 Summary ......................................... 55
   2.4 Building damage assessment ........................ 56
      2.4.1 Definition of structure deformation ............ 57
      2.4.2 Evaluation of risk of damage ................. 58
2.4.2.1 Preliminary assessment ........................................... 59
2.4.2.2 Second stage assessment ........................................ 59
2.4.2.3 Detailed evaluation ............................................. 60
2.4.3 Category of damage ................................................. 61
2.4.4 The concept of critical strain .................................... 63
2.4.5 Calculation of building strain .................................... 64
2.4.6 Soil structure interaction .......................................... 68
  2.4.6.1 Numerical studies of the interaction problem ............ 69
  2.4.6.2 The relative stiffness approach ............................ 72
  2.4.6.3 Conclusions ................................................... 77
2.4.7 Summary .......................................................... 79

3 Method of analysis ...................................................... 81
  3.1 Introduction ......................................................... 81
  3.2 The Finite Element Method ........................................ 81
    3.2.1 Requirements for a solution ................................... 81
    3.2.2 Finite Element formulation .................................. 83
      3.2.2.1 Element discretisation ................................... 83
      3.2.2.2 Displacement approximation ............................... 84
      3.2.2.3 Element formulation ..................................... 84
      3.2.2.4 Global equations ......................................... 84
    3.2.3 Non linear FEM ................................................ 85
    3.2.4 Geotechnical considerations ................................ 86
      3.2.4.1 Pore water pressure ...................................... 86
      3.2.4.2 Excavation ............................................... 87
      3.2.4.3 Construction ............................................. 87
      3.2.4.4 Soil-Structure interface ................................. 88
  3.3 Material models .................................................... 89
    3.3.1 Non linear elastic behaviour .................................. 90
    3.3.2 Mohr Coulomb yield surface .................................. 92
  3.4 Modelling of the soil structure interaction .................... 93
    3.4.1 Initial stress conditions ..................................... 94
    3.4.2 Geometry of the problem ..................................... 94
    3.4.3 Modelling of the building .................................... 95
    3.4.4 Modelling of tunnel construction ............................ 98
    3.4.5 Calculation of the building deformation criteria ........ 100
4 An evaluation of the relative stiffness method

4.1 Introduction .................................................. 102
4.2 Parameter of FE analysis .................................... 103
  4.2.1 Volume loss ............................................... 103
  4.2.2 Mesh width ............................................... 106
  4.2.3 Initial stress ............................................. 108
  4.2.4 Summary .................................................. 110
4.3 Influence of building stiffness ............................. 111
4.4 Influence of geometry ....................................... 114
  4.4.1 Building width .......................................... 115
    4.4.1.1 Influence on deformation criteria ................ 115
    4.4.1.2 Influence on ground deformation ............... 119
  4.4.2 Eccentricity ........................................... 122
    4.4.2.1 Influence on deformation criteria ............. 122
    4.4.2.2 Influence on ground deformation ............. 131
  4.4.3 Tunnel depth ........................................... 133
    4.4.3.1 Influence on building deformation ............. 134
    4.4.3.2 Influence on ground deformation ............. 140
  4.4.4 Summary ................................................ 142
4.5 An alternative formulation for the relative stiffness .... 143
  4.5.1 Deflection ratio ........................................ 143
  4.5.2 Maximum horizontal strain ............................. 146
4.6 Conclusions ................................................ 147

5 The influence of building weight ............................ 149

5.1 Introduction ................................................ 149
5.2 Finite Element Analysis ................................... 149
5.3 Stress state ............................................... 151
  5.3.1 Introduction ........................................... 151
  5.3.2 Behaviour at tunnel depth ............................ 153
  5.3.3 Behaviour under the foundation ..................... 155
5.4 Parametric study ........................................... 157
  5.4.1 Deflection ratio ........................................ 157
  5.4.2 Horizontal strain ...................................... 160
5.5 Conclusions ................................................ 164
6 The influence of the soil-structure interface ........................................... 166
6.1 Introduction .................................................................................. 166
6.2 Finite Element analysis ................................................................. 167
6.3 Interface parameters ..................................................................... 167
   6.3.1 Elastic parameters ................................................................. 167
      6.3.1.1 Normal interface stiffness $K_n$ ........................................ 168
      6.3.1.2 Shear interface stiffness $K_s$ ........................................... 172
   6.3.2 Plastic parameters ................................................................. 173
   6.3.3 Choice of parameters ............................................................ 179
6.4 Ground deformation ................................................................. 180
6.5 Building deformation ............................................................... 185
   6.5.1 Deflection ratio ................................................................. 186
   6.5.2 Horizontal strain ................................................................. 189
6.6 Conclusions .............................................................................. 190

7 The influence of out of plane geometry ........................................... 192
7.1 Introduction .............................................................................. 192
7.2 Finite Element analysis ............................................................... 192
7.3 Behaviour in longitudinal direction ............................................. 194
7.4 Influence of building out of plane geometry ............................... 199
   7.4.1 Displacement behaviour .................................................... 200
   7.4.2 Deformation Criteria .......................................................... 209
7.5 Conclusions .............................................................................. 214

8 Prediction of three-dimensional greenfield settlement ...................... 216
8.1 Introduction .............................................................................. 216
8.2 Finite Element analysis of 3D tunnel construction ....................... 217
8.3 Greenfield settlement predictions .............................................. 219
   8.3.1 FE analysis for St. James’s Park ......................................... 220
   8.3.2 Isotropic soil model ......................................................... 220
   8.3.3 Anisotropic soil model ..................................................... 226
      8.3.3.1 Anisotropic soil parameters ....................................... 227
      8.3.3.2 Results ................................................................. 233
8.4 Conclusions .............................................................................. 239

9 The influence of step-by-step tunnel excavation ................................ 242
9.1 Introduction .............................................................................. 242
9.2 Parameters for FE analysis ........................................... 243
9.2.1 Distances to mesh boundaries ................................. 243
9.2.2 Length of tunnel construction ................................. 246
9.2.3 Excavation length .............................................. 249
9.3 Parametric study .................................................... 253
9.3.1 Transverse behaviour ........................................... 255
  9.3.1.1 Deflection ratio ............................................. 257
  9.3.1.2 Strain ..................................................... 260
9.3.2 Longitudinal behaviour ......................................... 262
  9.3.2.1 Deflection ratio ............................................. 266
  9.3.2.2 Strain ..................................................... 268
9.3.3 Twist ............................................................ 271
  9.3.3.1 Definition ................................................... 271
  9.3.3.2 Case studies ............................................... 275
  9.3.3.3 Results ................................................... 282
9.4 Conclusions ......................................................... 287

10 Design Charts ....................................................... 290
10.1 Introduction ....................................................... 290
10.2 Summary of previous chapters .................................. 292
  10.2.1 Transverse geometry .......................................... 292
  10.2.2 Building weight ............................................... 292
  10.2.3 Soil-structure interface ....................................... 293
  10.2.4 Longitudinal geometry ........................................ 293
  10.2.5 Tunnel construction process .................................. 294
10.3 An alternative formulation for dimension less modification factors .... 295
  10.3.1 Deflection ratio ............................................... 295
  10.3.2 Horizontal strain .............................................. 299
10.4 Conclusions ......................................................... 304

11 Conclusions and recommendations .................................. 307
11.1 Introduction ....................................................... 307
11.2 Mechanisms of ground movement ............................... 308
11.3 Building deformation ............................................. 312
11.4 Three-dimensional analysis ..................................... 316
11.5 Recommendations for future research ........................... 319
A The Finite Element Method

A.1 Basic formulation .............................................. 322
A.2 Non linear solution method ................................. 327
  A.2.1 Modified Newton-Raphson method ................... 328
  A.2.2 Substepping stress point algorithm ................. 329
    A.2.2.1 Modified Euler integration .................... 330

B A non-linear transversely anisotropic soil model

B.1 Introduction .................................................. 332
B.2 Original model formulation ............................... 333
B.3 Model development ......................................... 337
B.4 Model evaluation ........................................... 338
  B.4.1 Constant anisotropic scale factor .................. 338
  B.4.2 Variable anisotropic scale factor .................. 344
B.5 Summary ..................................................... 347

References ......................................................... 349
List of Figures

2.1 Geometry of the tunnel induced settlement trough. ................................. 27
2.2 Transverse settlement trough. ................................................................. 28
2.3 Distribution of horizontal surface displacement and strain in the transverse
direction together with settlement trough. .................................................. 30
2.4 Longitudinal settlement profile. ................................................................. 31
2.5 Different relations between stability number $N$ and volume loss $V_L$. ....... 34
2.6 Relation between load factor and volume loss, determined from centrifuge tests
and finite element analysis. ........................................................................ 34
2.7 Correlation between position of surface point of inflection, $i$, and tunnel depth $z_0$. ................................................................. 35
2.8 Variation of trough width parameter $i$ of subsurface settlement troughs with
depth. .......................................................................................................... 37
2.9 Variation of $K$ of subsurface settlement troughs with depth. .................. 37
2.10 Subsurface settlement above tunnel centre line. ....................................... 38
2.11 Distribution of $i$ for subsurface settlement troughs with depth and focus of
vectors of soil movement. ........................................................................... 40
2.12 Surface settlement troughs obtained from different isotropic soil models. ... 45
2.13 Surface settlement predictions obtained from different input parameters of non-
linear elastic perfectly plastic model. .............................................................. 45
2.14 Layout of zone of reduced $K_0$ (after Potts & Zdravković, 2001). .......... 46
2.15 Surface settlement troughs obtained from different anisotropic soil models. 46
2.16 Longitudinal settlement profiles obtained for different excavation lengths $L_{exc}$. 50
2.17 Longitudinal settlement profiles for different excavation methods in the first
excavation step. ......................................................................................... 51
2.18 Definition of building deformation. ............................................................. 57
2.19 Schematic diagram of three-stage approach for damage risk evaluation. .... 57
2.20 Tunnel induced building settlement of Mansion House, London. ............. 61
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.21</td>
<td>Cracking of a simple beam in different modes of deformation.</td>
<td>65</td>
</tr>
<tr>
<td>2.22</td>
<td>Relationship between $\Delta/B_{\epsilon_{\text{crit}}}$ and $B/H$ for rectangular beams deflecting due to combined bending and shear. Neutral axis in the middle.</td>
<td>66</td>
</tr>
<tr>
<td>2.23</td>
<td>Relationship between $\Delta/B_{\epsilon_{\text{crit}}}$ and $B/H$ for rectangular beams deflecting due to combined bending and shear. Neutral axis at the bottom.</td>
<td>66</td>
</tr>
<tr>
<td>2.24</td>
<td>Relation of damage to angular distortion and horizontal extension.</td>
<td>68</td>
</tr>
<tr>
<td>2.25</td>
<td>Relation of damage category to deflection ratio and horizontal tensile strain for hogging.</td>
<td>69</td>
</tr>
<tr>
<td>2.26</td>
<td>Geometry of the problem and definition of deflection ratio.</td>
<td>73</td>
</tr>
<tr>
<td>2.27</td>
<td>Design curves for modification factors of (a) deflection ratio and (b) maximum horizontal strain.</td>
<td>77</td>
</tr>
<tr>
<td>2.28</td>
<td>Schematic diagram of the relative stiffness approach within the three-stage risk evaluation.</td>
<td>78</td>
</tr>
<tr>
<td>3.1</td>
<td>Principle of FE excavation simulation.</td>
<td>88</td>
</tr>
<tr>
<td>3.2</td>
<td>Interface element.</td>
<td>89</td>
</tr>
<tr>
<td>3.3</td>
<td>Typical London Clay unconsolidated undrained test data: (a) initial stress strain behaviour; (b) stiffness-strain characteristics.</td>
<td>91</td>
</tr>
<tr>
<td>3.4</td>
<td>Range of shear strain observed during construction of different geotechnical structures.</td>
<td>92</td>
</tr>
<tr>
<td>3.5</td>
<td>Mohr-Coulomb model: (a) failure criterion; (b) view on deviatoric plane.</td>
<td>93</td>
</tr>
<tr>
<td>3.6</td>
<td>Geometry of the problem.</td>
<td>95</td>
</tr>
<tr>
<td>3.7</td>
<td>Combinations of bending and axial building stiffness.</td>
<td>98</td>
</tr>
<tr>
<td>3.8</td>
<td>Finite Element mesh for 20m symmetrical building geometry.</td>
<td>99</td>
</tr>
<tr>
<td>3.9</td>
<td>Development of volume loss over increment of excavation and percentage of unloading.</td>
<td>99</td>
</tr>
<tr>
<td>4.1</td>
<td>Development of volume loss $V_L$ (a) and settlement (b) during simulation of excavation process.</td>
<td>104</td>
</tr>
<tr>
<td>4.2</td>
<td>Development of greenfield settlement with volume loss $V_L$.</td>
<td>105</td>
</tr>
<tr>
<td>4.3</td>
<td>Settlement data from JLE westbound tunnel construction at St. James’s Park plotted against $V_L$.</td>
<td>105</td>
</tr>
<tr>
<td>4.4</td>
<td>Relationship between building deformation criteria and volume loss.</td>
<td>105</td>
</tr>
<tr>
<td>4.5</td>
<td>Transverse settlement trough for greenfield analyses with different mesh widths.</td>
<td>107</td>
</tr>
<tr>
<td>4.6</td>
<td>$M_{\text{DR}}$ for different initial stress conditions compared with results by Potts &amp; Addenbrooke (1997).</td>
<td>109</td>
</tr>
</tbody>
</table>
4.7 $M^{\text{hec}}$ for different initial stress conditions compared with results by Potts & Addenbrooke (1997) ................................................................. 109
4.8 Vertical profile of horizontal and vertical soil displacement at $x = 6m.$ .... 112
4.9 Vertical profile of vertical soil displacement along a line above the tunnel CL, $x = 0m.$ ................................................................. 113
4.10 Point of inflection for surface and subsurface settlement troughs. .......... 113
4.11 Development of deflection ratio with $B.$ (a) Greenfield conditions from Gaussian settlement trough and from FE analysis; (b) $DR_{\text{sag}}$ in buildings. ... 116
4.12 $M^{\text{DR}}$ against $\rho^*$ for varying building width and stiffness for $z_0 = 20m.$ 117
4.13 Development of max. horizontal strain with $B.$ (a) Greenfield conditions from Gaussian settlement trough and from FE analysis; (b) $\epsilon_{\text{hc}}$ in buildings. 118
4.14 $M^{\text{hec}}$ against $\alpha^*$ for varying building width and stiffness. .......... 120
4.15 Position of point of inflection of surface and subsurface settlement troughs for different building widths. ................................................................. 120
4.16 Vertical profile of horizontal soil movement due to tunnel construction at $x = 6m$ for different building widths. ................................................................. 121
4.17 Vertical profile of vertical soil movement above the tunnel crown for different building widths. ................................................................. 121
4.18 Different position of a section (or building) in respect to the settlement trough 123
4.19 Variation of deflection ratio with eccentricity $e$ for different greenfield section widths $B.$ Results from spreadsheet calculation. ......................... 124
4.20 Variation of deflection ratio with eccentricity $e$ for different greenfield section widths $B.$ Results from FE analyses. ................................. 124
4.21 Deflection ratio with $e/B$ for different greenfield section widths $B.$ Results from FE analyses. ................................................................. 125
4.22 Deflection ratio with $e/B$ for different building widths $B$ of 5 storey structure. 125
4.23 Positions of points of inflection with eccentricity $e$ together with building edges and greenfield points of inflection. ................................................................. 126
4.24 Change of $M^{\text{DR}}$ with $e/B$ for a range of buildings of different widths $B.$ ... 126
4.25 Different position of a section (or building) in respect to the horizontal strain distribution. ................................................................. 128
4.26 Maximum horizontal strain with eccentricity $e$ for greenfield sections with different widths $B.$ Results from spreadsheet calculation. ......................... 129
4.27 Maximum horizontal strain with eccentricity $e$ for greenfield sections of different widths $B.$ Results from FE analyses. ................................. 129
4.28 Maximum horizontal strain with \( e/B \) for greenfield sections of different widths

4.29 Maximum horizontal strain with \( e/B \) for 5 storey buildings of different widths

4.30 Change of \( M^e \) with \( e/B \) for a range of 5 storey buildings with different widths

4.31 Vertical profile of horizontal soil movement due to tunnel construction at \( \pm 6m \) distance to tunnel centre line. (a): \( e = 12m \); (b): \( e = 48m \).

4.32 Vertical profile of settlement above tunnel crown.

4.33 Greenfield surface deformation depending on tunnel depth \( z_0 \).

4.34 Building deformation above tunnel of different \( z_0 \).

4.35 (a) \( DR_{sag} \) and (b) \( M^{DR}_{sag} \) plotted versus tunnel depth \( z_0 \) for different building geometries of 5 storey structures.

4.36 (a) \( DR_{hog} \) and (b) \( M^{DR}_{hog} \) plotted versus tunnel depth \( z_0 \) for different building geometries of 5 storey structures.

4.37 Modification factors \( M^{DR} \) for different tunnel depths \( z_0 \).

4.38 \( \epsilon_{hc} \) and \( M^{\epsilon_{hc}} \) plotted versus tunnel depth \( z_0 \) for different building geometries of 5 storey structures.

4.39 \( \epsilon_{ht} \) and \( M^{\epsilon_{ht}} \) plotted versus tunnel depth \( z_0 \) for different building geometries of 5 storey structures.

4.40 Modification factors \( M^{f_h} \) for different tunnel depths \( z_0 \).

4.41 Vertical profile of horizontal and vertical soil displacement at \( x = 6m \) for \( z_0 = 20m \) and 34m.

4.42 \( M^{DR} \) against \( \rho^* \) for buildings of no eccentricity.

4.43 \( M^{DR} \) against \( \rho_{m1}^* \) for buildings of no eccentricity.

4.44 \( M^{DR} \) against \( \rho^* \) for buildings with eccentricity.

4.45 \( M^{DR} \) against \( \rho_{m1}^* \) for buildings with eccentricity.

4.46 \( M^{f_h} \) against \( \alpha^* \) for buildings with varying eccentricity.

4.47 \( M^{f_h} \) against \( \alpha_{m1}^* \) for buildings with varying eccentricity.

5.1 Matrix of stiffness/stress combinations used in the parametric study.

5.2 Vertical profile of horizontal soil movement during tunnel construction.

5.3 Profile of normalized mean effective stress \( p' \) over depth on centre line of building prior to tunnel construction.

5.4 Vertical profile of horizontal soil movement during tunnel construction for stress scenario 1.
5.5 Vertical profile of horizontal soil movement during tunnel construction for stress scenario 2. ........................................ 155
5.6 Soil stiffness profile on centre line of 5-storey building prior to tunnel construction. ........................................ 156
5.7 Vertical profile of horizontal soil movement during tunnel construction for modified soil model used in top soil layer. ........................................ 156
5.8 Change of modification factor (a) $M^{DR}_{hog}$ and (b) $M^{DR}_{ass}$ with applied load. 158
5.9 Change of modification factor $M^{DR}_{hog}$ with stiffness. ........................................ 159
5.10 Modification factors $M^{DR}$ together with the design curves. ........................................ 159
5.11 Horizontal strain distribution in structure for standard soil model and modified top layer. ........................................ 161
5.12 Change of (a) $M^{\epsilon}_{hc}$ and (b) $M^{\epsilon}_{ht}$ with applied load. ........................................ 162
5.13 Change of tensile strain modification factor $M^{\epsilon}_{ht}$ with stiffness. ........................................ 162
5.14 Strain modification factors $M^{\epsilon}_{h}$ together with the design curves. ........................................ 163
5.15 Strain modification factors $M^{\epsilon}_{h}$ versus a modified relative axial stiffness taking into account the change of soil stiffness beneath the foundation. ........................................ 163

6.1 Settlement profiles for 100m wide 5-storeys building with different values of normal interface stiffness $K_n$. ........................................ 169
6.2 Distribution of (a) normal interface strain $\epsilon_{if}$ and (b) shear interface strain $\gamma_{if}$ along the building for different values of $K_n$. ........................................ 169
6.3 (a) Distribution of $\epsilon_{if}$ along building for a high value of $K_n$ using the original mesh (b). Maximum length of interface elements: 2m. ........................................ 171
6.4 (a) Distribution of $\epsilon_{if}$ along building for a high value of $K_n$ using refined 'mesh 2' (b). Maximum length of interface elements: 1m. ........................................ 171
6.5 (a) Distribution of $\epsilon_{if}$ along building for a high value of $K_n$ using refined 'mesh 3' (b). Maximum length of interface elements: 0.5m. ........................................ 171
6.6 Distribution of shear interface strain $\gamma_{if}$ along the building for different values of $K_n$. ........................................ 173
6.7 Distribution of shear interface strain $\gamma_{if}$ along the building for different values of $\epsilon_{if}$. ........................................ 173
6.8 Distribution of shear interface strain $\gamma_{if}$ along the building for different values of $\varphi_{if}$ for (a) no-load case and for (b) building load of 10kPa. ........................................ 174
6.9 Stress paths of interface elements at different position for elastic and elastoplastic analyses of non-load cases. ........................................ 176
6.10 Stress paths of interface elements at different position for elastic analysis of 10kPa load cases. ........................................ 176
6.11 Stress paths of interface element at $x = 0\text{m}$ for elastic and elasto-plastic analyses of no-load cases. .................................................. 176
6.12 Distribution of interface normal strain $\epsilon_{if}$ along the building analyses with different degree of separation between soil and building. ....................... 178
6.13 Distribution of interface shear strain $\gamma_{if}$ along the building analyses with different degree of separation between soil and building. ....................... 178
6.14 Building settlement for analyses with different degree of separation between soil and building. ............................................................. 179
6.15 Distribution of shear interface strain $\gamma_{if}$ along the building for elastic and elasto-plastic analyses. ......................................................... 180
6.16 Building settlement for elastic and elasto-plastic analyses. ....................... 180
6.17 Vertical profile of horizontal soil displacement below 100m-wide 5-storey buildings with and without interface elements. ................................. 182
6.18 Horizontal profile of horizontal displacement below 100m-wide 5-storey buildings with and without interface elements. ................................. 183
6.19 Horizontal profile of vertical settlement below 100m-wide 5-storey buildings with and without interface elements. ................................. 184
6.20 Position of point of inflection, $i$, with depth for interface-cases of different building stiffness. ............................................................. 185
6.21 Variation of (a) $DR_{sagif}$ and (b) $DR_{hogif}$ with $K_s$. ................................. 186
6.22 Comparison of (a) $M_{DR,sag}$ and (b) $M_{DR,hog}$ of interface and non-interface analyses for different geometries and building stiffnesses. ......................... 187
6.23 Modification factors (no eccentricity) $M_{DR}$ together with the design curves by Potts & Addenbrooke (1997). .......................................................... 188
6.24 Modification factors (building eccentricity) $M_{DR}$ together with the design curves by Potts & Addenbrooke (1997). .......................................................... 188
6.25 Variation of $\epsilon_{hclf}$ with $K_s$. ............................................................. 189

7.1 Finite element mesh for 100m $\times$ 100m building. Only one quarter of the problem is modelled ................................................................. 193
7.2 Vertical surface settlement around 5 storey building. ............................... 195
7.3 Comparison between 3D and plane strain settlement curves (transverse to tunnel). ................................................................. 196
7.4 Comparison between 3D and plane strain horizontal $S_{hr}$ displacement profile (transverse to tunnel). ................................................................. 196
7.5 Comparison of surface settlement on building centre line ($y = 0\text{m}$) and building edge ($y = -50\text{m}$). ................................................................. 197
7.6 Comparison of horizontal surface $S_{hz}$ displacement on building centre line $(y = 0m)$ and building edge $(y = -50m)$. 197
7.7 Comparison of longitudinal profiles of (a) surface settlement and (b) horizontal surface $S_{hz}$ displacement along a 5 storey building. 198
7.8 Comparison of longitudinal profiles of (a) surface settlement and (b) horizontal surface $x$-displacement along a 1 storey building. 198
7.9 Finite element mesh used in parametric study to investigate the influence of $L$. 199
7.10 Transverse settlement profiles for 5 storey structures of varying length $L$. 201
7.11 Transverse horizontal $x$-displacement profiles for 5 storey structures of varying length $L$. 201
7.12 Longitudinal distribution of $V_L$ (calculated from surface settlement) for different building lengths. 202
7.13 Longitudinal distribution of $V_L$ (calculated from soil movement along tunnel boundary) for different building lengths. 202
7.14 Vertical profile of horizontal soil movement beneath edge of buildings with varying length. 203
7.15 Comparison of transverse settlement profiles for 5 storey structure of $L = 8m$ and varying $y$-distance to mesh boundary. 204
7.16 Transverse settlement profiles for 1 storey structures of varying length $L$. 205
7.17 Transverse horizontal $x$-displacement profiles for 1 storey structures of varying length $L$. 205
7.18 Plane strain results of transverse (a) settlement and (b) horizontal $x$-displacement for structures with constant bending but different axial stiffnesses. 206
7.19 Normalized results of (a) settlement and (b) horizontal $x$-displacement for $L = 1m$ and $8m$ structures with constant bending but different axial stiffnesses. 207
7.20 Change of sagging modification factor $M_{DRass}$ with building length $L$. 209
7.21 Change of hogging modification factor $M_{DRass}$ with building length $L$. 209
7.22 Change of compressive modification factor $M_{Drhoc}$ with building length $L$. 210
7.23 Deflection ratio modification factors from 3D analyses plotted against relative bending stiffness calculated for 3D geometry. 211
7.24 Deflection ratio modification factors from 3D analyses plotted against their corresponding plane strain relative bending stiffness. 211
7.25 Compressive modification factors from 3D analyses plotted against relative bending stiffness calculated for 3D geometry. 212
7.26 Compressive modification factors from 3D analyses plotted against their corresponding plane strain relative bending stiffness. 212
8.1 FE mesh for tunnel excavation beneath St. James’s Park greenfield monitoring site. ............................................ 218
8.2 Sequence of tunnel excavation simulation. ......................................................... 218
8.3 Surface settlement trough obtained from St. James’s Park greenfield FE analysis. 221
8.4 Longitudinal settlement profiles from St. James’s Park greenfield FE analysis (isotropic soil model). ..................................................... 222
8.5 Transverse settlement profiles from St. James’s Park greenfield FE analysis (isotropic soil model). ..................................................... 223
8.6 Development of volume loss with advancing tunnel construction (St. James’s Park FE analysis, isotropic soil model). ......................... 224
8.7 Transverse settlement profiles from 3D and 2D FE analyses (St. James’s Park FE analysis, isotropic soil model) and field measurements. .............. 224
8.8 Transverse settlement troughs, shown in Figure 8.7b, normalized against maximum settlement. ...................................................... 225
8.9 Stress-strain curves for analyses of undrained triaxial extension test using isotropic and anisotropic soil models. .............................. 231
8.10 Stiffness-strain curves for isotropic and anisotropic soil models. ......................... 232
8.11 Transverse settlement profile from 2D FE analyses with isotropic and anisotropic soil behaviour, together with field measurements. ................. 233
8.12 Development of volume loss in isotropic (M1) and anisotropic (M2) 2D analyses. 235
8.13 Normalized transverse settlement profile of 2D FE analyses for isotropic and anisotropic soil behaviour together with field measurements. ......................... 235
8.14 Longitudinal settlement profiles from St. James’s Park greenfield 3D analysis (anisotropic soil model). ..................................................... 236
8.15 Hypothetical longitudinal settlement trough for further advanced tunnel construction. ........................................................................ 237
8.16 Normalized longitudinal settlement profiles from isotropic and anisotropic 3D analyses. ................................................................. 237
8.17 Normalized transverse settlement profiles of 2D and 3D analyses using isotropic (M1) and anisotropic (M2) soil models. ............................. 239

9.1 Influence of mesh dimension, varying $L_{\text{soil}}$ ................................................. 244
9.2 Influence of mesh dimension, varying $L_{\text{tunnel}}$ ................................................. 245
9.3 Development of longitudinal settlement profile with tunnel progress for greenfield conditions with $K_0 = 1.5$. ........................................ 247
9.4 Development of longitudinal settlement profile with tunnel progress for greenfield conditions with $K_0 = 0.5$. ........................................ 247
9.5 Development of $V_L$ in 1-storey building analyses with different $L_{exc}$. 250
9.6 Development of $DR_{sag}$ and $DR_{hog}$ with tunnel advance for different $L_{exc}$. 250
9.7 Development of $M^{DR}_{sag}$ and $M^{DR}_{hog}$ with $V_L$ for different $L_{exc}$. 250
9.8 Development of $\epsilon_{hc}$ with tunnel advance for different $L_{exc}$. 251
9.9 Development of $\epsilon_{hc}$ with $V_L$ for different $L_{exc}$. 251
9.10 Interpolation of deformation criteria to a common volume loss. 252
9.11 Finite element mesh for fully 3D analyses of tunnel-soil-building interaction. 254
9.12 Transverse settlement and horizontal displacement for different tunnel face position on transverse centre line of 3-storey building. 255
9.13 Development of $i$ with tunnel progress. 256
9.14 Development of $V_L$ with tunnel progress. 256
9.15 Development of $DR_{sag}$ and $DR_{hog}$ with volume loss and comparison with plane strain results. 258
9.16 Development of $M^{DR}_{sag}$ and $M^{DR}_{hog}$ with volume loss and comparison with plane strain results. 258
9.17 Development of $\epsilon_{hc}$ with volume loss and comparison with plane strain results. 260
9.18 Development of $M^{\epsilon_{hc}}$ with volume loss and comparison with plane strain results. 260
9.19 Comparison between $L = 20m$ and $30m$ geometry: (a) settlement trough, (b) horizontal displacement. 261
9.20 Longitudinal settlement profiles for different tunnel face positions. 262
9.21 Longitudinal settlement profiles for different tunnel face positions. 263
9.22 Longitudinal settlement profiles for different tunnel face positions. 264
9.23 Development of greenfield longitudinal $DR_{sag}$ and $DR_{hog}$ with tunnel progress.
Comparison between spread sheet calculation and FE results. 267
9.24 Development of longitudinal $DR_{hog}$ in $20m \times 100m$ buildings. 267
9.25 Development of greenfield longitudinal horizontal strain with tunnel progress. Results from spread sheet calculation. 269
9.26 Development of greenfield longitudinal horizontal strain with tunnel progress. Results from FE analysis. 269
9.27 Development of longitudinal horizontal strain with tunnel progress in 5-storey building. 270
9.28 (a) Torsion of a single rod; (b) definition of angle of twist per unit length. 272
9.29 Twist of a thin shell caused by moments $m_{xy}$. 274
9.30 Deformation of shell and definition of displacement of corner nodes. 274
9.31 Plan view of Elizabeth House with position of JLE tunnel. 278
9.32 Tunnelling sequence beneath Elizabeth House. 278
9.33 Settlement across Elizabeth House during construction of westbound and eastbound tunnels. 279
9.34 Development of twist along Elizabeth House during construction of westbound and eastbound tunnels. 280
9.35 Development of twist of Elizabeth House with time during construction of westbound and eastbound tunnel. 282
9.36 Development of twist of Elizabeth House with time between westbound tunnel construction and start of cross-over excavation. 282
9.37 Development of twist with tunnel progress. Results from FE analyses of 20m x 100 geometries. 283
9.38 Development of twist with tunnel progress for different greenfield and building geometries. 285
9.39 Development of rotation with tunnel progress for longitudinal building edges. 285
9.40 Development of twist within 20m x 100m geometry for different tunnel depths for greenfield conditions and 3-storey building. 287

10.1 $M^{DR}$ for cases with no eccentricity plotted against $\rho^*$. 296
10.2 $M^{DR}$ for cases with no eccentricity plotted against modified relative bending stiffness $\rho^*_{mod}$. 296
10.3 $M^{DR}$ for cases with no eccentricity but additional building features plotted against $\rho^*_{mod}$. 297
10.4 $M^{DR}$ for cases with eccentricity plotted against $\rho^*$. 299
10.5 $M^{DR}$ for cases with eccentricity plotted against modified relative bending stiffness $\rho^*_{mod}$. 299
10.6 $M^{DR}$ for eccentric cases with additional building features plotted against $\rho^*_{mod}$. 300
10.7 $M^f$ plotted against $\alpha^*$. 301
10.8 $M^f$ plotted against modified relative axial stiffness $\alpha^*_{mod}$. 302
10.9 $M^f$ for cases with additional building features plotted against $\alpha^*_{mod}$. 303
10.10 Proposed design curves for $M^{DR}$ adopting the modified relative bending stiffness $\rho^*_{mod}$. 305
10.11 Proposed design curves for $M^f$ adopting the modified relative bending stiffness $\alpha^*_{mod}$. 306

11.1 Relation between different elements of the tunnel-soil-building interaction problem. 311

A.1 Coordinate systems of parent element and global element. 323
A.2 Shape function for corner node of 8 node 2D element. ....................... 324
A.3 The Modified Newton-Raphson method. ........................................... 329
A.4 Sub-stepping stress point algorithm. .................................................. 331

B.1 Horizontal layers of transversely isotropic material. ......................... 334
B.2 Strain-stiffness curves for isotropic material behaviour. ................. 339
B.3 Settlement trough for isotropic soil behaviour. .............................. 339
B.4 Strain-stiffness curves for isotropic and anisotropic material. .......... 341
B.5 Stress path for isotropic and anisotropic material in triaxial extension. 341
B.6 Stress-strain behaviour and pore water response for isotropic and anisotropic material in triaxial extension. ......................... 342
B.7 Settlement troughs for isotropic and anisotropic soil behaviour. ....... 343
B.8 Development of Young’s moduli with strain for isotropic and anisotropic (constant and variable) material. ......................... 345
B.9 Development of shear moduli with strain for isotropic and anisotropic (constant and variable) material. ......................... 345
B.10 Stress paths for isotropic and anisotropic (constant and variable) material. .............................. 346
B.11 Settlement troughs for isotropic and anisotropic (constant and variable) material behaviour. .............................. 346
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Details of 3D FE analyses summarized in this chapter.</td>
<td>48</td>
</tr>
<tr>
<td>2.2</td>
<td>Classification of visible damage to walls with particular reference to ease of repair of plaster and brickwork masonry.</td>
<td>62</td>
</tr>
<tr>
<td>2.3</td>
<td>Relation between category of damage and limiting tensile strain.</td>
<td>63</td>
</tr>
<tr>
<td>3.1</td>
<td>Basic solution requirements satisfied by the various methods of analysis.</td>
<td>83</td>
</tr>
<tr>
<td>3.2</td>
<td>Material parameters used for non-linear elastic soil model.</td>
<td>92</td>
</tr>
<tr>
<td>3.3</td>
<td>Material parameters used in Mohr-Coulomb model.</td>
<td>93</td>
</tr>
<tr>
<td>3.4</td>
<td>Stiffness of buildings. A $m$-storey building consists of $m+1$ slabs.</td>
<td>97</td>
</tr>
<tr>
<td>4.1</td>
<td>Volume loss for different greenfield situations and different building stiffness.</td>
<td>106</td>
</tr>
<tr>
<td>4.2</td>
<td>Deflection ratio and compressive strain for different mesh widths and initial stress conditions.</td>
<td>108</td>
</tr>
<tr>
<td>4.3</td>
<td>Initial soil stiffness at different half tunnel depths $z_0/2$.</td>
<td>137</td>
</tr>
<tr>
<td>5.1</td>
<td>Stress state at $z = 34$m prior to tunnel construction depending on applied building load.</td>
<td>153</td>
</tr>
<tr>
<td>5.2</td>
<td>Stress state scenario 1: Constant lateral stress ratio, while $p'$ is varied.</td>
<td>153</td>
</tr>
<tr>
<td>5.3</td>
<td>Stress state scenario 2: Constant $p'$ while the lateral stress ratio is varied.</td>
<td>153</td>
</tr>
<tr>
<td>5.4</td>
<td>Deflection ratio and horizontal strain for greenfield conditions.</td>
<td>157</td>
</tr>
<tr>
<td>7.1</td>
<td>Reduction of axial stiffness while keeping bending stiffness constant.</td>
<td>206</td>
</tr>
<tr>
<td>8.1</td>
<td>Material parameters of tunnel lining.</td>
<td>219</td>
</tr>
<tr>
<td>8.2</td>
<td>Input parameters for the isotropic and anisotropic soil models used for analysing St. James’s Park greenfield site.</td>
<td>229</td>
</tr>
<tr>
<td>8.3</td>
<td>Stiffness ratios for the two sets of anisotropic soil parameters.</td>
<td>229</td>
</tr>
<tr>
<td>9.1</td>
<td>Mesh details and calculation time for analyses with different $L_{exc}$.</td>
<td>249</td>
</tr>
</tbody>
</table>
9.2 Summary of $DR_{sag}$ and $M^{DR}{}_{sag}$ of all 3D analyses. 259
9.3 Summary of $DR_{hog}$ and $M^{DR}{}_{hog}$ of all 3D analyses. 259
9.4 Summary of $\epsilon_{hc}$ and $M^{\epsilon}{}_{hc}$ of all 3D analyses. 261

B.1 Material parameters used for non-linear elastic soil model M1 (identical to Table 3.2). 340
B.2 Input parameter for anisotropic soil model and different cases of $\alpha$. 340
Chapter 1

Introduction

1.1 Background

The growth of many cities has resulted in the need for increased infrastructure. As urban space becomes more limited, subsurface structures such as tunnels are becoming more efficient in providing the required infrastructure. In 1863 the first underground railway line was opened in London. Since then over 100 cities worldwide have implemented underground transport systems and over 50% of them are undergoing development or expansion (Hellawell et al., 2001). The latest line which was added to the London Underground system was the Jubilee Line Extension, which opened in 1999.

As such, new tunnel projects have to be constructed beneath high density urban areas. The construction and operation of these systems can cause a restriction of services, and damage to surface or other subsurface structures. The prediction of tunnel induced building deformation therefore becomes a key issue in the planning process. Current design approaches are conservative and lead to unnecessary expenditure in the design and construction of protective measures. A better understanding into the mechanisms that control the tunnelling induced deformations could reduce costs and help avoid disputes and resolve claims.

Recently a new approach taking into account the interaction between building stiffness and soil was proposed to refine the prediction of tunnel induced building deformation. This approach by Potts & Addenbrooke (1997) can be incorporated in the current damage risk assessment and may reduce the number of buildings for which an expensive, detailed evaluation has to be performed.
1.2 Scope of research

The main objective of this thesis is to assess the relative stiffness method (Potts & Addenbrooke, 1997). This approach considers the stiffness of a structure when predicting its deformation and potential damage caused by tunnel construction. The approach provides a simple design tool for engineering practice and was employed during the planning process of the Jubilee Line Extension, London (Mair & Taylor, 2001). The simplicity of the design charts is, however, in contrast to the complex nature of the tunnel-soil-building interaction problem in which not only the tunnel construction causes deformation to the building but the presence of the building also influences the soil displacement around the tunnel.

This thesis investigates the mechanisms involved in this interaction problem and highlights the influence of different building features on both building and ground deformation. As 3D Finite Element (FE) analysis is employed in some of the studies presented in this thesis, greenfield settlement predictions obtained from 3D analyses are evaluated and the influence of soil properties such as anisotropy on the settlement behaviour is assessed.

This thesis focuses on the geotechnical aspects of the interaction problem. Damage within the building or the influence of non-linear building behaviour were not considered during this research.

**Relative stiffness method:** The relative stiffness method was based on a parametric study of plane strain FE analyses in which building stiffness and geometry were varied. However, building features such as building weight, the influence of the soil structure interface or the three dimensional effects of building geometry and tunnel construction were not included in the original work of Potts & Addenbrooke (1997). This thesis presents a number of parametric studies in which the influence of all these building features is studied independently to assess their influence on building deformation. The findings of the various parametric studies are used to propose a modified formulation of the relative stiffness expressions originally proposed by Potts & Addenbrooke (1997). These new expressions account for the influence of a wider range of building features and are dimension-less, whether used in two or three dimensional situations.
Mechanisms of interaction problem: The influence of the various building characteristics on the ground movement are also investigated individually. It is therefore possible to structure the interaction problem and to highlight the contribution of each building feature on the ground deformation. This improves our understanding of the tunnel-soil-structure interaction.

Assessment of 3D FE tunnel analysis As 3D FE analyses are employed during the research, greenfield settlement predictions from such simulations are assessed and compared with field data and results from plane strain analyses. The influence of different soil properties (anisotropy), initial stress conditions and the extent of the FE mesh are investigated.

1.3 Layout of thesis

Chapter 2: A literature review presents the currently used design approaches for predicting tunnel induced soil movement and summarizes numerical studies, both 2D and 3D, which have modelled tunnel construction in greenfield conditions. Furthermore, this chapter describes the damage risk evaluation for buildings subjected to tunnel construction. Numerical studies are presented which focus on soil-structure interaction. It is shown how the relative stiffness approach models the interaction problem and how it is included into the currently used risk assessment.

Chapter 3: The Finite Element Method (FEM) employed in this research project is briefly introduced and specific details related to the use of ICFEP (Imperial College Finite Element Programme) are described. The Chapter also introduces geotechnical consideration of the FEM. It also describes how the tunnel-building interaction is modelled and which soil models and initial stress profiles are applied throughout this thesis.

Chapter 4: This chapter investigates the influence of the geometric parameters which were considered in the original study of Potts & Addenbrooke (1997). By varying them independently the effect of the building features are studied uncoupled from each other. The chapter also assesses the influence of such parameters as volume loss and the initial stress conditions. The sensitivity of the relative stiffness expressions to the geometric input parameters is investigated.
Chapter 5: As an additional building feature, building weight is included in the plane strain analyses presented in this chapter. The influence on the relative stiffness method is quantified and the effects of building weight on the ground movement in the vicinity of the tunnel and of the building is investigated.

Chapter 6: This chapter investigates how changes in the soil-structure interface affect both building and ground deformation. It focusses on a situation where relative horizontal movement between the building and soil is allowed.

Chapter 7: The influence of the out of plane dimension of the building (in the longitudinal direction parallel to the tunnel axis) is studied in this chapter by performing a number of 3D FE analyses in which the tunnel is constructed simultaneously over the whole mesh while the building is represented by its width and its length.

Chapter 8: This chapter presents a number of 3D analyses of tunnel excavation under greenfield conditions. Tunnel construction is modelled by incremental excavation, progressing in the longitudinal direction. This chapter highlights how 3D modelling affects the transverse settlement profile by comparing these results with both 2D analyses and field data. The longitudinal settlement trough is also assessed. This chapter studies the influence of soil anisotropy on the settlement predictions from both 2D and 3D analyses.

Chapter 9: In this chapter the same tunnel excavation sequence as in the previous chapter is modelled and its influence on building deformation is assessed. The chapter also highlights the problem of boundary end effects on the results obtained from 3D analyses. Results of building deformation transverse to the tunnel axis are compared with corresponding plane strain analyses. Additionally the deformation of the building in the longitudinal direction, and the development of twist are studied.

Chapter 10: This chapter summarizes the influence of the various building parameters on the building deflection ratio and horizontal strain deformation criteria. It shows how the results from the previous chapters can be applied to modify the relative stiffness expressions to give dimension-less factors for use in design.
Chapter 1: Introduction

Section 1.3

Chapter 11: The conclusions of the previous chapters are summarized in this chapter and recommendations for future research are given.
Chapter 2

Tunnel induced ground and building deformation

2.1 Introduction

The construction of tunnels in soft ground lead inevitably to ground movement. In an urban environment this movement can affect existing surface or subsurface structures. This Chapter summarizes methods to estimate tunnel induced ground movement and to assess its potential damage on buildings.

While a semi-empirical method is adopted to predict tunnel induced soil movement under greenfield conditions (i.e. without the presence of any other structure) such an approach is not suitable to predict the behaviour of existing structures subjected to the influence of tunnel construction. Tunnel construction affects surface and subsurface structures (such as buildings or services) but their presence also alters ground movement around the tunnel. To analyse this interaction problem numerical methods, mainly the Finite Element Method, provide a flexible tool which has been adopted by many authors. This chapter will compare a number of numerical studies by different authors to model tunnel construction in soft soils.

The currently adopted design approach to assess potential building damage along a route of a constructed tunnel will be presented. It will be shown how numerical models can be used to refine this building damage assessment.

The terms defining the geometry and settlement and the coordinate system which will be
Figure 2.1: Geometry of the tunnel induced settlement trough (after Attewell et al., 1986).

adopted throughout the thesis are defined in Figure 2.1. The coordinate system is defined such as \( x \) denotes the distance from the tunnel centre line in the transverse direction, \( y \) is the coordinate in the longitudinal direction and \( z \) is the depth below the soil surface. In this chapter the origin of the coordinate system is above the tunnel face, as shown in Figure 2.1. However, when presenting results from 3D FE analyses (Chapters 8 and 9) the origin will be located at the front boundary of the FE mesh (as described in the appropriate chapters). The displacement variables are defined as \( S_v \) describing vertical displacement while \( S_{hx} \) and \( S_{hy} \) denote horizontal displacement in the transverse and in the longitudinal direction, respectively.

### 2.2 Ground movement in greenfield conditions

#### 2.2.1 Surface movement

##### 2.2.1.1 Transverse behaviour

Figure 2.1 shows the development of a surface settlement trough caused by tunnel construction. Peck (1969) stated that the transverse settlement trough can be described by a
Figure 2.2: Transverse settlement trough.

Gaussian error function and this mathematical description has been widely accepted since then. Following this approach, the vertical settlement in the transverse direction is given by

$$S_v(x) = S_{v,\text{max}} e^{-\frac{x^2}{2\sigma^2}}$$

(2.1)

where $S_{v,\text{max}}$ is the maximum settlement measured above the tunnel axis. The parameter $\sigma_x$, the trough width parameter, represents the standard deviation in the original Gaussian equation. A typical transverse settlement trough is shown in Figure 2.2. It can be seen that the trough has its maximum slope at the point of inflection which is located at the distance $\sigma_x$ from the tunnel centre line. It will be shown later that this point is crucial when determining building deformation criteria. As the position of maximum slope the point of inflection separates the sagging zone from the hogging zone as shown in Figure 2.2.

The area below a Gaussian error function is by definition equal to 1 as it represents the probability that the variable $x$ has a value between $-\infty$ and $\infty$. However, the parameters $S_{v,\text{max}}$ and $\sigma_x$ in the above equation are mathematically independent. Consequently, the area enclosed by the settlement trough can have a value expressed by

$$V_S = \int_{-\infty}^{\infty} S_v \, dx = \sqrt{\frac{2\pi}{\sigma_x}} S_{v,\text{max}}$$

(2.2)

where $V_S$ is the volume of the settlement trough per unit length.

In materials with a low permeability such as stiff clay the initial response of the ground to the tunnel construction can be considered to be undrained. The volume of the surface settlement trough therefore is equal to the volume of soil which is excavated in excess of the
theoretical volume of the tunnel. It is common to specify this excess volume as a proportion of the theoretical tunnel volume (per unit length):

\[ V_L = \frac{V_S}{\pi \frac{D^2}{4}} \]  

(2.3)

where \( V_L \) is the volume loss and \( D \) is the outer tunnel diameter. It is normally expressed as a percentage. After combining Equations 2.1 to 2.3 the transverse settlement profile can be expressed in terms of volume loss:

\[ S_v(x) = \sqrt{\frac{\pi}{2}} \frac{V_L D^2}{4i_x} e^{-\frac{x^2}{2i_x^2}} \]  

(2.4)

For a given tunnel diameter \( D \) the shape and the magnitude of the transverse settlement curve therefore only depends on the volume loss \( V_L \) and the trough width \( i_x \). These two crucial parameters will be discussed in more detail after deriving the horizontal displacement components in the following paragraphs.

O’Reilly & New (1982) showed that the horizontal surface soil displacement in the transverse direction can be derived from the above equations assuming that the resultant displacement vectors point towards the centre of the tunnel. The horizontal soil surface displacement in the transverse direction can then be expressed by

\[ S_{hz}(x) = -\frac{x}{z_0} S_v(x) \]  

(2.5)

Figure 2.3 shows the horizontal displacement together with the Gaussian settlement trough. It can be seen that the maximum horizontal displacement occurs at the point of inflection. The horizontal strain in the transverse direction is also shown in this graph. It is obtained by differentiating the horizontal displacement with respect to \( x \). This yields

\[ \epsilon_{hz}(x) = \frac{S_v(x)}{z_0} \left( \frac{x^2}{i_x^2} - 1 \right) \]  

(2.6)

This equation leads to compression being defined as negative while a positive value is assigned to tension. This sign convention, which is in contrast to the commonly used strain definition in soil mechanics, will be adopted throughout this thesis when describing horizontal strain within a building. It will be shown that horizontal strain is a crucial criterion when describing deformation of a building. In the following the index \( x \) will be omitted when describing horizontal strain in transverse direction and \( \epsilon_h \) will be used instead.
Figure 2.3 shows that there is a compression zone between the two points of inflection. Outside these points tensile strain develops. The maximum (absolute) values of $\epsilon_{hx}$ will be referred to as $\hat{\epsilon}_h$ and $\hat{\epsilon}_t$ for compression and tension, respectively (where the index $x$ is omitted). These values develop at $x = 0$ (compression) and $x = \sqrt{3} i_x$ (tension).

### 2.2.1.2 Longitudinal behaviour

Attewell & Woodman (1982) showed that the longitudinal settlement profile can be derived by considering a tunnel as a number of point sources in the longitudinal direction and by superimposing the settlement craters caused by each point source. If a Gaussian settlement profile is adopted for the settlement crater the longitudinal profile is described by

$$S_v(y)|_{x=0} = S_{v,max} \Phi \left( \frac{y}{y_i} \right)$$

where $\Phi(y)$ is the cumulative probability curve and $y$ is the longitudinal coordinate as shown in Figure 2.1. The cumulative probability function is defined as

$$\Phi(y) = \frac{1}{\frac{1}{y_i} \sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{x^2}{2y^2}}$$

and values of $\Phi(y)$ are listed in standard probability tables such as given by Attewell & Woodman (1982) or in most statistics text books.
Figure 2.4 shows a longitudinal settlement profile. Settlement increases in the positive $y$-direction and reaches $S_{v,\text{max}}$ at $y = \infty$ while $S_v = 0$ develops at $y = -\infty$. The settlement at $y = 0$ is equal to $S_{v,\text{max}}/2$. Attewell & Woodman (1982) showed that in stiff clay 30% - 50% of $S_{v,\text{max}}$ occurs above the tunnel face with an average value at about 40%. For convenience it is often assumed that the tunnel face is at $y = 0$ where 50% of $S_{v,\text{max}}$ is predicted, adopting the coordinate system in Figure 2.1.

The width of the longitudinal settlement profile is defined by $i_y$. It is often assumed that $i_x = i_y$. Attewell et al. (1986) compared the magnitudes of $i_x$ and $i_y$ for a range of case studies. Although the data indicated that the transverse settlement troughs were slightly longer than the longitudinal ones, they concluded that this relation is generally valid for most practical design problems. For tunnel construction of the Jubilee Line Extension beneath St. James’s Park, London, Nyren (1998) presented field data which confirm this trend as a ratio of $i_x/i_y = 1.3$ was obtained. However, despite this discrepancy it is common to consider $i_x = i_y$. In the following the trough width parameter $i$ will therefore be used instead of $i_x$ and $i_y$.

Assuming that the resultant displacement vectors point towards the centre point of the current tunnel face, Attewell & Woodman (1982) show that the horizontal soil displacement

![Figure 2.4: Longitudinal settlement profile.](image-url)
in the longitudinal direction at the ground surface is given by

\[ S_{hy}(y)_{x=0} = \frac{V_L D^2}{8\varepsilon_0} e^{-\frac{y^2}{2\varepsilon_0^2}} \]  

(2.9)

Differentiating \( S_{hy} \) with respect to \( y \) leads to the horizontal strain in the longitudinal direction above the tunnel centre line

\[ \epsilon_{hy}(y)_{x=0} = -\frac{V_L D^2}{8i^2\varepsilon_0} e^{-\frac{y^2}{2\varepsilon_0^2}} \]  

(2.10)

which describes tension (positive value) ahead of the tunnel face (negative \( y \)-coordinate) and compression behind.

All components of surface displacement and strain which were presented above depend on the trough width parameter \( i \) and the volume loss \( V_L \). The next two sections will focus on these two crucial design parameters.

### 2.2.1.3 Volume loss

The construction of a tunnel inevitably leads to a larger amount of soil to be excavated than to be replaced by the volume of the tunnel. The amount of this over excavation is quantified by the volume loss \( V_L \) which is the ratio of the difference between volume of excavated soil and tunnel volume (defined by the tunnel’s outer diameter) over the tunnel volume. The volume loss is a measure of the total ground disturbance. It causes the settlement trough at the surface and in undrained conditions, the volume of this settlement trough is equal to \( V_L \).

For shield tunnelling Attewell (1978) divides the sources of volume loss into 4 categories:

**Face loss:** Soil movement towards the unsupported tunnel face as a result of radial pre-yield soil movement which causes the development of surface settlements in front of the tunnel face.

**Shield loss:** The radial ground loss around the tunnel shield due to the cutting bead width void filling with soil. Additional volume loss can be caused by overcutting (due to poor workmanship) or by blocky soil leaving gaps around the tunnel boundary.

**Ground loss during and subsequent to lining erection:** If the lining is erected behind the shield there will be a gap of unsupported soil over which soil can squeeze into the tunnel. Once the lining is constructed soil movement might occur around the lining
which inadequately replaces the cross sectional area of the shield. To minimize this
effect, it is common practice to grout any voids between lining and soil.

**Ground loss after grouting:** Radial losses continue after grouting as the lining is de-
formed due to the transfer of overburden pressure to the new boundary.

Attewell (1978) presents an approach to calculate the different components of volume loss.
The calculation is based on the ratio of rate of soil movement into the excavation (determined
from laboratory tests) over the rate of tunnelling advance.

For tunnels using a Sprayed Concrete Lining (SCL)\(^1\) ICE (1996) gives the following rec-
ommendations to limit the surface settlement:

- Excavation stages must be sufficiently short both in terms of dimension and duration.
- The closure of the sprayed concrete ring must not be delayed.

Several relations have been proposed to estimate \(V_L\) from the stability ratio \(N\) which was
defined by Broms & Bennermark (1967) as:

\[
N = \frac{\sigma_v - \sigma_t}{s_u}
\]

(2.11)

where \(\sigma_v\) is the total overburden pressure at tunnel axis level (including any surcharge), \(\sigma_t\)
is the tunnel support pressure (if present) and \(s_u\) is the undrained shear strength of the clay.
For \(N\) less than 2 the response is likely to be elastic with the face being stable (Lake et al.,
1992). Between \(N = 2\) and 4 local plastic zones develop around the tunnel while between
\(N = 4\) and 6 plastic yielding is likely leading to face instability when \(N\) is greater than
6. For unsupported tunnels (i.e. \(\sigma_t = 0\)) Mair & Taylor (1993) show profiles of \(N\) with
depth indicating that the stability ratio in London Clay typically varies between 2.5 and
3. Lake et al. (1992) summarizes relations between \(V_L\) and \(N\) proposed by several authors
(Figure 2.5). It can be seen that a stability ratio of \(N = 2\) leads to a volume loss of between
1.5% and 3%.

\(^1\)The term *New Austrian Tunnelling Method* (NATM) is often used to describe tunnel construction using a
sprayed concrete lining. ICE (1996), however, points out that some of the key features of NATM do not apply
in tunnelling in soft ground, especially when employed in an urban environment. The term *Sprayed Concrete
Lining* (SCL) is therefore more appropriate.
Chapter 2. Tunnel induced ground and building deformation  

Section 2.2

Figure 2.5: Different relations between stability number \( N \) and volume loss \( V_l \) (after Lake et al., 1992).

Davis et al. (1980) showed that for shallow tunnels the stability ratio at collapse varies with depth. Mair et al. (1981) introduced the concept of load factor to take account for this effect. The load factor is defined by

\[
LF = \frac{N}{N_{TC}}
\]  

where \( N \) is the stability ratio of a tunnel under working conditions and \( N_{TC} \) is the stability ratio at collapse. Figure 2.6 shows results from their work including plane strain finite element analyses and centrifuge tests. The graph indicates that a volume loss of less than 3% is obtained for load factors smaller than \( LF = 0.5 \).

O’Reilly & New (1982) presented field data for open faced shield tunnelling in London Clay showing a typical range of \( V_l \) of 1 - 2%. The magnitude of \( V_l = 1.4\% \) reported by Attewell & Farmer (1974) for the construction of the Jubilee Line beneath Green Park, London, falls within this range. Significantly higher volume losses were, however, measured by Standing et al. (1996) during construction of the Jubilee Line Extension in St. James’s Park, London. Values of 3.3 and 2.9% were observed during construction of the westbound and eastbound tunnel, respectively. Barakat (1996) reported a volume loss between \( V_l = 1.0\% \) and 2.9% for the Heathrow Express tunnel construction using a tunnel shield.

For tunnels constructed with a sprayed concrete lining (SCL) the volume losses have a
similar magnitude. For the Heathrow trial tunnel, which was constructed in London Clay, volume losses of 1.0 - 1.3% are reported by New & Bowers (1994) and Deane & Basset (1995).

Burland et al. (2001) highlights the importance of volume loss on tunnel induced ground subsidence and he suggests the specification of limits to volume loss as a contractual requirement.

2.2.1.4 Trough width parameter

As explained above the trough width parameter \( i \) describes the width of the settlement trough. In a transverse settlement profile it is defined as the distance of the point of inflection (i.e. the point of maximum slope) from the tunnel centre line as shown in Figure 2.2. O’Reilly & New (1982) presented 19 case studies of tunnel construction in clay. Plotting the trough width parameter \( i \) against the corresponding tunnel depth \( z_0 \) revealed a trend as shown in Figure 2.7. From linear regression they obtained the following relation:

\[
i = 0.43z_0 + 1.1
\]  
(2.13)

where both \( i \) and \( z_0 \) are measured in metres. Figure 2.7 shows that the linear regression passes close to the origin. O’Reilly & New (1982) therefore simplified the above equation to

\[
i = Kz_0
\]  
(2.14)

From their data they concluded that for clay a value of \( K = 0.5 \) is appropriate for most design purposes although they pointed out that this value can vary between \( K = 0.4 \) and 0.7

\[ \text{Figure 2.7: Correlation between position of surface point of inflection, } i, \text{ and tunnel depth } z_0(\text{after O’Reilly & New, 1982}). \]
for stiff and soft clay, respectively. Rankin (1988) presented results from a similar study but with an enlarged data base. The results confirmed the value of $K = 0.5$ for clay leading to

$$i = 0.5z_0$$  \hspace{1cm} (2.15)

Kimura & Mair (1981) reported similar results from centrifuge tests. Furthermore their results indicated that a value of $K = 0.5$ is obtained independently of the degree of support within the tunnel. They concluded that the value of $K$ is independent of the tunnelling technique.

### 2.2.2 Subsurface movement

In the previous sections only tunnel induced ground deformation at ground surface level was described. Although surface settlement is the most straightforward way to describe ground deformation it only gives a limited picture of the mechanisms which control tunnel-soil and, if present, also the tunnel-soil-building interaction.

Mair & Taylor (1993) applied the plasticity solution for the unloading of a cylindrical cavity to describe vertical and horizontal subsurface movement. This solution predicts a linear relation when plotting ground movement as $S_v/R$ or $S_{h2}/R$ against $R/d$ where $R$ is the tunnel radius and $d$ is the vertical or horizontal distance from the tunnel centre. They presented field data which were in good agreement with the predicted trend.

In their work Mair & Taylor (1993) only focused on vertical soil movement above the tunnel centre line and on horizontal movement at tunnel axis level. When describing the shape of transverse subsurface settlement troughs, a Gaussian curve is commonly adopted. Mair et al. (1993) showed that this assumption is in reasonable agreement with field data.

To predict the settlement trough width parameter $i$ of subsurface settlement troughs it would be straightforward to apply Equation 2.15 substituting $(z_0 - z)$ for the tunnel depth $z_0$:

$$i = 0.5(z_0 - z)$$  \hspace{1cm} (2.16)

Mair et al. (1993), however, presented field and centrifuge data which showed that subsurface settlement troughs are proportionally wider with depth $z$. Figure 2.8 shows these measurements. The trough width $i$ normalized by $z_0$ is plotted against normalized depth $z/z_0$. The
**Figure 2.8:** Variation of trough width parameter $i$ of subsurface settlement troughs with depth (after Mair et al., 1993).

The dashed line represents Equation 2.16. It can be seen that it underpredicts $i$ with depth. In contrast, the solid line, described by

$$\frac{i}{z_0} = 0.175 + 0.325 \left(1 - \frac{z}{z_0}\right)$$

matches better the measurements. Changing Equation 2.14 to

$$i = K(z_0 - z)$$

(2.18)

and substituting into Equation 2.17 leads to

$$K = 0.325 + \frac{0.175}{1 - \frac{z}{z_0}}$$

(2.19)

It can be shown that for $z = 0$ the above equation yields $K = 0.5$ which is consistent with Equation 2.15 describing the trough width at soil surface level. Figure 2.9 plots $K$ against $z/z_0$ for the measurements shown in Figure 2.8. The curve, described by Equation 2.19 is also included. The graph shows that for large values of $z/z_0$ a constant value of $K = 0.5$ would underestimate the width of subsurface settlement troughs.
Combining Equations 2.2 and 2.3 with Equation 2.17, the maximum settlement of a subsurface trough can be expressed as

\[
\frac{S_{v,\text{max}}}{R} = \frac{1.25V_L R}{0.175 + 0.325 \left(1 - \frac{z}{z_0}\right) z_0}
\]

where \(R\) is the radius of the tunnel. Figure 2.10 shows the maximum settlement (normalized against tunnel radius) plotted against \(R/(z_0 - z)\). The term \((z_0 - z)\) is the vertical distance between settlement profile and tunnel axis. The graph includes field data from tunnel construction in London Clay together with curves derived from Equation 2.16 (curve A) and 2.20 (curves B and C). For the latter equation upper and lower bound curves are given for the range of tunnel depths and volume losses of field data (solid symbols) considered in the graph (see Mair et al. (1993) for more details). The straight solid line refers to the plasticity solution given by Mair & Taylor (1993). The figure shows that the field data are in reasonable agreement with both approaches. In contrast, Equation 2.16 (curve A) would overpredict \(S_{v,\text{max}}\) of subsurface settlement troughs.

Nyren (1998) presented greenfield measurements from the Jubilee Line Extension (St. James’s Park) and compared the data with the result of Mair et al. (1993). For the graph shown in Figure 2.8 his measurements were in good agreement. However when adding the data...
points obtained from the St. James’s Park greenfield site to the graph shown in Figure 2.10
the new measurements were significantly higher in magnitude than the data presented by
Mair et al. (1993). Nyren (1998) concluded that this difference was due to the high volume
loss of \( V_L = 3.3\% \) measured at St. James’s Park. By normalizing \( S_{v,max} \) in Figure 2.10
not only against tunnel radius \( R \) but also against \( V_L \) he demonstrated that all his data were in
good agreement with the measurements originally shown in the diagram.

Heath & West (1996) present an alternative approach to estimate the trough width and
maximum settlement of subsurface settlement troughs by applying the binomial distribution
rather than the Gaussian curve to describe the settlement trough. This approach yields

\[
\frac{i}{i_0} = \sqrt{\frac{z_0 - z}{z_0}}
\]

(2.21)

where \( i_0 \) is the trough width at ground surface level. The maximum subsurface settlement
\( S_{v,max} \) is then proportional to \( (z_0 - z)^{-\frac{1}{2}} \). They suggest that for design purposes the Gauss
curve should be adopted to describe settlement troughs but that the trough width should
be determined from the above equation. When comparing predictions of \( S_{v,max} \) with depth
obtained from their framework with the relation given by Mair et al. (1993) they conclude
that both approaches give similar predictions over depths between \( z/z_0 = 0 \) to approximately
0.8. Close to the tunnel, however, Heath & West (1996) predict larger values of \( S_{v,max} \) which
is in good agreement with the field data they present.

In Section 2.2.1.1 the horizontal surface soil displacement was derived from the vertical
surface settlement assuming that resultant displacement vectors point to the tunnel centre,
leading to Equation 2.5. Following the work of Mair et al. (1993), Taylor (1995) stated that
in order to achieve constant volume conditions displacement vectors should point to the point
where the line described by Equation 2.17 (i.e. the solid line in Figure 2.8) intersects with
the tunnel centre line. This point is located at \( 0.175z_0/0.325 \) below tunnel axis level.

The fact that the assumption of soil particle movement towards a single point at the tunnel
axis is not adequate in the vicinity of the tunnel was highlighted by New & Bowers (1994) by
presenting subsurface field measurements of the Heathrow Express trial tunnel. The results
show that predictions of subsurface troughs based on the point sink assumption were too
narrow and consequently the settlements above the tunnel centre line were over-predicted.
Instead they proposed to model the ground loss equally distributed over a horizontal plane
Figure 2.11: (a) Distribution of $i$ for subsurface settlement troughs with depth; (b) focus of vectors of soil movement (after Grant & Taylor, 2000).

at invert level, equal in width to the tunnel. Using this approach their predictions were in better agreement with the field data.

Grant & Taylor (2000) present data of subsurface settlement troughs obtained from a number of centrifuge tests. Their test results are in good agreement with Equation 2.17 apart from a zone in the vicinity of the tunnel where the test data show narrower subsurface troughs, and close to the surface where wider troughs were measured. This distribution of $i$ with $z$ is schematically shown in Figure 2.11a. Following the statement of Taylor (1995) that soil displacement vectors point to the interception of the distribution of $i$ with $z$, they conclude that vectors point in the direction of the tangent of the distribution shown in Figure 2.11a. The intersections of the tangents for different depths with the tunnel centre line are shown in Figure 2.11b. It follows from this framework that horizontal soil displacement close to the surface is underestimated when Equation 2.17 is adopted to determine the focus point of soil movement. Their laboratory data support this trend by showing high horizontal displacement near the surface.

Grant & Taylor (2000) conclude that the high values of $i$ close to the surface are associated with the free ground surface boundary in their tests. They point out that such a condition
is rare in an urban environment where even a thin layer of pavement may provide sufficient restraint to reduce or even to negate this free-surface effect.

Hagiwara et al. (1999) investigated the influence of soil layers of varying stiffness overlying clay in which a tunnel is being constructed. They performed a number of centrifuge tests in which sand layers of different density were placed over a clay layer. The thickness of the overlying soil and the level of the water table were chosen to maintain the same stress regime within the clay layer in all tests. Results from these tests were compared with a reference case containing clay only.

To quantify the stiffness of the top layer compared to the clay they calculated a relative shear capacity which related the shear stiffness (at very small strains) and the thickness of the top layer to the equivalent measures for the reference clay layer. For each test they determined \( i \) for subsurface troughs within the clay layer.

When comparing \( i \) of subsurface troughs from different tests with the results from the clay-only case they showed that settlement troughs become wider as the stiffness of the top layer increases. Their tests demonstrated that tunnel induced settlement behaviour is affected by the interaction of stiffness of an overlying material with the soil in which the tunnel is constructed.

### 2.2.3 Summary

This section presented the empirical framework that is widely used to describe tunnel induced ground movement. The main conclusions are:

- It is widely accepted and supported by field data that transverse surface settlement troughs can be described by a Gaussian Error curve. A Cumulative Error curve is adopted to describe the longitudinal settlement profile.

- The Gaussian curve applies also to subsurface transverse settlement troughs. Different relations between the trough width \( i \) and the depth of the subsurface settlement curve have been proposed.

- Centrifuge tests indicate that there is an interaction between subsurface settlement behaviour and overlying layers of various stiffness and thickness. It was shown that
there is a trend of increasing $i$ with increasing relative stiffness between the overlaying soil and the soil in which the tunnel is constructed.

2.3 Numerical analysis of tunnel construction

The previous section summarized empirical methods to predict tunnel induced surface and subsurface settlement. The approaches presented were restricted to greenfield situations. In engineering practice, however, problems often involve the interaction between tunnel construction and other structures. These situations can include existing surface structures (as buildings), existing subsurface structures (tunnels, piles) or construction of complex underground structures such as twin tunnels and underground stations. Clearly the empirical methods presented before are of restricted use in these cases.

Numerical modelling provides, in contrast, the possibility to accommodate the different elements of the interaction problem in one analysis. This section gives an overview of how numerical methods, mainly the Finite Element Method have been applied to predict tunnel induced subsidence. It will concentrate on greenfield analyses to demonstrate different approaches to simulate tunnel excavation.

2.3.1 Two dimensional analysis

Tunnel excavation is a three-dimensional problem (Swoboda, 1979; Gens, 1995). Fully three dimensional (3D) numerical analysis, however, often requires excessive computational resources (both storage and time). Therefore, tunnel excavation is often modelled two dimensionally (2D). Various methods have been proposed to take account of the stress and strain changes ahead of the tunnel face when adopting plane strain analyses to simulate tunnel construction.

**The Gap method:** This method proposed by Rowe et al. (1983) pre-describes the final tunnel lining position and size which is smaller than the initial size of the excavation boundary. Soil movement into the tunnel is allowed until the soil closes the gap between tunnel and initial excavation boundary position. The gap parameter is the difference between the diameters of the initially excavated boundary and final tunnel size.
The convergence-confinement method: This method which is also referred to as the \( \lambda \)-method was introduced by Panet & Guenot (1982). The parameter \( \lambda \) describes the proportion of unloading before the lining is installed. For \( 0 < \lambda < 1 \) the remaining radial stress on the lining is \( \sigma_r = (1 - \lambda)\sigma_r^0 \) where \( \sigma_r^0 \) is the initial stress in the radial direction.

The progressive softening method: This method was developed by Swoboda (1979) for modelling tunnel excavation using the New Austrian Tunnelling Method (NATM). It involves reducing the stiffness of the soil within the tunnel boundary before tunnel excavation is simulated, thus allowing soil to move towards the tunnel boundary.

The volume loss control method: This method described by Addenbrooke et al. (1997) prescribes a volume loss \( V_L \) (defined in Equation 2.3). Tunnel excavation is simulated over a number of increments. After each increment \( V_L \) is calculated. As soon the prescribed \( V_L \) is reached the lining is placed. If the analysis only focuses on the ground displacement (and no results of the lining stresses and moments are required) the analysis can be terminated after the required volume loss has been achieved.

The longitudinal-transverse method: Finno & Clough (1985) performed plane strain analyses of both longitudinal and transverse sections of the tunnel to account for stress changes and soil movements ahead of the tunnel face. In a transverse section stress changes were applied prior to tunnel excavation to obtain soil displacement similar to those obtained from the longitudinal analysis. Tunnel construction was then simulated adopting the gap method (see above).

For the last of the above methods it has to be noted that a plane strain analysis of the longitudinal tunnel basically models an infinite slot cut in the soil at some depth below the ground. Rowe & Lee (1992) compared longitudinal settlement profiles from such a plane strain approach with results from 3D analysis and concluded that the longitudinal plane strain approach significantly overestimates the ground displacements and the extent of the plastic zone compared with 3D results.

All of the above methods cause volume loss to develop during the excavation process. The gap method can be seen as a simulation of radial \( V_L \) along a tunnel shield (with the gap parameter being the gap between cutting bead and the lining). However, as volume...
loss also has a tunnel face component it is difficult to determine the gap parameter which accounts for the different sources of volume loss. In stiff clay, with its low permeability, tunnel construction can be considered undrained and the volume loss can be calculated from the surface settlement trough. As outlined above values of $V_L$ have been reported for a wide range of tunnelling projects. This measure is therefore suitable as a design parameter and can be directly adopted as done in the volume loss control method.

Using this method Addenbrooke et al. (1997) performed a suite of 2D tunnelling analyses investigating the effects of different pre-yield soil models on the results. These models were

1. linear elastic with Young’s modulus increasing with depth

2. non-linear elastic, based on the formulation by Jardine et al. (1986). Shear stiffness varies with deviatoric strain and mean effective stress while the bulk stiffness depends on volumetric strain and mean effective stress (referred to as J4 in the original publication). This model is outlined in more detail in Section 3.3.1, Page 90.

3. non-linear elastic with shear and bulk stiffness depending on deviatoric strain and mean effective stress level (referred to as L4). The model also accounts for loading reversals.

All pre-yield models were combined with a Mohr-Coulomb yield surface.

Addenbrooke et al. (1997) modelled tunnel construction of the Jubilee Line Extension (JLE) beneath St. James’s Park, London. Field data from this greenfield site are reported by Standing et al. (1996). Two tunnels were constructed and both were included in the numerical model. However, only results from the first tunnel (the westbound tunnel, $z_0 = 30.5m$) will be summarized here.

The soil profile consisted of London Clay beneath a top layer of Thames Gravel. The lateral earth pressure coefficient at rest adopted in the clay was $K_0 = 1.5$. This value was within the upper bound of $K_0$ values reported by Hight & Higgins (1995).

Figure 2.12 shows the surface settlement obtained from their analyses together with the field data. Their study demonstrated the necessity of including small strain stiffness into the pre-yield model as the predictions of the linear elastic model are inadequate. The responses of the two non-linear models are similar. However, both models predict a settlement trough which is too wide compared with the field data. As a consequence the maximum settlements are too small as the analyses were performed under volume loss control.
The fact that FE analyses predict too wide settlement troughs in a high \( K_0 \)-regime has been reported by many authors. Gunn (1993) presented results from analyses with \( K_0 = 1.0 \). He also applied a non-linear pre-yield model. Figure 2.13 compares the results with the Gaussian settlement trough, showing that the predicted settlement troughs are too wide. Similar conclusions were drawn by Grammatikopoulou et al. (2002) who adopted kinematic yield surface models in their analyses.

The role of \( K_0 \) in the prediction of tunnel construction was highlighted by Gens (1995). He stated that the importance of \( K_0 \) is often neglected in the published literature. Addenbrooke (1996) compared FE settlement predictions for tunnel construction of the Jubilee Line for both \( K_0 = 1.5 \) and 0.5 with field measurements. He concluded that the low \( K_0 \) cases showed a deeper and narrower settlement trough and consequently were closer to the field data. Similar results were presented more recently by other authors for 2D and 3D analyses (Guedes & Santos Pereira, 2000; Doležalová, 2002; Lee & Ng, 2002).

Addenbrooke (1996) performed axisymmetric analyses of a tunnel heading. The results showed a reduction in radial stresses while the hoop stresses around the tunnel boundary increased. Addenbrooke (1996) concluded that at tunnel springline this radial stress reduction causes the lateral stress ratio to reduce while it increases at the tunnel crown and invert. This
change in stress state can be represented in a plane strain analysis by a zone of reduced $K_0$ around the tunnel. The layout of such a zone is shown in Figure 2.14. Apart from this zone the lateral earth pressure coefficient at rest was kept at a global value of $K_0 = 1.5$.

Analyses which adopted such a zone showed an improved settlement profile compared with the global $K_0 = 1.5$ cases. All analyses adopted a non-linear elasto-plastic model (number 2 in the above list). The results also showed that the lateral extent of the $K_0$-reduced zone did not influence the surface settlement predictions. Potts & Zdravković (2001), however, pointed out that justifying a $K_0$-reduced zone from the stress changes in front of the tunnel face is inconsistent with the use of a non-linear elastic model assuming no strain has taken place ahead of the tunnel face.

In order to improve the FE predicted settlement profiles it has been suggested by Lee & Rowe (1989) to include anisotropic soil models in the analyses. In 1996 Simpson et al. presented results from FE analysis of the Heathrow Express trial tunnel. By comparison of results from a linear elastic transversely anisotropic soil model with those of a non-linear isotropic model they showed that soil anisotropy gives better surface settlement predictions for overconsolidated clay. However, only limited details about the applied soil-model were given.
and no information was provided about the initial stress profile adopted in their analyses.

In their study, Addenbrooke et al. (1997) included soil anisotropy in the 2nd of the 3 pre-yield models listed above. The transversely anisotropic soil parameters were derived from the small strain stiffness formulation of the original isotropic model and by defining ratios of $E_v/E_h$ and $G_{vh}/E_v$ where the index ‘v’ and ‘h’ refer to vertical and horizontal stiffness measures respectively. Two analyses were performed. The first one adopted anisotropic ratios observed in field measurements reported by Burland & Kalra (1986) ($E_v/E_h = 0.625$, $G_{vh}/E_v = 0.44$) while the second one reduced $G_{vh}/E_v$ to 0.2, thus making the clay very soft in shear.

Figure 2.15 shows the results from the analyses. When compared with Figure 2.12 it can be seen that the first parameter set (referred to as AJ4i) in the anisotropic analyses does not improve the settlement profile significantly. The second parameter set (AJ4ii) with its low value of shear modulus $G_{vh}$ yields a settlement trough which is closer to the field data. Addenbrooke et al. (1997), however, point out that the low value of $G_{vh}$ adopted in this 2nd parameter set is not appropriate for London clay. This led to their conclusion that unrealistic soil stiffness is required to achieve better settlement predictions when modelling tunnel excavation in plane strain with $K_0 > 1.0$.

It has been pointed out before that tunnel excavation is basically a 3D process. 3D analyses should therefore be able to predict the surface settlement more precisely. Due to advances in computational power over recent years 3D numerical analysis has become more widely applied. The next section summarizes how to simulate tunnel construction in 3D and presents results from various publications.

### 2.3.2 Three dimensional analysis

There are different methods to model 3D tunnel construction. Several authors adopt a ‘step-by-step’ approach (Katzenbach & Breth, 1981) in which tunnel excavation is modelled by successive removal of tunnel face elements while successively installing a lining at a certain distance behind the tunnel face. This distance will be referred to as the excavation length $L_{exc}$.

Some authors suggest to model 3D tunnel construction by applying volume loss control

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2Chapter 8 and Appendix B give more details about anisotropic soil models
<table>
<thead>
<tr>
<th>Author</th>
<th>Material</th>
<th>K₀</th>
<th>D</th>
<th>z₀</th>
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<th>Tunnel length</th>
<th>Lₑsc</th>
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<td>0.5, 1.0</td>
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<td>77.0*</td>
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*: Not specified in text of publication. Given value was determined from plots, graph etc.

**Table 2.1**: Details of 3D FE analyses summarized in this chapter.
methods as presented for 2D analysis. Another approach is to model details of the tunnelling machine such as grouting or slurry pressure. These different approaches will be presented in this section.

In 3D analysis a surface settlement trough as shown in Figure 2.1 develops - or at least should develop. At a certain distance behind the face (i.e. in the positive $y$-direction, adopting the coordinate system shown in the figure) the short term settlement (caused by the immediate undrained response) should not increase as the tunnel shield advances further. These conditions will be referred to as ‘steady-state’ (Vermeer et al., 2002) in this thesis.

The 3D studies which are discussed in this thesis are summarized in Table 2.1. Details of the soil model, mesh geometry etc. are listed there and will be given in the text only where appropriate.

In 1981 Katzenbach & Breth presented results from a 3D FE analysis of NATM tunnelling in Frankfurt Clay. The soil was represented by a non-linear elastic model. A coefficient of lateral earth pressure at rest of $K_0 = 0.8$ was adopted. Tunnel excavation was simulated by a step-by-step approach in which slices of tunnel face elements were removed and subsequently a tunnel lining was installed at a distance $L_{exc}$ behind the tunnel face. Although the authors pointed out that their analyses was only a first step in simulating 3D tunnel construction their results showed some typical 3D soil behaviour around the tunnel face, such as arching and the importance of early ring closure (i.e. keeping $L_{exc}$ small) on the stability of the tunnel face. Their transverse and longitudinal surface settlement profiles were in good agreement with the (wide) range of field measurements presented in their study.

The step-by-step approach has been adopted by various authors to model tunnel excavation, specifically when constructed with NATM (or Sprayed Concrete Lining Method, SCL). Desari et al. (1996) modelled NATM tunnel excavation of the Heathrow trial tunnel in London Clay. A step-by-step approach was used to simulate the advancing tunnel construction although the top half of the tunnel was excavated first, followed by the bottom half. With $K_0 = 1.0$ and using a non-linear elastic perfectly plastic soil model their transverse settlement profile was too wide when compared with field measurements. Their study also revealed that the time dependent Young’s modulus of the shotcrete tunnel lining has no great influence on the settlement trough.

Desari et al. (1996) only presented results in the transverse direction to the tunnel axis.
They stated, however, that stationary settlement conditions (i.e. steady-state) were established 2 diameters behind the tunnel face.

The influence of $L_{\text{exc}}$ on both transverse and longitudinal settlement profiles was investigated by Tang et al. (2000) by modelling a similar tunnel construction as Desari et al. (1996). In their study Tang et al. (2000) applied a stress regime with $K_0 = 1.5$. Tunnel construction was modelled coupled (with a coefficient of permeability for the clay of $k = 1 \times 10^{-9} \text{m/s}$) and the tunnel advance rate was 2.5m per day. The London Clay was represented by a transversely anisotropic linear-elastic perfectly plastic constitutive relationship.

In their study the tunnel was constructed with two excavation lengths of $L_{\text{exc}} = 5\text{m}$ and $10\text{m}$. As in the analyses described above, their transverse settlement profile was too wide regardless of the excavation length. The longitudinal settlement profiles obtained from their analyses are shown in Figure 2.16. For both excavation lengths (referred to as ‘Span’ in the figure) the longitudinal settlement profile becomes horizontal at a distance of approximately 20m behind the tunnel face, indicating a steady state of settlement behaviour in this region. The graph illustrates that the magnitude of the steady-state settlement increases with increasing excavation length. The plot also indicates that at the beginning of the tunnel, the tunnel was excavated over a longer distance (noting the distribution of data points which coincides with the mesh shown in their publication). Unfortunately, Tang et al. (2000) give no further information about the initial tunnel construction in their analysis.

The importance of the first excavation step in a 3D FE analysis of tunnel construction was highlighted by Vermeer et al. (2002). They modelled step-by-step tunnel excavation ($L_{\text{exc}} = 2\text{m}$) in soil described by a linear elastic perfectly plastic model with an initial stress regime of $K_0 = 0.67$. In their work both full face and sequential face excavation were analysed. 

![Figure 2.16: Longitudinal settlement profiles obtained for different excavation lengths $L_{\text{exc}}$ (here referred to as ‘Span’, settlement referred to as $u_z$, after Tang et al., 2000).](image)
Figure 2.17 shows longitudinal settlement profiles of the full face excavation of a circular tunnel with a diameter of $D = 8$ m. The two curves in the graph correspond to different initial excavation steps. The lower curve represents an analysis in which an unsupported excavation was performed in the first increment of the analysis before the lining was installed (i.e. the first increment was modelled in the same way as all subsequent steps). In the other analysis the lining was installed over the length of the first excavation step prior to excavation. The upper curve shows the results from this analysis. Vermeer et al. (2002) concluded that the first excavation phase has a tremendous influence on the whole analysis and steady-state conditions only develop at a certain distance from the start boundary. The settlement profiles shown in Figure 2.17 are for a tunnel face position at 80$m$. The graph indicates that steady state conditions establish approximately 40m ($5D$) behind the tunnel face.

When comparing transverse settlement results with a settlement trough obtained from a corresponding plane strain study their results showed that both analyses predicted the same settlement curve. From this observation Vermeer et al. (2002) proposed a fast settlement analysis. In this approach the step-by-step analysis is replaced by two stages only: In the first phase a complete tunnel is constructed up to a distance from the start boundary over which steady-state conditions can develop. After the lining is installed over the whole length all displacements are reset to zero. In the second phase a single excavation step of $L_{\text{exc}}$ is simulated without lining installation. This excavation induces a settlement crater on the surface. The volume of this crater represents the volume loss of a single excavation step. With this information a plane strain analysis is performed (using the convergence-confinement method) to predict the transverse settlement profile. This approach clearly reduces the calculation

![Figure 2.17: Longitudinal settlement profiles for different excavation methods in the first excavation step (after Vermeer et al., 2002).](image-url)
time as only two 3D steps have to be simulated in contrast to a step-by-step approach in which the number of steps depends on $L_{exc}$ and the tunnel length which is necessary to reach steady-state conditions. Vermeer et al. (2002) showed that the tunnel length to reach steady-state conditions increases when a sequential face excavation is considered instead of full face excavation. The fast settlement approach might be suitable for greenfield conditions. However, its applicability to more complex situations, such as tunnel-building interaction is debatable.

In order to reduce the number of steps in a 3D analysis it was suggested to adopt excavation techniques from plane strain situations. Lee & Rowe (1991) present results of a 3D analysis of the Thunder Bay sewer tunnel in Ontario, Canada. The soil was represented by a transverse anisotropic linear elastic perfectly plastic model. The coefficient of lateral earth pressure at rest was 0.85. In order to simulate the advance of the $D = 2.5m$ tunnel shield they applied the gap method. The radial volume loss was determined from the tunnelling machine while the potential face loss was estimated from 3D analysis. The tunnel was then excavated over the whole length. At the tunnel face full release of axial stress was allowed to simulate face volume loss. The physical gap of the tunnel machine was applied over the length of the shield while the total gap (radial + face loss) was applied behind the shield. The lining was installed when the gap was closed.

Their predictions were in reasonable agreement with the field data. In the longitudinal direction their analyses showed a horizontal settlement profile approximately 15 - 20m behind the tunnel face which was in good agreement with the field measurements. The transverse settlement profile was slightly too wide giving a ratio of calculated settlement trough width $i$ to measured $i$ of 1.1 to 1.2.

The subsurface settlement was investigated in their study. Vertical profiles of horizontal soil movement showed that the maximum value of lateral displacement occurs near the springline level (at approximately $1D$ distance from the tunnel centre line).

Another approach to model 3D tunnel excavation was described by Augarde et al. (1998). In their model, soil within a tunnel section of specified length is excavated and the tunnel lining, represented by shell elements, is installed simultaneously. A required volume loss $V_L$ is then achieved by subjecting the lining to uniform hoop shrinkage over the length of tunnel excavation. The number of stages to construct the whole tunnel depends on the problem.
under investigation. In their work Augarde et al. (1998) apply their model to investigate the influence of tunnel construction on existing buildings. They model 4 stages in which the tunnel face is in front, beneath and behind the building. In the forth stage the tunnel is completed over the whole mesh length. Greenfield settlements obtained from analyses using this method of tunnel construction were presented in Burd et al. (2000) (this publication will be addressed in more detail in Section 2.4.6.1). The maximum settlement of the transverse settlement trough is approximately 70% of the value expected from the Gaussian curve, calculated for the same volume loss. This indicates that the settlement trough predicted by their analyses is too wide.

In order to produce better predictions of transverse settlement profiles in 3D FE analyses the influence of the lateral earth pressure coefficient at rest $K_0$ and of soil anisotropy has been investigated in 3D studies. Guedes & Santos Pereira (2000) presented results from both 2D and 3D analyses with $K_0 = 0.5$ and 1.0. They conclude that 3D simulation does not change the trend of wider settlement trough with increasing $K_0$ observed in plane strain analyses. Similar conclusions were drawn by Doležalová (2002) who analysed construction of the Mrazovka Exploratory Gallery, near to Prague, using 2D and 3D analysis. She states that for given ground conditions the relation between $K_0$ and shape of the settlement trough is practically independent of the type of analysis (2D or 3D). Agreement with the Gaussian curve and with field observations were only achieved with $K_0 = 0.5$.

Both $K_0$ and the degree of soil anisotropy (expressed as $n' = E_v'/E_h'$) was varied by Lee & Ng (2002) in a suite of 3D FE analyses. Values of $K_0 = 0.5$ and 1.5 were applied in their analyses while $n' = 1.6$ was compared with isotropic cases ($n' = 1.0$). The ratio of $G_{vh}/E_v' = 0.44$ was kept constant during the study. The soil model was linear elastic perfectly plastic. The soil parameters given in their publication indicate that the analyses were performed coupled with a permeability of $k = 1 \times 10^{-9}$m/s, however, no information about the tunnelling rate is provided.

The degree of anisotropy is equivalent to that used by Addenbrooke et al. (1997)$^3$. However, the tunnel geometry of $D = 9$ m and $z_0 = 22.5$ m was different to that of Addenbrooke et al. (1997).

$^3$Note that $n'$ is inverse to the ratio Addenbrooke et al. (1997) used to quantify anisotropy. The value of $E_v'/E_h' = 0.625$ in their analyses is equivalent to $n' = 1.6$ adopted by Lee & Ng (2002).
Chapter 2. Tunnel induced ground and building deformation

Section 2.3

The analyses show that the transverse surface settlement trough becomes deeper as $K_0$ reduces and/or $n'$ increases. This trend is consistent with the results presented by Addenbrooke (1996) and Addenbrooke et al. (1997). However, when comparing the results of the different studies it emerges that settlement troughs from the 3D analyses by Lee & Ng (2002) are much more affected by changes in $n'$ than the 2D analyses were. Lee & Ng (2002) states that this difference is due to the change from 2D to 3D analysis. This conclusion is, however, in contrast to studies presented by Guedes & Santos Pereira (2000) and Doležalová (2002) who did not find great differences between 3D and plane strain analyses.

Lee & Ng (2002) present results in the transverse direction taken at a section 3 tunnel diameters behind the tunnel face (and approximately 50m from the start boundary). However, the longitudinal settlement profile shown in their publication, indicates that steady-state conditions might not have developed during the analysis.

Other authors suggest simulation of the tunnel construction procedure in more detail by modelling the tunnel boring machine. Such an approach is presented by Komiya et al. (1999). In their analyses the shield is represented by a rigid body of high stiffness. Body forces were applied to simulate the weight of the shield machine. The advance of the tunnel shield was modelled by applying external forces at the back of the shield, equivalent to the hydraulic jacks used in tunnelling practice. By applying these forces at different positions and in different combinations it was possible to simulate 3D movement (i.e. pitching and yawing) of the tunnelling machine. Due to this forward movement soil elements in front of the shield were deformed. At the begin of each tunnelling sequence the soil around the tunnelling machine was remeshed in order to obtain the same mesh geometry ahead of the shield in all excavation steps.

By comparing their predictions of soil movement 1m above the tunnel crown with field measurements Komiya et al. (1999) show that the effects of tunnel excavation on the adjacent soil was modelled with reasonable accuracy. As they used records of the hydraulic jacks it was possible to reproduce the measured 3D movement of the tunnel machine. Their data showed that surface settlement was negligible which was in agreement with field data.

Dias et al. (2000) presents 3D analyses in which the slurry shield tunnelling machine was simulated by applying pressure at the face and at the circumferential tunnel boundary to model slurry pressure and grouting injection, respectively. Apart from face support and
grouting they also modelled over cut and the conical shape of the shield. In their analyses which modelled construction of a tunnel in Cairo, drained conditions were applied to the soil. Their 3D results overestimated the measured surface settlement by nearly 100%. They also compare their 3D results with 2D analyses in which grouting pressure and soil deformation around the conical shape of the shield was modelled. The 3D analyses exhibit a narrower settlement trough than obtained from the 2D model.

2.3.3 Summary

This section presented a number of 2D and 3D numerical analyses (mainly FEM) of tunnel construction under greenfield conditions performed by other authors. The following points were highlighted in this literature review:

- It has been noted by many authors that the process of tunnel excavation is clearly a 3D problem and, thus, 3D analysis should be applied to model tunnel construction. However, due to limitations in computational power 2D analysis is widely used. For plane strain analyses there are different approaches to account for stress redistribution ahead of the tunnel face.

- Plane strain analyses show the importance of considering the small strain behaviour of the soil. These results also show that good agreement with field measurements in London Clay can be achieved when applying a stress regime with $K_0 = 0.5$ (either over the whole mesh or only as a local zone of reduced $K_0$). However, for $K_0 = 1.5$ predicted settlement troughs are too wide when compared with field data. Some authors stated that soil anisotropy can improve these settlement predictions, however, other publications show that the improvement is not significant.

- In recent years full 3D analysis has become more widely applied. Table 2.1 summarizes a number of 3D studies. Although the earliest publication dates back to 1981, most studies included in this overview were performed over the last 5 years. Analyses which applied a high value of $K_0$ showed that the transverse settlement profile is not improved by 3D analysis. Furthermore, many authors stated that there is no significant difference in the shape of the transverse settlement trough between 2D and 3D analyses.
• In longitudinal direction steady-state conditions (i.e. stationary settlement at the end of the undrained response) should be achieved at a certain distance behind the tunnel face. Different values for this distance were reported by different studies which adopted a wide range of constitutive models and initial stress profiles. Desari et al. (1996) (for $K_0 = 1.0$) stated that steady-state conditions develop $2D$ behind the face (albeit without presenting any longitudinal settlement profile) while for $K_0 = 0.67$ Vermeer et al. (2002) gives approximately $5D$ as such a distance.

• The 3D analyses presented in this section focused on different aspects, such as tunnelling technique, initial stress conditions or soil anisotropy. However, the mesh dimensions in the longitudinal direction are a crucial parameter in 3D analysis. Vermeer et al. (2002) demonstrated that boundary effects alter the settlement profile close to the vertical start boundary. It is interesting to note, that no publication addressed this problem (apart from Vermeer et al. (2002) although the mesh dimension was no variable in their study). Furthermore, Table 2.1 reveals that full information about the mesh dimensions are not provided by all authors.

Despite the fact that some numerical predictions do differ from field measurements the variety of problems analysed demonstrates the flexibility of numerical simulation. This will become more evident when investigating the effect of tunnel construction on existing buildings.

\section*{2.4 Building damage assessment}

It has been pointed out in previous sections that tunnel construction in soft ground causes ground movement, most notably in the form of a surface settlement trough. In urban areas this ground subsidence can affect existing surface and subsurface structures. Predicting tunnel induced deformation of such structures and assessing the risk of damage is an essential part of planning, design and construction of tunnels in an urban environment (Mair et al., 1996). This section will summarize the widely used design approach to predict and assess potential building damage. It will be shown that the damage assessment presented in this section does not account for building characteristics such as building stiffness. More detailed assessment strategies have therefore been developed which will be summarized in Section 2.4.6.
2.4.1 Definition of structure deformation

In 1974 Burland & Wroth noted that a variety of symbols and measures were used to quantify building deformation. They proposed a set of nine parameters to define building distortion. The following deformation parameters, shown in Figure 2.18, are defined:

1. **Settlement** defines the vertical movement of a point. Positive values indicate downwards movement (Figure 2.18a).

2. **Differential or relative settlement** $\delta S_v$ is the difference between two settlement values (Figure 2.18a).

3. **Rotation or slope** $\theta$ describes the change in gradient of the straight line defined by
two reference points embedded in the structure (Figure 2.18a).

4. **Angular strain** $\alpha$ produces sagging or upward concavity when positive while hogging or downward concavity is described by a negative value (Figure 2.18a).

5. **Relative deflection** $\Delta$ describes the maximum displacement relative to the straight line connecting two reference points with a distance $L$. Positive values indicate sagging (Figure 2.18b).

6. **Deflection ratio** $DR$ is defined as the quotient of relative deflection and the corresponding length: $DR = \frac{\Delta}{L}$ (Figure 2.18b).

7. **Tilt** $\omega$ describes the rigid body rotation of the whole superstructure or a well-defined part of it. It is difficult to determine as the structure normally flexes itself (Figure 2.18c).

8. **Relative rotation or angular distortion** $\beta$ is defined as the rotation of the straight line joining two reference points relative to the tilt (Figure 2.18c).

9. **Average horizontal strain** $\epsilon_h$ develops as a change in length $\delta L$ over the corresponding length $L$: $\epsilon_h = \frac{\delta L}{L}$.

The above definitions only describe ‘in-plane’ deformation. Three-dimensional behaviour such as twisting is not included.

Since their introduction by Burland & Wroth (1974) the above definitions have become widely accepted. Rankin (1988) noted, however, that a large number of observation points are required in order to adequately quantify the deformation parameters. He pointed out that such information is seldom available in engineering practice.

It will be shown in the following sections that the deflection ratio $DR$ and the horizontal strain $\epsilon_h$ are of significant importance when assessing potential building damage. These measures will be referred to as deformation criteria in this thesis.

### 2.4.2 Evaluation of risk of damage

This section summarizes the design approach which is currently used to assess potential building damage for tunnelling projects in London (Mair et al., 1996). The three-stage assessment which will be outlined below was adopted for the recently constructed Jubilee
Chapter 2. Tunnel induced ground and building deformation

Section 2.4

Line Extension, for the Channel Tunnel Rail Link (currently under construction) and for the proposed CrossRail project.

The design approach consists of three stages, schematically shown in Figure 2.19 which are referred to as preliminary assessment, second stage assessment and detailed evaluation.

2.4.2.1 Preliminary assessment

In this stage the presence of the building is not considered. Instead the greenfield settlement profile (normally as a contour plot along the proposed tunnel route) is evaluated. Rankin (1988) provided guidelines of how the maximum settlement and the maximum slope of a building affect its potential damage. He showed that for a slope of less than \( \theta = 1/500 \) and \( S_v,\text{max} \) of less than 10mm the risk of building damage is negligible. Buildings which are located within a zone in which greenfield predictions give lower values than the above thresholds are assumed to experience negligible damage risk. Such buildings are not considered further in order to avoid a large number of unnecessary calculations.

This approach is very simple and conservative as only greenfield settlement is considered. It does not provide any information about the distortion of a building. The above values of maximum slope and settlement might be reduced when assessing the risk for structures of higher sensitivity. If the greenfield settlement associated with a building exceeds the maximum slope and settlement a second stage assessment has to be carried out.

2.4.2.2 Second stage assessment

In this stage of the risk assessment the building is represented as an elastic beam whose foundation is assumed to follow the settlement profile described by the empirical greenfield trough given in Equation 2.4. For this greenfield situation the portion of the settlement trough below the building is used to calculate the deflection ratios \( DR^{\text{GF}} \) (both sagging and hogging) and the maximum horizontal strains \( \epsilon_{h}^{\text{GF}} \) (both compression and tension). With this information the strain within the beam is evaluated following an approach described in more detail in Sections 2.4.4 and 2.4.5. Categories of damage, defined in Section 2.4.3 can then be obtained from the magnitude of strain.

Although this approach is more detailed than the preliminary assessment it is still conservative as the building is assumed to follow the greenfield settlement trough. However, it has
been shown by case studies (Frischmann et al., 1994) that the building’s stiffness interacts with the ground such that deflection ratio and horizontal strain reduce. Burland (1995) points out that the category of damage obtained from this assessment is only a possible degree of damage and that in the majority of cases the actual damage will be less than the predicted category.

The damage categories, defined in Table 2.2 and described in more detail in Section 2.4.3 distinguish between levels of aesthetical (categories 0 - 2), serviceability (3,4) and stability (5) damage. For building cases which exceed damage category 2 (i.e. damage potentially affects serviceability) a detailed evaluation has to be performed.

2.4.2.3 Detailed evaluation

In this stage details of the building and of the tunnel construction should be taken into account. This includes the three-dimensional process of tunnel construction and the orientation of the building with respect to the tunnel. Building features such as the foundation design and structural continuity as well as any previous movement a building may have experienced in the past should also be accounted for (Burland, 1995).

The interaction between soil and structure is a key factor as the influence of the building’s stiffness is likely to reduce its deformation. Figure 2.20 shows the settlement of Mansion House, due to tunnel construction for the Docklands Light Railway, London. The figure compares the building settlement with the predicted greenfield trough. This figure demonstrates that the building’s presence reduces both slope and maximum settlement compared to the greenfield situation. A similar effect reduces the horizontal strain within the structure (Geddes, 1991). Potts & Addenbrooke (1997) showed how the influence of soil-structure interaction can be incorporated into the second stage assessment in order to reduce the number of cases for which a detailed evaluation has to be carried out. Their approach will be discussed in more detail in Section 2.4.6.2.

Burland (1995) points out that because of the conservative assumption of the second stage assessment, the detailed evaluation will usually predict lower categories of damage than obtained from the previous stage. If the risk of damage remains high it has to be considered whether protective measures are necessary. Protective measures will only be required for buildings which after detailed evaluation remain in damage category 3 (Table 2.2) or higher.
2.4.3 Category of damage

Burland *et al.* (1977) summarized several approaches to quantify building damage. They distinguished between three criteria when considering building damage:

1. Visual appearance

2. Serviceability or function

3. Stability

They concluded that the visual appearance of a building might be affected when structural elements show deviations of 1/250 from the vertical or horizontal. Deviations of 1/100 and/or deflection ratios of 1/250 would be clearly visible. However, they pointed out that visual damage is difficult to quantify as it depends on subjective criteria. They proposed a system of damage categories based on the ease of repair. This classification scheme is presented in Table 2.2.

In this list crack width are given as an additional indicator rather than a direct measure as the emphasis is on the ease of repair. It has also to be noted that this classification was developed for brickwork or stone masonry. The degree of severity given in the table only applies to standard domestic or office buildings. For buildings with sensitive finishes the degree of severity might not apply. Burland *et al.* (1977) also points out that more stringent
### Table 2.2: Classification of visible damage to walls with particular reference to ease of repair of plaster and brickwork masonry (after Burland, 1995).

<table>
<thead>
<tr>
<th>Category of damage</th>
<th>Normal degree of severity</th>
<th>Description of typical damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Negligible</td>
<td>Hairline cracks less than about 0.1 mm</td>
</tr>
<tr>
<td>1</td>
<td>Very Slight</td>
<td>Fine cracks which are easily treated during normal decoration. Damage generally restricted to internal wall finishes. Close inspection may reveal some cracks in external brickworks or masonry. Typical crack widths up to 1 mm.</td>
</tr>
<tr>
<td>2</td>
<td>Slight</td>
<td>Cracks easily filled. Re-decoration probably required. Recurrent cracks can be masked by suitable linings. Cracks may be visible externally and some repointing may be required to ensure weathertightness. Doors and windows may stick slightly. Typical crack width up to 5 mm.</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>The cracks require some opening up and can be patched by mason. Repointing of external brickwork and possibly a small amount of brickwork to be replaced. Doors and windows sticking. Service pipes may fracture. Weathertightness often impaired. Typical crack widths are 5 to 15 mm or several up to 3 mm.</td>
</tr>
<tr>
<td>4</td>
<td>Severe</td>
<td>Extensive repair work involving breaking-out and replacing sections of walls, especially over doors and windows. Windows and door frames distorted, floor sloping noticeably. Walls leaning or bulging noticeably, some loss of bearing in beams. Service pipes disrupted. Typical crack widths are 15 to 25 mm but also depends on the number of cracks.</td>
</tr>
<tr>
<td>5</td>
<td>Very severe</td>
<td>This requires a major repair job involving partial or complete rebuilding. Beams lose bearing, walls lean badly and require shoring. Windows broken with distortion. Danger of instability. Typical crack widths are greater than 25 mm but depends on the number of cracks.</td>
</tr>
</tbody>
</table>

Note: Crack width is only one factor in assessing category of damage and should not be used on its own as a direct measure of it.

1 Note: Local deviation of slope, from the horizontal or vertical, of more than 1/100 will normally be clearly visible. Overall deviations in excess of 1/150 are undesirable.
Section 2.4

Table 2.3: Relation between category of damage and limiting tensile strain (after Boscardin & Cording, 1989 and Burland (1995)).

<table>
<thead>
<tr>
<th>Category of damage</th>
<th>Normal degree of severity</th>
<th>Limiting Tensile strain [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Negligible</td>
<td>0 - 0.05</td>
</tr>
<tr>
<td>1</td>
<td>Very slight</td>
<td>0.05 - 0.075</td>
</tr>
<tr>
<td>2</td>
<td>Slight</td>
<td>0.075 - 0.15</td>
</tr>
<tr>
<td>3</td>
<td>Moderate*</td>
<td>0.15 - 0.3</td>
</tr>
<tr>
<td>4 to 5</td>
<td>Severe to Very Severe</td>
<td>&gt;0.3</td>
</tr>
</tbody>
</table>

*Note: Boscardin & Cording (1989) describe the damage corresponding to the tensile strain in the range 0.015 - 0.3% as ‘moderate to severe’. However, none of the cases quoted by them exhibit severe damage for this range of strains. There is therefore no evidence to suggest that tensile strains up to 0.3% will result in severe damage.

criteria might be necessary when initial cracks can lead to corrosion, penetration of harmful liquids etc.

The 6 categories in this classification can be subdivided into the above mentioned three groups of damage level. Categories 0 to 2 correspond to aesthetical damage. Serviceability damage occurs in categories 3 and 4 while the stability of the structure is affected by damage of category 5. As mentioned in Section 2.4.2.2 the division between category 2 (visual damage) and 3 (serviceability) represents an important threshold. Burland (1995) points out that damage related to categories 0 to 2 can result from several causes within the structure such as thermal effects. In contrast damage of category 3 or higher is frequently associated with ground movement.

2.4.4 The concept of critical strain

Burland & Wroth (1974) showed that tensile strain is the fundamental parameter when determining the onset of cracking. They summarized the results from a number of large scale tests on masonry panels and walls. Their results showed that the onset of visible cracking is associated with a well defined value of average tensile strain. This value was found not to be sensitive to the mode of deformation. They defined this strain, measured over a length of 1m or more, as critical strain $\epsilon_{\text{crit}}$.

For brick work they reported values of $\epsilon_{\text{crit}} = 0.05\% - 0.1\%$ while for concrete lower values
of $\epsilon_{\text{crit}} = 0.03 - 0.05\%$ were determined. Burland & Wroth (1974) noted that these values are larger than the local tensile strain corresponding with tensile failure.

In 1977 Burland et al. replaced the notation of $\epsilon_{\text{crit}}$ by the $\epsilon_{\text{lim}}$ which they referred to as the limiting tensile strain in order to take account of different materials and serviceability limit states.

Boscardin & Cording (1989) extended the framework of limiting tensile strain by linking strain values to building damage observed from case studies subjected to excavation induced subsidence. In their work they linked values of limiting strain with the categories of damage shown in Table 2.2. This relation is summarized in Table 2.3. It provides a link between building deformation and potential building damage and therefore is essential for building damage assessment.

### 2.4.5 Calculation of building strain

Burland & Wroth (1974) and Burland et al. (1977) applied the concept of limiting tensile strain to elastic beam theory to study the relation between building deformation and onset of cracking. Although modelling a building as an elastic beam clearly is a simplification it was found that predictions from this model were in good agreement with case records of damaged and undamaged buildings. Furthermore, this simple approach demonstrates the mechanisms which control the onset of cracking within a structure.

The elastic beam in their model is described by a width, $B$ and a height, $H$, see Figure 2.21$^4$. The figure shows two extreme modes of deformation: In bending (Figure 2.21c) cracking is caused by direct tensile strain while in shear (Figure 2.21d) diagonal cracks appear, caused by diagonal tensile strains. For a centrally loaded beam subjected to both shear and bending deformation the total central deflection is given by Timoshenko (1955):

$$\Delta = \frac{PB^3}{48EI} \left( 1 + \frac{18EI}{B^2HG} \right) \quad (2.22)$$

where $E$ is the Young’s modulus and $G$ is the shear modulus. $P$ is the point load which is applied at the centre of the beam. For an isotropic elastic material $E/G = 2(1 + \nu)$.

$^4$Note that the original nomenclature of Burland & Wroth (1974) describe the beam by the length $L$ instead of the width $B$ used in this thesis. The nomenclature was changed to be consistent with the work of Potts & Addenbrooke (1997) and with building dimensions used in later chapters of this thesis.
Figure 2.21: Cracking of a simple beam in different modes of deformation (after Burland & Wroth, 1974).

Assuming a Poisson’s ratio of $\nu = 0.3$ one obtains $E/G = 2.6$. In the case where the neutral axis is in the middle of the beam, Burland & Wroth (1974) expressed Equation 2.22 in terms of deflection ratio $\Delta/B$ and the maximum extreme fibre strain $\epsilon_{b,\text{max}}$:

$$\frac{\Delta}{B} = \left( 0.167 \frac{B}{H} + 0.65 \frac{H}{B} \right) \epsilon_{b,\text{max}}$$  \hspace{1cm} (2.23)

and for the maximum diagonal strain $\epsilon_{d,\text{max}}$:

$$\frac{\Delta}{B} = \left( 0.25 \frac{B^2}{H^2} + 1 \right) \epsilon_{d,\text{max}}$$  \hspace{1cm} (2.24)

Both equations are plotted in Figure 2.22 (with $\epsilon_{\text{lim}} = \epsilon_{\max}$). It is obvious that for $B/H < 0.5$
the diagonal strain is critical. As \( B/H \) increases above this value bending becomes the more critical mode of deformation.

For the equations plotted in Figure 2.22 it was assumed that the neutral axis was in the middle of the beam. In real buildings, however, the foundations offer a significant restraint to their deformation. Therefore, it may be more realistic to assume the neutral axis to be at the lower extreme fibre. With this assumption Burland & Wroth (1974) changed Equations 2.23 and 2.24 to:

\[
\frac{\Delta}{B} = \left( 0.083 \frac{B}{H} + 1.3 \frac{H}{B} \right) \epsilon_{b,\max}
\]

\[
\frac{\Delta}{B} = \left( 0.064 \frac{B^2}{H^2} + 1 \right) \epsilon_{d,\max}
\]

respectively. As the neutral axis is at the lower extreme fibre, Equation 2.25 only applies for a hogging deformation mode. In the case of sagging there are no tensile strains. Equations 2.25 and 2.26 are plotted in Figure 2.23. Comparing Figure 2.23 with Figure 2.22 shows that for any given value of \( \Delta/(B\epsilon_{\text{lim}}) \) the value of \( B/H \) in Figure 2.23 is twice that in Figure 2.22.

In their original studies Burland & Wroth (1974) and Burland et al. (1977) focused on building weight induced settlement. In this case the movement of structure and underlying soil develops mainly in a vertical direction and therefore little consideration is given to horizontal displacement or strain. In contrast, it was shown in Section 2.2 that ground movements
induced by tunnelling not only involve sagging and hogging but also develop horizontal strain. Geddes (1978) showed that this horizontal strain can have a significant influence on existing buildings. Boscardin & Cording (1989) included horizontal strain in the above framework by superimposing building strain developed due to deflection deformation with the horizontal ground strain $\epsilon_h$. The resultant extreme fibre strain $\epsilon_{br}$ is then given by

$$\epsilon_{br} = \epsilon_{b,\text{max}} + \epsilon_h$$

(2.27)

and the resultant diagonal tensile strain can be derived from the Mohr’s circle of strain:

$$\epsilon_{dr} = \epsilon_h \frac{1 - \nu}{2} + \sqrt{\epsilon_h^2 \left(\frac{1 - \nu}{2}\right)^2 + \epsilon_{d,\text{max}}^2}$$

(2.28)

where $\nu$ is the Poisson’s ratio of the beam.

Geddes (1991) by referring to Boscardin & Cording (1989) pointed out that the assumption of the development of equal horizontal strain in the ground and in the building is not generally true. Shear strain or horizontal slip might develop at the soil-structure interface. Consequently, the strain in the structure may differ considerably from the ground strain. Geddes (1991) concludes, that the approach of Boscardin & Cording (1989) generally overestimates a structure’s horizontal strain.

Applying deep beam theory Boscardin & Cording (1989) related limiting strain to angular distortion $\beta$ and horizontal strain as shown in Figure 2.24. Each contour line represents a value of limiting strain as listed in Table 2.3. The figure also includes case studies subjected to tunnel construction, shallow mines or braced cuts. Boscardin & Cording (1989) concluded that the level of recorded damage for most cases fell within the boundaries described by the curves of limiting strain.

Burland (1995) presented similar plots for horizontal strain and deflection ratio. From these plots interaction diagrams were developed showing the relationship between $DR$ and $\epsilon_h$ for a particular value of $B/H$. Such a diagram is presented in Figure 2.25. Each contour line in this plot represents a value of limiting strain, listed in Table 2.3. For $DR = 0$ (i.e. points on the horizontal axis) limiting values of horizontal strain are the same as $\epsilon_{\text{lim}}$ given in Table 2.3.
2.4.6 Soil structure interaction

It has been pointed out that the preliminary and the second stage assessment in the three stage risk assessment presented in Section 2.4.2 uses the greenfield settlement profile to evaluate potential damage caused by tunnel construction. This approach can be highly conservative as it assumes that any structure follows the greenfield settlement profile. However, the structure’s stiffness is likely to alter ground induced soil movement and, hence, to reduce the building’s deformation. Burland (1995) concludes that by incorporating the soil-structure interaction into the detailed evolution, this assessment stage will usually result in a reduction in the possible degree of damage predicted by the second stage assessment.

The influence of soil-structure interaction was highlighted by building measurements during the construction of the Jubilee Line Extension (JLE). The case studies showed that building stiffness can substantially reduce building deformation. Burland et al. (2001) reported that load bearing walls behave more flexibly in hogging than in sagging. Standing (2001) presents horizontal strain measurements close to the foundations of Elizabeth House (near Waterloo Station, London). They show that the strain development during the construction of the JLE was negligible and that those induced by thermal effects exceeded them.

The effect the construction of a tunnel may have on adjacent structures is difficult and often impossible to estimate using conventional methods of analysis (Potts, 2003). Numerical

![Figure 2.24: Relation of damage to angular distortion and horizontal extension (after Boscardin & Cording, 1989).](image-url)
analysis, in contrast, can model such an interaction problem. The next sections show how numerical modelling can be adopted to improve the understanding of the tunnel-soil-structure interaction.

2.4.6.1 Numerical studies of the interaction problem

To investigate the soil-structure interaction, FE modelling has been adopted by several authors. These models include the building and its interaction with tunnel induced ground movement can be evaluated. Different approaches have been used to represent the building with varying level of details included in these models:

**Fully 3D modelling:** In such an analysis details of a buildings such as the layout of the façades, the position of windows and doors, etc. can be modelled. The advantage of such a 3D model is that the building can be considered in any geometrical configuration with respect to the tunnel axis.

**Plane strain/stress analysis of structure:** This approach models the in-plane geometry of the structure transverse to the tunnel. The building is described by its width and height and details such as windows and doors can be incorporated in the model. The advantage of 2D modelling is the small amount of computational resources required compared to 3D analysis. It is therefore possible to perform parametric studies including a wide range of different parameters.
Deep beam model: This model is similar to the approach adopted by Burland & Wroth (1974). The structure is represented by an elastic beam with bending stiffness \( (EI) \) and axial stiffness \( (EA) \) representing the overall stiffness of the structure. The deformation can be imposed on the beam by incorporating it into a FE tunnelling analysis (either 2D or 3D, in the latter case the term ‘shell’ should be used instead of ‘beam’) or by predescribing the displacement of the beam. The advantages of this method are – when used in 2D conditions – the small amount of computational resources required and therefore the ability to perform extensive parametric studies. Furthermore this approach is consistent to the risk assessment outlined by Burland (1995).

Burd et al. (2000) presented results of a study employing the first approach. In the fully 3D FE analyses the building consisted of four masonry façades modelled by plane stress elements. Details such as windows and doors were included in the model. The masonry was represented by a constitutive model giving high strength in compression but relatively low tensile strength. The building was assigned to have self weight. Burd et al. (2000) presented results for both symmetric geometries (where only half of the mesh had to be modelled) and asymmetric situations in which the building had a skew angle in plan with respect to the tunnel centre line.

The soil was described by a multi-surface plasticity model which accounts for the variation of tangent shear stiffness with strain. The initial stress profile is controlled by a bulk unit weight and a coefficient of earth pressure at rest of \( K_0 = 1.0 \). The geometry of the mesh is summarized in Table 2.1, Page 48.

Tunnelling was modelled by specifying the soil parameters to achieve a volume loss of \( V_L = 2\% \). Tunnel construction was modelled in only four stages by using the same approach adopted by Augarde et al. (1998), described on Page 52.

In their study Burd et al. (2000) compared analyses in which building and soil movement were coupled with greenfield analyses whose settlement and horizontal displacements were then imposed directly on the building (referred to as uncoupled analysis)\(^5\).

From their analyses they draw the following conclusions:

\(^5\)The meaning of the terms coupled and uncoupled in this context has to be distinguished from their use in consolidation analysis.
• The stiffness of the building reduces differential settlement although significant tilting was observed for a building with an eccentricity with respect to the tunnel centre line.

• In sagging deformation the building behaves stiff and develops substantially less damage during a coupled analysis compared to the uncoupled one. They suggest that the ground provides a certain amount of lateral restraint when the building is subjected to sagging deformation. Similar conclusions were drawn by Burland & Wroth (1974).

• In hogging such a restraint is not provided and the structure behaves more flexibly leading to higher degrees of damage than in sagging. In such a case the coupled analysis developed more damage than the uncoupled one. Burd et al. (2000) related this behaviour to the imposition of building weight which alters the settlement behaviour compared to greenfield situations adopted for the uncoupled settlement predictions.

Similar conclusions were presented by Liu et al. (2000) who performed plane strain analyses including both symmetric and eccentric building cases. The geometry of the building façade, the initial stress conditions and the material models describing the soil and the masonry structure were similar to those employed by Burd et al. (2000). By varying the eccentricity and the weight of the structure Liu et al. (2000) concluded that the application of building weight increases the tunnel induced building deformation. This behaviour was due to the fact that prior to tunnel construction a number of yield surfaces of the multisurface soil model were activated after the application of building load whereas such an effect was not present in greenfield conditions. In their model this behaviour leads to lower soil stiffness in building cases which then exhibit larger deformation caused by subsequent tunnel construction.

Another approach using 2D analyses was proposed by Miliziano et al. (2002). They pointed out that in urban areas tunnel construction often follows the route of existing streets and that old masonry buildings are often characterized by a modular and repetitive structural arrangement on either side of the street. They modelled the structure in plane strain and assigned a reduced equivalent stiffness to the masonry which accounts for the spacing between different structural elements of such a ‘terrace house’ configuration in the longitudinal direction. The masonry itself is represented following a discontinuous approach by connect-
ing linear elastic elements\textsuperscript{6} by elasto-plastic interfaces. Their results not only indicate that building deformation decreases with increasing building stiffness but also that it decreases with reducing soil stiffness.

An approach to relate the building’s stiffness to that of the soil was proposed by Potts & Addenbrooke (1997). They modelled the building as an elastic beam with a stiffness representing the overall behaviour of the structure. With this simple model it was possible to perform an extensive parametric study to investigate the influence of both building stiffness and geometry on the interaction problem. From this study they developed a design approach which can be incorporated into the three stages risk assessment. The next section will present this relative stiffness approach in more detail.

\subsection{The relative stiffness approach}

In 1997 Potts & Addenbrooke presented an approach which considers the building’s stiffness when predicting tunnel induced building deformation. Their study included over 100 2D plane strain analyses in which buildings were represented by elastic beams with a Young’s modulus $E$, a second moment of area $I$ and a cross-sectional area $A$. The bending stiffness of the structure is described by $EI$ whereas $EA$ represents the axial stiffness. The geometry of their model was described by the building width $B$, the tunnel depth $z_0$ and the eccentricity $e$ which is the offset between building and tunnel centre lines as shown in Figure 2.26. Their parametric study included a wide range of different bending and axial stiffnesses and also varied the geometry. The stiffness of the structure was related to that of the soil by defining relative stiffness expressions:

\begin{align}
\rho^* &= \frac{EI}{E_s \left( \frac{B}{2} \right)^4}; \\
\alpha^* &= \frac{EA}{E_s \left( \frac{B}{2} \right)} \tag{2.29}
\end{align}

where $E_s$ is the secant stiffness of the soil that would be obtained at 0.01\% axial strain in a triaxial compression test performed on a sample retrieved from half tunnel depth. In their analyses they included tunnel depths of $z_0 = 20m$ and $34m$ leading to a soil stiffness of $E_s = 103MPa$ and $163MPa$, respectively, for the modelled soil profile. The parameter $\rho^*$ is referred to as relative bending stiffness whereas $\alpha^*$ describes the relative axial stiffness. Potts

\textsuperscript{6}Note that the term ‘elements’ is not used in the sense of finite elements as the finite difference code FLAC was used.
Addenbrooke (1997) applied these relative stiffness expressions to plane strain analyses. In such a context $I$ is expressed per unit length and, consequently, $\rho^*$ has the dimension $[1/\text{length}]$. In contrast, $\alpha^*$ is dimension-less in plane strain situations. $EI$ controls the bending behaviour of the beam while $EA$ governs both shear and axial behaviour.

The above relative bending stiffness expression $\rho^*$ is similar to that introduced by Fraser & Wardle (1976) to describe the settlement behaviour (induced by vertical loading) of rectangular rafts. Similar factors were also adopted for retaining wall analyses by Potts & Bond (1994). The definition of relative axial stiffness $\alpha^*$ is similar to the normalized grade beam stiffness used by Boscardin & Cording (1989).

It should be noted that the above relative stiffness expressions are of an empirical nature. They were not derived in the same fashion as the similar expressions for retaining walls (Rowe, 1952; Potts & Bond, 1994). A consequence of this empirical approach are the different
dimensions of the relative stiffness expressions when used in 2D and 3D situations. The advantage of this approach, however, is that it describes building deformation with only two parameters.

The soil profile in their analyses consisted of London Clay represented by a non-linear elastic pre-yield model, described by Jardine et al. (1986) (and also adopted by Addenbrooke et al. (1997), see Page 44) and a Mohr-Coulomb yield and plastic potential surface. The soil was modelled undrained as only the short term response was investigated. An initial hydrostatic pore water pressure profile was modelled with a water table located at 2m below ground surface. The coefficient of earth pressure at rest was $K_0 = 1.5$. A zone of reduced $K_0$ was included in the analyses in order to obtain better predictions for greenfield surface settlement (see Figure 2.14).

The building deformation criteria adopted in their study were deflection ratio and horizontal strain. The building deformation was related to greenfield situations (denoted by a superscript ‘GF’) by defining modification factors as

$$M^{DR_{sag}} = \frac{DR_{sag}}{DR_{sag}^{GF}}$$

$$M^{DR_{hog}} = \frac{DR_{hog}}{DR_{hog}^{GF}}$$

for sagging and hogging, respectively. Similar factors are defined for horizontal compressive and tensile strain:

$$M^{\epsilon_{hc}} = \frac{\epsilon_{hc}}{\epsilon_{hc}^{GF}}$$

$$M^{\epsilon_{ht}} = \frac{\epsilon_{ht}}{\epsilon_{ht}^{GF}}$$

To calculate the greenfield deformation criteria the part of the surface settlement trough below the building must be extracted. This part of the greenfield settlement profile will be referred to as the greenfield section and is described by the same geometry as the building. This situation is shown for deflection ratio in Figure 2.26. If this section contains the point of inflection sagging and compressive horizontal strain occur between one edge of the section and the point of inflection whereas hogging and tension can be found between the point of inflection and the other edge.\(^7\) If the position of the point of inflection is outside the section either sagging and compression or hogging and tension develop within the section.

\(^7\)For long structures both points of inflection may be within the greenfield section. For symmetric cases both hogging zones give the same results while for eccentricities one hogging zone (normally the longer one) is the more critical one.
The horizontal strains $\epsilon_{GF hc}$ and $\epsilon_{GF ht}$ represent the maximum (absolute) value of horizontal compressive and tensile strain over the greenfield section and $\epsilon_{hc}$ and $\epsilon_{ht}$ are the same measure for the building. These expressions have to be distinguished from the maximum horizontal compression and tension, $\hat{\epsilon}_{hc}$ and $\hat{\epsilon}_{ht}$, respectively, found over the entire greenfield profile (compare with Figure 2.3).

Potts & Addenbrooke (1997) performed a parametric study in which $\rho^*$ and $\alpha^*$ were varied independently over a range. While this variation sometimes resulted in an unrealistic combination of bending and axial stiffness it allowed them to investigate the extreme limits of the interaction problem. Additional stiffness combinations were also included which represented 1, 3 and 5-storey structures which consisted of 2, 4 and 6 slabs, respectively, with a vertical spacing of 3.4m. Each slab had a bending stiffness, $EI_{slab}$ and a axial stiffness $EA_{slab}$. The bending stiffness of the entire building was calculated from $EI_{slab}$ of the individual slabs by employing the parallel axis theorem (Timoshenko, 1955). The neutral axis was assumed to be at the middle of the building. Axial stiffness of the entire structure was obtained by assuming axial straining along each slab’s full height. The resulting overall stiffness represents a rigidly framed structure and therefore can be considered to over-estimate the building stiffness.

No vertical load was applied in the analyses. The interface between structure and soil was assumed to be rough. Tunnel construction was modelled by controlling the volume loss with a value of $V_L = 1.5\%$.

For deflection ratio their parametric study revealed the following behaviour:

- For low values of $\alpha^*$ the settlement is equal to those obtained in greenfield conditions regardless of the value of $\rho^*$.

- For low values of $\rho^*$ but high values of $\alpha^*$ the deflection ratio modification factors for both sagging and hogging are higher than unity.

- As $\rho^*$ increases from a low value both $M^{DR_{sag}}$ and $M^{DR_{hog}}$ remain at unity for low values of $\alpha^*$ but decreases for higher values of $\alpha^*$ showing a greater reduction with increasing $\alpha^*$.

- As $\alpha^*$ increases from a low value both $M^{DR_{sag}}$ and $M^{DR_{hog}}$ increases for extremely low values of $\rho^*$ but significantly decrease for higher values of $\rho^*$.
The increase of both $M_{\text{DR}_{\text{sag}}}$ and $M_{\text{DR}_{\text{hog}}}$ with $\alpha^*$ for small values of $\rho^*$ can lead to modification factors exceeding unity. This behaviour is due to the change in surface boundary conditions and due to the uncoupled influence of $\rho^*$ and $\alpha^*$ (for high values of $\alpha^*$) on deflection ratio and horizontal strain, respectively.

When considering realistic combination of axial and bending stiffness the results for different geometries indicated a unique trend when plotting $M_{\text{DR}_{\text{sag}}}$ and $M_{\text{DR}_{\text{hog}}}$ against $\rho^*$. Different upper bound curves were fitted to different degrees of eccentricity, expressed as $e/B$. Modification factors for sagging reduce with eccentricity whereas those for hogging increase. These upper bound curves for $M_{\text{DR}}$ are shown in Figure 2.27a.

For axial strain the parametric study exhibited the following trends:

- For low values of $\alpha^*$ the settlement is equal to those obtained in greenfield conditions regardless of the value of $\rho^*$.
- As $\alpha^*$ increases from a low value the modification factors for compression and tension decrease. Their value does not depend on the magnitude of $\rho^*$ and, consequently, all results (for a given geometry) lie on a unique curve.

When realistic stiffness combination were considered the results for different geometries indicated trends of reducing $M_{\epsilon h}$ with increasing $\alpha^*$. It was possible to set upper bound curves for different degrees of eccentricity. These curves are shown in Figure 2.27b. The figure indicates that for realistic stiffness combinations the strain modification factors are small compared to deflection ratio modification factors.

Potts & Addenbrooke (1997) proposed to include these design curves into a design approach to predict tunnel induced building deformation and assess potential damage. The key steps of this design approach are:

1. The greenfield settlements and horizontal strains for the geometrical section of the building are predicted using Equations 2.4 and 2.6, respectively.
2. The bending and axial stiffness of the structure must be evaluated. Using Equations 2.29 both relative bending and axial stiffness can be calculated.
3. The design curves in Figure 2.27a and b are used to estimate the modification factors.
Chapter 2. Tunnel induced ground and building deformation

Section 2.4

Figure 2.27: Design curves for modification factors of (a) deflection ratio and (b) maximum horizontal strain (after Potts & Addenbrooke, 1997).

4. The deformation criteria of the building can be calculated by multiplying the greenfield deformation criteria with the corresponding modification factors:

\[
DR_{sag} = M^{DR_{sag}} DR_{sag}^{GF},
\]

\[
\epsilon_{hc} = M^{\epsilon_{hc}} \epsilon_{hc}^{GF};
\]

\[
DR_{hog} = M^{DR_{hog}} DR_{hog}^{GF}
\]

\[
\epsilon_{ht} = M^{\epsilon_{ht}} \epsilon_{ht}^{GF}
\]

5. Combinations of \(DR_{sag}\) and \(\epsilon_{hc}\), and \(DR_{hog}\) and \(\epsilon_{ht}\) are used as input parameters in damage category charts such as that shown in Figure 2.25 to evaluate the damage category (as listed in Table 2.2) and to assess the potential damage.

This design approach can be incorporated into the second stage risk assessment as shown in Figure 2.28. Considering the effects of soil-structure interaction in this stage rather than in the third stage reduces the number of cases for which a detailed evaluation has to be carried out.

2.4.6.3 Conclusions

The previous subsections presented different approaches to estimate tunnel induced ground and building deformation. While it is generally accepted that greenfield ground surface settlement can be described by a simple mathematical expression, such a method is not suitable for more complex situations involving existing surface structures. Finite Element
analysis provides a powerful tool to simulate these scenarios. Such analyses, however, can be time consuming, specially when the 3D process of tunnel construction is modelled. When a large number of buildings has to be assessed, the use of Finite Element analyses to model each individual building becomes prohibitively expensive in engineering practice.

The relative stiffness approach by Potts & Addenbrooke (1997) is an alternative as it is based on the results of a large number of Finite Element analyses. By showing that the bending behaviour of a building is governed by its bending stiffness (within a certain range of axial stiffnesses), while the axial stiffness controls the horizontal building strain, it is possible to describe the building’s behaviour by only two parameters. This approach therefore provides simple design charts which can easily be used in engineering practice. Furthermore it can be incorporated into the currently used three stage building damage assessment and reduce the number of buildings for which a detailed evaluation has to be carried out. Because of its advantages it has been applied in engineering practice (Mair & Taylor, 2001).

There were, however, substantial simplifications in the parametric study undertaken by Potts & Addenbrooke (1997). No building load was included in the analyses and it was not possible to model 3D effects of the building geometry and of the tunnelling progress in the plane strain FE study. The following chapters will address these shortcomings and will show
how further building and tunnel features can be incorporated in this design approach.

2.4.7 Summary

Tunnelling in an urban area can affect existing buildings and can lead to damage. This section outlined the currently used design approach and showed how numerical analysis can be employed to refine these design methods.

- The currently applied evaluation of risk of damage consists of three stage. The first stages (preliminary assessment) considers greenfield settlement and its slope while the second stage imposes greenfield values of deflection ratio and horizontal strain on a deep beam model which represents the building. Further details of the building and of the tunnel construction are only considered – if necessary – in the third stage (detailed evaluation).

- The first two stages of this risk assessment are very conservative as they neglect the stiffness of the building and therefore overestimate the tunnel induced building deformation. As the third stage can be very cost and time consuming more building details should be considered in the second stage in order to reduce the number of building cases for which a detailed evaluation is required.

- Case studies of tunnelling projects in London have shown that the building’s own stiffness interacts with tunnel induced ground movement. This interaction leads to a reduction in the building’s deflection ratio. Case studies also indicated that the development of horizontal strain in buildings was negligible during tunnel construction.

- To study the interaction between tunnel construction, soil and building numerical analyses were performed by various authors. In some studies the building was modelled fully 3D allowing any building geometry with respect to the tunnel axis to be modelled. These analyses, however, require a large amount of computational resources. Plane strain analysis models building behaviour in the transverse direction to the tunnel. The computational costs for these analyses are substantially lower than for 3D analyses.
• An extensive parametric study was presented by Potts & Addenbrooke (1997) using plane strain FE analysis in which the building was represented by an elastic beam. By defining modification factors, they related the deflection ratio and horizontal strain for a wide range of different stiffnesses and geometries to corresponding greenfield situations. Relative stiffness expressions were defined for bending and axial stiffness taking into account the building’s stiffness and geometry as well as the soil stiffness. They showed that modification factors correlated with relative stiffness expressions and developed new design charts to assist in the assessment of building damage in response to tunnelling.
Chapter 3

Method of analysis

3.1 Introduction

This chapter gives an overview of the Finite Element Method (FEM) and summarizes key aspects of the analyses presented in the following chapters. In the first part the basic FE formulation is summarized and details about its application in the Imperial College Finite Element Program (ICFEP) (Potts & Zdravković, 1999, 2001) are discussed. A more detailed summary of the governing equations of the FEM can be found in Appendix A. The second and third parts of this chapter introduce features which apply to most analyses in this thesis. Firstly soil models are introduced and, secondly, details about the modelling of a building, tunnel excavation etc. are presented.

3.2 The Finite Element Method

3.2.1 Requirements for a solution

Any theoretical solution of a static boundary value problem must satisfy the following three requirements:

1. **Equilibrium:** This can be expressed by the following equations:
   \[
   \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0; \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0; \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \gamma = 0 \quad (3.1)
   \]
   where the bulk unit weight \( \gamma \) only in the z-direction. Inertia effects are not considered in the above equations.
Chapter 3. Method of analysis

Section 3.2

An equivalent way of expressing equilibrium is by the principle of minimum potential energy.

2. **Compatibility**: This requires the continuity of the displacement field. No gaps or overlaps must occur in the problem domain during the deformation process. Compatibility is expressed mathematically by defining strain in terms of the differentials of displacement which is described by a continuous function.

3. **Material behaviour**: This links strains (and therefore displacements) with stresses. It is expressed by the constitutive equation \( \sigma = D \epsilon \) where \( D \) is the constitutive matrix. The stresses \( \sigma \) and the strains \( \epsilon \) can both be expressed as symmetrical \( 3 \times 3 \) matrix. They are, however, often written in vector form. The constitutive matrix then has \( 6 \times 6 = 36 \) entries. Under consideration of symmetry conditions the number of independent material parameters reduces to 21 and for isotropic linear-elastic materials only 2 independent constants are required.

4. In addition the **boundary conditions** given for both force and displacement must be fulfilled.

Table 3.1 summarizes how different solution techniques satisfy the requirements described above. It can be seen that closed form solutions are normally restricted to linear elastic behaviour. Other solution techniques listed in Table 3.1 are \textit{Limit equilibrium}, \textit{Stress field} and \textit{Limit analysis} which are all based on simplifying assumptions and lead to crucial limitations when applied to complicated boundary value problems. The strength of a \textit{Full numerical analysis} is that is able to satisfy all solution requirements albeit approximately. The most widely used numerical techniques in geotechnical engineering are the \textit{Finite Element Method} and the \textit{Finite Difference Method} (Potts & Zdravković, 1999).

The Finite Element Method was developed during the 1960s mainly for the aeroplane industry but soon became widely used in other areas because of its flexibility and diversity (Huebner & Thornton, 1982). The method is based on the idea of discretisation of the geometry of a problem into several smaller regions, termed elements, over which a solution can be approximated. This approximation is done by assigning a primary variable (e.g. displacement, stresses) at several discrete nodes and using shape functions to determine the value of the variable at any point within the element from these nodal values. The element
Chapter 3. Method of analysis

3.2 Finite Element formulation

3.2.1 Element discretisation

The problem domain is subdivided into several small regions called finite elements. The geometry of these elements is expressed in terms of discrete nodes. For the 2D analyses presented in this thesis quadrilateral elements with 8 nodes (4 corner and 4 mid-side nodes) were used to model the soil while 20 node hexadron elements were used in the 3D analyses. The use of mid-side nodes allows the element boundaries to be curved (in contrast to linear 4 node 2D and 8 node 3D elements). Curved boundaries were used during the analyses when modelling the tunnel geometry.

3.2.2 Limit equilibrium

3.2.2.1 Equilibrium

3.2.2.2 Compatibility

3.2.2.3 Constitutive behaviour

3.2.2.4 Boundary conditions

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Equilibrium</th>
<th>Compatibility</th>
<th>Constitutive behaviour</th>
<th>Force</th>
<th>Disp.</th>
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</thead>
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<td>NS</td>
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<td>NS</td>
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<td>S</td>
<td>NS</td>
<td>Ideal plasticity with associated flow rule</td>
<td>S</td>
<td>NS</td>
</tr>
<tr>
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<td>S</td>
<td>NS</td>
<td>Soil modelled by springs of elastic interaction factors</td>
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<td>Upper bound</td>
<td>NS</td>
<td>S</td>
<td>Soil modelled by springs of elastic interaction factors</td>
<td>NS</td>
<td>S</td>
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Table 3.1: Basic solution requirements satisfied by the various methods of analysis (after Potts & Zdravković (1999)).

equations are then derived using the principle of minimum potential energy which essentially invokes equilibrium. These steps and their application in ICFEP are briefly described in the following paragraphs. A more detailed description of the FE theory can be found in Appendix A.
The analyses presented in the subsequent chapters also included some structural components such as the tunnel lining and a building. These structures were modelled using 3 node isoparametric curved Mindlin beam elements (Day & Potts, 1990) in the 2D case or 8 node isoparametric shell elements for 3D analysis (Schroeder, 2003).

3.2.2.2 Displacement approximation

The displacement is the primary variable normally chosen for the FE formulation in geotechnical engineering. It is described over each element using shape functions which derive the displacement for any point from the nodal displacement values. Isoparametric elements (i.e. applying the same shape functions for the descriptions of geometry and displacement field) were used in the analyses presented in this thesis. The shape functions are defined in a way that displacement and geometry on an element side only depend on their nodal values on this boundary, thus satisfying continuity between two elements, see Potts & Zdravković (1999).

3.2.2.3 Element formulation

Using the principle of minimum potential energy the element equations can be derived as

\[
\mathbf{K}_E \Delta \mathbf{d}_E = \mathbf{R}_E \quad \text{(see Equation A.21)}
\]

where \( \mathbf{K}_E \) is the element stiffness matrix, \( \Delta \mathbf{d}_E \) contains the nodal displacements of nodes connected to an element and \( \mathbf{R}_E \) is called the right hand side load vector. \( \mathbf{K}_E \) is formed by integrating the product \( \mathbf{B}^T \mathbf{D} \mathbf{B} \), where \( \mathbf{B} \) contains the derivatives of the shape functions, over each element. This integration is evaluated using a Gaussian integration scheme. In this work an integration of the order 2 was used.

3.2.2.4 Global equations

The global equations

\[
\mathbf{K}_G \Delta \mathbf{d}_G = \mathbf{R}_G \quad \text{(see Equation A.23)}
\]

are formed by assembling all element equations of the problem domain. The global stiffness matrix \( \mathbf{K}_G \) is singular unless the boundary conditions are invoked. Prescribed nodal forces
are included into the right hand side vector $\mathbf{R}_G$ while $\Delta d_G$ incorporates the prescribed nodal displacements.

The global stiffness matrix is sparse and shows a band structure with the band width depending on the numbering of the nodes. ICFEP uses a Cuthill and McKee algorithm to minimize this bandwidth and therefore to reduce the storage required for this matrix.

The global equilibrium equation can then be solved using Gaussian elimination with matrix decomposition.

### 3.2.3 Non linear FEM

The material behaviour of the soil is incorporated into the FE formulation via the constitutive matrix $\mathbf{D}$ (Equation A.13). If the soil behaves linearly elastic this matrix remains constant during the calculation and the global equilibrium equation can be solved using a Gaussian elimination solution technique. If, however, the soil is modelled to behave non-linear elastic and/or elasto-plastic the constitutive matrix $\mathbf{D}$ depends on the stress and/or strain level and the solution algorithm has to include this change of material behaviour. Several different solution strategies exist to deal with this problem. In general the loading history is divided into several increments which have to be solved consecutively.

Three common solution techniques have been implemented into ICFEP, namely the tangent stiffness method, the visco-plastic method and the modified Newton-Raphson method (MNR). Potts & Zdravković (1999) give an overview of these methods and compare results of analyses using all three solution algorithms. It is shown that both the tangent stiffness approach and the visco-plastic method can lead to significant errors depending on the increment size and on the complexity of the soil model used. The best results were obtained using the MNR method. This approach has therefore been applied in all analyses presented in this thesis. During the application of the MNR algorithm the constitutive equations have to be integrated along the applied strain path to obtain the stress changes. ICFEP provides different kinds of stress point algorithms to perform this integration: namely the substepping algorithm and the return algorithm. Comparing both approaches Potts & Ganendra (1994) concluded that the substepping algorithm gives better results and consequently it was applied in all the analyses performed for this thesis.

The MNR method and the substepping algorithm are explained in more detail in Ap-
The tolerances controlling the MNR scheme for the analyses presented in the following chapters were set to 2%. The substepping tolerance $SSTOL = 0.01\%$ (see in Appendix A.2.2).

### 3.2.4 Geotechnical considerations

The previous sections give a general overview of the standard finite element method. However, the simulation of geotechnical problems such as tunnelling requires additional features to be modelled. These are described in the following sections.

#### 3.2.4.1 Pore water pressure

Appendix A derives the FE formulation in terms of total stress. In order to fully simulate seepage and consolidation behaviour a coupled formulation must be used incorporating the fluid pressure as a primary unknown together with the displacement.

However, in this thesis only the short term response to tunnelling in clay with a relative low permeability is investigated in which case the problem can be regarded to behave undrained. In order to calculate the excess pore fluid pressure $\Delta p_f$ under such conditions a pore fluid vector

$$\Delta \sigma_f = [\Delta p_f \Delta p_f \Delta p_f 0 0 0]^T$$

is introduced and incorporated using the principle of effective stress

$$\Delta \sigma = \Delta \sigma' + \Delta \sigma_f$$

The constitutive matrix (Equation A.13) can then be expressed as

$$D = D' + D_f$$

where $D_f$ is the pore fluid stiffness which is related to the bulk modulus of the single phase\(^1\) pore fluid $K_f$. This stiffness has the form

$$D_f = K_e \begin{bmatrix} 1^{3 \times 3} & 0^{3 \times 3} \\ 0^{3 \times 3} & 0^{3 \times 3} \end{bmatrix}$$

\(^1\)This does not include two phase problems such as partly saturated soils.
where each entry of the matrix is a $3 \times 3$ matrix. $K_e$ is the equivalent bulk modulus of the pore fluid and is related to $K_f$ and $K_s$ (the bulk modulus of the solid particles) by
\[
K_e = \frac{1}{\frac{n}{K_f} + \frac{1-n}{K_s}} \tag{3.6}
\]
where $n$ is the soil porosity. It is shown by Potts & Zdravković (1999) that the relative magnitudes of $K_f$ and $K_s$ are not of great importance as both moduli are much bigger than the stiffness of the soil skeleton $K_{skel}$. Hence, the above expression reduces to $K_e \approx K_f$.

In an undrained analyses $K_e$ must be assigned to a high value compared with $K_{skel}$. Values which are too large, however, lead to numerical instability as the equivalent undrained Poisson’s ratio $\mu_u$ approaches 0.5 (Potts & Zdravković, 1999). Within this thesis a value of $K_e = 100K_{skel}$ was adopted.

3.2.4.2 Excavation

The simulation of excavation is required when modelling the construction of a tunnel. In the event of excavation a new boundary is introduced into the FE mesh. Figure 3.1 summarizes the excavation procedure modelled by ICFEP.

1. The part which is to be excavated (the shaded tunnel face) is determined and removed from the active mesh.

2. The equivalent nodal forces which act at the internal boundary of excavation before the elements are removed are calculated and imposed as tractions $T$ along this boundary. There are neither displacements nor stress changes within the remaining soil at this stage.

3. Surface tractions of magnitude $-T/i$ are applied to the system over $i$ increments.

Excavation over $i$ increments enables the simulation of the construction of a lining at a certain increment of excavation which allows a specified volume loss $V_L$ to be achieved, as further explained in Section 3.4.4.

3.2.4.3 Construction

As mentioned in the section above the tunnel lining is constructed during the excavation process. The beam elements representing the tunnel lining are present in the original FE
mesh but are deactivated at the beginning of the analysis. They are then reactivated at a chosen increment. During this increment the stiffness of the material to be constructed is assigned to a low value. In all the analyses presented in this thesis the tunnel lining was assumed to have no weight. No self weight body forces were therefore applied to the constructed elements. At the end of the increment the material parameters of the lining are changed to represent the material behaviour of the installed lining.

3.2.4.4 Soil-Structure interface

When investigating soil-structure interaction the behaviour of the actual interface between both the soil and any structure is of significance. The compatibility condition for adjacent elements prevents these elements from moving relative to each other. In order to simulate such a relative movement interface elements were introduced. There are several approaches to model such elements:

- **Continuum elements** applied with standard constitutive laws.
- **Linkage elements** using discrete springs to connect opposite nodes.
- **Special interface elements** with either zero or finite thickness.

Interface elements with zero thickness have been implemented into ICFEP (Day & Potts, 1994). The elements incorporated in the study presented in Chapter 6 have 6 nodes and are compatible with the 8 node solid elements and the 3 node beam elements used in the 2D analyses.
The strain within an interface element is defined as the difference in displacement between the bottom and the top of the element, see Figure 3.2:

\[ \gamma_{if} = u_{bot} - u_{top} \]  
\[ \epsilon_{if} = v_{bot} - v_{top} \]

(3.7)  
(3.8)

It follows from this definition that the dimension of interface strain is \([length]\) rather than being dimension-less as the strain is defined for the adjacent soil and beam elements.

Day & Potts (1994) showed that the use of interface elements can lead to numerical instability depending on the difference in stiffness between the interface and adjacent soil and/or beam elements and on the element size. Consequently care was taken in selecting the stiffness values assigned to these elements.

### 3.3 Material models

The choice of a appropriate soil model is a key step in any FE analysis. In order to be consistent with the work carried out by Potts & Addenbrooke (1997) the same soil model and parameters as used in their analyses were applied. These are a non-linear elastic pre-yield formulation together with a yield surface described by a Mohr-Coulomb model. These models will be described in the following sections. Chapter 8 will investigate the influence of anisotropy on FE tunnel analyses. The soil model and parameters used for this purposes will be discussed in that chapter and in Appendix B.

The material models are formulated in terms of stress and strain invariants. These are
Chapter 3. Method of analysis

Section 3.3

defined as

Mean effective stress:
\[ p' = \frac{1}{3} (\sigma'_1 + \sigma'_2 + \sigma'_3) \]  (3.9)

Deviatoric stress:
\[ J = \sqrt{\frac{1}{6} \left( (\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2 \right) } \]  (3.10)

Lode’s angle
\[ \theta = \tan^{-1} \left( \frac{1}{\sqrt{3}} \left( 2 \frac{(\sigma'_2 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)} - 1 \right) \right) \]  (3.11)

Volumetric strain
\[ \epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3 \]  (3.12)

Deviatoric strain
\[ E_d = 2 \sqrt{\frac{1}{6} \left( (\epsilon_1 - \epsilon_2)^2 + (\epsilon_1 - \epsilon_3)^2 + (\epsilon_2 - \epsilon_3)^2 \right) } \]  (3.13)

where \( \sigma'_1, \sigma'_2, \sigma'_3 \) are the principal stresses and \( \epsilon_1, \epsilon_2, \epsilon_3 \) the principal strains.

3.3.1 Non linear elastic behaviour

Using local strain measurements on triaxial samples (Burland & Symes, 1982) it has been found that the behaviour of clays at small strains is highly non-linear. Test results presented by Jardine et al. (1984) show an initially stiff response of London Clay when sheared in an undrained triaxial test. Figure 3.3a presents typical results from such tests. It can be seen that results based on external measurements show initially a linear stress-strain relationship. This linear behaviour, however, is not evident when using more accurate local instrumentation. The non-linearity can also be seen when plotting the normalized undrained soil stiffness \( E_u \) versus axial strain \( \epsilon_{ax} \). Although in Figure 3.3b the soil stiffness is normalized against the undrained shear strength \( S_u \) Jardine et al. (1986) concluded that a normalization against mean effective stress \( p' \) would be preferable as it can easily be determined in the laboratory while the undrained strength depends on many factors such as rate, stress path and sample disturbance. However, although the calculation of \( p' \) in laboratory testing is without ambiguity it is more difficult to determine in the field as it depends on \( K_0 \).

The normalization of soil stiffness against stress level is in agreement with critical state theory (Wroth, 1971). Jardine (1995) summarizes results for a variety of soil types showing that for very small strains the stiffness depends on the stress level raised to a power \( N \). This exponent depends on strain level and tends to unity for moderate strains. The stiffness therefore becomes progressively more linearly dependent on \( p' \) as strain increases.

Jardine et al. (1986) developed trigonometric expressions to describe the non-linear elastic
strains determined from local instrumentation
strains determined from external strains corrected for load cell compliance
Apparent linear modulus from external measurements \( E = 300 \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.3.pdf}
\caption{Typical London Clay unconsolidated undrained test data: (a) initial stress strain behaviour; (b) stiffness-strain characteristics (after Mair (1993)).}
\end{figure}

behaviour. From their original formulation, following variations of the tangent shear modulus \( G \) and the tangent bulk modulus \( K \) with deviatoric strain \( E_d \) and volumetric strain \( \epsilon_v \) respectively were derived:

\[
\frac{G}{p'} = A + B \cos (\beta X) - \frac{B \beta \gamma X^{-1}}{2.303} \sin (\beta X) \quad \text{with} \quad X = \log_{10} \left( \frac{E_d}{\sqrt{3C}} \right) \tag{3.14}
\]

\[
\frac{K}{p'} = R + S \cos (\delta Y) - \frac{S \delta \mu Y^{-1}}{2.303} \sin (\delta Y) \quad \text{with} \quad Y = \log_{10} \left( \frac{\epsilon_v}{T} \right) \tag{3.15}
\]

where \( A, B, C, R, S, T, \beta, \gamma, \delta, \) and \( \mu \) are material parameters which define the shape of the trigonometric curves. Minimum and maximum strains are defined in order to limit the range over which the mathematical expressions defined above are used. Below \( E_d, \text{min}, \epsilon_v, \text{min} \) or above \( E_d, \text{max}, \epsilon_v, \text{max} \) the tangent shear and bulk moduli only depend on mean effective stress \( p' \) and not on strain level.

This model has been implemented into ICFEP for some time and has been used for tunnelling analyses in the past. As shown in Section 2.3.1 Addenbrooke et al. (1997) demonstrated the importance of using such a model when predicting tunnel induced greenfield settlement troughs. Mair (1993) relates the range of strain experienced during the construction of different geotechnical structures to a typical stiffness-strain curve. Figure 3.4 shows that the strains developed during tunnel construction lie well within the range of stiffness reduction.

Table 3.2 summarizes the parameters used for this model in this thesis. As mentioned
Figure 3.4: Range of shear strain observed during construction of different geotechnical structures (after Mair (1993)).

Table 3.2: Material parameters used for non-linear elastic soil model.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C [%]</th>
<th>β</th>
<th>γ</th>
<th>$E_{d,\min}$ [%]</th>
<th>$E_{d,\max}$ [%]</th>
<th>$G_{\min}$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>373.3</td>
<td>338.7</td>
<td>$1.0 \times 10^{-4}$</td>
<td>1.335</td>
<td>0.617</td>
<td>$6.66 \times 10^{-4}$</td>
<td>0.69282</td>
<td>2333.3</td>
</tr>
<tr>
<td>R</td>
<td>S</td>
<td>T [%]</td>
<td>δ</td>
<td>µ</td>
<td>$\epsilon_{v,\min}$ [%]</td>
<td>$\epsilon_{v,\max}$ [%]</td>
<td>$K_{\min}$ [kPa]</td>
</tr>
<tr>
<td>549.0</td>
<td>506.0</td>
<td>$1.00 \times 10^{-3}$</td>
<td>2.069</td>
<td>0.42</td>
<td>$5.00 \times 10^{-3}$</td>
<td>0.15</td>
<td>3000.0</td>
</tr>
</tbody>
</table>

above these values are identical to the parameters used by Potts & Addenbrooke (1997).

3.3.2 Mohr Coulomb yield surface

The failure criterion on which the Mohr-Coulomb model is based is shown in Figure 3.5a and can be expressed as

$$\tau_f = c' + \sigma'_{\text{nf}} \tan \varphi'$$  \hspace{1cm} (3.16)

with $\tau_f$ and $\sigma'_{\text{nf}}$ being the shear and the normal effective stress on the failure plane respectively. In terms of stress invariants the Mohr-Coulomb yield surface is defined as:

$$F (\sigma', \mathbf{k}) = J - \left( \frac{c'}{\tan \varphi'} + p' \right) g(\theta) = 0 \quad \text{with} \quad g(\theta) = \frac{\sin \varphi'}{\cos \theta + \frac{\sin \theta \sin \varphi'}{\sqrt{3}}}$$  \hspace{1cm} (3.17)

The vector $\mathbf{k}$ contains the hardening parameters which are the cohesion $c'$ and the angle of shearing resistance $\varphi'$. As they are assumed to be constant there is no hardening rule required. The yield function forms an irregular hexagonal cone in principal stress space. Figure 3.5b shows the shape of the hexagon in the deviatoric plane (i.e. normal to the space diagonal with $\sigma'_1 = \sigma'_2 = \sigma'_3$).

If an associated flow rule is applied the plastic potential has the same form as the yield surface leading to dilatant plastic volumetric strains. The magnitude of these volumetric
strains are often unrealistic. To overcome this problem the plastic potential has to be formulated separately. This is done by applying the same equation as for the yield surface but using the angle of dilation $\nu$ rather than the angle of shearing resistance $\varphi'$ to define the shape of the cone. The plastic potential has then to be defined in a way that it always intersects with the yield surface at the current stress point.

However, even with the use of such a non-associated flow rule ($0 < \nu < \varphi'$) the model predicts finite plastic volumetric strains which continue to occur as long as the soil is yielding. This is in contrast to real soil behaviour where a constant volume is observed for large strains (critical state). This problem could be addressed by allowing the angle of dilation (and the other soil parameters) to depend on the level of strain. In this thesis the Mohr-Coulomb model was, however, used with constant values of $\nu$, $\varphi'$ and $c'$ (see Table 3.3).

<table>
<thead>
<tr>
<th>$c'$</th>
<th>$\varphi'$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>25.0</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Table 3.3: Material parameters used in Mohr-Coulomb model.

### 3.4 Modelling of the soil structure interaction

This section contains general information which applies to all the analyses presented in the following chapters. Descriptions of particular aspects of certain analyses, such as soil-structure
interface characteristics or the simulation of building weight, are addressed in the relevant chapters.

### 3.4.1 Initial stress conditions

Before any geotechnical process can be modelled in a FE analyses the initial stress conditions in the ground must be prescribed. This is done by calculating the vertical total stress \( \sigma_v \) from the bulk unit weight of the material(s). From a given pore water pressure profile the vertical effective stress \( \sigma'_v = \sigma_v - p_f \) is then evaluated. The horizontal stresses in the ground are finally computed from the given lateral earth pressure coefficient at rest: \( \sigma'_h = K_0 \sigma'_v \).

In the studies performed in this thesis the ground profile consisted of London Clay with a bulk unit weight of \( \gamma = 20 \text{kN/m}^3 \). A hydrostatic pore water pressure distribution was prescribed with a water table 2m below the ground surface. Above the water table pore water suctions were specified. The lateral earth pressure coefficient \( K_0 = 1.5 \) was applied to the whole FE mesh, in contrast to the use of a \( K_0 \)-reduced zone around the tunnel in order to obtain better results for the greenfield settlement prediction. This latter approach was used by Potts & Addenbrooke (1997) (see Section 2.3.1).

This thesis, however, presents studies where other geotechnical processes, such as the construction of a building are simulated before the tunnel construction starts. The modelling of a zone of reduced \( K_0 \) would not be reasonable in such a context and was therefore not included in the present study. Section 4.2.3 investigates how this change in initial conditions affects the results.

### 3.4.2 Geometry of the problem

Figure 3.6 shows the geometry of the building with respect to the tunnel. The coordinate system included in this figure will be used throughout this thesis: the origin is on the ground surface above the tunnel centre line, the \( x \)-axis refers to the horizontal direction transverse to the tunnel axis, \( z \) refers to the depth and is positive downwards. In 3D analyses the longitudinal horizontal direction is described by \( y \) with the mesh extending in negative \( y \)-direction.

In their original study Potts & Addenbrooke (1997) varied the building width \( B \), the eccentricity \( e \) and included two tunnel depths \( z_0 \) for a wide range of building stiffnesses.
They expressed the relative stiffness using the half building width \( H = \frac{B}{2} \). In this thesis only \( B \) will be used to describe the width of the building. The building dimension longitudinal to the tunnel axis is described by the length \( L \). It only applies to 3D analyses presented in Chapters 7 and 9.

The tunnel diameter was not a variable but was set to \( D = 4.146 \text{m} \). This value is typical for running tunnels on the London underground system and was applied throughout this thesis.

In their study Potts & Addenbrooke (1997) applied a tunnel depth of either \( z_0 = 20 \text{m} \) or \( 34 \text{m} \). Most analyses presented in this thesis are based on this geometry. Section 4.4.3, however, includes \( z_0 = 15 \text{m} \) and \( 20 \text{m} \) in order to investigate the influence of tunnel depth on building deformation.

Figure 3.6: Geometry of the problem

3.4.3 Modelling of the building

The building was modelled by an elastic beam (in 2D analyses) or shell (in 3D) located on the ground surface. It had a Young’s modulus \( E \), a second moment of area \( I \) and cross sectional area \( A \). The building was considered to be a concrete frame structure consisting of a certain number of storeys. For a building with \( m \) storeys the properties of the elastic beam were calculated assuming that the building consists of \( m + 1 \) slabs with a vertical spacing of 3.4m. The thickness of each slab was taken to be \( t_{\text{slab}} = 0.15 \text{m} \). With \( L \) being the out of plane
dimension of the slab the second moment of area $I$ and the area $A$ are defined as

$$I_{\text{slab}} = \frac{t_{\text{slab}}^3 L}{12}; \quad A_{\text{slab}} = t_{\text{slab}} L \quad (3.18)$$

leading in plane strain conditions to an area of $A_{\text{slab}} = 0.15 \text{m}^2/\text{m}$ and an second moment of area of $I_{\text{slab}} = 0.028 \text{m}^4/\text{m}$. For concrete a Young’s modulus of $E_c = 23.0 \times 10^6 \text{kN/m}^2$ was used. The second moment of area for the equivalent single beam was then calculated using the parallel axis theorem (Timoshenko, 1955) assuming the neutral axis to be at the mid-height of the building:

$$(E_c I)_{\text{struct}} = E_c \sum_{1}^{m+1} (I_{\text{slab}} + A_{\text{slab}} h_m^2) \quad (3.19)$$

where $h_m$ is the vertical distance between the structure’s and the $m^{th}$ slab’s neutral axis. Axial straining is assumed along each structure’s full height to give the axial stiffness:

$$(E_c A)_{\text{struct}} = (m + 1) (E_c A)_{\text{slab}} \quad (3.20)$$

The properties for the beam elements used in plane strain analyses are the Young’s modulus $E_{fe}$, Area $A_{fe}$ and the second moment of area $I_{fe}$. In order to get a consistent set of parameters, the stiffness expressions above have to be converted to:

$$(E_c I)_{\text{struct}} = \frac{E_{fe} t_{fe}^3 L}{12}; \quad (E_c A)_{\text{struct}} = E_{fe} t_{fe} L \quad (3.21)$$

where $t_{fe}$ is the thickness of the beam used in the FE analysis and $L$ is the out of plane dimension of the structure. Rearranging Equations 3.21 leads to

$$t_{fe} = \sqrt{\frac{12 (E_c I)_{\text{struct}}}{(E_c A)_{\text{struct}}}} \quad (3.22)$$

For plane strain analyses the out of plane dimension $L$ is unity and the properties of the beam elements are obtained from

$$E_{fe} = \frac{(E_c A)_{\text{struct}}}{t_{fe}}; \quad A_{fe} = t_{fe}; \quad I_{fe} = \frac{t_{fe}^3}{12} \quad (3.23)$$

The input parameters for the shell elements used in 3D analysis are $t_{fe}$ and $E_{fe}$.

In this study 1-, 3-, 5- and 10-storey buildings are considered while greenfield conditions are modelled using a beam with a negligible stiffness. The stiffness parameters for the
Table 3.4: Stiffness of buildings. A m-storey building consists of m+1 slabs.
and the ground. In the other chapters the building is assumed to be weightless. Without the use of interface elements the building is assumed to be connected to the soil such that the full soil strength can be mobilised at the interface. The implications of this assumption are investigated in Chapter 6 where interface elements are incorporated into the analysis.

3.4.4 Modelling of tunnel construction

Tunnel construction is simulated with the soil behaving undrained (see Section 3.2.4.1). The mesh for a symmetrical plane strain geometry with a 20 m deep tunnel is presented in Figure 3.8. The beam representing the building is not shown in this figure. This beam is activated at the beginning of an analysis. The initial stress and stiffness profile in the soil
are not altered as long the building is assumed to be weightless.

In the 2D analyses the tunnel is excavated over 15 increments. The increment in which
the lining is installed is chosen in order to obtain a certain volume loss $V_L$ as defined in
Equation 2.3 (Page 29). Under undrained conditions $V_L$ can be established by determining
the volume $V_S$ of the surface settlement trough. This value is calculated numerically. Figure
3.9 shows an example of development of volume loss against increments of excavation and
percentage of unloading. The results presented in this graph are for a greenfield situation
above a 20m deep tunnel. No lining was installed in this analysis in order to demonstrate the
increase of $V_L$ with number of excavation increments. To achieve a typical value of $V_L = 1.5\%$,
the lining is installed on completion of the 7th excavation increment as marked in the graph.

**Figure 3.8:** Finite Element mesh for 20m symmetrical building geometry.

**Figure 3.9:** Development of volume loss over increment of excavation and percentage
of unloading.
Chapter 3. Method of analysis

This volume loss is detected on this particular increment when a greenfield excavation is analysed. As long not stated otherwise the results presented in this work are taken from this increment. It should be noted that with different building scenarios the volume loss varies. Section 4.2.1 will focus on the implications of this change in volume loss.

When modelling 3D tunnel construction the excavation process is not volume loss controlled. The $V_L$ depends on the excavation length $L_{exc}$ that is the longitudinal distance over which soil is excavated in one step of the analysis. This procedure will be further described in Section 8.2.

### 3.4.5 Calculation of the building deformation criteria

When investigating building deformation most of the results will be presented in terms of deflection ratio $DR$ and maximum horizontal strain $\epsilon_h$ and their corresponding modification factors. For the calculation of the deflection ratio (defined in Figure 2.26, Page 73) the point(s) of inflection had to be determined. It was shown in Section 2.2.1.1 that the point of inflection separates zones of sagging/compression from hogging/tension. When buildings are included in the analysis the change from sagging to hogging can occur at a different position to the change from compression to tension. In this thesis the term point of inflection will refer to the change from sagging to hogging (i.e. the point of maximum slope of the settlement trough). These points were found by calculating the rate of change of the slope of the surface settlement trough (i.e. its 2nd differentiation) numerically in a spread sheet and locating the change of sign. For this purpose the nodal displacement of the beam elements were used. The deflection ratio was then calculated for both sagging and hogging by dividing the maximum deflection $\Delta$ by the length $L$ connecting the points of inflection with the end of the structure or with each other. $DR_{sag}$ is defined to be positive while $DR_{hog}$ is expressed by negative values.

This procedure is in contrast to the approach adopted by Potts & Addenbrooke (1997). They determined the positions of the points of inflection graphically. It has been found that the spread sheet calculation increases the accuracy and thus the consistency of the results.

The horizontal strain $\epsilon_h$ was obtained directly from the ICFEP output. It was given as the maximum compressive or tensile horizontal strain of the neutral axis of the beam elements and therefore does not include any bending effects. ICFEP assigns tensile and compressive
strain with a positive and negative value respectively.

When comparing compressive strains and hogging magnitudes in this work their absolute values are taken into consideration. In this context a *higher* compressive strain has mathematically a lower value.

Some of the deflection ratio results show a scatter. This is specially the case for hogging in situations of short buildings and/or deep tunnels (for example $B < 60\text{m}$ with $z_0 = 20\text{m}$). The reason can be found in the relatively short hogging zone. A small change in position of the point of inflection can result a significant change in $DR_{\text{hog}}$. Even more significant scatter can occur when $M^{DR_{\text{hog}}}$ is calculated as both building and greenfield measures are sensitive to the determination of each point of inflection.
Chapter 4

An evaluation of the relative stiffness method

4.1 Introduction

The relative stiffness method proposed by Potts & Addenbrooke (1997) was based on a parametric FE-study in which buildings were modelled as elastic weightless beams. In addition the beams were assumed to always remain in contact with the soil along their width. A wide range of building stiffnesses represented by the axial stiffness $E_A$ and the bending stiffness $EI$ were included in their analyses. The geometry of the system (the building width $B$, the eccentricity $e$ and the tunnel depth $z_0$, see Figure 3.6, Page 95) was also varied. Relative stiffness expressions, which take account of the building geometry when relating building stiffness to soil stiffness, were defined. The deformation data were then plotted as deflection ratio against relative bending stiffness $\rho^*$ and as maximum horizontal strain versus relative axial stiffness $\alpha^*$ as explained in Section 2.4.6.2.

While in their work the influence of both bending and axial stiffness on deformation behaviour was investigated independently the effect of different geometric parameters was not uncoupled from each other. Furthermore, their study only focused on the deformation of the structure itself in order to compare it with the corresponding greenfield surface settlement profile. While this approach was justified in order to describe the building deformation it did not provide information about the mechanics of the tunnel-soil-structure interaction.
Chapter 4. An evaluation of the relative stiffness method

This chapter investigates these aspects by assessing the influence of different geometric parameters independently. The results of this study will be used to modify the relative stiffness expressions in order to reduce the scatter observed when plotting modification factors against relative stiffness.

In addition it is not only demonstrated how the building’s geometry and stiffness affects its deformation but also how the soil displacement field is affected by the variation of these parameters. This approach will lead to a better understanding of the tunnel-soil-structure interaction.

The building is modelled in the same way as in the original work by Potts & Addenbrooke (1997). It is therefore analysed in plane strain and considered to be weightless and always remains in contact with the soil.

In the original analyses the volume loss was chosen to be close to $V_L = 1.5\%$ and was not varied. Other constants were the initial stress conditions and the mesh geometry. These presumption influence the results. Before investigating the effect of building parameters the following section first assesses how these analysis characteristics affect the results.

4.2 Parameter of FE analysis

4.2.1 Volume loss

As explained in Section 3.4.4 a certain volume loss $V_L$ has to be specified to simulate tunnel construction under plane strain conditions. The results for the settlement are then taken from the increment where this particular volume loss was reached. A volume loss of $V_L = 1.5\%$ was adopted in the study of Potts & Addenbrooke (1997) and is therefore used in all 2D analyses presented in this thesis. Although this is a realistic value for London Clay (O’Reilly & New, 1982) higher volume losses have been observed. Standing et al. (1996) for example presents greenfield measurements at St. James’s Park, London, indicating a high volume loss of $V_L = 3.3\%$.

In the 3D tunnel analyses presented in Chapter 9 higher values of $V_L$ were obtained. In order to compare 2D and 3D results and to estimate the building deformations caused by higher volume losses the influence of $V_L$ on the modification factors has to be investigated. This section presents results from a set of analyses in which no tunnel lining was installed.
Figure 4.1: Development of volume loss $V_L$ (a) and settlement (b) during simulation of excavation process. Data for greenfield analysis without tunnel lining, $z_0 = 20$m.

over the 15 increments of tunnel excavation.

Figure 4.1a shows the development of volume loss with percentage of unloading for greenfield conditions above a 20m deep tunnel. The corresponding increment numbers of the excavation process are given along the top axis. The volume loss increases with unloading. As described in Section 3.4.4 a value of $V_L \approx 1.5\%$ is reached in increment 7 of 15 excavation increments; this is equivalent to 47% of unloading. Figure 4.1b plots the maximum surface settlement for this case against percentage of unloading. The curve has a similar shape as the one in the previous graph.

The combination of these two plots leads to the graph shown in Figure 4.2. The maximum settlement shows a linear response to $V_L$. This trend predicted by the FE analysis is in good agreement with field data presented by Nyren (1998) for the above mentioned greenfield site at St. James’s Park. The field data shown in Figure 4.3 are for the construction of the westbound running tunnel of the Jubilee Line Extension. A linear trend of settlement developing with $V_L$ is indicated by the dashed trendline. The gradient of this line is, however, different to the slope of the curve in Figure 4.2. This is due to the predicted settlement curve being too wide as discussed in Section 2.3.1.

In order to investigate the development of building deformation criteria with $V_L$ a 100m wide 1-storey building has been analysed. The results are shown in Figure 4.4a and b. The deflection ratios for both sagging and hogging exhibit an approximately linear response to changes in volume loss; in contrast the development of maximum horizontal compression is not linear. No tension developed in this particular building case with no eccentricity.
Chapter 4. An evaluation of the relative stiffness method  

Section 4.2

As stated all results presented in this section were taken from analyses where no tunnel lining had been installed. This leads to an increased volume loss of about 5% at the end of excavation. In all analyses presented in the following sections results were taken from the 7th excavation increment in which a volume loss of approximately 1.5% was determined for a greenfield analysis. Table 4.1 summarizes the volume loss obtained for a range of analyses with 20m and 34m deep tunnel and different building stiffnesses. It can be seen that the volume loss for these cases varies between 1.51 and 1.41%. This is a small range compared

**Figure 4.2:** Development of settlement with volume loss $V_L$. Greenfield analysis without tunnel lining, $z_0 = 20$m

**Figure 4.3:** Settlement data from JLE westbound tunnel construction at St. James’s Park plotted against $V_L$. Data taken from Nyren (1998).

**Figure 4.4:** Relationship between building deformation criteria and volume loss: (a) deflection ratio, (b) horizontal compressive strain. Results for 100m wide 1 storey building, $z_0 = 20$m.
with the variation considered in Figures 4.1 to 4.4. It is therefore justified to linearize the graphs presented in Figure 4.4 over this range of volume loss in order to adjust results to a common volume loss of 1.5%.

This study shows that the linear trend seen for maximum settlement developing with $V_L$ also applies to deflection ratio. For maximum horizontal strain such a relationship is not evident. However for small variations in volume loss it is possible to linearly interpolate these magnitudes to a common value of volume loss. This procedure will be applied in Chapter 9 when dealing with high volume losses obtained from 3D tunnel modelling.

### 4.2.2 Mesh width

In their analyses, Potts & Addenbrooke (1997) placed the vertical mesh boundary at a distance of 70m from the centre line of the tunnel. For the studies presented in this thesis this dimension was increased to 100m to include wide buildings and cases with a large eccentricity. This section investigates how this change in the distance to the mesh boundary affected the results. In all analyses presented in this section the tunnel axis depth $z_0$ was 20m.

Figure 4.5 shows the surface settlement curves obtained from two greenfield analyses adopting 70m and a 100m wide FE meshes. The volume losses detected (for the same excavation increment) were 1.49% and 1.50% respectively. These results indicate that the mesh width does not affect the simulation of the excavation process itself.

Addenbrooke et al. (1997) demonstrated that by choosing a realistic soil stiffness the computed settlement trough is too wide, with settlement occurring at the vertical mesh boundaries (compare with Figure 2.12, Page 45). In the analyses carried out for this thesis the same effect was observed even though a larger distance between the vertical mesh boundary and tunnel centre line was chosen.

The volume loss is calculated by integrating the area of the (plane strain) surface settlement trough over the whole mesh width. The 70m wide mesh therefore has to show a deeper

<table>
<thead>
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<th>Stiffness</th>
<th>GF</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0=20m$</td>
<td>1.51</td>
<td>1.48</td>
<td>1.46</td>
<td>1.44</td>
<td>1.41</td>
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<tr>
<td>$z_0=34m$</td>
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<td>1.49</td>
<td>1.49</td>
<td>1.48</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Table 4.1: Volume loss for different greenfield situations and different building stiffness (in storeys). Building width $B=100m$. 

Section 4.2
Chapter 4. An evaluation of the relative stiffness method  

Section 4.2

Figure 4.5: Transverse settlement trough for greenfield analyses with different mesh widths, \( z_0 = 20 \text{m} \).

settlement trough than the corresponding 100m wide system in order to give the same volume loss. This effect can be seen in Figure 4.5 where the greenfield surface settlement troughs for both analyses are plotted. The use of the 70m mesh results in a deeper settlement curve although the difference is small.

The effect of mesh width on building deformation criteria was assessed by analysing a 60m wide building of varying stiffness. A 60m wide structure was chosen because this was one of the geometries adopted by Potts & Addenbrooke (1997) in their studies; the results will be compared in the next section. Table 4.2 summarizes the deflection ratios (\( DR_{\text{hog}} \) and \( DR_{\text{sag}} \)) and the maximum compressive strain (\( \epsilon_{hc} \)) calculated in this parametric study. The corresponding modification factors are given in the last three columns. No tensile strain was detected in the 60m wide structure during these analyses.

The first two lines of each deformation criterion (\( DR_{\text{hog}}, \ DR_{\text{sag}} \) and \( \epsilon_{hc} \)) compare the analysis using a 100m wide mesh (1st line) with the corresponding 70m case in the 2nd line (the 3rd and 4th line will be addressed in the following section). It can be seen that (the absolute value of) \( DR_{\text{hog}} \) increases when the mesh width is reduced. This results from the deeper and narrower settlement trough. The change is, however, very small. For \( DR_{\text{sag}} \) and \( \epsilon_{hc} \) a small decrease with decreasing mesh width was observed. The effect on the modification factors for all criteria, shown in the last three columns, remains negligible.
4.2.3 Initial stress

As described in Sections 2.3.1 and 3.4.1 Potts & Addenbrooke (1997) used a zone of reduced $K_0$ around the tunnel to obtain better predictions of the greenfield surface settlement trough. In order to consider more building characteristics (for example the weight of a building and the corresponding consolidation of the soil before tunnel construction) such a zone is not included in the analyses presented in this thesis. This section investigates the effect of this change in initial conditions.

Table 4.2 summarizes results for a set of analyses including two different initial stress scenarios:

1. A coefficient of lateral earth pressure at rest of $K_0 = 1.5$ over the whole mesh. This scenario is referred to as 1.5.

2. A zone of reduced $K_0 = 0.5$ around the tunnel, otherwise $K_0 = 1.5$. In vertical direction the zone extends to a distance of 6m (approximately 1.5 tunnel diameter) from the

<table>
<thead>
<tr>
<th>Mesh Width</th>
<th>$K_0$</th>
<th>Deflection Ratio</th>
<th>$M^{DR}$</th>
<th>$M^{PC}$</th>
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<td>3 storey</td>
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<td>$-6.29 \times 10^{-6}$</td>
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</tr>
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<td>$-6.38 \times 10^{-6}$</td>
<td>$-1.09 \times 10^{-6}$</td>
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<td>$4.96 \times 10^{-5}$</td>
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<tr>
<td>70</td>
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<td>$5.81 \times 10^{-5}$</td>
<td>$4.97 \times 10^{-5}$</td>
<td>$2.80 \times 10^{-5}$</td>
</tr>
<tr>
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<td>$1.03 \times 10^{-4}$</td>
<td>$6.96 \times 10^{-5}$</td>
<td>$3.85 \times 10^{-5}$</td>
</tr>
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<td>$K_0 = 0.5$</td>
<td>$1.11 \times 10^{-4}$</td>
<td>$6.98 \times 10^{-5}$</td>
<td>$3.84 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 4.2: Deflection ratio and compressive strain for different mesh widths and initial stress conditions for analyses with 60m wide building, $z_0 = 20m$. 

Width is the dimension of the FE mesh 

$K_0 = 1.5$ - global $K_0$ over entire mesh 

reduced - reduced zone around the tunnel, otherwise $K_0 = 1.5$
Chapter 4. An evaluation of the relative stiffness method  
Section 4.2

Figure 4.6: $M_{\text{DR}}$ for different initial stress conditions compared with results by Potts & Addenbrooke (1997). Data are for 60m wide building, $z_0 = 20m$.

Figure 4.7: $M_{\text{hc}}$ for different initial stress conditions compared with results by Potts & Addenbrooke (1997). Data are for 60m wide building, $z_0 = 20m$.

tunnel centre line, see Figure 2.14, Page 46. The height of the zone is identical with the diameter of the tunnel. This case is referred to as reduced in the table.

Both initial stress scenarios were analysed for 100m a 70m wide meshes. This was done in order to compare the results with the data by Potts & Addenbrooke (1997), who adopted a similar geometry. Results for the first stress situation can be found in Table 4.2 in lines 1 and 2 (for the 100m and 70m mesh respectively) of each deformation criterion; data for the second initial stress profile are given in lines 3 and 4.

The data reveal an increase (in terms of absolute value) of all deformation criteria when a zone of reduced $K_0$ is introduced. This trend results from the narrower settlement trough observed when such a zone was used. The corresponding modification factors exhibit the opposite trend and reduce (apart from some scatter in hogging cases) when a $K_0$-reduced zone is applied. The use of a global $K_0$ in this work results on the one hand in a less accurate
greenfield surface settlement prediction, while on the other hand it yields a more conservative estimate of the modification factors.

This trend is further demonstrated when plotting the deflection ratios versus relative bending stiffness $\rho^*$. Figure 4.6 shows this plot for the results of the 70m wide mesh. The data points obtained by Potts & Addenbrooke (1997) are also given for comparative purposes.

For the $\text{DR}_{\text{sag}}$ results it can be seen that the data points for a global $K_0$ lie above their counterparts for a $K_0$-reduced zone. For the 3 and the 5 storey building the reduced $K_0$ points of this study coincide well with the results by Potts & Addenbrooke (1997). For the 1-storey building (lowest $\rho^*$) there is some scatter which can be explained by the different approaches adopted to determine the point of inflection and thus the deflection ratio (explained in Section 3.4.5): In this thesis a spread sheet calculation has been used while a graphical approach was chosen by Potts & Addenbrooke (1997). The modification factors $M^{\text{DR}_{\text{hos}}}$ are low and therefore all data points lie close together when plotted on a scale that enables the design curves to be included.

A similar picture emerges when plotting $M^{\text{the}}$ against $\alpha^*$, shown in Figure 4.7. As in the previous figure, results from a 70m wide mesh and different initial stress situations are compared with the results by Potts & Addenbrooke (1997). For all three cases of different relative stiffness, the data points of the analyses which applied a global zone of $K_0 = 1.5$ lie above those for a zone of reduced $K_0$. It can be seen that the change in initial stress conditions shifts the results slightly outside the Potts & Addenbrooke (1997) design curve.

This study shows that the use of a reduced zone of $K_0$ influences the deformation behaviour of greenfield and building situations. However, it does not change the trend observed when varying the building stiffness. All $M^{\text{DR}}$ data points lie below the design curves provided by Potts & Addenbrooke (1997) while the compressive strain modification factors lie slightly outside of the design curves.

### 4.2.4 Summary

This section has investigated how some of the key factors in the FE-analyses of Potts & Addenbrooke (1997) affected their results. These parameters were the volume loss (which was chosen to be 1.5%), the FE mesh width (70m) and the initial stress conditions (where a zone of reduced $K_0$ was used around the tunnel). The effects of varying these factors have
been assessed.

- It has been found that for an increase in volume loss there is a corresponding increase in deflection ratio (both hogging and sagging); this relationship is approximately linear. It is therefore possible to linearly adjust these criteria to a common volume loss. The response of strain on $V_L$ is not linear. However, results can be linearly interpolated, when small variations of $V_L$ are considered.

- Increasing the FE mesh width from 70m to 100m has only a minor influence on the results. In general the greenfield surface settlement trough becomes deeper and narrower when the mesh width is reduced. The development of volume loss over excavation increments is not affected by the change in mesh width.

- The use of a global coefficient of earth pressure at rest of $K_0 = 1.5$ instead of a zone of reduced $K_0$ around the tunnel leads to smaller values of deformation criteria. This can be explained by the wider settlement trough obtained from an analysis in a high $K_0$-regime. The modification factors, however, increase. The modification factors presented in this study are therefore more conservative in comparison to those obtained by Potts & Addenbrooke (1997), who used a $K_0$-reduced zone in order to obtain more realistic greenfield surface settlement troughs.

### 4.3 Influence of building stiffness

Building stiffness was the fundamental parameter examined in the original work of Potts & Addenbrooke (1997). In their study they varied both bending stiffness $EI$ and axial stiffness $EA$ independently of each other and investigated the influence of these variations on the deformation behaviour of a surface structure. The main results of their work were discussed in Section 2.4.6.2. This section presents the results of tunnel induced soil movements beneath a surface structure of variable stiffness. The aim was to achieve a better understanding of the tunnel-soil-structure interaction.

The nature of this interactive problem can be seen in Figure 4.8a and b which profiles horizontal and vertical soil movement, respectively, for a section at 6m offset from the tunnel centre line ($z_0 = 20$m). Results are presented for 100m wide buildings with 1, 3, 5 and 10
Figure 4.8: Vertical profile of horizontal (a) and vertical (b) soil displacement at $x = 6m$. Data are for greenfield and for 100m wide structures, $z_0 = 20m$. Negative horizontal displacement indicates movement towards the tunnel, positive vertical displacement indicates downwards movement.

storeys. In addition soil movements obtained from a greenfield analysis are included.

The maximum horizontal greenfield displacement (in terms of absolute value, the negative sign refers to movement towards the tunnel) along the vertical line is reached just above the tunnel axis depth of $z_0 = 20m$. From there it reduces towards the surface but increases over the top 6m and shows a surface horizontal displacement of $S_{hx} = -1.8mm$. At the surface the horizontal soil movement is altered drastically by the presence of a structure; for example it changes to $S_{hx} = -0.08mm$ for a 1-storey building. For this case the displacement then increases with depth and becomes larger than that obtained for greenfield conditions at a depth of approximately 5m. Below this depth a higher building stiffness leads to higher horizontal displacements, although the difference remains small. This trend continues well below tunnel axis depth.

The vertical greenfield displacement (Figure 4.8b) shows an increase from its surface value of $S_v = 3.9mm$ to $S_v = 6.1mm$ at a depth of $z = 15m$. It then reduces before changing to up-
Figure 4.9: Vertical profile of vertical soil displacement along a line above the tunnel CL, $x = 0\text{m}$. Positive values refer to downwards movement.

However, the magnitude of settlement reduces with increasing building stiffness over approximately the top 18m of soil. Below this depth the influence of building stiffness becomes less significant. The vertical displacement along a vertical line above the tunnel centre line ($x = 0\text{m}$) shown in Figure 4.9 displays the same trend. Near to the surface the settlement reduces with building stiffness. Immediately above the tunnel crown the influence of building stiffness on the vertical soil movement is not significant.

Comparing the horizontal soil movement profiles of Figure 4.8a with the profiles of vertical movements shown in Figure 4.8b and Figure 4.9 demonstrates that horizontal soil movement close to the soil surface is much more influenced by the building’s stiffness than the vertical movement is in this zone.

The previous two graphs showed the variation of the magnitude of vertical settlement with depth. In order to investigate how the shapes of subsurface settlement troughs are altered by the presence of a building the position of the point of inflection, $i$, is shown against depth $z$ in Figure 4.10. This figure includes the same greenfield and building cases as plotted in the previous figures. In general $i$ decreases with depth. This shows that subsurface settlement troughs become narrower and (considering the increasing settlement found in the previous graph) deeper. The distribution of $i$ with depth for greenfield conditions has been discussed.

Figure 4.10: Point of inflection for surface and subsurface settlement troughs. Data for greenfield and 100m wide buildings, $z_0 = 20\text{m}$
in Section 2.2.2. The trend of an over proportional decrease of $i$ near to the tunnel and an over proportional increase next to the ground surface as described by Grant & Taylor (2000), see Figure 2.11 on Page 40, can be seen in this figure. The greenfield surface settlement trough shows a width of $i = 11.5m$. This is in good agreement with the empirical formula (Equation 2.15, Page 36), which predicts $i$ to be $i = 0.5 \times z_0 = 10m$.

Building stiffness changes this distribution over approximately the upper 8.5m of soil by increasing $i$. Between this depth and $z \approx 2m$ the results for the 1, 3, 5 and 10-storey buildings coincide but then diverge towards the surface with higher stiffnesses showing larger values of $i$. This widening of the settlement trough with increasing overlying stiffness (in this case due to the building) is in agreement with centrifuge tests performed by Hagiwara et al. (1999) and discussed in Section 2.2.2. In their tests they investigated the influence of the stiffness of a soil layer overlying a clay strata in which a tunnel excavation was investigated.

The results presented in this section demonstrate how the stiffness of a surface structure alters the soil displacements. These changes have been found to be significant close to the ground surface. In the vicinity of the tunnel the influence of building stiffness on vertical ground movement is insignificant. The horizontal displacement, however, changes close to the tunnel although the differences between greenfield and building cases remain small.

### 4.4 Influence of geometry

The geometry of the tunnel-soil-structure interaction problem is a key feature when assessing building deformation due to tunnel induced ground subsidence. The building width, $B$, the eccentricity of the building with respect to the tunnel centre line, $e$, and the tunnel depth, $z_0$ are all represented in the relative stiffness method: $B$ is included directly in the formulation of relative stiffness (see Equations 2.29, Page 72) while the eccentricity (in terms of $e/B$) is represented by a range of different design curves. The soil stiffness $E_S$ in both equations depends on $z_0$. The tunnel depth also affects the greenfield settlement profile all building criteria are related to.

In their study, Potts & Addenbrooke (1997) included a wide range of different building widths and eccentricities and two tunnel depths ($z_0 = 20m$ and $34m$). This variation allowed them to fit upper bound curves to their results and to include the building geometry into the relative stiffness approach. The influence of each geometric parameter was, however,
not investigated independently. The following sections assess the effect of each of these parameters uncoupled from each other. In addition it will be shown how the geometry affects the mechanisms involved in the tunnel-soil-structure interaction problem.

4.4.1 Building width

4.4.1.1 Influence on deformation criteria

The building width $B$, defined in Figure 3.6 (Page 95), is a significant parameter within the relative stiffness approach. In the formulation of the relative bending stiffness $\rho^*$ (Equation 2.29a on Page 72) $B$ is represented in the denominator with an exponent of 4. Hence an increase in building width leads to a vast reduction in $\rho^*$. The reduction of $\alpha^*$ with increasing $B$ is less significant as the building width has no exponent in the definition of relative axial stiffness.

The definition of modification factors for any building geometry include the corresponding greenfield deformation criteria. This chapter therefore will first investigate the behaviour of a greenfield situation where different widths $B$ are considered. The term building width is obviously not appropriate for this case as no building is present. Instead section width is used.

Firstly the change in $DR$ and $\epsilon_h$ were analysed by applying a Gaussian settlement trough, described in Section 2.2.1.1. This calculation was performed using a spreadsheet. These results were then compared with data obtained from a greenfield FE study. Finally a building was introduced by performing a set of FE analyses with variation in $B$ and number of storeys. Only geometries with no eccentricities are considered in this section.

**Deflection Ratio:** Both the deflection $\Delta$ and the corresponding length $L$, defined in Figure 2.26 on Page 73, change with the length of a greenfield section. Figure 4.11a shows the development of $DR^{GF}$ with section width $B$ over a 20m deep tunnel. The solid lines represent results calculated from a Gaussian settlement trough. The dotted lines are similar results using the settlement trough obtained from a greenfield FE analysis. The Gaussian trough was calculated using Equation 2.1 (Page 28) adopting a volume loss of $V_L = 1.50\%$. The position of the point of inflection was set to be $i = \frac{z_0}{2} = 10$ m. In the FE analysis a volume loss of $V_L = 1.51\%$ was achieved.
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

Figure 4.11: Development of deflection ratio with $B$ above a 20m deep tunnel. (a) Greenfield conditions from Gaussian settlement trough and from FE analysis; (b) $D_{\text{GF}}^{\text{Sag}}$ in buildings with different stiffness.

The sagging ratio $D_{\text{GF}}^{\text{Sag}}$ only changes with $B$ as long as the section lies between the points of inflections, eg. $B/2 < i$. This can be seen in the solid curve representing $D_{\text{GF}}^{\text{Sag}}$ for the Gaussian settlement trough. The curve increases for small $B$ but remains constant for $B > 20m = 2i$. In contrast no hogging develops within this section; outside this zone, however, $D_{\text{GF}}^{\text{Hog}}$ increases (in terms of absolute value) until reaching a maximum for $B \approx 70m$.

The corresponding results for the greenfield FE analysis show a similar trend. However, the dotted curves are much flatter reaching maximum values of about a third of those obtained from the Gaussian settlement calculation. This effect can be explained by the wider and shallower settlement trough obtained from the FE simulation (as discussed in Section 2.3.1).

Figure 4.11b shows how $D_{\text{Sag}}$ changes with $B$ when buildings are included in the study. Each data point on this graph represents one FE analysis. The building widths, $B$, were 16, 32, 60, 100 and 120m and for each of these building stiffnesses of 1, 3, 5 and 10 storeys were analysed. The dotted line represents the FE greenfield results (sagging) from Figure 4.11a.

The curves for the 1 and 3-storey structure show an increase in $D_{\text{Sag}}$ until $B = 60m$. For wider structures $D_{\text{Sag}}$ remains essentially constant and therefore shows a similar trend as found for greenfield conditions. For the stiffer 5-storey building $D_{\text{Sag}}$ increases until $B = 100m$. The 10-storey building shows an increasing $D_{\text{Sag}}$ over the whole range of building widths analysed. These results indicate that there is a certain width for each building stiffness above which $D_{\text{Sag}}$ does not increase further. This building width increases with increasing...
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

Figure 4.12: \( M^{DR} \) against \( \rho^* \) for varying building width and stiffness for \( z_0 = 20 \text{m} \). The arrow in the \( DR_{sag} \) graph indicates the 120m, 100m and 60m wide 1 storey buildings which have a similar \( DR_{sag} \) while \( \rho^* \) varies.

![Diagram showing \( M^{DR} \) and \( \rho^* \) for varying building width and stiffness.]

building stiffness.

The modification factors \( M^{DR_{sag}} \) for the results presented in Figure 4.11b are obtained by dividing the building’s \( DR_{sag} \) by the corresponding greenfield \( DR_{sag}^{GF} \). As \( DR_{sag}^{GF} \) does not change significantly for \( B > 32 \text{m} \) the modification factors \( M^{DR_{sag}} \) for structures longer than \( B = 32 \text{m} \) are proportional to their \( DR_{sag} \). Hence, \( M^{DR_{sag}} \) for 1-storey buildings of \( B > 60 \text{m} \) are similar in magnitude. However, the relative stiffness \( \rho^* \) between these buildings varies as \( B \) is included in \( \rho^* = \frac{EI}{E_S (B/2)^4} \). This effect is demonstrated in Figure 4.12a which plots \( M^{DR_{sag}} \) versus \( \rho^* \). The three data points mentioned above are indicated by the arrow. Similar values of \( M^{DR_{sag}} \) but with \( \rho^* \) increasing from \( 1.5 \times 10^{-5} \) (\( B = 100 \text{m} \)) to \( 2.4 \times 10^{-4} \) (\( B = 60 \text{m} \)) produces a significant scatter which can also be found for data points of 3 and 5-storey buildings. All results (apart from one exception) lie below the design curves proposed by Potts & Addenbrooke (1997) which thus continue to provide a conservative estimation of \( DR_{sag} \).

Such a scatter is not evident in Figure 4.12b where \( DR_{hog} \) is plotted against \( \rho^* \). There are less data points as hogging was not obtained for all analyses. The remaining data lie close
This study shows that by including \( B^4 \) in the expression for the relative bending stiffness \( \rho^* \) the influence of \( B \) can be overestimated for some building geometries. Although this can lead to a significant scatter the design curves by Potts & Addenbrooke (1997) still provide an upper bound for these data points. The scatter for \( M_{D^{\text{DRhog}}} \) is much smaller indicating that the development of \( M_{D^{\text{DRhog}}} \) with \( B \) is in good agreement with the relative bending stiffness expression proposed by Potts & Addenbrooke (1997).

**Maximum horizontal strain:** Figure 4.13a shows the development of maximum horizontal strain with width \( B \) of a greenfield section. As described above curves are obtained from a spreadsheet calculation assuming a Gaussian settlement trough (solid line) and using results from a greenfield FE calculation (dashed line).

It can be seen that the maximum horizontal compressive strain \( \epsilon_{\text{GF}}^{\text{hc}} \) calculated for a greenfield section remains constant as \( B \) increases. This behaviour is due to the fact that any section with no eccentricity experiences \( \epsilon_{\text{hc}}^{\text{GF}} \) which is the maximum horizontal compressive strain which develops above the centre line of the tunnel (see Figure 2.3 on Page 30). Hence it is \( \epsilon_{\text{hc}}^{\text{GF}} = \epsilon_{\text{hc}}^{\text{GF}} \) for sections of all widths and no eccentricity. As shown in Section 2.2.1.1 no tensile horizontal strain develops within the zone between the points of inflection. In the case of the Gaussian surface settlement trough calculation tensile strain occurs as soon as

![Figure 4.13](image)

**Figure 4.13:** Development of max. horizontal strain with \( B \) above a 20m deep tunnel. (a) Greenfield conditions analysed for Gaussian settlement trough and with FE analysis; (b) \( \epsilon_{\text{hc}} \), calculated in buildings with different stiffness.
$B/2 > i = 10\text{m}$ and increases with $B$ until the section reaches the point of $\epsilon_{\text{ht}}^{\text{GF}}$ at $x = \sqrt{3}i$ (see Figure 2.3). It then remains constant for further increases in $B$. As in Figure 4.11a the curves obtained from the FE analyses are flatter and show lower values (in absolute terms) then those from the Gaussian calculation: For $\epsilon_{\text{ht}}$ the FE gives results which are 57% lower than those from the Gaussian settlement. The value for $\epsilon_{\text{hc}}$ obtained from the FE study is only 14% below the spread sheet calculation.

Figure 4.13b presents the change in $\epsilon_{\text{hc}}$ with $B$ for buildings with different numbers of storeys. The FE greenfield response is, in contrast to Figure 4.11b, not included into this graph as its absolute value ($\epsilon_{\text{hc}}^{\text{GF}} = -3.5 \times 10^{-4}$) is more than 1 order of magnitude greater than those obtained for building scenarios.

The curves show an increase (in terms of absolute value) of $\epsilon_{\text{hc}}$ with $B$. In all cases, the gradient of the curve becomes flatter as $B$ increases. As for $\text{DReag}$ this trend is most evident for buildings of a low stiffness. For the 1-storey stiffness case, $\epsilon_{\text{hc}}$ does not increase further after $B$ reaches 100m.

Dividing each $\epsilon_{\text{hc}}$ by the corresponding $\epsilon_{\text{hc}}^{\text{GF}}$ gives $M_{\epsilon_{\text{hc}}}$. As $\epsilon_{\text{hc}}^{\text{GF}}$ remains constant over the whole range of $B$ the distribution of modification factors with $B$ is proportional to the $\epsilon_{\text{hc}}$ curves shown in Figure 4.11b. When plotting these modification factors\(^1\) against relative axial stiffness $\alpha^*$ as shown in Figure 4.14 the results lie close together along the design curve postulated by Potts & Addenbrooke (1997). This shows that the incorporation of $B$ (without any exponent) into $\alpha^*$ provides a good estimation of $M_{\epsilon_{\text{hc}}}$ over a wide range of building widths.

### 4.4.1.2 Influence on ground deformation

In the previous section it was demonstrated how building deformation is affected by the width of a structure. While the above results are important for assessing building damage they only describe one aspect of the tunnel-soil-structure interaction. The nature of this problem is that not only does the tunnel affect the building but the building can also alter the ground deformation induced by tunnel construction. This section investigates the influence of the building width on this interaction.

Figure 4.15 presents the distribution of $i$ of subsurface settlement troughs with depth for

\(^1\)No tension was obtained in the analyses with no eccentricity
32m, 60m and 100m wide 5-storey buildings above a $z_0 = 20m$ deep tunnel. For comparative purposes the results of a greenfield analysis are included. The curves for greenfield and for the 100m wide building have been previously presented in Figure 4.10 and were described on page 113.

It can be seen that the curves for all building widths are approximately coincident between the tunnel crown and a depth of approximately 6m below the ground surface. Below 8m depth they also follow the greenfield distribution of $i$. In the top 6m of soil they not only diverge from the greenfield case but also from each other giving higher values of $i$ for wider buildings\(^2\). This indicates that with increasing building width the point of inflection of the surface structure is dragged away from the tunnel centre line. Below a depth of 6m, the influence of the width of the surface structure on $i$ for a subsurface settlement trough is negligible.

Figure 4.16 shows that the horizontal soil displacement adjacent to the tunnel is marginally affected by $B$. In this figure horizontal ground movement is plotted against depth at a distance of 6m from the tunnel centre line. Negative displacement indicates movement towards the tunnel. The displacement is plotted for the cases of 32m, 60m and 100m wide 5-storey

\(^2\)It was found that shallow subsurface troughs have two points of inflection on each side of the tunnel centre line. This effect arises from the disturbed settlement beneath the edge of a building. In all cases the inner point was chosen for representation in this graph. For the $B = 32m$ case only one point was detected close to the surface. This point was outside the edge of the structure.
buildings. The results of the greenfield case are also included. The general pattern of the curves was previously presented in Figure 4.8a. The graph indicates that the horizontal movement increases with building width, eg. the curve for the 60m building scenario shows a higher response than the 32m building case. However, the change in horizontal soil movement is negligible when $B$ is increased further from 60m to 100m. This trend also can be found when comparing the vertical soil movement along a vertical line above the tunnel centre line, shown in Figure 4.17. The results for 60m and 100m wide surface structures coincide while the data for the 32m building case lies between them and the greenfield results. However, this difference becomes less significant towards the crown of the tunnel.

This study has demonstrated the interaction between the surface structure and tunnel. While the distribution of $i$ near the tunnel shows that the width of the subsurface settlement trough is not affected by the overlying surface buildings the magnitude of the horizontal soil movements around and above the tunnel are influenced by the presence of the building.
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

However, the differences are small in engineering terms.

4.4.2 Eccentricity

4.4.2.1 Influence on deformation criteria

In the previous section the behaviour of surface structures with no eccentricity but varying width was investigated. Under these conditions no tensile strain developed in these structures. Now, it will be shown that tensile strain and hogging deformation become more significant when the surface structure is located eccentrically with respect to the tunnel centre line. As tension and hogging are more critical deformation criteria than compression and sagging (Burland & Wroth, 1974; Boscardin & Cording, 1989), the eccentricity $e$, defined in Figure 3.6 (Page 95), is a crucial parameter. This section will investigate its influence on deformation behaviour.

As in the previous section the surface behaviour under greenfield conditions will be assessed first by imposing a Gaussian settlement trough and by using FE greenfield results. Subsequently the behaviour of buildings will be investigated.

Deflection ratio: Figure 4.18 shows three distinct positions for a greenfield section which have to be considered when investigating the change in $DR$ with eccentricity $e$. The first position, a greenfield section with no eccentricity, has been analysed in the previous section. The geometry covers the entire sagging zone and $DR_{sag}^{GF}$ does not change until $e$ increases to position 2. As $e$ further increases $DR_{sag}^{GF}$ decreases until the section reaches position 3 where $DR_{sag}^{GF} = 0$.

This behaviour is illustrated in Figure 4.19 which plots the development of $DR_{sag}^{GF}$ with $e$ for different section widths of $B = 32m$, $60m$ and $100m$. The deflection ratios in this plot are calculated from a Gaussian settlement trough with $V_L$ set to 1.5%, $i = 0.5z_0$ and where $z_0 = 20m$. It can be seen that $DR_{sag}^{GF}$ remains constant until position 2 (marked in the figure for the 100m wide geometry) is reached. For $B = 100m$ this point is located at $e = \frac{B}{2} - i = 40m$. The sagging then reduces to $DR_{sag}^{GF} = 0$ over $2i = 20m$ (position 3).

The trend in $DR_{sag}^{GF}$ with $e$ depends on $B$. Previously, it has been shown in Figure 4.11a

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3 Initially two hogging zones can be found as $e$ increases from position 1 to 2. Higher $DR_{hog}$ are obtained in the increasing hogging zone, in case of Figure 4.18 the right hand side. Only results from this side are
that a maximum $DR_{hog}$ is obtained for sections of $B \approx 70$m (assuming a Gaussian settlement distribution). The edge of such a section is located 35m from the tunnel centre line. The maximum $DR_{hog}$ for sections of varying eccentricity can be found for those cases which extend from the point of inflection to a distance of approximately 35m from the tunnel centre line. If the eccentricity is increased from this position, $DR_{hog}$ will reduce. This behaviour can be found in Figure 4.19 for the 32m wide section. Its hogging deformation is small for $e = 0$m. $DR_{hog}$ increases with increasing eccentricity. It maximum value is reached at $e \approx 19$m. In this situation one edge of the section is at a distance of approximately 35m from the tunnel centre line. From this eccentricity $DR_{hog}$ reduces as $e$ is increased further. The 100m section with $e = 0$m, in contrast, extends beyond this position of maximum $DR_{hog}$. Its hogging deformation reduces therefore over the whole range of eccentricities. The behaviour of the 60m wide section is between these two extremes. In all 3 cases there is a vast reduction in $DR_{hog}^{GF}$ as soon the geometry is located beyond position 3.

Figure 4.20 shows a similar graph presenting results from a greenfield FE analysis. As before, see Figure 4.11a, the FE study shows the same trend as the Gaussian calculation but the results are lower in magnitude (the vertical axes in Figures 4.19 and 4.20 have different
Figure 4.19: Variation of deflection ratio with eccentricity $e$ for different greenfield section widths $B$. Results from spreadsheet calculation of Gaussian settlement trough. The numbers refer to positions (for the 100m section) defined in Figure 4.18.

Figure 4.20: Variation of deflection ratio with eccentricity $e$ for different greenfield section widths $B$. Results from FE analyses.

Figure 4.21 re-plots the data from the previous figure but with the eccentricity $e$ normalized by $B$. The graph shows that the deflection deformation for short greenfield sections extends to larger $e/B$ ratios than it does for long geometries.

How this behaviour changes when a building is included in the study is shown in Figure 4.22 (note that this and the previous graph have different scales on the vertical axes). The eccentricity is again expressed as $e/B$. The graph includes the same 3 widths as before. All buildings have a stiffness equivalent to 5 storeys. In general $DR_{sag}$ reduces with increasing
eccentricity while $DR_{\text{hog}}$ increases until a maximum value is reached. This general trend is in agreement with the behaviour found for greenfield situations. There are, however, some significant differences. For both $DR_{\text{sag}}$ and $DR_{\text{hog}}$ their magnitude increases with increasing building width. This trend has already been shown when investigating the influence of $B$ (Figure 4.11b). There is no constant $DR_{\text{sag}}$ over a range of low $e/B$ ratios but $DR_{\text{sag}}$ reduces as $e/B$ increases from 0. This reduction is evident for all building widths over a similar range of $e/B$. The sagging deflection is small when $e/B \geq 0.5$ in all cases while for the greenfield scenario the small deflections do not occur until $e/B = 0.6$ and $\approx 0.75$ for the 100m and 32m section respectively. For the hogging case all included building widths show a low $DR_{\text{hog}}$ for $e/B = 0$. $DR_{\text{hog}}$ then increases and its maximum is reached at $e/B \approx 0.5$ for the 100m structure and approximately 0.75 for the 32m wide building.

One reason for the different behaviour between building and greenfield deflection can be seen when plotting the positions of the points of inflection against eccentricity. Figure 4.23 shows such a graph for a 60m wide 5-storey building. The dashed lines represent the building edges which linearly change their position with $e$ while the solid lines mark the position of the points of inflection. Each pair of symbols (for equal $e$) on those lines represents a FE analysis. The vertical distance between both solid lines symbolizes the sagging zone of the building while the two hogging zones lie between the solid and the dashed lines, e.g. between

**Figure 4.21:** Deflection ratio with $e/B$ for different greenfield section widths $B$. Results from FE analyses.

**Figure 4.22:** Deflection ratio with $e/B$ for different building widths $B$ of 5 storey structure.
the points of inflection and the edges of the structure. The pair of horizontal dotted lines represent the positions of the greenfield points of inflections ($i = 11.5$ m, obtained from FE analysis) which remain constant.

The main difference between building and greenfield behaviour is that the positions of the points of inflection in a structure change with its eccentricity while under greenfield conditions they are constant regardless for which geometrical section the deflection ratio is calculated. As a result the sagging zone of the building reduces as $e$ increases. This explains the immediate reduction in $DR_{sag}$ with $e$ as shown in Figure 4.22.

The different positions of the point of inflection between greenfield and building scenar-
An evaluation of the relative stiffness method

Section 4.4

ios leads to cases where sagging occurs only for the building case but not under greenfield conditions. This can be seen in Figure 4.23 for $e > 41m$ ($e/B = 0.68$). $M^{DR_{sag}}_{sag}$ is not defined in these cases. For eccentricities just below this value greenfield $DR_{sag}^{GF}$ are very small in magnitude. Dividing the building $DR_{sag}$ by the low magnitude of $DR_{sag}^{GF}$ can lead to high values of $M^{DR_{sag}}_{sag}$.

This effect can be seen in Figure 4.24 where the modification factor $M^{DR}$ are plotted against $e/B$ for the same buildings considered previously in Figure 4.22. For $e/B < 0.4$, $M^{DR_{sag}}_{sag}$ (upper graph) reduces for all building widths; the curves show the same shape as those describing the development of $DR_{sag}$ with $e/B$ given in Figure 4.22. However, as $e/B$ increases further, $M^{DR_{sag}}_{sag}$ increases again. In the case of the 100m building and $e/B = 0.6$, a $M^{DR_{sag}}_{sag} = 2.58$ is calculated (and because of its magnitude is not included on the graph). The actual magnitude of $DR_{sag} = 4.96 \times 10^{-7}$ for this building is, however, very low.

The hogging modification factors, shown in the lower graph, increase with $e/B$. The value of $e/B$ at which $M^{DR_{hogg}}$ reaches its maximum depends on $B$; for the 100m wide structure it occurred at $e/B \approx 0.5$ ($e = 50m$) while for $B = 60m$ it is located at $e/B \approx 0.6$ ($e = 36m$). $M^{DR_{hogg}}$ for the 32m wide structure remains small ($M^{DR_{hogg}} < 0.1$) over the range of eccentricities investigated.

The design charts of Potts & Addenbrooke (1997) (shown in Figure 2.27a, Page 77) include curves of various $e/B$ ratios. In the sagging mode, these curves reduce $M^{DR_{sag}}_{sag}$ with increasing $e/B$. For relative stiffnesses between $\rho^* = 10^{-5}$ to $10^{-1}$ a vast reduction in $M^{DR_{sag}}_{sag}$ with $e/B$ is predicted by the suite of design curves. The 5-storey buildings analysed in this section have relative stiffnesses between $\rho^* = 1.0 \times 10^{-1}$ and $1.1 \times 10^{-3}$ for the 32m and 100m wide cases respectively. The decrease in $M^{DR_{sag}}_{sag}$ shown in Figure 4.24 is therefore in good agreement with the sagging design charts proposed by Potts & Addenbrooke (1997).

For hogging their design curves give higher $M^{DR_{hogg}}$ with increasing $e/B$. The highest eccentricity represented by a curve is $e/B = 0.6$. This is in good agreement with the maximum $M^{DR_{hogg}}$ seen in this study and shown in Figure 4.24. The present study indicates that a further increase of $e/B$ does not lead to higher values of $M^{DR_{hogg}}$.

Maximum horizontal strain: The maximum compressive horizontal strain $\epsilon_{GF}^{hc}$ (in terms of absolute value) within a greenfield surface settlement trough develops above the centre line.
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

Figure 4.25: Different position of a section (or building) in respect to the horizontal strain distribution.

of the tunnel. Assuming a Gaussian distribution the zone of horizontal compression extends to the point of inflection at a distance $i$ from the centre line. Beyond this point horizontal tensile strain increases to a maximum value $\epsilon_{ht}^{GF}$ at $\sqrt{3}i$ as shown in Figure 4.25.

When investigating the maximum compressive and tensile horizontal strains occurring in greenfield sections with varying $e$, the four positions presented in Figure 4.25 are important. A section with no eccentricity (position 1) covers the point of $\epsilon_{hc}^{GF}$ and therefore $\epsilon_{hc}^{GF} = \epsilon_{hc}^{GF}$. This situation remains with increasing $e$ until position 2 is reached. From there on $\epsilon_{hc}^{GF}$ reduces until the section shows no compressive strain at position 3.

Section 4.4.1.1 demonstrated how it depends on the value of $B$ whether or not a greenfield section of no eccentricity experiences any tensile strain. It was shown that for $B/2 < i$ there is no horizontal tension. On the contrary, for $B/2 > \sqrt{3}i$ the section shows the maximum tensile strain ($\epsilon_{hc} = \epsilon_{ht}^{GF}$). This value remains constant until $e$ is increased so that the greenfield section does not cover the point of $\epsilon_{ht}^{GF}$ (position 4) anymore.

This behaviour is further demonstrated in Figure 4.26 which shows the maximum horizontal strain with $e$ for greenfield sections of $B = 32m$, $60m$ and $100m$ obtained from a Gaussian settlement trough. Initially all sections show a constant $\epsilon_{hc}$ until position 2 is reached. This

128
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

Figure 4.26: Maximum horizontal strain with eccentricity \( e \) for greenfield sections with different widths \( B \). Results from spreadsheet calculation of Gaussian settlement trough. The numbers refer to positions (of the 60m wide section) defined in Figure 4.25.

Figure 4.27: Maximum horizontal strain with eccentricity \( e \) for greenfield sections of different widths \( B \). Results from FE analyses.

point is marked for the 60m geometry. It can be seen that the horizontal compression then reduces within 10m (= \( i \)) leading to position 3.

All section widths in this figure are long enough to show tensile strain for \( e = 0 \)m. The point of maximum horizontal tension at a distance of \( \sqrt{3}i = 17.3 \)m is initially covered by the 60m and 100m section. The 32m geometry reaches this point for \( e = 1.3 \)m. From there \( \epsilon_{ht} \)
remains constant until \( e = B/2 + \sqrt{3}i \) (position 4). From there on \( \epsilon_{ht} \) reduces.

Figure 4.27 shows a similar graph based on the greenfield settlement trough calculated in a FE analysis. As previously seen in Figure 4.13 the FE studies exhibit the same trends as the Gaussian calculations but the strains are lower in magnitude. The point where the horizontal strain at the surface changes from compression to tension is at a distance of approximately 13m from the tunnel centre line and does not coincide with the point of inflection, calculated from the change from sagging to hogging at \( i = 11m \).

Figure 4.28 presents the same results plotted against \( e/B \). In this context, position 2 is at \( e/B = 0.5 \) for all section widths \( B \). The graph shows that for smaller sections both compressive and tensile strains extend to higher \( e/B \) ratios compared to wider sections.

Figure 4.29 shows how \( \epsilon_{ht} \) in a building depends on its eccentricity. The graph is similar to the previous one plotting \( \epsilon_{ht} \) versus \( e/B \). The results are from the same set of FE analyses presented in Figure 4.22 incorporating 32m, 60m and 100m wide 5-storey buildings. It can be seen that \( \epsilon_{hc} \) (whose magnitude depends upon \( B \)) decreases as soon as \( e/B \) increases from 0. This is in contrast to the greenfield case where \( \epsilon_{hc} \) was initially constant. At approximately \( e/B = 0.5 \) to 0.6 \( \epsilon_{hc} \) reduces to 0.

None of the 3 building widths show any \( \epsilon_{ht} \) for \( e/B = 0 \). Tensile strain develops within the 100m wide structure from \( e/B \approx 0.2 \). The 32m wide building shows no \( \epsilon_{ht} \) until approximately
$e/B = 0.4$. The strain $\epsilon_{ht}$ then increases until a peak value is reached. In the case of the 100m structure this peak occurs at $e/B \approx 0.6$.

The corresponding modification factors are plotted in Figure 4.30. For compression (upper graph) $M^{hc}$ reduces as $e/B$ increases. By $e/B = 0.6$ all factors have reduced to very small values ($M^{hc} < 0.003$). The lower graph shows an increase of $M^{ht}$ with $e/B$. Peak values are found for approximately $e/B = 0.6$ for the 100m and 60m buildings. The 32m wide structures which shows lower values of $M^{hc}$ exhibit a peak for larger eccentricities.

The design charts by Potts & Addenbrooke (1997) (shown in Figure 2.27b on Page 77) include a suite of curves for different eccentricities. For compression higher values of $e/B$ are represented by lower curves. In the tension chart, higher modification factors are obtained when using curves of higher $e/B$. The highest and lowest curves for compression and tension respectively are for $e/B = 0.6$. This value has been confirmed by this study.

### 4.4.2.2 Influence on ground deformation

In Section 4.4.1.2 the influence of building width on ground movement was analysed. This section presents a similar study that investigates how the eccentricity of a structure affects the tunnel-soil-structure interaction problem.

Figure 4.31a shows the horizontal soil movement along vertical profiles at 6m to either side.
Figure 4.31: Vertical profile of horizontal soil movement due to tunnel construction at ±6m distance to tunnel centre line. \( z_0 = 20 \) m, 5 storey building with \( B = 60 \) m and different eccentricity. (a): \( e = 12 \) m; (b): \( e = 48 \) m. For sign convention, see Figure 4.8.

Figure 4.32: Vertical profile of settlement above tunnel crown.
of the tunnel centre line for the case of a 60m wide, 5-storey building with an eccentricity of 12m. The displacements obtained for greenfield conditions are plotted for comparison. Negative displacement indicates movement towards the tunnel. Referring to the coordinate system defined in Figure 3.6 (Page 95), the building has an offset to the right of the tunnel centre line. The $x = -6m$ and $+6m$ profiles are to the left and to the right of the tunnel respectively.

It was shown in Figure 4.16 that the horizontal soil movements in the vicinity of the tunnel increase with the imposition of a building. This trend can also be seen in Figure 4.31a. The asymmetrical nature of the building geometry leads to asymmetrical soil deformation, with higher movement on the left hand side of the tunnel ($x = -6m$). This asymmetry becomes less distinct with increasing eccentricity as shown in Figure 4.31b. In this case neither vertical profile is beneath the structure. Therefore, the soil movement at the surface shows a similar trend to that displayed for the greenfield displacement.

Figure 4.32 demonstrates how the vertical soil movement changes with building eccentricity. The results are for a vertical profile above the tunnel crown ($x = 0m$). As in the previous plot it can be seen that the movement for the $e = 48m$ case lies close to the greenfield displacement profile. For $e = 12m$ the building reduces the settlement in the top 15m of the soil profile. The difference in vertical soil movement close to the tunnel crown remains negligible.

These results demonstrate that the eccentricity of a surface structure affect the horizontal soil movements adjacent to a tunnel. An asymmetrical building geometry leads to an asymmetrical soil displacement response. The difference, however, remains small. As the building moves away from the tunnel (i.e. $e$ increases) its influence on soil displacement in the vicinity of the tunnel reduces. The vertical settlement close to the tunnel crown is not altered by the presence of a building.

### 4.4.3 Tunnel depth

The tunnel depth $z_0$ is a geometric parameter which influences both building and greenfield deformation. In terms of the relative stiffness approach it is represented by the numerator and denominator of the modification factors (see Equations 2.30 and 2.31 on Page 74). The first part of this section investigates how the tunnel depth affects these modification factors.
while the effect of tunnel depth on the tunnel induced ground movement is studied in the second part.

### 4.4.3.1 Influence on building deformation

In their work Potts & Addenbrooke (1997) only included two tunnel depths: $z_0 = 20$m and 34m. In this section consideration is also given to $z_0 = 15$m and 28m. The transverse greenfield surface settlement and the horizontal strain distribution obtained from the FE for these tunnel depths are shown in Figure 4.33. The volume loss in all analyses was close to $V_L = 1.5\%$. The construction of the shallowest tunnel results in a maximum settlement of approximately $S_{v,\text{max}} = 5.6$mm. This value reduces to $S_{v,\text{max}} = 1.6$mm for the 34m deep

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**Figure 4.33:** Greenfield surface deformation depending on tunnel depth $z_0$.  
**Figure 4.34:** Building deformation above tunnel of different $z_0$. 

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**Chapter 4. An evaluation of the relative stiffness method**

**Section 4.4**

Figure 4.35: (a) $D_{Rsag}$ and (b) $M^{DR_{sag}}$ plotted versus tunnel depth $z_0$ for different building geometries of 5 storey structures.

The maximum compressive strain observed for the 15m deep tunnel is $\epsilon_{hc} = -5.7 \times 10^{-4}$ while the maximum tensile strain is $\epsilon_{ht} = 1.3 \times 10^{-4}$ at $x = 18.0m$. For the 34m deep tunnel the corresponding magnitudes reduce to $\epsilon_{hc} = -1.3 \times 10^{-4}$ and $\epsilon_{ht} = 3.0 \times 10^{-5}$ (at $x = 40m$) respectively.

Figure 4.34 shows similar plots for a 100m wide 5-storey building with an eccentricity of $e = 20m$. Comparison with the previous figure indicates the same trend for building and greenfield scenarios: the deeper the tunnel the smaller the deformation, for both vertical displacement and horizontal strain. The following sections will investigate the change of deflection ratio and horizontal strain and the associated modification factors in more detail.

**Deflection ratio:** The trend of decreasing building deformation with increasing $z_0$ can be seen in Figure 4.35a. This graph plots $D_{Rsag}$ versus $z_0$ for a range of 5-storey buildings of different width. Solid lines represent buildings with no eccentricity, the $e = 20m$ and $30m$ cases are indicated by dashed lines. For all cases $D_{Rsag}$ decreases as $z_0$ increases. The 100m building with no eccentricity shows the steepest slope. Its deflection ratio reduces by 42% (between $z_0 = 15m$ and to 34m). The 16m wide building has the lowest $D_{Rsag}$ and the flattest gradient. It reduces, however, by 77%.

When the corresponding modification factors $M^{DR_{sag}}$ are plotted against $z_0$ (Figure 4.35b)
an increase in those factors with depth is evident. The 100m and 16m wide structures with no eccentricity show increases of 189% and 114% (from $z_0 = 15$m to 34m) respectively.

Figure 4.36a and b presents similar graphs for $DR_{\text{hog}}$ and $M^{DR_{\text{hog}}}$ respectively. The hogging deflection ratio exhibits a similar trend as $DR_{\text{sag}}$ in Figure 4.35a: a reduction (in terms of absolute value) with increasing $z_0$. $DR_{\text{hog}}$ for the 100m wide structure changes by 66% over the tunnel depths analysed. The 60m structure has the flattest slope with a reduction of 57%. Comparison of Figures 4.36b and 4.35b indicates that the $M^{DR_{\text{hog}}}$ is less affected by changes in $z_0$ than $M^{DR_{\text{sag}}}$. The curves for the 100m and 120m wide buildings given in Figure 4.36b show a small or negligible increase in $M^{DR_{\text{hog}}}$ with increasing tunnel depth.

The reasons for the significant increases in $M^{DR_{\text{hog}}}$ evident for the 32m and 60m wide structures are the small hogging zones found in the corresponding greenfield geometries. With increasing tunnel depth $DR_{\text{GF}}^{\text{hog}}$ becomes very small in these relatively short sections. Dividing the hogging deflection of these structures by these low $DR_{\text{GF}}^{\text{hog}}$ values, results in correspondingly high $M^{DR_{\text{hog}}}$. A similar effect was found for small building widths when investigating the effects of eccentricity in Section 4.4.2.1 (see page 127).

The formulation of relative bending stiffness $\rho^*$ (Equation 2.29a, Page 72) includes $z_0$ indirectly by incorporating the soil stiffness $E_S$ obtained at half tunnel depth in the denominator of $\rho^*$. Table 4.3 summarizes these values of initial soil stiffness at $z_0/2$ for different tunnel
Chapter 4. An evaluation of the relative stiffness method

Section 4.4

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<td>$E_S$ [MPa]</td>
<td>76.8</td>
<td>103.0</td>
<td>135.8</td>
<td>163.0</td>
</tr>
</tbody>
</table>

Table 4.3: Initial soil stiffness at different half tunnel depths $z_0/2$.

Table 4.4: Initial soil stiffness at different tunnel depths $z_0$.

As $E_S$ increases linearly with depth, $\rho^*$ reduces with increasing tunnel depth. The deflection ratio modification factors for the building cases analysed above are plotted versus $\rho^*$ in Figure 4.37. For each building geometry in the top graph four data points, representing the four tunnel depths, can be found. The increase in $M_{DR_sag}$ with $z_0$ shown in Figure 4.35b leads to a variation in corresponding modification factors within each quadruple. However, the change of $\rho^*$ within each building geometry remains small. This leads to a vertical scatter of data points, most notably for the non-eccentric 100m and 120m wide building cases (represented by solid circle and hollow triangle symbols, respectively) for which the tunnel depth is indicated. The scatter becomes less distinct with decreasing building width. For clarity only the design curve for $e/B = 0$ is included on the graph. All data points lie beneath this curve and the results for $B = 60$m, 32m and 16m coincide well with it.

The results for hogging show less scatter. The design curves shown are for $e/B < 0.2$ and 0.4. The results for the two $B = 100$m situations with $e/B = 0.2$ and 0.3 lie between
these curves and follow their pattern. All other building cases show small $M_{DR_{sag}}$ lying well below the design curve except for the case of the 60m wide building and $z_0 = 34$m.

The above study shows that although an increase in $z_0$ leads to a flatter and wider settlement trough the corresponding modification factors increase. For $M_{DR_{sag}}$ this results in significant scatter. The inclusion of $z_0$ directly in the expression for relative bending stiffness $\rho^*$ may reduce this scatter. Such a formulation will be proposed in Section 4.5. For hogging the scatter is less significant.

**Maximum horizontal strain:** A similar trend for $\epsilon_{hc}$ and $M^{\epsilon_{hc}}$ with $z_0$ as seen for $DR_{sag}$ and $M_{DR_{sag}}$ emerges from Figure 4.38; $\epsilon_{hc}$ reduces as $z_0$ increases. The case of the 120m wide building with no eccentricity gives the highest compressive strain (in absolute values). Its magnitude changes by 22% between $z_0 = 15$m and 34m while the 32m structure shows a change of 50%. The corresponding modification factors, however, increase with tunnel depth, in the case of the non-eccentric 120m structure by 238% and for the 32m wide building by 116% (from $z_0 = 15$m to 34m).

The corresponding plots for tensile strain and modification factors, shown in Figure 4.39 only include the two eccentric building cases as no tension was developed for most of the non-eccentric geometries. Again, a reduction of the horizontal strain with increasing tunnel depth is observed.

**Figure 4.38:** (a) $\epsilon_{hc}$ and (b) $M^{\epsilon_{hc}}$ plotted versus tunnel depth $z_0$ for different building geometries of 5 storey structures.
depth is evident. In the building with $e = 30\text{m}$, $\epsilon_{ht}$ reduces continuously over the whole range of tunnel depths by 203%. The maximum tensile strain in the $e = 20\text{m}$ case does not change significantly between $z_0 = 15\text{m}$ and 20m but it reduces as $z_0$ increases further. When plotted as modification factor $M^{\epsilon_{ht}}$ versus $z_0$ this 30m eccentricity shows, in contrast, a steady increase of 213% while the $e = 20\text{m}$ building does not show much change with $z_0$.

Despite the high relative increases observed for some modification factors in compression and tension it can be seen from Figures 4.38b and 4.39b that the factors remain relatively small. The highest value found in these two graphs is $M^{\epsilon_{hc}} = 0.038$ for the case of the 120m wide building with no eccentricity and $z_0 = 34\text{m}$. This is further illustrated in Figure 4.40 which plots strain modification factors against relative axial stiffness $\alpha^*$. In the upper graph $M^{\epsilon_{hc}}$ shows some scatter although all data points lie close to the $e/B = 0$ design curve by Potts & Addenbrooke (1997). $M^{\epsilon_{hc}}$ for the 60m, 100m ($e = 0\text{m}$) and 120m wide structures show a steeper decrease with increasing $\alpha^*$ than described by the design curve. Their scatter, however, is much less than that observed in the corresponding $M^{\text{DRass}}$ versus $\rho^*$ graph (Figure 4.37).

The $M^{\epsilon_{ht}}$ versus $\alpha^*$ plot (lower graph) only includes the 100m wide building cases with eccentricity. The Potts & Addenbrooke (1997) design curves for both $e/B < 0.2$ and 0.4 are given for comparison. The results for the building with 20m eccentricity coincide well with the $e/B < 0.2$ design curve. In the case of the building with $e = 30\text{m}$, the results show a steeper decrease with increasing $\alpha^*$ than predicted by the design curves; some of the results lie outside the $e/B = 0.4$ curve although the difference is small.
4.4.3.2 Influence on ground deformation

When investigating the influence of $B$ and $e$ on the tunnel-soil-structure interaction it was found that a variation in these building characteristics has some effect on the horizontal ground movements around the tunnel excavation. These studies were carried out assuming a tunnel depth of $z_0 = 20m$. This section compares these results with analyses performed using a 34m deep tunnel.

Figure 4.41a shows the horizontal displacement along a vertical profile at an offset of from the tunnel centre line. The depth along the vertical axis of this graph has been normalized against tunnel depth $z_0$ in order to include data for $z_0 = 20m$ and 34m. For both tunnel depths, the results for greenfield (solid lines) and 100m wide, non-eccentric 5-storey building cases (dashed lines) are shown. The data for the 20m deep tunnel were already presented in
Figure 4.8a (Page 112). The differences in $V_L$ between the four analyses were small and all curves were adjusted to $V_L = 1.5\%$. Towards the surface the displacement induced by the 34m deep tunnel is less than that found for $z_0 = 20m$. This trend was expected given the results presented in the previous section.

The important difference between the data sets for the two different tunnel axis depths is that the results for the 34m deep tunnel are much less affected by building stiffness than the equivalent results for the shallow tunnel (indicated by cross symbols). This shows that the influence of the surface structure on the soil displacement field around a tunnel excavation reduces as $z_0$ increases. The difference between greenfield and 5-storey building cases for $z_0 = 34m$ remain small close to the ground surface. This indicates that the wider a settlement trough (and therefore the entire ground displacement field) the smaller is the influence of a surface structure of certain width $B$ on the ground movement.

Similar behaviour can be seen from the vertical profile of settlement (Figure 4.41b). At tunnel depth all curves are of similar magnitude. Towards the surface the $z_0 = 34m$ results are lower than those for the shallower tunnel. The two curves for the 34m deep tunnel lie

![Figure 4.41](image)

**Figure 4.41**: Vertical profile of horizontal (a) and vertical (b) soil displacement at $x = 6m$ for $z_0 = 20m$ and 34m. For sign convention, see Figure 4.8.
closer together than the equivalent results for the $z_0 = 20m$ tunnel.

In previous sections it was shown that a surface structure has a small effect on the soil displacement around a tunnel under excavation. This study demonstrates that this influence reduces even further with increasing tunnel depth. While for shallow tunnel an interaction between tunnel and building can be found, the influence of the building on the tunnel becomes less important with increasing tunnel depth.

### 4.4.4 Summary

The previous sections investigated how tunnel induced building deformation depends on the geometry of the system under consideration. By separately varying the building width $B$, eccentricity $e$ and tunnel depth $z_0$ their influence on building deformation and ground movement was assessed. The following trends were observed:

- A variation in $B$ can lead to a significant scatter in corresponding $M^{DR_{sag}}$ when plotted against $\rho^*$. This trend is most evident for structures of low stiffness. $DR_{sag}$ does not increase for certain widths $B$ while $\rho^*$ reduces. It is therefore arguable that the influence of $B$ in the relative bending stiffness expression is overestimated. The scatter of $M^{DR_{hog}}$ and $M^{\epsilon_{hc}}$ against $\rho^*$ and $\alpha^*$ respectively was smaller and the results were in general agreement with the design curve provided by Potts & Addenbrooke (1997).

- An increase in $e$ reduces $DR_{sag}$ and $\epsilon_{hc}$ over the range from $e/B = 0$ to approximately 0.6. In contrast, $DR_{hog}$ and $\epsilon_{ht}$ increase with increasing $e$ reaching maxima at $e/B \approx 0.6$. This value is higher for short buildings which show low values of $DR_{hog}$ and $\epsilon_{ht}$. These boundaries of $e/B$ are well represented in the design charts of Potts & Addenbrooke (1997).

- A deeper tunnel depth $z_0$ generally induces smaller deformations at the surface and correspondingly increases in the modification factors. There are cases, however, where this increase is not well represented by the relative stiffness, most notably for $M^{DR_{sag}}$. It may be prudent to include the tunnel depth directly into the expression for relative stiffness to account for this effect.

- The effect of building geometry on soil movement has been found to decrease with depth. When comparing cases with a surface structure to greenfield situations a different pat-
tern of horizontal ground displacement was observed beneath the surface structure. This effect reduces the greater the value of $z_0$.

4.5 An alternative formulation for the relative stiffness

In the previous sections the influence of various parameters on tunnel induced building deformation have been investigated. It has been shown that the variation of some of these factors can lead to significant scatter when plotting modification factors against relative stiffness. This section comprises the results of over 100 FE analyses varying $z_0$, $B$ and $e$ in order to investigate how this scatter can be reduced by alternative formulations for relative stiffness.

4.5.1 Deflection ratio

Figure 4.42 shows the distribution of $M_{DR}^{sag}$ with $\rho^*$ for buildings of various $z_0$ and $B$ but with no eccentricity. Most results lie below the design curves postulated by Potts & Addenbrooke (1997) which therefore define an upper bound for the wide range of building cases analysed \(^4\). Sections 4.4.1.1 and 4.4.3.1 concluded that variations in $B$ and $z_0$ can lead to scatter in $M_{DR}^{sag}$ especially for structures with a low stiffness; this trend is evident in the upper graph. The hogging data in the lower plot, in contrast, follow the design curves more closely. It has been shown in the previous sections that the formulation of $\rho^*$ overestimates the influence of $B$ on $M_{DR}^{sag}$ while it under-predicts the importance of $z_0$ in some cases. It is therefore arguable to include the tunnel depth directly in the expression for relative stiffness and to reduce the exponent of the building width $B$ used in the original formulation. By redefining $\rho^*$ to

$$\rho_{m1}^* = \frac{EI}{\frac{z_0^2}{2}E_s \left( \frac{B}{2} \right)^2} \quad (4.1)$$

where $E_s$ is the soil stiffness at $z_0/2$ (as defined by Potts & Addenbrooke (1997)) the scatter in $M_{DR}^{sag}$ can be reduced to that shown in Figure 4.43. Unfortunately the redefined relative bending stiffness $\rho_{m1}^*$ results in an increased scatter for $M_{DR}^{sag}$ although this scatter remains smaller than that previously observed in $M_{DR}^{sag}$. The original design curves are given for

\(^4\)It can be seen in Figure 4.42 that some modification factors exceed unity. Such a behaviour was found by Potts & Addenbrooke (1997) for low values of $\rho^*$ combined with high values of $\alpha^*$, as discussed on page 76.
comparison. It can be seen that the $M^{\text{DR}_{\text{sag}}}$ results are in good agreement with the curve while $M^{\text{DR}_{\text{hog}}}$ lie outside the design curve which was fitted to data using the original definition of $\rho^*$.

Figures 4.44 and 4.45 present similar plots of $M^{\text{DR}}$ against $\rho^*$ and $\rho^*_{m1}$ respectively for the eccentric cases. The original design curves for different $e/B$ ratios are also included. Figure 4.44 shows that, when using the original formulation $\rho^*$, $M^{\text{DR}_{\text{sag}}}$ data lie close together regardless of their $e/B$ ratio. A clearer pattern of behaviour, with three bands for each different eccentricity, was observed in the case of $M^{\text{DR}_{\text{hog}}}$. The $e/B = 0.66$ results lie on the corresponding design curve while the $e/B = 0.3$ points follow the $e/B = 0.4$ curve. The data for the $e/B = 0.2$ case lie slightly outside of the corresponding line.

When using the modified expressions for relative stiffness $\rho^*_{m1}$ the picture is similar to the centric cases. While for sagging the scatter decreases, it increases for hogging. There is

**Figure 4.42:** Modification factors $M^{\text{DR}}$ against relative bending stiffness $\rho^*$ for buildings of no eccentricity.

**Figure 4.43:** Modification factors $M^{\text{DR}}$ against modified relative bending stiffness $\rho^*_{m1}$ for buildings of no eccentricity.
a clearer distribution of the sagging results over the variety of $e/B$ ratios studied and these data points coincide well with the original design curves (defined for the use with the original $\rho^*$). However, for $M^{DR}_{hog}$ the trend between the different $e/B$ ratios becomes less clear when plotted against $\rho^*_{m1}$.

The incorporation of $z_0$ into the expression for the relative bending stiffness and the reduced influence of $B$ give a clearer relationship between $M^{DR}_{sag}$ and relative stiffness. This is in agreement with the results shown in the previous sections where the influence of both $B$ and $z_0$ on building deformation was investigated separately. In contrast, the results of $M^{DR}_{hog}$ show an increased scatter when plotted versus $\rho^*_{m1}$.

![Figure 4.44: Modification factors $M^{DR}$ against relative bending stiffness $\rho^*$ for buildings with eccentricity.](image)

![Figure 4.45: Modification factors $M^{DR}$ against modified relative bending stiffness $\rho^*_{m1}$ for buildings with eccentricity.](image)
Chapter 4. An evaluation of the relative stiffness method

Section 4.5

4.5.2 Maximum horizontal strain

Figure 4.46 shows $M^{ch}$ for both concentric and eccentric geometries. The results follow generally the trends indicated by the original design curves, with higher eccentricities leading to smaller $M^{chc}$ and higher $M^{cht}$. There is more scatter for the compressive strain graph (top). Separate groupings for each eccentricity are evident in the bottom tensile strain plot. All tensile results, however, lie outside the corresponding design curves although they remain relatively small in magnitude.

Figure 4.38b demonstrated that $M^{hc}$ increases with tunnel depth. Following the redefining of $\rho^*$ by including the tunnel depth in the denominator of the expression for the relative bending stiffness, a new relative axial stiffness can be expressed as

$$\alpha^{*}_{mi} = \frac{EA}{z_0E_k \frac{E}{2}}$$

(4.2)
which changes the dimension of the expression to $[1/m]$ under plane strain conditions. Figure 4.47 shows the distribution of $M^{\epsilon h}$ against this new expression. Note that the boundaries of the horizontal axes are reduced by one order of magnitude compared to the previous graph. The design curves are therefore not plotted in this context. The graph presenting $M^{\epsilon h c}$ shows a slightly smaller scatter than in the previous figure, especially between the $e/B = 0.0$ and $0.2$ ratios (although they lie close together). The distribution for different eccentricities in the tension graph is, however, less distinct than it was in the $M^{\epsilon t h}$ versus $\alpha^*$ formulation.

In general, the hogging tensile mode of deformation are the more critical for any structure (Burland & Wroth, 1974; Boscardin & Cording, 1989). The previous figures showed that the original relative stiffness terms give better predictions for these deformation criteria then the modified expressions $\rho_{m1}$ and $\alpha_{m1}$. It was, however, found, that some data points lie outside the design curves. The results from this chapter, together with conclusions of the following chapters will be used in Chapter 10 to improve the design charts.

### 4.6 Conclusions

This chapter investigated the influence of various parameters adopted in the study of Potts & Addenbrooke (1997). The parameters were either constants in their FE analyses (volume loss $V_L$, initial stress conditions and mesh width) or geometric variables (building width $B$, eccentricity $e$ and tunnel depth $z_0$). The influence of the geometric parameter together with the building stiffness on the soil movement beneath the building and around the tunnel was also studied.

It has been found that the deflection ratio increases approximately linearly with volume loss. Results can therefore be adjusted to a common $V_L$. For maximum horizontal strain such a relationship is not evident. Instead strain can be linearly interpolated to a common value of $V_L$. The influence of change of initial stress profile around the tunnel and of mesh width has been found to have little influence on the modification factors. It is therefore justified to change these conditions in order to investigate the behaviour of more building details.

The study revealed that variation in $B$ and $z_0$ can lead to significant scatter when plotting $M^{\text{DR}_{\text{hog}}}$ against $\rho^*$ and $M^{\text{thc}}$ against $\alpha^*$. When modifying the relative stiffness expressions by including $z_0$ and reducing the influence of $B$ this scatter reduced while it increased slightly in the corresponding $M^{\text{DR}_{\text{hog}}}$ and $M^{\text{tht}}$ graphs. The variation of modification factors with
eccentricity obtained in this study was in good agreement with the representation of different curves for different $e/B$ ratios in the relative stiffness design charts.

Apart from a few exceptions discussed in the previous sections most results were below the upper bound curves introduced by Potts & Addenbrooke (1997). This study therefore provides further confirmation to adopt this approach as a design method.

To get a better understanding of the mechanisms controlling this tunnel-soil-structure interaction problem the influence of the above mentioned geometric parameters and of building stiffness on the soil displacement was investigated separately.

The study demonstrated that these building characteristics have only a small effect on the horizontal ground movement around the tunnel under excavation. This effect becomes even smaller when $z_0$ increases. The variation of vertical ground displacement above the tunnel crown has been found to be negligible.

The presence of a building changes the displacement field significantly near to the soil-structure interface although this effect also reduces with increasing tunnel depth. The influence of the geometry on the horizontal and vertical soil movement is small compared with the effect of increasing building stiffness. This confirms that the bending and axial stiffness of the structure is the key parameter of this interaction problem.
Chapter 5

The influence of building weight

5.1 Introduction

In the previous chapter the building was modelled in the same way as in the original work of Potts & Addenbrooke (1997). In particular a weightless elastic beam was adopted to represent the structure. This chapter presents the results of analyses that consider building weight in order to investigate its influence on the tunnel induced deformation of a structure. By investigating the mechanisms that control this interaction problem it is demonstrated how the application of building load changes the stress regime in the ground and how this stress change alters tunnelling induced ground and building deformation. The results of a parametric study are then used to quantify the effect of the building’s self weight.

5.2 Finite Element Analysis

Most of the results presented in this chapter are for a 100m wide building with its centre line coinciding with that of the tunnel. However, when considering the horizontal strain induced in the structure, results from analyses of a building with an eccentricity $e = 20$ m are included. The tunnel had a diameter of $D = 4.146$ m and a depth of either $z_0 = 20$ m or 34 m. The initial stress conditions are described in Section 3.4.1 and the soil model and parameters in Section 3.3.

The building is modelled as an elastic beam as explained in Section 3.4.3 and the load it imposes on the ground was simulated by applying a uniform stress over several increments
to this beam. The stress values were 10kPa, 30kPa, 50kPa and 100kPa corresponding to a 1-, 3-, 5- and 10-storey building respectively. In addition zero-load cases were considered.

Combining the 5 load options with the 5 stiffness values gives 25 variations. These are represented in a $5 \times 5$ matrix in Figure 5.1. The matrix contains some unrealistic cases, for instance structures with a low stiffness but loaded with a high stress. On the other hand, the leading diagonal to this matrix represents realistic cases: 10kPa applied to a 1-storey building, 30kPa applied to a 3-storey building etc. If basement construction were considered for a given stiffness the net loading would be reduced. This, arguably, could be represented by the cases below the leading diagonal. The stiffness-load combinations in the first column are referred to as zero-load cases (index '0'). These combinations are equivalent to the cases investigated by Potts & Addenbrooke (1997). The flexible cases can be found in the first row denoted with an index ‘fl’. Combining the flexible case with the zero-load represents greenfield conditions.

Figure 5.1: Matrix of stiffness/stress combinations used in the parametric study.

The weightless beam was constructed in the first increment of each FE-analyses. Over the next increments the uniform stress was applied. As this stage was modelled as being fully drained it was not necessary to simulate any consolidation period in order to reach pore water equilibrium conditions. In reality during the consolidation time ageing of the soil may affect its soil stiffness properties. For this reason the high initial soil stiffness to $p'$ ratio was reset prior to tunnel excavation. This was achieved by zeroing the accumulated strains in the soil at the beginning of the first increment of the tunnel excavation. As a result the initial soil stiffness before tunnel excavation depends only on the stress level $p'$ in the soil and
therefore on the applied building load. The tunnel excavation was then simulated as outlined in Section 3.4.4.

5.3 Stress state

5.3.1 Introduction

The construction and subsequent loading of the building changes the stress conditions in the soil. While the building stiffness changes the boundary conditions at the ground surface, the load affects the effective stresses in the soil down to a depth where the tunnel will be constructed at a later stage. The ageing process ongoing within the soil between the end of building construction and the start of tunnel excavation might also affect the soil behaviour.

The response of the soil to the tunnel excavation depends on its stress state as this controls its stiffness. It will be demonstrated later how the stress state at tunnel depth controls the soil displacement while the change in soil stiffness beneath the building influences the building deformation.

These different zones of influence can be seen in Figure 5.2 which shows the horizontal soil displacement in response to tunnelling plotted against depth for a vertical line at an offset from the centre line of the building and tunnel of 6m. Different curves are presented for different load applied by a 5-storey building. It can be seen how the building load influences the displacement behaviour near to the tunnel depth of \( z_0 = 34m \). As the load increases the soil movement towards the tunnel reduces (a negative sign describes movement towards the tunnel). A similar pattern can be seen close to the building where the higher load reduces the horizontal soil displacement. At the very surface the soil movement is restricted by the axial stiffness of the structure.

Figure 5.3 shows the effect of the building load on the mean effective stress \( p' \) prior to tunnel construction. The mean effective stress \( p' \) in this graph is normalized by \( p'_0 \) which is the mean effective stress of the corresponding zero-load case. The initial value of \( p' = 26.16 \text{kN/m}^2 \) at the surface for the zero-load case is due to the negative pore water pressure assumed above the water table.

It can be seen that the increase in stress becomes more significant towards the surface where the overburden pressure does not dominate the stress regime. This is most marked for
Section 5.3

the 100kPa case where the stress is increased to $p' = 71.94\text{kN/m}$ (275% of the initial stress profile) at the soil surface but only to $28.00\text{kN/m}$ (107%) at a depth of 34m. As the soil stiffness is modelled to be directly proportional to the mean effective stress $p'$, the profiles in Figure 5.3 therefore also indicate the increase of soil stiffness due to the application of the load. As the overburden pressure of the soil increases with depth, the effect of the building load becomes less significant. This effect can be seen in Table 5.1 which summarizes the stress state at 34m depth prior to tunnel construction under a 5-storey structure. At this depth the mean effective stress $p'$ (and the soil stiffness) increases by 7% when a load of 100kPa is applied. The values for no building load (0kPa) represent the initial conditions in the soil at the beginning of each analysis with a lateral stress ratio set to $K_0 = \sigma'_h/\sigma'_v = 1.5$. This ratio, however, decreases when load is applied to the structure: The ratio decreases to 1.21 when a building load of 100kPa is imposed.

The soil behaviour in both zones, i.e. at tunnel depth and in the vicinity of the structure, are investigated in the following two sections.
Chapter 5. The influence of building weight  
Section 5.3

5.3.2 Behaviour at tunnel depth

The stress state at tunnel depth controls directly the deformation field caused by tunnel construction as it defines the loads removed from the soil during the excavation process. The soil movements around the tunnel, however, also depend on the soil stiffness which is stress level dependent. In order to separate these effects, further analyses were undertaken with no building load applied but with different soil unit weights creating the following stress scenarios at tunnel depth:

1. $\sigma_h'/\sigma_v' = 1.5$ as though there were no building, $p'$ (and soil stiffness) varying as though there were, see Table 5.2

2. $\sigma_h'/\sigma_v'$ varying as though there were a building, $p' = 487\,\text{kPa}$ as though there were not, see Table 5.3.

The analyses were then carried out applying a 5-storey building with no load. The results of these analyses are presented as vertical profiles of horizontal displacement at a distance of 6m from the tunnel axis. Figure 5.4a shows the displacement profile for the first stress situation (constant $K_0$). Each curve represents a different value of mean effective stress which is equal to the stress observed under the corresponding building load. The pattern at the

<table>
<thead>
<tr>
<th>Load [kPa] (applied)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p'$ [kPa]</td>
<td>486.9</td>
<td>489.7</td>
<td>497.2</td>
<td>504.0</td>
<td>518.7</td>
</tr>
<tr>
<td>$p'/p'_b$</td>
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<td>1.01</td>
<td>1.02</td>
<td>1.04</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma_h'/\sigma_v'$</td>
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<td>1.46</td>
<td>1.40</td>
<td>1.34</td>
<td>1.21</td>
</tr>
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<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>$V_L$ [%]</td>
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<td>1.47</td>
<td>1.45</td>
<td>1.43</td>
<td>1.40</td>
</tr>
</tbody>
</table>

**Table 5.2:** Stress state scenario 1: Constant lateral stress ratio, while $p'$ is varied.

<table>
<thead>
<tr>
<th>Load [kPa] (not applied)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p'$ [kPa]</td>
<td>486.9</td>
<td>486.9</td>
<td>486.9</td>
<td>486.9</td>
<td>486.9</td>
</tr>
<tr>
<td>$\sigma_h'/\sigma_v'$</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
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</tr>
<tr>
<td>$\gamma_{soil}$ [kN/m$^3$]</td>
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<td>20.03</td>
<td>20.19</td>
<td>20.34</td>
<td>20.67</td>
</tr>
<tr>
<td>$V_L$ [%]</td>
<td>1.50</td>
<td>1.48</td>
<td>1.46</td>
<td>1.45</td>
<td>1.42</td>
</tr>
</tbody>
</table>

**Table 5.3:** Stress state scenario 2: Constant $p'$ while the lateral stress ratio is varied.
level of the tunnel axis is similar to that shown in Figure 5.2. As the mean effective stress increases (in Figure 5.2 caused by a higher building load) the horizontal displacement reduces. The displacement in Figure 5.4a only depends on \( p' \), in contrast to Figure 5.2 where both the lateral stress ratio and the mean effective stress were varied. The increase of soil stiffness due to the increased mean effective stress \( p' \) around the tunnel explains the smaller movement and leads to smaller volume losses as shown in Table 5.2.

It was demonstrated in Section 4.2.1 that the soil displacement is linearly related to the volume loss. Figure 5.4b shows the curves of Figure 5.4a linearly adjusted to a volume loss of \( V_L = 1.5\% \). The fact that the curves for different load cases coincide in this plot demonstrates that the different displacement behaviour observed for different values of mean effective stress in Figure 5.4a is simply a consequence of the different values of volume loss.

In the second stress scenario, see Table 5.3, the mean effective stress \( p' \) at tunnel axis level is constant (so the soil stiffness is constant) and only the lateral stress ratio \( \sigma_h' / \sigma_v' \) varies according to the corresponding values when the building load is applied. In Figure 5.5a it can be seen that, again, the horizontal displacement decreases with stress states representing a higher building load. The variation in volume loss for these cases is small as shown in Table 5.3. When the results are adjusted to a volume loss of \( V_L = 1.5\% \) there are still differences (Figure 5.5b). The reason for the different horizontal displacements is the change
Figure 5.5: (a): Vertical profile of horizontal soil movement during tunnel construction for stress scenario 2. (b): Horizontal movement adjusted to volume loss.

of deformation mode of the tunnel boundary when subjected to different $\sigma'_h/\sigma'_v$ regimes.

Figure 5.2 showed that the application of building load altered the ground response to tunnelling. This is because building load changes both the mean effective stress and the $\sigma'_h/\sigma'_v$ ratio at tunnel depth. Figures 5.4 and 5.5 have revealed that the change in mean effective stress affects the volume loss, and can be neglected when the results are adjusted to a constant volume loss. But crucially such adjustment cannot account for the effect of different lateral stress ratios.

5.3.3 Behaviour under the foundation

Figure 5.2 shows that the tunnelling induced horizontal ground movement near the surface is affected by the applied building load. Prior to tunnel construction all strains in the soil are set to zero. Consequently, the initial soil stiffness immediately prior to tunnel construction only depends on the level of mean effective stress $p'$. The dotted lines in Figure 5.6 show this situation. For the zero-load case the stiffness increases linearly with depth. The soil stiffness for the 100kPa load is higher due to the increased stress level.

To investigate how the soil movement in response to tunnelling is influenced by the soil stiffness in close proximity to the foundation further analyses were undertaken using a linear
elastic soil model for the uppermost 6m of soil. A depth of 6m was chosen as it corresponds to
the depth where the maximum variation of horizontal ground movement with building load
was found (shown in Figure 5.2). The elastic soil stiffness parameters in this layer are chosen
to increase linearly with depth to match the stiffness distribution given by the zero-load case.
The analyses then undertaken for a 5-storey building comprises different load cases of 0kPa,
50kPa and 100kPa. For the 100kPa case the stiffness profile prior to tunnel construction is
shown by the solid line in Figure 5.6.

When using the non-linear elastic model in the upper 6m, the soil stiffness reduces with
increasing strain level during tunnel excavation. This effect is reproduced in the modified soil
layer by changing the elastic properties during each excavation increment. The soil stiffness
in that zone therefore represents, at each stage of the excavation, the soil stiffness found in a
zero-load analysis in the corresponding excavation increment. By this modification the soil
stiffness in the uppermost 6 m is not influenced by the building load.

Figure 5.7 presents the results of the analyses plotted as a profile of horizontal displacement
with depth for a distance from the tunnel centre line of $x = 6m$. There is a uniform
horizontal displacement for all load cases in the top 6m of soil which coincides with the

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**Figure 5.6:** Soil stiffness profile on centre line of 5-storey building prior to tunnel construction.

**Figure 5.7:** Vertical profile of horizontal soil movement during tunnel construction for
modified soil model used in top soil layer.
zero-load case (without modified soil layer) shown in Figure 5.2. Below this depth the dis-
placement follows the pattern observed in Figure 5.2. These analyses show that it is the soil
stiffness in the zone beneath the foundation which controls the horizontal displacements in
this region, and not the change in $\sigma_h'/\sigma_v'$ ratio.

5.4 Parametric study

5.4.1 Deflection ratio

For the 100 m building without eccentricity all stiffness-load combinations shown in Figure 5.1
were analysed in order to obtain an overall picture of the influence of building load on the
deflection ratio.

For all 50 cases the deflection ratios $\text{DR}_{\text{hog}}$ for hogging and $\text{DR}_{\text{sag}}$ for sagging were deter-
mined. Dividing these values by the corresponding greenfield values (given in Table 5.4) gives
the modification factors $M^{\text{DR}}$. The results of each stiffness are then normalized against the
corresponding zero-load modification factor $M^{\text{DR}}_0$ (see Figure 5.1). The data are presented
in graphs showing $(M^{\text{DR}}/M^{\text{DR}}_0)$ versus building load. Figure 5.8a and b show these plots
for hogging and sagging respectively. In order to obtain these results the deflection ratio is
adjusted in each case to a common volume loss of $V_L = 1.5\%$.

For hogging Figure 5.8a shows for each stiffness (given by the number of storeys) a steady
increase in the modification factor $M^{\text{DR}_{\text{hog}}}$ with increasing building load. This effect is small
for structures with a low stiffness (0 and 1 storey) and a high stiffness (10 storeys) but is
more significant for the 3 and 5-storey cases. It is appropriate to focus on the realistic cases
(i.e. those on the leading diagonal in Figure 5.1). The data for these cases are marked with
a thick line and black squares. The biggest increase is 42% for the 5-storey building and a
tunnel depth $z_0 = 20$ m. For the sagging case shown in Figure 5.8b, the change of modification

<table>
<thead>
<tr>
<th>Geometry: $e = 0$ m, $B = 100$ m</th>
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<tr>
<td>$z_0 = 20$ m</td>
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<td>$z_0 = 34$ m</td>
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<table>
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<tr>
<th>Geometry: $e = 20$ m, $B = 100$ m</th>
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<td>$z_0 = 20$ m</td>
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<td>$z_0 = 34$ m</td>
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Table 5.4: Deflection ratio and horizontal
strain for greenfield conditions.
Chapter 5. The influence of building weight

Section 5.4

The maximal increase is 20% for the 10-storey building and a 20m deep tunnel. Although the increases for both cases seems to be significant, it must be noted that the deflection ratio decreases rapidly as building stiffness increases. Figure 5.9 gives a clear picture of this as the modification factor for each magnitude of applied load is normalized against the corresponding result for the flexible structure, $M_{\text{fl}}^{\text{DR}}$. The data for hogging above the 34m deep tunnel are plotted against the structure’s stiffness. The data lie in a very narrow range and show a significant decrease of between 70% and 85% as the stiffness increases from 1 storey up to 3 storeys. For higher stiffness values the results are stable and the deflection ratio is about 10% of the corresponding value for the flexible structure. This clearly shows that building stiffness dominates the problem, not building weight.

This is further illustrated in Figure 5.10 where the modification factors for hogging and sagging are plotted against the relative bending stiffness $\rho^*$. For all stiffness values the results for the zero-load (square symbols) and for the ‘realistic’ case (on the leading diagonal, triangle symbols) lie almost on top of each other. Even the 42% increase of modification factor mentioned above is small when plotted in this context. The increase is from $M_{\text{fl}}^{\text{DR}_{\text{hog}}}=0.053$ for the zero-weight case to $M_{\text{fl}}^{\text{DR}_{\text{hog}}}=0.076$ for the case considering self weight. Following
the design approach of Potts & Addenbrooke (1997) this means for engineering practice that $M_{DR_{hog}}$ for a 5-storey building affected by tunnelling induced ground subsidence is only 0.076 times the hogging ratio for the corresponding greenfield situation.

The corresponding design curves suggested by Potts & Addenbrooke (1997) which were an upper bound to their design data, are shown for comparison and they are an upper bound to the data from these analyses, too.

The soil stiffness $E_S$ used for describing the relative bending stiffness (Equation 2.29a, Page 72) is taken at a depth half way between the tunnel axis and the ground surface. In the analyses of Potts & Addenbrooke (1997) this value is only dependent on the depth of the tunnel. In the data presented in Figure 5.10 the increase in $E_S$ due to the increased effective stress $p'$ under the building load is taken into account. The difference in relative stiffness for each data couple in this graph is, however, small.
Chapter 5. The influence of building weight

Section 5.4

5.4.2 Horizontal strain

When investigating the horizontal strain $\epsilon_h$ in the building it was found that hardly any tensile strain occurs in a 100m wide building with no eccentricity. Therefore additional analyses were performed in which the building had an eccentricity of $e = 20m$ with respect to the centre line of the tunnel. Figure 5.11a shows the strain distribution over the length of the 5-storey building. The results for the zero-load, the 50kPa and the 100kPa case are presented. The increase (in terms of absolute value) in both compressive (negative sign) and tensile (positive sign) strain can be seen. In a previous section it was demonstrated how the soil stiffness in close proximity to the building controls the horizontal displacement of the soil in this region. A set of analyses using the modified soil model for the uppermost 6m of soil (see Figure 5.6) was undertaken with the eccentric building geometry. Figure 5.11b shows the strain distribution from these analyses. The lines for different load cases lie close together and coincide well with the zero-load case using the conventional soil model. This result reveals how the stiffness in the uppermost soil controls the strain development in the structure caused by tunnelling: On the one hand the higher soil stiffness reduces the horizontal soil movement whereas on the other hand it means the soil is more able to transfer this movement to the structure. The net result is an increase in horizontal strain.

This effect can be observed for all stiffness-load combinations. Figure 5.12a and b show the development of horizontal strain when the building load is increased. The plots are of a similar format to those shown in Figure 5.8 with the modification factors $M^{\epsilon_h}$ for each load case normalized against the corresponding zero-load case. For the flexible structure the modification factors reduce slightly with increasing load. This is due to the absence of any stiff structure restraining the horizontal movement of the soil. Hence the reduction in modification factors is caused by the reduction in horizontal soil movement with increasing soil stiffness. If, however, a stiff structure is included into the analyses this picture changes: For the 1-, 3-, 5- and 10-storey buildings the horizontal strain (both compression and tension) in the structure increases steadily with building load. For both tensile and compressive strain the behaviour for each tunnel depth is very similar. The tensile strain is increased by approximately 100% when a load of 100kPa is applied compared to the zero-load case. For the compressive strain this increase is smaller giving a compressive strain that is 50% greater than in the corresponding zero-load case when 100kPa are applied.
Figure 5.11: Horizontal strain distribution in structure (B = 100 m, e = 20 m, z0 = 34 m) for (a) standard soil model and (b) modified top layer.

These graphs, however, give no indication about the absolute magnitude of horizontal strain developed in the structure. Figure 5.13 shows that the strain drops significantly when normalizing the modification factors against the corresponding flexible case (note that in contrast to Figure 5.9 this graph has a logarithmic scale on the ordinate): As an example the data for tension and for the 34m deep tunnel are given, showing a significant reduction to values between 0.017 and 0.05 for the 1-storey stiffness compared to the flexible case. Combining these results with the curves in Figure 5.12b reveals that although there is a clear trend of increasing modification factor with load the modification factors themselves remain at a very low level. This becomes clear when plotting the modification factors against the relative axial stiffness $\alpha^*$. Figure 5.14 includes all the load cases. The zero-load cases are marked with square symbols while the ‘realistic’ cases are represented by triangle symbols. All other cases are shown with a cross symbol. Because of the small scale chosen in this graph the increase in modification factors for all cases when load is considered can clearly be seen. The realistic cases, however, remain close to the corresponding zero-load square symbols.

Figure 5.14 also shows the design curves suggested by Potts & Addenbrooke (1997). It can be seen that some ‘weightless’ results lie above the curves for an eccentricity of $e/B = 0.2$. 

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Chapter 5. The influence of building weight

Section 5.4
Figure 5.12: Change of (a) $M^{\infty c}$ and (b) $M^{\infty h}$ with applied load. $M^{\infty h}$ normalized against corresponding zero-load case.

Figure 5.13: Change of tensile strain modification factor $M^{\infty h}$ with stiffness. $M^{\infty h}$ normalized against corresponding flexible case (0 storeys).

This is due to the changed initial conditions (i.e. no zone of reduced $K_0$) used in these analyses, as discussed in Section 4.2.3. It is therefore not possible to compare the results of the present work directly with the study of Potts & Addenbrooke (1997). However, as the difference between the current data and their upper bound curves remains small it is still possible to use their design approach.

This graph reveals that the modification factors remain small with a maximum value of $M^{\infty c} = 0.08$ for the 1-storey building under a load of 10kPa, $z_0 = 34$m. According to the design approach by Potts & Addenbrooke (1997) this means that the horizontal compressive strain in this type of structure is only 0.08 times the compressive strain found in the corre-
Chapter 5. The influence of building weight

Section 5.4

Figure 5.14: Strain modification factors $M^{ht}$ together with the design curves by Potts & Addenbrooke (1997).

Figure 5.15: Strain modification factors $M^{ht}$ versus a modified relative axial stiffness taking into account the change of soil stiffness beneath the foundation.

sponding greenfield situation. These greenfield values are given in Table 5.4. For the tensile strain (which is more critical in respect to building damage) the modification factor is even smaller: $M^{ht} = 0.02$ for the 1-storey structure mentioned above.

For calculating the relative axial stiffness $\alpha^*$ the soil stiffness $E_S$ considered in Equation 2.29b is taken at half tunnel depth. For the results shown in Figure 5.14 the increase of $E_S$ due to the increase of $p'$ at this depth is taken into account. It can be seen that the difference in relative axial stiffness for each range of load cases is very small. Previously it was shown that it is the soil stiffness in the zone immediately beneath the building which controls the horizontal strain behaviour in the building. It is therefore straightforward to include this soil stiffness into the relative axial stiffness $\alpha^*$. For this particular geometry the best curve fit can be achieved when taking the soil stiffness from a depth of $x = 5.8m$. Figure 5.15 shows the strain modification factors for all stiffness-load combination plotted...
against the modified relative axial stiffness incorporating the soil stiffness at $z = 5.8\text{m}$. It can be seen that for each tunnel depth the data points follow a uniform line with a very small scatter.

Although choosing the soil stiffness at $z = 5.8\text{m}$ seems to be an arbitrary choice the result demonstrates the significant influence of the soil stiffness below the structure’s foundation. Figure 5.15 shows different patterns for each tunnel depth. This is because the tunnel depth $z_0$ is not included anymore in the modified relative stiffness $\alpha^*$ while it was incorporated into the original formulation when the soil stiffness was taken from half tunnel depth.

In an engineering context the increase of modification factors for realistic stiffness-load combinations remains very small. These results lie very close to the design curves provided by Potts & Addenbrooke (1997). The scatter shown in Figure 5.14 is small compared to the potential error associated with the estimation of the structure’s stiffness.

5.5 Conclusions

This chapter shows the influence of the weight of a structure on its deformation behaviour caused by tunnelling induced ground subsidence. Loads of up to 100kPa were applied to structures with a stiffness representing 1 to 10-storey buildings. By varying the material properties of the soil profile the mechanisms which control this aspect of the soil-structure interaction problem were investigated. It was found that the load of the building alters the deformation behaviour of the soil in two distinct zones: At tunnel depth and in close proximity to the foundation of the building. At tunnel depth the effect of increasing mean effective stress $p'$ has been uncoupled from the change in lateral stress ratio $\sigma'_h/\sigma'_v$ and the consequences of both were analysed separately. It was found that the increase in $p'$ affects directly the volume loss $V_L$. Adjusting the soil displacement to a common volume loss leads to a uniform soil movement for different levels of mean effective stress while $\sigma'_h/\sigma'_v$ was kept constant. The lateral stress ratio $\sigma'_L/\sigma'_v$ in contrast influences the deformation field of the soil which consequently affects any structure above the tunnel. This demonstrates the complex character of the interaction problem: The load of the building changes the stress regime which influences the deformation mode of the soil around the tunnel which then affects the response of the building to the tunnelling induced subsidence. The increase in soil stiffness in close proximity to the structure has been found to influence the building response significantly. It
has been shown that the development of horizontal strain in the structure can be directly related to the soil stiffness beneath the structure.

The influence of building load on the deflection ratio and on horizontal strain has been investigated with a parametric study involving 50 nonlinear plane strain FE analyses for two different building geometries. It has been shown that in general the modification factors increase with increasing load. This effect is, however, small compared to the decrease of deflection ratio and horizontal strain with increasing building stiffness. Since the latter effect dominates, the graphs plotting modification factor against relative stiffness show little change when realistic building weight scenarios are included into the analyses. The results therefore lie close to the upperbound curves provided by Potts & Addenbrooke (1997) and provide further confidence in the use of these curves for practical design.
Chapter 6

The influence of the soil-structure interface

6.1 Introduction

In all analyses presented in the previous chapters it was assumed that the building and the underlying soil are rigidly connected and consequently no relative movement was modelled between the foundation and adjacent soil. This chapter focuses on the nature of this contact between soil and building. Interface elements, described in Section 3.2.4.4, were included into the 2D analysis to control the behaviour of the connection between soil and structure.

It will be shown that a separation between building and soil occurs when weightless buildings are modelled. If building load is included in the analyses no gap between soil and foundation develops. This chapter will therefore highlight the role of relative horizontal displacement within the soil-structure interface on tunnel induced building settlement. Using interface elements with low shear but high normal stiffness properties, it will be investigated how this movement affects ground and building deformation. It will be shown that horizontal soil movement close to the building foundation increases drastically, compared to conventional no-interface analysis, while horizontal strain within the building nearly vanishes. The results of a parametric study will conclude that the deflection ratios in various building geometries reduce when relative horizontal movement at the soil-structure interface is allowed.
6.2 Finite Element analysis

To vary the nature of the contact between building foundation and soil, interface elements were included into the FE analysis. They were placed between the elastic beam elements representing the building and the adjacent solid elements which model the soil. Day & Potts (1994) demonstrated that the use of interface elements can lead to numerical instability. Consequently, care was taken when choosing interface properties. A suite of FE analyses was performed to study the influence of various interface parameters. A 100m wide 5-storey building with no eccentricity subjected to tunnel excavation at $z_0 = 20m$ was used for this purposes. The tunnel diameter was $D = 4.146m$. For the parametric study, presented in Section 6.5, a tunnel depth of $z_0 = 34m$ and building eccentricity, defined in Figure 3.6 on Page 95, were also included. The initial stress profile is described in Section 3.4.1 and the soil model and parameters are given in Section 3.3.

Tunnel construction was modelled as undrained. As not stated otherwise, results were taken from the 7th increment (of the 15 excavation increments) in which a volume loss of approximately $V_L = 1.5\%$ was achieved. Further details of modelling the excavation process are given in Section 3.4.4. A building load was applied in some cases. The loading process was modelled fully drained as described in Section 5.2.

6.3 Interface parameters

The implementation of interface elements into ICFEP provides various material models for these type of elements. This section presents results from analyses in which the soil-structure interface was modelled elastically and elasto-plastically.

6.3.1 Elastic parameters

For the elastic model the interface normal stiffness $K_{n,if}$ and shear stiffness $K_{s,if}$ describe the material behaviour. In the following the index ‘if’ will be omitted for simplification when describing these two moduli. Interface strains and stresses are linked by

$$
\begin{bmatrix}
\Delta \tau_{if} \\
\Delta \sigma_{if}
\end{bmatrix} = 
\begin{bmatrix}
K_s & 0 \\
0 & K_n
\end{bmatrix}
\begin{bmatrix}
\Delta \gamma_{if} \\
\Delta \epsilon_{if}
\end{bmatrix}
$$

(6.1)
where $\tau_{if}$ and $\sigma_{if}$ are the shear and normal interface stress, respectively. It was pointed out in Section 3.2.4.4 that the interface shear strain $\gamma_{if}$ and normal strain $\epsilon_{if}$ are defined as the difference of displacement on opposite sides of the interface elements (shown in Figure 3.2, Page 89). Consequently, interface strain is not dimensionless but adopts the dimension [length]. As the stress has the dimension [force/length$^2$] it follows from the above equation that $K_s$ and $K_n$ have the dimension [force/length$^3$]. It is therefore not possible to assign the interface elements the same elastic parameters as the adjacent solid and beam elements which have elastic stiffness parameters of the dimension [force/length$^2$].

This section presents results from analyses in which both $K_n$ and $K_s$ were varied over a wide range. The studies performed in Sections 6.4 and 6.5 will focus on the interface characteristics which allow relative horizontal movement while restricting vertical interface strain. The interface shear stiffness $K_s$ is therefore of main interest. However, in order to choose a value of $K_n$ which can be adopted in all subsequent analyses the following subsection will investigate the influence of $K_n$ on the soil-structure interaction.

### 6.3.1.1 Normal interface stiffness $K_n$

The following results are from analyses in which the interface shear stiffness was assigned a low value of $K_s = 5$ kPa/m while $K_n$ was varied. The tunnel depth was $z_0 = 20$ m. Figure 6.1 shows the surface settlement troughs obtained from these plane strain analyses which modelled tunnel excavation beneath a 100 m wide building with a stiffness equivalent to 5 storeys. No building load was applied. Values of $K_n$ varied between $10^3$ kPa/m and $10^7$ kPa/m. The settlement profile of a corresponding building without interface elements is also presented together with the settlement trough obtained from a greenfield analysis.

The graph shows that a low value of $K_n = 10^3$ kPa/m compensates the tunnel induced settlement leading to a wide and shallow settlement trough. This effect reduces as $K_n$ increases to $10^5$ kPa/m. The settlement curves for $K_n = 10^5$ kPa/m to $10^7$ kPa/m coincide, indicating that an increase above $K_n = 10^5$ kPa/m does not change the settlement profile. The settlement troughs obtained for high values of $K_n$ are, however, slightly wider than the one calculated for a building without interface elements. This shows, that the low value of $K_s$, adopted in the above interface analyses, has an influence on the settlement trough.

The normal strains $\epsilon_{if}$ obtained in the interface elements during these analyses are shown
in Figure 6.2a. Normal strains reduce as $K_n$ increases. Increasing $K_n$ from $10^3$ kPa/m to $10^4$ kPa/m reduces the strain on the building centre line from $\epsilon_{if} = 1.77 \times 10^{-3}$ m to $4.79 \times 10^{-4}$ m (27% of the original value). A further increase of $K_n$ by an order of magnitude reduces $\epsilon_{if}$ to $5.86 \times 10^{-5}$ m. This is approximately one order of magnitude less than the previous $\epsilon_{if}$. For the increase from $K_n = 10^6$ kPa/m to $10^7$ kPa/m, $\epsilon_{if}$ reduces by one order of magnitude from $6.00 \times 10^{-6}$ m to $6.02 \times 10^{-7}$ m.

All distributions of $\epsilon_{if}$ along the building have the same pattern (although the $\epsilon_{if}$-variations for the $K_n = 10^6$ kPa/m and $10^7$ kPa/m cases are not visible in the scale of this graph). It can be seen that tensile normal strains develop around the centre line of the building while compression can be found towards the edge of the structure. The position where the normal
interface strain changes from tension to compression lies between 19.8m (for $K_n = 10^3$ kPa/m) and 16.1m (for $K_n = 10^7$ kPa/m) from the centre line of the building. Comparing these values with the settlement results (Figure 6.1) reveals that the change from tension to compression occurs only slightly outside the point where building settlement has the same magnitude as the corresponding greenfield settlement (approximately 16m). Consequently, tension develops where the building settles less than the greenfield site while compression is obtained where the building shows larger settlement than greenfield conditions.

Figure 6.2b shows how the variation of $K_n$ affects the development of interface shear strain $\gamma_{if}$ along the building. As pointed out above, $K_s$ was assigned a low value, hence allowing the development of relative horizontal displacement between soil and structure. The figure demonstrates that $\gamma_{if}$ is only slightly influenced by the change of $K_n$ over 5 orders of magnitude. As $K_n$ increases from $K_n = 10^3$ kPa/m to $10^7$ kPa/m, the maximum value of $\gamma_{if}$ shows an increase of 16% (from $2.67 \times 10^{-3}$ m to $3.09 \times 10^{-3}$ m). Further increase of $K_n$ does not increase $\gamma_{if}$.

As the horizontal displacement between soil and structure is of main interest a low value for $K_s$ will be adopted while $K_n$ has to be assigned to a high value. Figures 6.1 and 6.2b indicate that any value of $K_n > 10^5$ does not change the behaviour of soil and structure. One could argue that a higher value of $K_n$ should be adopted in order to be consistent with the results of previous chapters where the building was rigidly connected to the soil. However, a high interface stiffness can result in numerical instability. This effect is shown in Figure 6.3a which re-plots $\epsilon_{if}$ for $K_n = 10^7$ kPa/m from Figure 6.2a. Due to the small scale on the vertical axis it can be seen that the distribution of $\epsilon_{if}$ over the width of the building has a similar pattern as those observed in Figure 6.2a for lower values of $K_n$. However, at a distance of approximately 25m from the tunnel centre line the graph starts to exhibit oscillation which increases towards the building edge. Potts & Zdravković (2001) showed that such behaviour can result due to the use of elements which are too large to model the steep stress gradient that occurs when interface elements of high stiffness, compared with the adjacent beam and/or solid elements are used. They showed that by refining the mesh better results can be obtained. The analysis was therefore repeated with the refined meshes, shown in Figures 6.4b and 6.5b and referred to as ‘mesh 2’ and ‘mesh 3’, respectively.
Chapter 6. The influence of the soil-structure interface

Section 6.3

\[ K_s = 5 \text{kPa/m} \]
\[ K_n = 10^7 \text{kPa/m} \]

Figure 6.3: (a) Distribution of \( \epsilon_{if} \) along building for a high value of \( K_n \) using the original mesh (b). Maximum length of interface elements: 2m.

Figure 6.4: (a) Distribution of \( \epsilon_{if} \) along building for a high value of \( K_n \) using refined ‘mesh 2’ (b). Maximum length of interface elements: 1m.

Figure 6.5: (a) Distribution of \( \epsilon_{if} \) along building for a high value of \( K_n \) using refined ‘mesh 3’ (b). Maximum length of interface elements: 0.5m.
Figures 6.4a and 6.5a present the $\epsilon_{if}$-distributions from these analyses. The graphs show that finer meshes lead to a smaller zone of oscillation at the building edge. For mesh 2 oscillation starts at a distance of approximately 30m from the tunnel centre line while for mesh 3 no oscillation occurs before 45m. To compare results between the different meshes, Figure 6.5 superimposes the $\epsilon_{if}$ curve from mesh 1 (shown in Figure 6.3) over the graph for mesh 3. The plot shows that both meshes give the same results until the curve for mesh 1 starts to oscillate around the curve for mesh 3. Towards the building edge, the curve for mesh 3 shows oscillation with increasing amplitude towards the building edge. At the edge the amplitude of $\epsilon_{if}$ for mesh 3 is larger than that of mesh 1 leading to a peak tensile strain.

It should be noted that the oscillation seen in the previous figures is relatively minor when compared with the results presented in Figure 6.2a (noting the different scales on the vertical axis between the figures). The numerical instabilities do not therefore affect the settlement and horizontal shear strain results plotted in Figures 6.1 and 6.2b, respectively. However, when incorporating plastic behaviour for the interface elements, the ‘tension peak’ at the edge of the building can generate plastic strain. These instabilities should therefore be reduced.

In order to be consistent with previous chapters it was decided to perform all further analyses presented in this chapter with the original mesh (mesh 1). To keep the oscillation to a minimum, a value of $K_n = 10^5$ kPa/m was adopted throughout all of the following analyses.

6.3.1.2 Shear interface stiffness $K_s$

Figure 6.6 shows the interface shear strain $\gamma_{if}$ along the building obtained from a suite of analyses with varying $K_s$. A value of $K_n = 10^5$ kPa/m was adopted. The building included into the analyses was the same 5-storey structure as used in the previous analyses. It can be seen that increasing $K_s$ from 5 kPa/m to $10^5$ kPa reduces $\gamma_{if}$. As for the reduction of $\epsilon_{if}$ with increasing $K_n$, the decrease of $\gamma_{if}$ with increasing $K_s$ becomes more severe for high values of $K_s$: increasing $K_s$ from $10^4$ to $10^5$ reduces the maximum value of $\gamma_{if}$ by approximately one order of magnitude (from $4.24 \times 10^{-5}$ m to $4.31 \times 10^{-6}$ m). The increase from $K_s = 10$ kPa/m to 100 kPa/m only reduces $\gamma_{if}$ by 4% (from $3.08 \times 10^{-3}$ m to $2.95 \times 10^{-3}$ m). The results for $K_s = 5$ kPa/m coincide with the curve for $K_s = 10$ kPa indicating that $\gamma_{if}$ does not increase when $K_s$ reduces below 10 kPa/m.
6.3.2 Plastic parameters

ICFEP provides the possibility to analyse the interface behaviour with an elasto-plastic material model. This model adopts a Mohr-Coulomb failure criterion. Consequently, the input parameters required are the cohesion $c_{if}$, angle of friction $\varphi_{if}$ and the angle of dilation $\nu_{if}$. This section presents results from studies in which $c_{if}$ and $\varphi_{if}$ were varied separately. The dilation was set to $\nu_{if}=0^\circ$ in all analyses.

Figure 6.7 shows the interface shear strain $\gamma_{if}$ obtained from analyses with $\varphi_{if}=0^\circ$ and $c_{if}$ varying between 0kPa and 7.5kPa\(^1\). The elastic parameters were set to $K_n=10^5$kPa/m and $K_s=10^4$kPa/m. A 100m wide 5-storey structure with no building load was analysed.

The graph demonstrates that $\gamma_{if}$ increases as $c_{if}$ reduces. The lowest shear strain curves are obtained for $c_{if}=5.0$kPa and $7.5$kPa. Both curves coincide and are in good agreement with the strain distribution found in the elastic analysis, shown in the previous plot with $K_s=10^4$kPa/m. The fact that the two curves for different values of $c_{if}$ show the same magnitude of strain and coincide with the results of the elastic analysis with same elastic stiffness parameters, indicates that no plastic strain develops for cases of $c_{if} \geq 5.0$kPa.

\(^1\)Note that instead of $c_{if}=0$kPa and $\varphi_{if}=0^\circ$ a low value of $c_{if}=10^{-3}$kPa and $\varphi_{if}=(10^{-3})^\circ$ was adopted in the FE analyses.
Plastic strain, however, develops as $c_{\text{if}}$ reduces to 2.5 kPa with a zone of plastic strain approximately between 4 m and 30 m distance from the tunnel centre line. As $c_{\text{if}}$ decreases further to $c_{\text{if}} = 0$ kPa plastic strain develops along the whole building width. The results for the $c_{\text{if}} = 0$ kPa case are similar in pattern and magnitude to those obtained for $K_n = 5$ kPa in the elastic analyses, shown in Figure 6.6.

In the next set of analyses, $c_{\text{if}}$ was kept constant at 2.5 kPa while $\varphi_{\text{if}}$ was increased from 0° to 25° (the latter value is equivalent of the angle of shearing resistance $\varphi'$ adopted in the non-linear elasto-plastic soil model). Figure 6.8a shows the magnitude of $\gamma_{\text{if}}$ along the building. One would expect that an increase in $\varphi_{\text{if}}$ leads to a reduction in plastic strain. However, the opposite trend occurs. As $\varphi_{\text{if}}$ increases in increments of 6.25° the peak interface shear strain increases by increments between $2 \times 10^{-4}$m ($\varphi_{\text{if}} = 0°$ to 6.25°) and $3 \times 10^{-4}$m ($\varphi_{\text{if}} = 18.75°$ to 25.0°). During this increase the peak strain occurs closer towards the tunnel centre line (from 14 m, for $\varphi_{\text{if}} = 0°$ to 11 m distance for $\varphi_{\text{if}} = 25°$). In contrast, strain reduces with increasing $\varphi_{\text{if}}$ between a distance of $x = 20$ m and 30 m.

To investigate the influence of $\varphi_{\text{if}}$ further, additional analyses were performed with a load of 10 kPa being applied to the building. The results from these analyses, presented in Figure 6.8b, show that the $\varphi_{\text{if}} = 18.75°$ and $25.0°$ cases give identical curves, hence no plastic strain develops. Small amounts of plastic strain occur for $\varphi_{\text{if}} = 12.5°$ between approximately

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**Figure 6.8:** Distribution of shear interface strain $\gamma_{\text{if}}$ along the building for different values of $\varphi_{\text{if}}$ for (a) no-load case and for (b) building load of 10 kPa.
Section 6.3

$x = 6m$ and $12m$. This zone increases as $\varphi_{if}$ decreases until plastic strain develops over the full building width for $\varphi_{if} = 0^\circ$. This behaviour is opposite to the trend found for the no-load case shown in Figure 6.8a where the $\varphi_{if} = 0^\circ$ case showed less plastic strain than the situations with an increased value of $\varphi_{if}$.

The reason for this behaviour can be explained when plotting the stress paths developing in the interface during tunnel excavation. Such a diagram is given in Figure 6.9 for the no-load scenario. The horizontal axis represents interface normal stress while the vertical axis denotes interface shear stress. Two suites of stress paths are given for points at $x = 10m$ and $25m$ distance from the tunnel centre line. Each data point represents the result of one increment of the FE analysis.

For each position, stress paths are given for an elastic analysis with $K_n = 10^5$kPa/m and $K_s = 10^4$kPa/m and for elasto-plastic analyses with the same elastic parameters, a cohesion of $c_{if} = 2.5kPa$ and $\varphi_{if} = 0^\circ$, $12.5^\circ$ and $25^\circ$. The corresponding failure criteria are also plotted.

The stress paths for $x = 10m$ show the development of tensile normal stress, as expected from Figure 6.2a where tensile strain was found over approximately 20m distance from the tunnel centre line. For the elasto-plastic case with $\varphi_{if} = 0^\circ$ plastic strain occurs as the path reaches the $c_{if} = 2.5kPa$ failure criterion in the 6th excavation increment. Before this increment the results coincide with the elastic stress path. As $\varphi_{if}$ increases plastic strain develops at earlier stages of the analysis. For $\varphi_{if} = 12.5^\circ$ plastic strain occurs from the 5th increment while the $\varphi_{if} = 25.0^\circ$ case exhibits plastic strain already in the 4th excavation increment. The fact that with increasing $\varphi_{if}$ the failure criterion is reached in earlier stages of the analysis explains the behaviour found in Figure 6.8a. Normal tensile interface strain and stress develops over 20m distance from the tunnel centre line. In this zone an increase in $\varphi_{if}$ reduces the distance a stress path has to travel until it reaches the failure criterion. In the compressive zone, starting approximately beyond $x = 20m$, the opposite trend can be found. The $x = 25m$ stress paths in Figure 6.9 illustrate this behaviour. The stress path of the elasto-plastic analysis with $\varphi_{if} = 0^\circ$ has the shortest distance to the failure criterion (although it reaches it only in the last increment) followed by the $\varphi_{if} = 12.5^\circ$ case (which reaches the failure line just at the end of the analysis). The $\varphi_{if} = 25.0^\circ$ case does not reach the failure criterion and, consequently, no plastic strain develops.
Chapter 6. The influence of the soil-structure interface

Section 6.3

Figure 6.9: Stress paths of interface elements at different position for elastic and elasto-plastic analyses of non-load cases.

Figure 6.10: Stress paths of interface elements at different position for elastic analysis of 10kPa load cases.

Figure 6.11: Stress paths of interface element at $x = 0$m for elastic and elasto-plastic analyses of no-load cases.
Chapter 6. The influence of the soil-structure interface

Section 6.3

The application of building load shifts the initial stress conditions prior to tunnel construction into the compressive normal stress regime. This situation is shown in Figure 6.10 which plots stress paths due to tunnel construction for a 5-storey building, subjected to a load of 10kPa. The building loading process is not included in the diagram. Three paths are given for a position of \( x = 0 \) m, 10m and 25m from the tunnel centre line. For simplification only results from elastic analyses are presented. The \( x = 10 \) m and 25m stress paths show similar shapes as observed for the no-load case in the previous figure. For \( x = 10 \) m the stress path moves to the left hand side, showing a reduction in compressive strain (in terms of absolute value). However, no tensile strain develops as the initial stress situation is far enough in the compressive stress regime. Even on the tunnel centre line (\( x = 0 \) m) where Figure 6.2a showed the maximum tensile normal interface strain in the no-load case, the stress state remains compressive. In this stress regime an increase in \( \varphi_{if} \) leads to a larger distance the stress path has to travel before reaching the failure criterion. This explains the behaviour found in Figure 6.8b were plastic strain decreased as \( \varphi_{if} \) was increased.

For reasons of clarity Figure 6.9 did not include stress paths for \( x = 0 \) m. Such paths are presented in Figure 6.11 for the weightless 5-storey building. The horizontal axis only shows tensile normal interface stress and only stress paths for elastic and for plastic parameters with \( \varphi_{if} = 25^\circ \) are given. The graph shows that the stress path of the elastic analysis does not reach the yield surface for \( \varphi_{if} = 12.5^\circ \) and, consequently, no plastic strain would develop if plastic interface properties with such an angle of friction were adopted. As pointed out above, increasing \( \varphi_{if} \) reduces the way the stress paths have to travel to reach the yield surface. This can be seen for the \( \varphi_{if} = 25^\circ \) path which reaches the corresponding yield surface in the fifth increment. As the tensile stress increases the path travels down along the yield surface until it reaches the tip of the cone described by the yield surface in the sixth increment. The normal interface stress at this point has the value \( c_{if}/\tan(\varphi_{if}) \) and it is not possible to exceed this magnitude. Instead the joint between structure and soil opens while the normal interface stress is maintained and the shear stress remains zero. Strains occurring while the joint is open are accumulated as plastic strains.

To investigate the behaviour of a building which partly separates from the soil, an additional analysis was performed with a cohesion of \( c_{if} = 1.25 \) while \( \varphi_{if} \) was kept to \( 25^\circ \). No building load was applied to the 5-storey building. Figure 6.12 shows the distribution of nor-
Figure 6.12: Distribution of interface normal strain $\varepsilon_{if}$ along the building analyses with different degree of separation between soil and building.

Figure 6.13: Distribution of interface shear strain $\gamma_{if}$ along the building analyses with different degree of separation between soil and building.

normal interface strain $\varepsilon_{if}$ induced by tunnel construction. As in previous plots the horizontal axis denotes the distance from the tunnel and building centre line. The graph includes results from the elasto-plastic interface analyses together with a curve from an elastic interface analysis with the same elastic parameters as adopted in the elasto-plastic calculation. For the plastic analyses it can be seen that, as $\varepsilon_{if}$ reduces, the normal interface strain increases towards the centre line. This effect is due to opening of the joint between building and soil. Outside this zone all three analyses give similar values of $\varepsilon_{if}$. Figure 6.13 presents a similar graph for the interface shear strain of the three analyses. Comparing the curves for the elasto-plastic interface properties with those of elastic parameters shows that the zone of plastic strain increases with reducing $c_{if}$ (as already seen in Figure 6.7). Comparison of Figures 6.12 and 6.13 demonstrates that the plastic zone is larger than the zone in which a gap between soil and foundation develops.

Figure 6.14 presents the tunnel induced building settlement obtained from the three analyses included in the two previous figures. It can be seen that the settlement trough of the building becomes slightly flatter when opening of a gap between building and soil is modelled.

It was noted earlier that application of building weight shifts the stress regime at the soil-building interface towards a compressive stress state. Figure 6.10 revealed that a stress
path for $x = 0\, \text{m}$ does not show any tensile stress when a building load of 10kPa is introduced. In such a case, no gap beneath the foundation develops. An open joint between building and soil therefore is only a result of assuming no building load - clearly an unrealistic scenario. The following section will therefore concentrate on situations where horizontal movements between building and soil are allowed while normal relative movements are restricted.

6.3.3 Choice of parameters

The purpose of the following sections is to investigate the ground and building deformation when relative horizontal movements between soil and building foundations are allowed while relative vertical movement is restricted. To simulate this extreme situation either an elastic interface model with a low value of $K_s$ and a high value of $K_n$ or an elasto-plastic model with low values of $c_{if}$ and $\phi_{if}$ and a small angle of dilation $\nu_{if}$ can be adopted. The analyses presented previously included these extreme scenarios when varying the different elastic and plastic parameters separately. Figure 6.15 compares the interface shear strain obtained from an elastic and a plastic analysis with following interface properties:

- **Elastic**: $K_n = 10^5\, \text{kPa/m}$, $K_s = 5\, \text{kPa/m}$.
- **Elasto-plastic**: $K_n = 10^5\, \text{kPa/m}$, $K_s = 10^4\, \text{kPa/m}$, $c_{if} = 0\, \text{kPa}$, $\phi_{if} = 0^\circ$.

The graph shows that the elasto-plastic model yielded slightly higher interface shear strain $\gamma_{if}$ with a peak value of $2.9 \times 10^{-3}\, \text{m}$ compared to $2.8 \times 10^{-3}\, \text{m}$ (a difference of approximately
Chapter 6. The influence of the soil-structure interface

Section 6.4

As both analyses model an extreme scenario the small difference between both curves is negligible. Both analyses essentially describe the same behaviour at the soil-structure interface.

The settlement of the surface structure for both analyses together with that from the no-interface case is shown in Figure 6.16. Similar to Figure 6.1 the settlement troughs from both the analyses with interface elements are wider and shallower than that obtained from the no-interface scenario. The elasto-plastic model produces the widest trough while the curve of the elastic analysis lies between the elasto-plastic and the no-interface case but remains closer to the elasto-plastic analysis.

Clearly, both elastic and elasto-plastic interface properties are capable to describe a low shear friction between soil and building. For its simplicity the elastic model will be used when investigating the influence of interface behaviour on ground and building deformation in the following sections.

6.4 Ground deformation

This section investigates how the presence of a soil-structure interface with low shearing stiffness but high normal stiffness affects the soil movement beneath the building.
Figure 6.17 presents a vertical profile of horizontal displacement. The profile is at a distance of \( x = 6 \text{m} \) from the tunnel centre line. The tunnel depth is \( z_0 = 20 \text{m} \). Negative displacement indicates movement towards the tunnel. Results are given for a 5-storey building subjected to either no load or a load of 50kPa. The elastic parameters of the interface elements (referred to as ‘IF’) were \( K_n = 10^5 \text{kPa/m} \) and \( K_s = 5 \text{kPa/m} \). Both results are compared with corresponding scenarios without interface elements and with the greenfield case.

The no-interface cases show the typical pattern of such a horizontal displacement profile which was shown in Figure 5.2 (Page 152). The maximum horizontal soil displacement can be found at approximately tunnel axis depth with the peak horizontal displacement reducing as load is applied. Below a depth of approximately \( z = 10 \text{m} \) the interface-cases coincide well with these no-interface results. Close to the surface structure, however, they develop larger displacement than found in the no-interface analyses.

The horizontal displacement found at the ground surface for the interface case (with no load) is \( S_{hx} = 2.1 \times 10^{-3} \text{m} \). In the case when the soil is rigidly connected to the building, this value reduced to \( S_{hx} = 3.4 \times 10^{-5} \text{m} \) (for the no-interface, no-load case) showing that the surface structure restricts horizontal soil movement in this zone. Due to the low interface shear stiffness, such a restriction is not given for the interface-scenario. The horizontal surface soil-displacement for these cases is approximately of the same magnitude as \( S_{hx} \) obtained from a greenfield analysis (\( S_{hx} = 1.8 \times 10^{-3} \text{m} \)).

This graph reveals that the restrictions, a surface building with rigid soil-structure connection imposes on the horizontal soil movement, only affects soil displacement to a certain depth. In this particular case, no influence of interface elements can be seen below a depth of approximately \( z = 10 \text{m} \).

The greenfield results, in contrast, show a similar horizontal surface movement as the interface analyses but then give slightly lower displacement values until a depth of approximately \( z = 15 \text{m} \). As interface and greenfield cases exhibit similar horizontal displacement behaviour at the ground surface this difference must be due to their different vertical settlement profiles. It will be shown later, that a soil-structure interface with a low shear stiffness has only a small effect on the vertical settlement of a building. The fact that the horizontal displacement results of buildings with same load conditions (i.e. load or no-load) but different interface situation (i.e. IF or no IF) coincide from a depth of \( z = 10 \text{m} \) but the results

181
of buildings with interface and the greenfield case do not coincide in this zone, shows, that vertical surface settlement influences horizontal soil movement up to greater depth.

At a tunnel depth of \(z_0 = 20\) m, greenfield and both no-load cases coincide. In Chapter 5 it was concluded that the displacement field in the vicinity of the tunnel is governed by the stress state (mainly by the ratio \(\sigma'_h/\sigma'_v\)). The results shown in Figure 6.17 support this conclusion showing that the displacement boundary, imposed by the surface structure, has no influence on the horizontal soil displacement in this zone.

Figure 6.18 shows horizontal profiles of horizontal displacement from the same analyses included in the previous graph. Profiles are given for the ground surface (a) and for a depth of \(z = 14\) m (b). The ground surface plot demonstrates how the presence of a structure rigidly fixed to the soil reduces the horizontal surface movement (in soil and structure) drastically compared to the greenfield situation. Such a decrease in soil movement cannot be found for the interface analyses. In contrast, they show slightly higher soil movement than the greenfield case. The application of building load to the interface-scenario reduces the horizontal surface soil movement slightly. This effect is due to the increased soil stiffness in the top soil layer resulting from the increase in mean effective stress \(p'\), as discussed in Section 5.4.2.

The horizontal displacements obtained in the buildings with interfaces are not included in
Figure 6.18: Horizontal profile of horizontal displacement below 100m-wide 5-storey buildings with and without interface elements. (a) at surface, (b) at 14m depth.

this graph. For load and no-load cases their maxima have a magnitude of $S_{hx,max} = -4.8 \times 10^{-7} m$ and $-4.5 \times 10^{-7} m$, respectively, which is more than 2 orders of magnitude smaller than the maximum horizontal displacement calculated for the no-interface cases ($S_{hx,max} = -1.2 \times 10^{-4} m$ and $-1.6 \times 10^{-4} m$).

For a depth of $z = 14m$ both interface and no-interface cases are in good agreement. The greenfield case shows slightly smaller values of horizontal displacement supporting the above conclusion that the difference in vertical surface settlement boundaries affects soil movement to a greater depth than different horizontal displacement boundary conditions do.

Figure 6.19 shows similar horizontal profiles of vertical ground settlement. Compared to the greenfield settlement trough all four building cases included in this graph show a similar settlement response to tunnel excavation (Figure 6.19a). This indicates that the vertical soil displacement is practically independent of changes in the horizontal displacement boundary condition. For a depth of $z_0 = 14m$ (Figure 6.19b) all 5 curves show a similar
response. It should be noted that the results are not adjusted to a common volume loss. The interface analyses show lower volume losses (1.44% and 1.37% for the no-load and load cases, respectively) than the no-interface cases (1.47% and 1.41%).

When discussing the horizontal displacement results presented in the previous figures it was concluded that changes in vertical ground surface settlement affect the horizontal soil movement to a greater depth than changes in the horizontal soil movement at the ground surface do. To investigate how changes in the horizontal ground surface displacement boundary conditions influence vertical movement, the positions of the point of inflection, $i$, for surface and subsurface settlement troughs are plotted against depth in Figure 6.20.

Results of analyses with weightless 1-, 3-, 5-, and 10-storey buildings (100m wide) with interface elements are included in the graph together with the corresponding greenfield case and a 5-storey building without interface elements (taken from Figure 4.10, Page 113).

The graph shows that the width of surface settlement troughs increases with building
stiffness as $i$ increases with the number of storeys. A similar trend was found for buildings without interface elements (Figure 4.10). Comparing the curves for the 5-storey building with and without interfaces reveals that $i$ at the surface is only marginally affected by the reduction of shear stiffness in the soil-structure interface. However, as $z$ increases the two curves diverge with the no-interface case showing higher values of $i$ until a depth of approximately $z = 9m$. The curves of all interface cases reduce over the top 2m of soil. From this depth on their values of $i$ are in good agreement with results obtained from the greenfield analysis.

These results indicate that a change in the horizontal displacement boundary conditions at the ground surface has only a small effect on the width of surface settlement troughs while it has a significant influence on the width of subsurface settlement troughs. In contrast, the different vertical displacements imposed by different building stiffnesses have only a small effect on the width of subsurface settlement troughs.

### 6.5 Building deformation

The previous section showed how a soil-structure interface with a low shear stiffness influences tunnel induced ground movement below a building. This section will focus on the deformation of the building itself.
6.5.1 Deflection ratio

A 100m wide 5-storey building above a $z_0 = 20$m deep tunnel was analysed with a range of different values of interface shear stiffness $K_s$. The interface normal stiffness was $K_n = 10^5$kPa/m. Buildings without and with load (50kPa) were included in this study and the deformation criteria for all cases were calculated. Figure 6.21a shows the results for $D\!R_{\text{sag}}$. The sagging deflection ratio $D\!R_{\text{sag},\text{if}}$ for each $K_s$-case was normalized against $D\!R_{\text{sag},\text{no if}}$ obtained from a similar analysis without interface elements. These normalized values are plotted against $K_s$ using a log scale. It can be seen that cases with no load and with load give a similar response. For $K_s = 10^6$kPa/m the ratio of $D\!R_{\text{sag},\text{if}}/D\!R_{\text{sag},\text{no if}}$ is close to unity, showing that a high interface shear stiffness gives a similar sagging deflection ratio as a building without interface elements. As $K_s$ decreases $D\!R_{\text{sag},\text{if}}/D\!R_{\text{sag},\text{no if}}$ (and therefore $D\!R_{\text{sag},\text{if}}$) reduces until reaching a ratio of approximately 0.8 for low values of $K_s < 100$kPa/m.

Figure 6.21b shows a corresponding plot for $D\!R_{\text{hog}}$. Both, no-load and load cases, again, show similar curves. For high values of $K_s$ the ratio of $D\!R_{\text{hog},\text{if}}/D\!R_{\text{hog},\text{no if}}$ is approximately 0.8. The ratio then reduces until reaching a level of approximately 0.6 for low values of $K_s$.

When discussing the surface settlement profiles in Figure 6.19a it was concluded that the settlement trough is little affected by the introduction of interface elements. The study presented in Figure 6.21 gives a more detailed picture. It confirms that the change in $D\!R_{\text{sag}}$
and \( DR_{\text{hog}} \) is small considering the wide range of \( K_s \)-values included into the analysis. Furthermore, the graphs show that not including interface elements is on the conservative side as \( DR_{\text{sag}} \) and \( DR_{\text{hog}} \) reduce with decreasing \( K_s \).

To include more building cases into this parametric study, analyses with a low interface shear stiffness of \( K_s = 5 \text{kPa/m} \) were performed for different building stiffnesses of 100m wide buildings above 20m and 34m deep tunnels. For all stiffness cases no-load and load scenarios were considered. Loads of 10kPa, 30kPa, 50kPa and 100kPa were applied to 1-, 3-, 5-, and 10-storey buildings respectively (these stiffness-load combination were referred to as ‘diagonal’ cases in Chapter 5, compare with Figure 5.1 on Page 150). Additional analyses were carried out with an eccentricity of 20m with respect to the tunnel centre line. For all building scenarios, modification factors \( M_{\text{if}}^{DR} \) were calculated. These modification factors were then normalized against the corresponding modification factors \( M_{\text{no-if}}^{DR} \) from a no-interface analysis. It has to be noted that the same value of greenfield deflection ratio \( DR_{\text{GF}} \) was used to calculate modification factors of interface and no-interface cases. The ratios of \( M_{\text{if}}^{DR}/M_{\text{no-if}}^{DR} \) are therefore equivalent to \( DR_{\text{if}}/DR_{\text{no-if}} \) for \( K_s = 5 \text{kPa/m} \) presented in the previous graphs.

Figure 6.22a presents these results for \( M_{\text{DR-sag}}^{\text{if}} \) by plotting the ratios of modification factors against number of storeys. All curves in this graph lie close together regardless of the different geometries and load scenarios. The modification factors from interface cases are between 75% and 82% of the modification factors of corresponding no-interface analyses. For

![Figure 6.22](image-url)

**Figure 6.22:** Comparison of (a) \( M_{\text{DR-sag}}^{\text{if}} \) and (b) \( M_{\text{DR-hog}}^{\text{if}} \) of interface and non-interface analyses for different geometries and building stiffnesses. Interface modification factors are normalized against corresponding non-interface results.
Figure 6.23: Modification factors (no eccentricity) $M^{DR}$ together with the design curves by Potts & Addenbrooke (1997).

Figure 6.24: Modification factors (building eccentricity) $M^{DR}$ together with the design curves by Potts & Addenbrooke (1997).

$M^{DR_{hog}}$, shown in Figure 6.22b the results do not lie in such a narrow range but the ratios of modification factors are all below 80% with a minimum of 55%. The scatter is typical for the hogging results as the calculation of $DR_{hog}$ is very sensitive to the determination of the point of inflection, as described in Section 3.4.5. This explanation is supported by the fact that the eccentric building cases with their larger hogging zone show less scatter than the concentric cases.

Figures 6.23 and 6.24 plot the modification factors from these analyses onto the design charts proposed by Potts & Addenbrooke (1997). Only no-load cases are included in these graphs as the previous figures showed that both load and no-load results lied close together. Figure 6.23 shows $M^{DR}$ from the concentric analyses plotted against relative bending stiffness $\rho^*$. The design curves which are upper bounds to the study performed by Potts & Addenbrooke (1997) are also given. The square symbols mark the interface cases while triangle
symbols represent results from the no-interface scenarios. For both sagging and hogging the graphs show that low interface shear stiffness moves the data points further away from the design curves. Similar behaviour can be found for the eccentric cases plotted in Figure 6.24. For hogging (lower graph) some no-interface data points lie outside the design curve. Introducing interface elements moves these results closer to or even inside the curve.

The interface results in the above graphs were based on an extremely low value of $K_s = 5\text{kPa/m}$. Therefore the above parametric study essentially models free horizontal relative movement between soil and building foundation. For this extreme situation the study revealed that modification factors reduce to values between 82% and 55% compared to analyses were the building was rigidly connected with the ground. Results of intermediate $K_s$ values lie between these two extreme situations.

### 6.5.2 Horizontal strain

To investigate the horizontal strain in the building a 100m wide 5-storey structure with an eccentricity of 20m was analysed. Eccentric situations were chosen in order to obtain tensile strains within the structure. However, when analyses were performed it was found that with $K_s$ reducing from $10^6\text{kPa/m}$ to $10^4\text{kPa/m}$ tensile stresses within the structure vanished. Therefore only compressive strain results are presented in Figure 6.25. The graph is similar to Figure 6.21 with the interface shear stiffness on the horizontal axis and each maximum horizontal compressive strain $\epsilon_{hc,if}$ normalized against $\epsilon_{hc,no if}$ on the vertical axis. The curves for load and no load coincide. Both curves reduce drastically as $K_s$ decrease from $10^4\text{kPa/m}$ to $100\text{kPa/m}$. For a stiffness of $10\text{kPa/m}$ $\epsilon_{hc,if}$ has reduced to less than 1% of $\epsilon_{hc,no if}$ showing

![Figure 6.25](image-url)

Figure 6.25: Variation of $\epsilon_{hc,if}$ with $K_s$. Results are normalized against $\epsilon_{hc,no if}$ of corresponding non-interface analysis.
that horizontal strain vanishes within the structure when an extremely low interface shear stiffness is chosen.

A similar behaviour has been found when including other geometries into the study. For the extreme value of $K_s = 5 \text{kPa/m}$ 100m wide buildings with no eccentricity and with an eccentricity of 20m were analysed for a 20m and 34m deep tunnel. When normalizing the modification factors $M^{\text{fib}}$ against the corresponding $M^{\text{fib, nofi}}$ all ratios were below 1%.

In previous chapters it was shown that strain modification factors $M^{\text{e}}$ have generally smaller values than corresponding deflection ratio modification factors. Chapter 4, however, concluded that $M^{\text{e}}$-results from some building cases might lie outside the design curves proposed by Potts & Addenbrooke (1997). Figure 4.46 (Page 146) showed such a situation. The study presented in this section reveals that these modification factors can reduce significantly if relative movement occurs on the soil-structure boundary. This mechanism would shift the modification factors closer or even inside the design curves.

### 6.6 Conclusions

This chapter studied the effect of the nature of the contact between the building foundations and the soil on tunnel induced ground subsidence and building deformation. Interface elements were included between solid elements representing the soil and beam elements which model the building.

A set of 2D plane strain analyses was carried out to choose appropriate interface properties which allow relative horizontal movement between soil and building while restricting movements normal to the interface. Adopting an elastic model, the normal and the shear interface stiffness, $K_n$ and $K_s$, respectively, were varied. It was shown that an increase of $K_n$ above a certain value does not further influence the interface shear strain but can lead to numerical instabilities. A value of $K_s$ was established below which the interface shear strain does not increase further. Such a situation represents practically free horizontal movement between soil and building.

Similar situations were achieved by using an elasto-plastic interface model with the cohesion, the angle of friction and the angle of dilation being close to zero. For cases in which the cohesion was assigned to a value of $c_{\text{eff}} = 2.5 \text{kPa}$ it was found that an increase in the angle of friction, $\varphi_{\text{eff}}$, from $0^\circ$ to $25^\circ$ increased shear strain between soil and structure. The reason for
this behaviour was found in the tensile normal interface stress state which established during
tunnel construction between soil and structure. This tensile zone was found to be around the
tunnel centre line, extending approximately to a distance of $x = 20\text{m}$ from the tunnel and
building centre line for the concentric building geometry modelled.

An elastic interface model with a low value of $K_s$ was then chosen to investigate the
effects of this low interface shear stiffness on ground and building deformation. It was found
that horizontal displacement of the building reduces drastically and that, consequently, the
horizontal strain in the structure nearly vanishes. In contrast, horizontal soil movement just
beneath the building increases, leading to higher values than obtained in greenfield situations.
Below a depth of approximately $z = 10\text{m}$ the results from buildings with interface elements
coincided with data from conventional no-interface studies. This shows that the influence
of the horizontal displacement boundary condition at the surface influences horizontal soil
movement only to a certain depth. Furthermore, it was found that different vertical displace-
ment boundary conditions at the surface (as given between greenfield and building scenarios)
affect horizontal soil movement to greater values of $z$.

In contrast it was shown that the nature of the horizontal displacement boundary condi-
tions at the ground surface affects the width of settlement subsurface troughs to a deeper level
than changes in the vertical displacement boundary conditions at the surface do. The width
of the surface settlement trough of the building was not influenced by the use of interface
elements.

A parametric study with different building geometries and stiffnesses was performed to
quantify the effect of interfaces with low shear stiffness on the deflection ratio and corre-
sponding modification factors. It was shown that $DR_{sag}$ and $DR_{hog}$ reduce with decreasing
$K_s$. For low values of $K_s$, $DR_{sag}$ was approximately 20\% lower than the corresponding no-
interface case. The deflection ratio for hogging was reduced by 20 - 45\% when interface
elements with low shear stiffness were introduced. This reduction can also be seen when
plotting the corresponding modification factors against relative bending stiffness $\rho^*$. All data
points from analyses with interface elements lie below the results of buildings which were
rigidly fixed to the ground. Together with the vast reduction of horizontal strain within the
building, the results show that analyses which consider full friction between soil and building
are conservative.
Chapter 7

The influence of out of plane geometry

7.1 Introduction

This chapter presents a suite of 3D FE analyses, which investigate how a building’s behaviour changes when an out of plane dimension is included in the analysis. The geometry of the surface structure is described by its width $B$ (as used in the previous chapters) and by its out of plane length $L$, longitudinal to the tunnel axis. The purpose of this chapter is to examine the influence of the dimension $L$ on the transverse deformation behaviour of the building. The tunnel is excavated simultaneously over the whole length of the FE mesh, effectively representing a plane strain tunnel construction beneath a 3D structure. Firstly, this chapter demonstrates how the building deformation varies along its length $L$ and the change in soil movement from building to greenfield conditions is investigated. Then the building dimension $L$ is varied to investigate its influence on the transverse settlement profile and on the horizontal transverse displacement and strain behaviour.

7.2 Finite Element analysis

Figure 7.1 shows the 3D FE mesh which was adopted to investigate the behaviour of a 100m $\times$ 100m in plan structure with no eccentricity to the tunnel. As the problem is symmetric with respect to the two centre lines of the building, only one quarter of the geometry was
Figure 7.1: Finite element mesh for 100m × 100m building. Only one quarter of the problem is modelled. The figure therefore only shows a 50m × 50m surface structure which is indicated by the dotted pattern. The displacement boundary conditions were chosen such that no horizontal movement was allowed normal to the vertical boundary planes and no movements at all were allowed on the base. For the elastic shell elements, which represented the surface structure, the rotation about the axes of symmetry was restricted.

The tunnel depth for all analyses presented in this chapter was $z_0 = 20$ m. The dimension of the mesh in $x$-direction was 100m which is the same dimension as adopted for the plane strain mesh shown in Figure 3.8 on Page 99.

The coordinate system shown in Figure 7.1 is adopted in all 3D analyses presented in this
thesis. The longitudinal dimension is described by the y-coordinate. The mesh was generated in the x-z plane (as for plane strain analyses) and then extended in the y-direction. Therefore, the mesh geometry of the plane transverse to the tunnel does not change in the y-direction. The building was represented by elastic shell elements.

In order to reduce calculation time the plane strain mesh shown in Figure 3.8 was simplified for use in 3D analysis (Figure 7.1). By performing a set of plane strain FE analyses involving both meshes with a 100m wide building with varying stiffness it was shown that the difference in deflection ratio and horizontal strain between both meshes was below 3%. For the corresponding modification factors the biggest difference obtained was 1%.

The tunnel was constructed simultaneously over the whole mesh length. As in plane strain analyses, the excavation was carried out over 15 increments and all results were taken from the 7th excavation increment to obtain a volume loss of approximately 1.5% (as explained in Section 3.4.4). The initial stresses prescribed prior to tunnel construction were the same as those adopted in the earlier plane strain analyses (see Section 3.4.1).

The displacement variables used in this thesis were shown in Figure 2.1 on Page 27 and are here summarized for clarity: $S_v$ is the settlement in vertical direction while $S_{hx}$ and $S_{hy}$ denote horizontal displacement in the transverse and in the longitudinal direction, respectively.

### 7.3 Behaviour in longitudinal direction

When tunnel excavation was analysed using 2D plane strain meshes either building or greenfield situations were modelled, both scenarios could not be analysed together. Modelling a 3D building geometry subjected to plane strain tunnel construction includes both cases. This can be seen in Figure 7.2, which shows the tunnel induced surface settlement for the case of a 100m $\times$ 100m in plan 5-storey building. As mentioned above only one quarter of the problem is plotted in the graph.

The figure demonstrates how the building stiffness affects the settlement profile between $y = 0$m and -50m. Greenfield conditions, in contrast, can be observed at $y = -100$m. The graph shows that the transition from building to greenfield settlement occurs over a narrow zone adjacent to the $y = -50$m edge of the building. The following figures will investigate this behaviour in more detail.

Figure 7.3 compares the transverse settlement profiles for $y = 0$m and -100m with the
Section 7.3

Figure 7.2: Vertical surface settlement around 5 storey building. The grid of this surface plot does not coincide with the FE mesh.

Corresponding plane strain results for a building and greenfield analysis respectively. It can be seen that both pairs of curves are in good agreement. For the 2D greenfield data and the 3D $y = -100$ m profile, this confirms that the boundaries in the 3D analysis were placed at a sufficient distance away from the building. The same behaviour is observed when comparing horizontal $S_{hx}$ displacement of the 3D analysis with corresponding 2D results as shown in Figure 7.4.

Figures 7.5 and 7.6 show how these transverse displacement profiles change over the building length $L$. The first graph plots the building settlement profile transverse to the tunnel for $y = 0$ m and $-50$ m (i.e. on the transverse centre line and edge of the structure respectively). The small difference between both curves indicates that the variation of building settlement
Chapter 7. The influence of out of plane geometry

Section 7.3

Figure 7.3: Comparison between 3D and plane strain settlement curves (transverse to tunnel).

Figure 7.4: Comparison between 3D and plane strain horizontal $S_{hx}$ displacement profile (transverse to tunnel).

over $L$ is negligible. The change in horizontal $S_{hx}$ displacement with $L$, shown in Figure 7.6, is slightly greater. The horizontal movement at the edge of the structure is approximately 17% larger than the equivalent movement at the centre line.

This trend is further demonstrated in the following two graphs which plot longitudinal profiles of vertical ($S_v$) and horizontal ($S_{hx}$) surface displacement for the same 5-storey building examined in the previous graphs. Figure 7.7a presents such settlement profiles for
Chapter 7. The influence of out of plane geometry  

Section 7.3

Figure 7.5: Comparison of surface settlement on building centre line \((y = 0\text{m})\) and building edge \((y = -50\text{m})\).

Figure 7.6: Comparison of horizontal surface \(S_{\text{hx}}\) displacement on building centre line \((y = 0\text{m})\) and building edge \((y = -50\text{m})\).

\(x = 0\text{m}\) (along tunnel centre line), \(x = 26\text{m}\) and \(50\text{m}\) (longitudinal edge of structure). The graph confirms that there is little development of vertical settlement over the building length. For all 3 profiles there is a sharp change of settlement next to the edge of the structure at \(y = -50\text{m}\). Within a narrow zone of less than 10m greenfield displacement is reached. All 3 profiles show no significant development from \(y = -60\text{m}\) to the boundary of the FE mesh at \(-100\text{m}\). A similar picture emerges from Figure 7.7b, which presents similar data for \(S_{\text{hx}}\) horizontal displacement (because of symmetry there is no transverse horizontal displacement at \(x = 0\text{m}\)). There is a drastic increase in soil movement adjacent to the edge of the building and greenfield conditions are established within a narrow zone of approximately 10m. This narrow zone could lead to substantial deformation to structures (or building parts or services) of a small geometry located adjacent to larger buildings.

All results shown in the previous figures were for a 5-storey building. Figure 7.8a and b presents similar results for a 1-storey building. The longitudinal settlement profile (Figure 7.8a) demonstrates that the relatively low building stiffness affects the surface settlement less than the 5-storey structure (see Figure 7.7a). Over the building length \(L\) there is little change in settlement and transition from building to greenfield deformation occurs over a zone of less than 5m adjacent to the building edge. Figure 7.8b illustrates that the horizontal surface displacement is much more affected by the 1-storey structure than the settlement in
Figure 7.7: Comparison of longitudinal profiles of (a) surface settlement and (b) horizontal surface $S_{hx}$ displacement along a 5 storey building. Profiles shown are on the tunnel centre line ($x = 0m$), near the middle of the building ($x = 26m$) and at the building edge ($x = 50m$).

Figure 7.8: Comparison of longitudinal profiles of (a) surface settlement and (b) horizontal surface $x$-displacement along a 1 storey building. Profiles shown are on the tunnel centre line ($x = 0m$), near the middle of the building ($x = 26m$) and at the building edge ($x = 50m$).
Figure 7.8a. Consequently, the transition zone from building to greenfield condition extends further from the edge of the building than it does for the case of surface settlement.

The studies presented in this section have shown that there is little variation in building deformation in the longitudinal direction. The buildings analysed were, however, structures with a large geometry. The transverse settlement and horizontal movement profiles therefore were in good agreement with results from corresponding plane strain calculations. The next section will investigate how this transverse building deformation changes when $L$ is reduced leading to smaller building dimensions.

### 7.4 Influence of building out of plane geometry

This section presents results from a parametric study of buildings of different lengths $L$ and stiffnesses. The purpose of this study is to investigate the influence of $L$ on the deformation

![Figure 7.9: Finite element mesh used in parametric study to investigate the influence of $L$.](image)
Chapter 7. The influence of out of plane geometry  

Section 7.4

behaviour of a surface structure. For all analyses the building width was kept constant at \( B = 100\text{m} \) as was the tunnel depth at \( z_0 = 20\text{m} \). The structure’s length was varied as follows: \( L = 8\text{m}, 4\text{m}, 2\text{m} \) and \( 1\text{m} \).

In the previous section it was shown that in the longitudinal direction greenfield conditions were established within a narrow zone adjacent to the edge of the building and that the change in soil displacement was negligible between \( y = -60\text{m} \) and \( y = -100\text{m} \), the boundary of the FE mesh. On this basis it can be justified to reduce the \( y \)-distance between building and mesh boundary in order to save calculation time. For the parametric study presented in the following sections this distance was reduced from \( 50\text{m} \) to \( 30\text{m} \). The mesh for the case of a building of \( L = 8\text{m} \) structure is shown in Figure 7.9. Using symmetry only one quarter of the problem was modelled. The \( y \)-distance from the edge of the building to the remote mesh boundary was kept constant at \( 30\text{m} \) over the parametric study while the longitudinal dimension of the surface structure was varied.

Firstly, the mechanisms which control the behaviour of a surface structure, with stiffness equivalent to that of a 5-storey building, and of varying longitudinal dimension was investigated. Subsequently, other stiffness cases were considered to quantify the effect of \( L \) on the deformation criteria. The results in the following sub-sections are presented for the transverse surface profile along the \( y = 0\text{m} \) centreline of the structure.

### 7.4.1 Displacement behaviour

The transverse settlement and horizontal \( S_{hx} \) displacement profiles for 100m wide 5-storey structures of varying length \( L \) are presented in Figures 7.10 and 7.11. For comparative purposes, the results of a corresponding plane strain analysis are also included. Comparing the two graphs it can be seen that the influence of \( L \) on settlement behaviour is much less than on horizontal displacement. The maximum settlement for the \( L = 1\text{m} \) structure is \( S_v = 3.08\text{mm} \) while for the \( 8\text{m} \) long case it is \( S_v = 2.75\text{mm} \). The difference between these and the corresponding plane strain results, which gives a value of \( S_v = 2.91\text{mm} \), is +5.8% and -5.5% respectively. This is in contrast to the maximum horizontal displacement (measured at \( x = 50\text{m} \)), where the movement determined for the \( L = 1\text{m} \) case is \( S_{hx} = 0.35\text{mm} \), in comparison with \( 0.12\text{mm} \) for the plane strain case, an increase of 183%. In the horizontal displacement case the magnitude of movement decreases from the \( L = 1\text{m} \) structure towards
the \( L = 8 \text{m} \) case and then drops further to the plane strain result, while in the case of the vertical settlement the magnitude decreases slightly from the 1m long to the 8m long structure but then increases for the plane strain situation.

This pattern is surprising as one would expect that as \( L \) increases, settlement curves approach the plane strain solution, as shown for horizontal displacement. It has been found that a reason for this behaviour is the longitudinal variation of volume loss. Figure 7.12 plots this distribution for the above analyses of \( L = 1 \text{m}, 2 \text{m}, 4 \text{m} \) and \( 8 \text{m} \). Each data point represents one transverse settlement profile from which the volume loss was calculated. The longitudinal direction is expressed as distance from the building edge. The different building lengths can be seen on the positive side on the horizontal axis. Two horizontal lines mark the volume losses obtained from the plane strain greenfield and 5-storey building analyses.

For all building cases the volume loss decreases towards the building and remains on a relatively low value within the structure. For \( L = 8 \text{m} \) the building shows the lowest volume loss of \( V_L = 1.37\% \). At a longitudinal distance of 30m from the building edge (i.e. at the remote mesh boundary) the volume loss of this analysis increases to 1.52\%. As \( L \) decreases the longitudinal variation of \( V_L \) reduces. On the centre line of the \( L = 1 \text{m} \) building, \( V_L = 1.42\% \) was calculated while the value on the remote boundary for this case was 1.50\%. These two
Values of the $L = 1\text{m}$ case coincide well with the two corresponding greenfield results.

Integrating each curve along the longitudinal dimension and dividing over the mesh length provides an average volume loss $V_L$ for each analyses. It was found that the four values lie close together (between $V_L = 1.48\%$ and $1.49\%$ for the 8m and 1m long building, respectively).
This result is confirmed by Figure 7.13 which shows a similar graph as Figure 7.12 apart from the volume loss being calculated from the displacement along the circumferential tunnel boundary. Note that the different methods to calculate $V_L$ can lead to slightly different values which can be seen when comparing the corresponding plane strain volume losses in Figures 7.12 and 7.13. However, it is the variation within each graph, which is of interest, not the absolute values. The distribution of the curves in Figure 7.13 show a much narrower range to the variation of $V_L$ in the longitudinal direction. This result shows that the tunnel excavation process is not greatly affected by the presence of the (weightless) surface structure. The range of $V_L$ when calculated from the tunnel displacements is between $V_L = 1.49\%$ ($L = 8\text{m}$) and $1.50\%$ ($L = 1\text{m}$) which is in good agreement with the average volume losses, calculated from the surface settlement.

As a consequence of the small longitudinal change in $V_L$ when calculated from the tunnel displacements, the undrained analysis must exhibit longitudinal soil movement causing the larger variation of $V_L$ when calculated from the surface settlements. Figure 7.14 shows this behaviour by plotting longitudinal horizontal soil displacement $S_{h_y}$ against depth. The results are taken along a line above the tunnel axis ($x = 0\text{m}$) and below the edges of the structures (i.e. the position of the vertical line in Figure 7.13). Positive displacement values represent soil movement towards the building. The graph shows larger longitudinal horizontal soil movement beneath edge of buildings with varying length. Positive values indicate movement towards the building.
movement with increasing $L$. For the $L = 1m$ case the values are small as the results are taken from only 0.5m distance from the vertical $y = 0m$ boundary where horizontal movement in the longitudinal direction is restricted. As $L$ increases the vertical profile along which the results are taken moves further away from the $y = 0m$ boundary and larger longitudinal horizontal soil movements develop.

The results of Figures 7.12 to 7.14 show that it is the greenfield condition surrounding the structure in the $y$-direction which influences the structure’s settlement behaviour. This is further illustrated when reducing the longitudinal distance (i.e. in the $y$-direction) between the building edge and the remote boundary. Figure 7.15 shows the result of such a study for a 8m long structure. The distance between the building edge and boundary was reduced from 30m to 10, 5 and 0.5m. The graph shows that with reducing boundary distance the settlement approaches the plane strain results, again indicating that the greenfield situation adjacent to the building has an influence, albeit small, on the building settlement.

The main question emerging from Figures 7.10 and 7.11 is why the vertical settlement shows so little response to the variation of $L$ while the transverse horizontal displacement is clearly affected. Figures 7.16 and 7.17 show results of a similar study for a building of 1-storey stiffness. The variation in vertical settlement (Figure 7.16) is smaller than for the 5-storey case (Figure 7.10). The maximum settlement of the $L = 8m$ structure reduces by 4.6% with
Chapter 7. The influence of out of plane geometry

Section 7.4

Figure 7.16: Transverse settlement profiles for 1 storey structures of varying length $L$.

Figure 7.17: Transverse horizontal $x$-displacement profiles for 1 storey structures of varying length $L$.

respect to the 1 storey plane strain results, while the $L = 1m$ case decreases by only 1.0%. The corresponding horizontal $S_{hx}$ displacement profile (Figure 7.17) shows a wide variation, as observed for the 5-storey stiffness in Figure 7.11. The $L = 1m$ structure increases its maximum horizontal displacement by 144% with respect to the corresponding plane strain value. This increase is smaller than that observed for the 5-storey situation (183%).

Comparing the 1-storey results with the 5-storey cases indicates that the variations in both vertical and horizontal displacement increase with increasing building stiffness. In terms of relative stiffness (defined by Potts & Addenbrooke (1997), see Equations 2.29 on Page 72) the relative bending stiffness $\rho^*$ has much lower values than the relative axial stiffness $\alpha^*$ for the building cases considered here. It is of course questionable to compare values of $\rho^*$ and $\alpha^*$ as they refer to different modes of deformation. However, when considering the magnitude of modification factors for which strain modification factors have generally much lower values than deflection ratio modification factors it can be said that the building cases considered here behave more stiffly in the axial direction than in bending. This may explain the different behaviour in vertical and horizontal displacement observed when the structure’s length $L$ is varied.

The effect of the structure’s stiffness on the change of deformation behaviour with varying $L$ will be further demonstrated by changing the axial stiffness independently from the bending
Building Structure FE input

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Poisson’s ratio: $\nu = 0.15$

Table 7.1: Reduction of axial stiffness while keeping bending stiffness constant.

Figure 7.18: Plane strain results of transverse (a) settlement and (b) horizontal $x$-displacement for structures with constant bending but different axial stiffnesses.

stiffness. Table 7.1 summarizes the stiffness values for the following analyses. The bending stiffness $EI$ was kept constant to a value corresponding to a 5-storey building (compare with Table 3.4, Page 97) while the axial stiffness $EA$ was reduced. The case referred to as ‘soft2’ has only 10% of the 5-storey axial stiffness. $EA$ of the ‘soft1’ case lies between the two values.

The transverse vertical and horizontal displacement profiles for plane strain analyses, including the three stiffness cases outlined above are plotted in Figure 7.18. The vertical settlement plot shows that, although $EI$ was kept constant for all three cases, the buildings behave more flexibly in bending with decreasing $EA$. This behaviour was discussed by Potts & Addenbrooke (1997) by varying $\rho^*$ and $\alpha^*$ independently over a wide range (see Section 2.4.6.2). Also, the transverse horizontal displacement shows a more flexible response
Figure 7.19: Normalized results of (a) settlement and (b) horizontal $x$-displacement for $L=1m$ and 8m structures with constant bending but different axial stiffnesses.
Chapter 7. The influence of out of plane geometry

Section 7.4

to tunnel construction, most notably in the ‘soft2’ scenario, which gives a maximum value at approximately $x = 38m$ in contrast to the two other cases, where the maximum horizontal movement is obtained at the building edge ($x = 50m$).

Geometries of $L = 1m$ and $8m$ were then analysed with the above stiffness variation to investigate the effect of the structure’s axial stiffness on the change in deformation with varying $L$. The results, normalized against the corresponding maximum plane strain values (this is the vertical settlement at $x = 0$ and horizontal displacement at $x = 50m$ (38m for the ‘soft2’ case) of the curves shown in the previous graphs), are plotted in Figure 7.19. The curves of normalized vertical settlement (Figure 7.19a) show a similar shape as the plane strain results given in Figure 7.18a with the low axial stiffness case (‘soft2’) behaving most flexibly for both the $L = 1m$ and $8m$ case. At the tunnel centre line, however, cases with the same $L$ but different axial stiffness coincide. Although the change in $EA$ affects the overall shape of the settlement curves, it does not effect the dependency between settlement and building length $L$. This is not the case for the transverse horizontal displacement, plotted in Figure 7.19b. Not only does the shape of the curves change with decreasing $EA$ but also the relative position between each $L = 1m$ and $8m$ pair. As mentioned above for the 5-storey stiffness case, there is an increases of 183% in its maximum horizontal displacement from plane strain to $L = 1m$. This difference drops to 152% for the ‘soft1’ case and reduces further to 45% (at $x = 50m$) for the ‘soft2’ stiffness. The results for the $L = 8m$ structure are much less affected by the change in $EA$.

This study confirms the above observation that the influence of $L$ on building deformation increases with increasing building stiffness. The fact that the buildings considered in this thesis behave much more stiffly in the axial direction than in bending (as seen from the different magnitude in modification factors) explains why the horizontal building deformation is more sensitive to changes in the out of plane dimension $L$ than vertical settlement.

By only focusing on maximum vertical and horizontal displacement this study investigated the overall magnitude of the displacement curves for different stiffness cases. The shape of these displacement curves is described by the deformation criteria deflection ratio and maximum horizontal strain. The following section will investigate if these criteria follow a similar trend.
Chapter 7. The influence of out of plane geometry

Section 7.4

7.4.2 Deformation Criteria

For the geometries described in the previous section additional analyses were performed including 1, 3, 5 and 10-storey structures. The deflection ratio and horizontal strain results presented below are for the transverse surface profile at $y = 0m$. As mentioned in previous chapters no tensile strain was obtained for this non-eccentric geometry.

Figure 7.20 shows how $M^{DR_{sag}}$ changes with length $L$. In this graph the modification factors are normalized against the corresponding plane strain modification factor for the same building stiffness. $M^{DR_{sag}}$ decreases with increasing $L$. This trend is most distinct for the 10-storey structure which $L = 1m$ result is 53% higher than its corresponding plane strain result. The differences between $L = 1m$ and plane strain for the other building cases are all less than 15%, with the 1-storey structure showing the flattest curve. These data are in agreement with the results obtained in the previous section, which indicated that the increase in deformation with decreasing length $L$ becomes more distinct for high building stiffnesses.

The picture that emerges from the hogging modification factors in Figure 7.21 (also plotted as normalized values against the corresponding plane strain results) is, however, not so clear. All stiffness cases show a decrease with increasing $L$. The 5-storey structure decreases most
drastically. The $L = 1\text{m}$ result exceeds the plane strain $M_{\text{DR hog}}$ value by 243%. The 3-storey structure exhibits the second biggest difference (145% for $L = 1\text{m}$) followed by the 10-storey results (59%). For the 1-storey case the curve shows a similar flat gradient as in the previous graph. The largest difference found when comparing $L = 8$ results with plane strain is 28% for the 3-storey structure.

When the compressive strain modification factors $M_{\text{tc}}$ (normalized against the corresponding plane strain results) are plotted in Figure 7.22 for all stiffness cases considered a similar response to changes in $L$ is exhibited. The $L = 1\text{m}$ results are between 181% (3 storeys) and 160% (10 storeys) higher than the corresponding plane strain data. The maximum difference of any $L = 8\text{m}$ result with respect to the corresponding plane strain analysis is 31% for 10 storeys. Although the variation between the different stiffness curves remains small it is interesting to note that a higher stiffness tends to produce a smaller increase in strain with reducing $L$. This is in contrast to the displacement behaviour described above.

This study has shown that the sagging modification factors follow the trend observed for maximum vertical and horizontal displacement, with high building stiffness leading to a wider variation with $L$. This behaviour was not found with $DR_{\text{hog}}$ which showed the highest decrease with increasing $L$ for the 5 and 3-storey cases. The compressive strain modification factors in contrast gave a uniform relation between $M_{\text{tc}}$ and $L$ for all stiffness cases; lower stiffnesses producing slightly higher increases in $M_{\text{tc}}$ with reducing $L$.

It is possible to include the building length into the calculation of $\rho^*$ and $\alpha^*$ when plotting
the modification factors for the above analyses against relative stiffness. The results are given in Figure 7.23 for deflection ratio modification factors, $M_{DR}$. By including the building length $L$ into the second moment of area $I$ instead of calculating it per metre run the dimension of $\rho_{3D}^*$ becomes dimensionless (compare with Equation 2.29a, Page 72). The black symbols in this figure represent the 3D FE results. For each building stiffness there are four data points, referring to $L = 1\text{m}, 2\text{m}, 4\text{m}$ and $8\text{m}$, with the last case showing the highest relative stiffness. The hollow symbols are the corresponding plane strain results (whose relative stiffness has the same value as the $L = 1\text{m}$ case although the units of the relative stiffness expressions are different). The design curves, derived by Potts & Addenbrooke (1997) from plane strain results, are also included for comparison.
For each stiffness case, the $L = 1\text{m}$ sagging and hogging modification factors are higher than the corresponding plane strain data points (as previously seen in Figures 7.20 and 7.21). $M^{\text{DR}}$ reduces as $\rho^*_{3\text{D}}$ increases with $L$. Apart from the hogging results for 3 and 5 storeys the gradient of the decrease of $M^{\text{DR}}$ with increasing $\rho^*_{3\text{D}}$ is relatively flat. The difference in $M^{\text{DR}_{\text{Sag}}}$ within each stiffness case remains small when plotted in this context. This trend becomes clearer when all modification factors are plotted against the corresponding plane strain relative stiffness $\rho^*$. These graphs are shown in Figure 7.24. All results, apart from the hogging factors for the 3-storey structures, lie within the boundaries described by the Potts & Addenbrooke (1997) design curves. These graphs show that the description of relative bending stiffness by using plane strain measures for 3D structures provides a good estimate for predicting $M^{\text{DR}}$.

Figures 7.25 and 7.26 present the corresponding graphs for compressive strain modification factors plotted against relative axial stiffness, defined in Equation 2.29b (Page 72), which changes its dimension to $[\text{length}]$ when building length is considered (referred to as $\alpha^*_{3\text{D}}$). The steady increase in $M^{\epsilon_{hc}}$ with reducing $L$, previously shown in Figure 7.22 can be clearly seen in the first graph. All the 3D results lie outside of the Potts & Addenbrooke (1997) design curve. The gradient of changing $M^{\epsilon_{hc}}$ with $\alpha^*_{3\text{D}}$ is steeper than the slope of the design curve. When plotting the results against the plane strain relative stiffness (Figure 7.26) the
increase of $M^{\text{e hc}}$ looks severe. It should, however, be noted that the strain modification factors are low in magnitude compared to the corresponding $M^{\text{DR}}$. The highest compressive strain modification factor obtained from this parametric study is $M^{\text{e hc}} = 0.11$ for the 1-storey structure with $L = 1\text{m}$. Furthermore, the values of $L = 1$ and $2\text{m}$ are not realistic when considering entire buildings represented by shell elements. When focusing on realistic dimensions it can be concluded that the $L = 8\text{m}$ results lie close to the Potts & Addenbrooke (1997) design curves.

For the deflection ratio modification factors it was shown that the change in $M^{\text{DR}}$ over $L$ is small such that the relative bending stiffness can be expressed in plane strain geometry. On consideration of the magnitudes of the strain modification factors, and including only realistic building lengths, a similar conclusion can be drawn for the axial strain modification factors.

This section has highlighted that the relative stiffness expressions have different dimensions when calculating them for plane strain and 3D data respectively. For plane strain situations $\rho^*$ has the unit $[1/\text{length}]$ while the axial relative stiffness is dimensionless. The use of 3D geometry, in contrast, leads to a dimensionless relative bending stiffness but a relative axial stiffness measured in $[\text{length}]$. The reason for these changes in dimensions is that the numerator of both $\rho^*$ and $\alpha^*$ contains an out of plane dimension ($I$ and $A$ respectively) while the denominator does not. To overcome this problem either plane dimensions should not be incorporated into the definition of $\rho^*$ and $\alpha^*$ or $L$ must be present in both numerator and denominator. Following the latter approach, the original relative bending stiffness (Equation 2.29a) can be re-arranged to

$$\rho^{*m2} = \frac{EI}{ES \left( \frac{B}{2} \right)^4 L}$$

(7.1)

where the second moment of area $I$ has the dimension $[\text{length}^4]$ if the out of plane geometry is included. Dividing over $L$, however, always reduces it to its plane strain expression. If plane strain conditions apply, $I$ has the dimension $[\text{length}^4/\text{length}]$ and $L$ is $1\text{m}/\text{m}$ and, hence, the magnitude of $\rho^{*m2}$ remains the same as the corresponding plane strain expression $\rho^*$. These results together with the conclusions drawn from previous chapters will be used in Chapter 10 to derive a modified relative bending stiffness which is always dimensionless.

For relative axial stiffness $\alpha^*$ (defined in Equation 2.29b) the same approach can be
followed by introducing

$$\alpha^*_{m2} = \frac{EA}{E_S \left( \frac{B^2}{4} \right) L}$$

(7.2)

with $A$ having the dimension [\text{length}^2] in 3D analysis but [\text{length}^2/\text{length}] in plane strain situations. Dividing by $L$ (with the dimension [\text{length}] or [\text{length}/\text{length}] respectively) yields a dimensionless relative axial stiffness expression regardless of whether applied to 3D or to plane strain conditions.

### 7.5 Conclusions

This chapter has investigated how the building length $L$ parallel to the tunnel axis affects tunnel induced building deformation. While the building was modelled three dimensionally the tunnel was excavated simultaneously over the whole mesh.

It was found that building deformation varies little in the longitudinal direction. The transition from building to greenfield conditions for settlement and horizontal displacement takes place over a narrow zone adjacent to the building which could lead to substantial deformation of adjacent buildings or services.

It was shown by reducing the building length $L$ from 8m to 4m, 2m and 1m that the effects on the settlement profile remain small while the horizontal deformation of the structure changes significantly. It was found that the reason for this contrasting behaviour is that the buildings modelled in this study behave much more stiffly in axial than in bending deformation. In general, the variation in building deformation with varying building length $L$ increases with increasing stiffness.

Visual inspection of the settlement profile indicated small changes as $L$ increased. However, calculation of the deflection ratio modification factors showed an increase with reducing $L$. These increases were more significant for $M^{DR_{hag}}$ than for $M^{DR_{gaq}}$. The corresponding compression strain modification factors exhibited a steady increase for all building stiffnesses as $L$ reduced.

Plotting these modification factors against relative stiffness showed that the variation in $M^{DR}$ is small compared with their reduction with increasing stiffness. When focusing only on realistic geometries the compressive strain modification factors also lie close to the plane strain results. The results obtained from this 3D FE study are therefore in reasonable
agreement with the design charts proposed by Potts & Addenbrooke (1997), which are based on plane strain FE analyses.

Finally it has been shown how the out of plane dimension $L$ can be included in the relative stiffness expressions in order to obtain the same dimension for plane strain and 3D relative stiffness expressions.
Chapter 8

Prediction of three-dimensional greenfield settlement

8.1 Introduction

In all the analyses presented in the previous chapters, tunnel excavation was modelled as a plane strain event. Chapters 4 to 6 presented results of 2D analyses while in Chapter 7 a 3D mesh was used to study the influence of the building geometry in more detail. The tunnel was, however, constructed simultaneous over the whole mesh length.

Tunnel excavation is clearly a 3D process with the settlement trough extending in both the transverse and longitudinal direction. To investigate this effect, fully 3D FE analysis is required. This chapter presents the results of a number of 3D greenfield analyses which were performed before studying the tunnel-soil-building interaction (Chapter 9). In either case, tunnel construction was simulated by a step-by-step approach (Katzenbach & Breth, 1981) in which soil elements in front of the tunnel boundary are successively removed while tunnel lining elements are successively constructed.

It has been shown in Section 2.3.1 that the predicted surface settlement trough obtained from 2D plane strain FE analysis is too shallow and too wide (for the soil conditions applied in this thesis). To investigate how fully 3D modelling affects these results, the construction of the westbound Jubilee Line Extension (JLE) tunnel beneath St. James’s Park (London) was modelled both in 2D and 3D. This study will show that the use of fully 3D analysis brings...
only marginal improvement compared with corresponding results from plane strain studies. It had been suggested by many authors that including soil anisotropy into the constitutive model leads to more realistic predictions of tunnel induced greenfield settlement. The analyses for St. James’s Park are therefore repeated incorporating soil anisotropy. For this purpose the model described by Graham & Houlsby (1983) is combined with a small strain stiffness formulation (based on Jardine et al. (1986)). Details of this new soil model are given in Appendix B. This study concludes that neither 3D modelling nor soil anisotropy lead to a reduction in the width of the ground surface settlement trough in a high $K_0$-regime.

8.2 Finite Element analysis of 3D tunnel construction

This section describes how fully 3D tunnel excavation was modelled. Further details of the analyses (e.g. mesh size and geometry) are given where applicable.

Figure 8.1 shows the FE mesh used for a fully 3D analysis of tunnel construction at St. James’s Park, London. The tunnel is excavated in the negative $y$-direction, starting from $y = 0$ m. Only half of the problem is modelled as the geometry is symmetrical. On all vertical sides normal horizontal movements were restrained while for the base of the mesh movements in all directions were restricted.

As described in Section 7.2 the mesh was generated in the $x$-$z$ plane and then extended in the longitudinal direction. Figure 8.2 illustrates how tunnel construction was modelled. In Figure 8.2a the tunnel is already excavated over a length of four elements (in the longitudinal direction) with the lining being installed over the same length. In the next step (b) the next section of elements within the tunnel boundary is excavated. The longitudinal length of this section will be referred to as the excavation length $L_{exc}$ as defined in Figure 8.2b. Afterwards the lining is installed over the excavated length (c). This sequence is then repeated until the tunnel has progressed to the desired position. The process is analysed as undrained, therefore no time has to be considered between the different steps. As explained in Section 3.2.4.3, material which is being constructed during the FE analysis is assigned a low stiffness value at the beginning of the increment of construction. At the end of this increment the stiffness is changed to its real value. It was therefore possible to combine step b and c within one increment of the analysis: When a new section is excavated the lining is directly installed. As it has a low stiffness soil movement into the tunnel is not restricted. The stiffness of the
Chapter 8. Prediction of three-dimensional greenfield settlement

Section 8.2

Figure 8.1: FE mesh for tunnel excavation beneath St. James’s Park greenfield monitoring site.

Figure 8.2: Sequence of tunnel excavation simulation.
lining is changed at the end of this increment before the next section is excavated in the following increment.

The tunnel lining was modelled by elastic shell elements. For symmetry reasons their horizontal movement (x-direction) and rotation around the longitudinal direction (y-axis) was restricted on the plane of symmetry. The material parameters for the lining are summarized in Table 8.1.

The excavation sequence, described above, is not volume loss controlled in contrast to the 2D analyses presented earlier where results were taken for a specific volume loss. In fully 3D tunnel excavation the volume loss depends mainly on the excavation length $L_{exc}$. The choice of this excavation length has, however, serious consequences on the computational resources (storage and time) needed. For a given tunnel length to be modelled a reduction in $L_{exc}$ not only increases the number of elements within the entire mesh but also requires more increments of excavation. A comparison of analyses with different $L_{exc}$ will be presented in the next chapter (Section 9.2.3).

<table>
<thead>
<tr>
<th>Young's Modulus [kN/m$^2$]</th>
<th>Poisson's ratio</th>
<th>Thickness [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$28 \times 10^6$</td>
<td>0.15</td>
<td>0.168</td>
</tr>
</tbody>
</table>

Table 8.1: Material parameters of tunnel lining.

### 8.3 Greenfield settlement predictions

Before investigating the behaviour of buildings subjected to 3D tunnel excavation a set of greenfield analyses were performed. A mayor point of interest was wether the prediction for transverse settlement profiles can be improved by applying fully 3D FE analysis. Following the 2D work of Addenbrooke *et al.* (1997) (described in Section 2.3.1) the JLE westbound tunnel excavation beneath St. James’s Park (London) was modelled. Nyren (1998) presented field measurements from this site which will be used for comparison.

Nyren (1998) gives details of the tunnelling method used for this part of the JLE. The tunnel was excavated using an open-faced shield. The shield had a length of about 4.2m from cutting edge to shield tail. The maximum reach of the backhoe was about 1.9m in advance of the shield cutting edge.
8.3.1 FE analysis for St. James’s Park

The westbound tunnel was the first of the twin tunnels of the Jubilee Line Extension to be constructed beneath St. James’s Park. Its diameter was $D = 4.75\text{m}$ and its depth was approximately $z_0 = 30.5\text{m}$. The subsequent construction of the eastbound tunnel was not included in the analysis. For this study the whole soil profile was modelled as London Clay. In a first step the non-linear elasto-plastic model, described in Section 3.3, was used to model the soil behaviour. In a second set of analyses soil anisotropy was included into the study. In all cases the soil was modelled to behave undrained.

The mesh is shown in Figure 8.1. The dimensions in the $x$-$z$-plane was chosen to be identical with the plane strain mesh used by Addenbrooke et al. (1997) with a horizontal distance to the remote vertical boundary of 80m ($16.8 \times D$, transverse to the tunnel). In the longitudinal direction the tunnel was constructed over 100m ($21.0 \times D$) with an excavation length of $L_{exc} = 2.5\text{m}$. The analysis was therefore carried out over 40 increments. The distance in the longitudinal direction from the tunnel face in the last increment to the remote vertical boundary was 55m ($11.5 \times D$). These distances are significantly larger than the mesh dimensions of recently published analyses, summarized in Table 2.1 on Page 48. The mesh used in the following analyses consisted of 10125 20 node solid elements and had 45239 nodes.

It was pointed out previously that the excavation length $L_{exc}$ affects both computational time and storage. It is also the main parameter controlling the volume loss in a fully 3D analysis. The choice of $L_{exc} = 2.5\text{m}$ was made based on a compromise between calculation time and volume loss developing during the analyses. It was not the aim to simulate the actual tunnelling technique applied during the construction of this part of the Jubilee Line Extension.

The same initial stress conditions as for the 2D analyses presented in previous chapters were applied. They are described in Section 3.4.1.

8.3.2 Isotropic soil model

The non-linear pre yield model, described in Section 3.3.1, together with a Mohr-Coulomb yield (Section 3.3.2) surface, were applied to analyse a fully 3D tunnel excavation. The material parameters were the same as those used for the 2D analyses presented in earlier chapters of this thesis and they are listed in Tables 3.2 and 3.3 on Pages 92 and 93, respectively.
Figure 8.3: Surface settlement trough obtained from St. James’s Park greenfield FE analysis. The grid of the surface plot does not represent the Finite Element mesh.

Figure 8.3 shows the surface settlement obtained from this analysis. The results are for a tunnel face position of \( y = -75 \) m. Contour lines of settlement are given for increments of \( S_v = 0.5 \) mm. The 3D nature of the settlement trough can clearly be seen.

The following graphs will present data along two monitoring sections in order to study the settlement behaviour in more detail. Longitudinal surface profiles are plotted on the symmetry axis above the tunnel crown while transverse surface profiles are taken at \( y = -50 \) m as indicated in Figure 8.3.

The development of settlement along the longitudinal section is plotted in Figure 8.4. Different curves are given for every 10 m of tunnel progress. The position of the tunnel face for each curve is indicated by an arrow. The thick solid line marks the situation where the tunnel face is beneath the transverse monitoring section.

The graph demonstrates that during the first increments the settlement trough has a
similar shape as the cumulative error curve (compare with Figure 2.4 on Page 31), often used to describe longitudinal settlement profiles. However, as the tunnel face passes behind approximately $y = -70m$ the settlement does not continue to follow this anticipated trend. Instead the profile indicates that some hogging is developing at approximately $y = -60m$.

However, the main problem shown in this graph is that for all increments additional settlement occurs over the whole mesh length. One would expect that from a certain distance behind the tunnel face settlement does not change (if only the short term response is considered which is the case for undrained FE analysis). This end of the immediate settlement response will be referred to as ‘steady state’ conditions.

It can be seen from Figure 8.4 that such a condition is not established during the analysis. There is still additional settlement at the $y = 0m$ boundary when the tunnel is excavated from $y = -90m$ to -100m. Also the remote boundary at $y = -155m$ settles over the whole analysis, however, settlements for the first few increments are negligible on an engineering scale. The additional settlement for the last 4 increments (tunnel face from $y = -90m$ to -100m) has approximately the same magnitude as the additional settlement measured over
Figure 8.5: Transverse settlement profiles from St. James’s Park greenfield FE analysis (isotropic soil model).

the same increments at the $y = 0$ m boundary. This indicates that the longitudinal distance from the last excavation step to both boundaries is too small to obtain steady state conditions.

Figure 8.5 shows the development of transverse settlement with tunnel progress. Settlement curves are shown for the same increments as in the previous graph and correspond to a section at $y = -50$ m. The thick solid line indicates the settlement trough obtained when the tunnel face is beneath this transverse section. The graph shows that the biggest increase in settlement takes place when the tunnel face is approaching this monitoring section ($y = -40$ m to -50 m). There is still additional settlement when the tunnel face moves from $y = -90$ m to -100 m, again showing that there are no steady state conditions reached.

This is further illustrated in Figure 8.6 which plots the volume loss calculated at the transverse monitoring section against position of tunnel face. The curve has its steepest increase when it passes beneath the profile at $y = -50$ m. As the tunnel face moves away the increase in volume loss becomes less but the curve does not become horizontal which would indicate a steady state condition.

It is possible to compare the transverse settlement results with data obtained from plane
strain analyses as well as with field data. For this purpose the mesh shown in Figure 8.1 was used in the $x$-$z$ plane only. The tunnel construction was modelled over 15 increments (without tunnel lining) and the volume loss was calculated for each increment.

Figure 8.7a shows these results together with field measurements which were taken at that moment when the tunnel face was passing beneath the greenfield monitoring site (referred by Nyren (1998) to as ‘set22’). The volume loss of the measured settlement curve is $V_L = 1.6\%$. The settlement trough obtained from the 3D analysis is taken from the increment where the tunnel face is just beneath the monitoring section. The volume loss calculated\(^1\) for this curve

\(^1\)To be consistent with the field measurements the volume loss of the FE analysis was calculated over a width of 52m only. This is the width over which field data were collected. The $V_L$ given in Figure 8.7 differ from the values plotted in Figure 8.6 where the $V_L$ was calculated for the mesh width of 80m.

\[ \begin{align*}
\text{Vertical settlement [m]} & \quad \text{x-Coordinate (transverse) [m]} \\
2D, \text{inc. 8} & \quad 3D, \text{face at -50m} \\
\text{Field data, set 22} & \\
\end{align*} \]

\[ \begin{align*}
\text{Vertical settlement [m]} & \quad \text{x-Coordinate (transverse) [m]} \\
2D, \text{inc. 12} & \quad 3D, \text{face at -100m} \\
\text{Field data, set 29} & \\
\end{align*} \]

\textbf{Figure 8.7:} Transverse settlement profiles from 3D and 2D FE analyses (St. James’s Park FE analysis, isotropic soil model) and field measurements (Nyren, 1998).

\[ \begin{align*}
\text{Position of tunnel face [m]} & \quad \text{Volume loss [%]} \\
0.0 & \quad 0.0 \\
0.5 & \quad 0.5 \\
1.0 & \quad 1.0 \\
1.5 & \quad 1.5 \\
2.0 & \quad 2.0 \\
2.5 & \quad 2.5 \\
3.0 & \quad 3.0 \\
\end{align*} \]

\[ \begin{align*}
\text{Position of tunnel face [m]} & \quad \text{Volume loss [%]} \\
-100.0 & \quad 0.0 \\
-80.0 & \quad 20.0 \times 10^{-3} \\
-60.0 & \quad 10.0 \times 10^{-3} \\
-40.0 & \quad 5.0 \times 10^{-3} \\
-20.0 & \quad 0.0 \\
0.0 & \quad 0.0 \\
\end{align*} \]

\textbf{Figure 8.6:} Development of volume loss with advancing tunnel construction (St. James’s Park FE analysis, isotropic soil model).
is $V_L = 1.2\%$. For the plane strain analysis the settlement curve with the volume loss closest to the field data was chosen. For the 8th excavation increment (53\% of unloading) a volume loss of $V_L = 1.7\%$ was obtained.

The graph shows that both the FE troughs are too shallow and too wide. The difference between the 2D and 3D data is mainly because of the different values of volume loss. This difference, however, is small compared to the narrow settlement profile obtained from the field measurements.

Figure 8.7b presents a similar graph for troughs with higher volume losses. The field data are taken for a distance between greenfield monitoring site and tunnel face of 41m. Nyren (1998) reports no further short term settlement after this survey (referred to as ‘set29’). The results therefore represent the end of the immediate settlement response. The volume loss for this data set was $V_L = 3.3\%$. For the 3D analysis, results were taken from the largest distance between monitoring section and tunnel shield, i.e. 50m in the last increment. The volume loss calculated for this situation was $V_L = 2.1\%$. The plane strain results are taken from increment 12 (80\% of unloading) with a volume loss of $V_L = 3.3\%$.

The trend shown in this graph is the same as found in the previous plot. The difference between the 2D and 3D result can be explained due to the different volume loss. Both troughs are, however, too wide with settlement occurring at the $x = 80m$ boundary. This is demonstrated in Figure 8.8 which normalizes the settlement troughs from Figure 8.7b against maximum settlement. The two FE results practically coincide with the 3D data showing a slightly wider settlement trough. Both settlement troughs are much too wide when comparing

**Figure 8.8:** Transverse settlement troughs, shown in Figure 8.7b, normalized against maximum settlement.
them with the corresponding field data.

This study showed that modelling tunnel excavation in fully 3D FE analysis does not improve the surface settlement predictions. Furthermore the settlement trough is not only too wide in the transverse direction but also in the longitudinal direction with the consequence that no steady state conditions develop at the end of analysis. Increasing the mesh dimension is neither possible due to computer resources nor does it seems appropriate when considering the large dimensions of the used FE mesh, compared to those summarized in Table 2.1 on Page 48. Other model parameter must therefore be taken into account in order to obtain better results. The next section investigates whether the use of an anisotropic non-linear pre-yield constitutive soil model results in better settlement predictions.

8.3.3 Anisotropic soil model

The use of anisotropy in tunnel excavation analyses has been proposed by various authors. In 1989, Lee & Rowe showed that deformations around a tunnel are highly sensitive to stiffness anisotropy. They concluded that the ratio of $G_{hv}/E_v$ governs the shape of the tunnel induced settlement trough. Simpson et al. (1996) presented a 2D FE study of the Heathrow Express trial tunnel indicating that incorporating anisotropy into the analysis improves the surface settlement predictions. Addenbrooke et al. (1997), however, concluded that applying anisotropic parameters, which can be justified for London Clay, do little to improve the predicted settlement trough. More recently, Lee & Ng (2002) presented 3D results varying both $K_0$ and $E'_h/E'_v$. They demonstrated that the influence of anisotropy reduces with increasing $K_0$.

To get a better picture about the influence of anisotropy on tunnel induced settlement, the analyses presented in the previous section were repeated with an anisotropic non-linear pre-yield soil model.

The model is based on the elastic transversely anisotropic stiffness formulation by Graham & Houlsby (1983). A transversely anisotropic material shows different properties horizontally and vertically. This behaviour often applies to soils which were deposited over areas of large lateral extent leading essentially to one-dimensional deposition. With the vertical direction being an axis of symmetry, the elastic material can be described by only five independent parameters (Pickering, 1970) which are $E_v$, the vertical Young’s modulus; $E_h$, the horizontal
Young’s modulus; \( \nu_{vh} \), the Poisson’s ratio for horizontal strain due to vertical strain; \( \nu_{hh} \), the Poisson’s ratio for horizontal strain due to horizontal strain in the orthogonal direction; and \( G_{vh} \), the shear modulus in the vertical plane.

Graham & Houlsby (1983) showed that only three of these material parameters can be obtained from triaxial tests as no shear stress can be applied to the sample. They introduced a material model which only uses three parameters to describe transversely anisotropic stiffness: \( E_v \), \( \nu_{hh} \) and an anisotropic scale parameter \( \alpha \) from which the remaining material constants can be calculated by:

\[
E_h = \alpha^2 E_v \\
\nu_{vh} = \frac{\nu_{hh}}{\alpha} \\
G_{hv} = \frac{\alpha E_v}{2(1 + \nu_{hh})}
\]

The nature of the anisotropic scale parameter \( \alpha \) becomes clearer when rewriting the above equations to give

\[
\alpha = \sqrt{\frac{E_h}{E_v}} = \frac{\nu_{hh}}{\nu_{vh}} = \frac{G_{hh}}{G_{hv}}
\]

where \( G_{hh} \) is the shear modulus in the horizontal plane.

To implement this form of anisotropy into ICFEP, it was combined with a small strain formulation reducing the vertical Young’s modulus \( E_v \) with increasing deviatoric strain \( E_d \) (between strain limits defined by \( E_{d,\text{min}} \) and \( E_{d,\text{max}} \)). As an additional option the value of \( \alpha \) can be varied with strain level from its anisotropic value at \( E_{d,\text{min}} \) to the isotropic case of \( \alpha = 1.0 \) at \( E_{d,\text{max}} \) where \( E_{d,\text{min}} \) and \( E_{d,\text{max}} \) are lower and upper bounds of deviatoric strain used in the small strain formulation. The result is a non-linear anisotropic pre-yield model which is described in more detail in Appendix B.

8.3.3.1 Anisotropic soil parameters

Undrained triaxial extension tests and tunnel excavation were analysed to evaluate the new model in single element and boundary problem analyses respectively. The results of this study are presented in Appendix B. Different values of \( \alpha \) were applied during this study. When increasing \( \alpha \) from 1.0 (isotropic case) to 2.0, \( E_v \) was reduced in order to get similar strain-stress curves in undrained triaxial extension tests. Results for the anisotropic pre-yield
model combined with a Mohr-Coulomb yield surface were compared with analyses performed with the isotropic non-linear elasto-plastic model which was used in all analyses presented previously in this thesis.

The analyses of greenfield tunnel excavations showed that an increase of $\alpha$ (from 1.0 to 2.0) improves the surface settlement prediction in the vicinity of the tunnel centre line. Close to the remote vertical boundary, however, the anisotropic model shows similar settlement behaviour as found for the isotropic material model. Reducing the degree of anisotropy with increasing deviatoric strain (from $\alpha = 2.0$ at $E_{d,\text{min}}$ to 1.0 at $E_{d,\text{max}}$) brings little improvement close to the tunnel centre line. The settlement trough, however, is still too wide when comparing with a Gaussian curve. It was concluded in Appendix B that only a high degree of anisotropy might improve settlement predictions.

When using anisotropic parameters, the ratios

\[
\begin{align*}
n' &= \frac{E_h'}{E_v'} \\
n &= \frac{E_h}{E_v}
\end{align*}
\]

\[
\begin{align*}
m' &= \frac{G_{vh}}{E_v'} \\
m &= \frac{G_{vh}}{E_v}
\end{align*}
\]

are often used\(^2\). To calculate the undrained ratios from the drained expressions, Hooper (1975) gives the relationship

\[
\begin{align*}
n &= \frac{2n'(1 - \nu''_{hh} - 2n'\nu''_{vh})}{1 - 2n'\nu''_{vh} + n' - n'^{2}\nu'^{2}_{vh} - 2n'\nu'_{vh}\nu''_{hh} - \nu''_{hh}} \\
m &= \frac{2m'n(1 + \nu''_{hh})}{n'(4 - n)}
\end{align*}
\]

Lee & Rowe (1989) showed that the ratio of $G_{vh}/E_v$ influences the shape of settlement troughs. For $K_0 = 0.5$ they concluded that a ratio of $m = 0.2$ to 0.25 produced a reasonable match between FE results and centrifuge tests. Field data for London Clay, summarized by Gibson (1974), however, give a ratio of approximately $m = 0.38$. In the same publication a ratio of undrained Young’s moduli of $n = 1.84$ is given. When Addenbrooke et al. (1997) analysed the Jubilee Line Extension tunnelling work at St. James’s park, they included a transversely anisotropic model in their 2D study. The anisotropic parameters were chosen to match field data reported by Burland & Kalra (1986). There, drained ratios of $n' = 1.6$

\(^2\)Note, that in their publication Addenbrooke et al. (1997) defined $n' = E_v'/E_h'$, the inverse ratio of above definition.
Chapter 8. Prediction of three-dimensional greenfield settlement

Section 8.3

Isotropic model

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C [%]</th>
<th>β</th>
<th>γ</th>
<th>E_{d,min} [%]</th>
<th>E_{d,max} [%]</th>
<th>G_{min} [kPa]</th>
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</thead>
<tbody>
<tr>
<td>373.3</td>
<td>338.7</td>
<td>1.0 × 10^{-4}</td>
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<td>0.617</td>
<td>8.66 × 10^{-4}</td>
<td>0.69282</td>
<td>2333.3</td>
</tr>
</tbody>
</table>

R S T [%] β γ ε_{v,min} [%] ε_{v,max} [%] K_{min} [kPa]

549.0 506.0 1.00 × 10^{-4} 2.069 0.42 5.00 × 10^{-4} 0.15 3000.0

Anisotropic model

Parameter set 1

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C [%]</th>
<th>β</th>
<th>γ</th>
<th>E_{d,min} [%]</th>
<th>E_{d,max} [%]</th>
<th>E_{min} [%]</th>
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<td>666.0</td>
<td>1.0 × 10^{-4}</td>
<td>1.335</td>
<td>0.617</td>
<td>8.66 × 10^{-4}</td>
<td>0.69282</td>
<td>5558.8</td>
</tr>
</tbody>
</table>

α ν_{hh} hh

Anisotropic model

Parameter set 2

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C [%]</th>
<th>β</th>
<th>γ</th>
<th>E_{d,min} [%]</th>
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<tbody>
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<td>280.2</td>
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<td>0.617</td>
<td>8.66 × 10^{-4}</td>
<td>0.69282</td>
<td>5558.8</td>
</tr>
</tbody>
</table>

α ν_{vh} hh

Table 8.2: Input parameters for the isotropic and anisotropic soil models used for analysing St. James’s Park greenfield site.

Table 8.3: Stiffness ratios for the two sets of anisotropic soil parameters

<table>
<thead>
<tr>
<th>Isotropic model</th>
<th>Anisotropic model set 1</th>
<th>Anisotropic model set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>n'</td>
<td>1.00</td>
<td>1.60</td>
</tr>
<tr>
<td>m'</td>
<td>0.55</td>
<td>0.46</td>
</tr>
<tr>
<td>n</td>
<td>1.00</td>
<td>1.18</td>
</tr>
<tr>
<td>m</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

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<td>0.46</td>
</tr>
<tr>
<td>n</td>
<td>1.00</td>
<td>1.18</td>
</tr>
<tr>
<td>m</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

and m' = 0.44 were given. With $\mu'_{vh} = 0.125$ and $\mu'_{hh} = 0.0$ the undrained ratios can be calculated to be $n = 1.41$ and $m = 0.30$.

Two parameter sets, listed in Table 8.2 and referred to as ‘set 1’ and ‘set 2’, were chosen for the anisotropic model. Both sets were applied with α reducing to 1.0 as the deviatoric strain increased from $E_{d,min}$ to $E_{d,max}$. Table 8.3 summarizes the ratios $n'$, $m'$, $n$ and $m$ for these sets. They were calculated for small strains (i.e. $E_d < E_{d,min}$) as they change with strain level. The first set produces approximately the same drained ratios of $n'$ and $m'$ as adopted by Addenbrooke et al. (1997). This leads to an anisotropic scale factor of $\alpha = \sqrt{n'} = 1.265$. The undrained ratio of $n = 1.18$ is lower than $n = 1.41$ which was calculated from their parameters. It is also below the value of $n = 1.84$ given by Gibson (1974). In contrast the
ratio \( m = 0.33 \) is higher than in their work and closer to the value of 0.38 reported by Gibson (1974) for London Clay.

The second set was chosen in order to reduce the undrained ratio \( m \) close to a value adopted by Lee & Rowe (1989) and bringing \( n \) close to the ratio reported by Gibson (1974). This was achieved by increasing the anisotropy factor to \( \alpha = 2.5 \). This high value for London Clay is not supported by any literature. Both Simpson et al. (1996) and Jovičić & Coop (1998) report ratios of approximately \( \frac{G_{hh}}{G_{hv}} = \alpha = 1.5 \). The results of this parameter set can be seen, however, as an extreme example of how anisotropy affects tunnel induced settlement predictions.

As described in Section B.4.1 the parameters \( \tilde{A} \) and \( \tilde{B} \) were calculated to give approximately the same modified shear modulus \( G^* \) (defined in Equation B.11) as \( G \) obtained for the isotropic model. These calculations were carried out for small strains, i.e. \( E_d \leq E_{d,\text{min}} \).

These parameters were then applied to single element analyses simulating an undrained triaxial extension test. The initial stress within the sample was isotropic with \( p' = 750 \) kPa and the test was simulated by prescribing displacement at the top of the element (i.e. they were strain controlled). The failure criterion was described by a Mohr-Coulomb model as outlined in Section 3.3.2. When comparing the results\(^3\) of these single element analyses with laboratory data it was found that the predictions could be improved by adjusting \( \tilde{A} \) and \( \tilde{B} \) while keeping the other parameters unchanged. The input parameters presented previously in Table 8.2 are these adjusted parameters and the results of the triaxial extension study are presented in Figure 8.9a and b. The triaxial deviatoric stress, defined as

\[
q = \sigma_{ax} - \sigma_r \tag{8.5}
\]

where \( \sigma_{ax} \) and \( \sigma_r \) are the axial and radial stress in the sample respectively; is plotted against \( \epsilon_{ax} \). The results of the isotropic model are referred to as ‘M1’ while ‘M2’ denotes data from the anisotropic analyses.

The first plot shows the strains up to \( \epsilon_{ax} = 0.01\% \). The results of a similar analysis using the isotropic model are also included together with laboratory results reported by Addenbrooke et al. (1997). For this strain range the results of parameter set 1 are in better

\(^3\)Note that the soil mechanics sign convention with compression defined positive is adopted when presenting data of these tests.
agreement with the test data than the curve for the isotropic model. The stresses calculated from parameter set 2 over-predict the test data.

Figure 8.9b shows the results for a strain range up to $\epsilon_{ax} = 0.4\%$ which corresponds to the upper limit of non-linear elastic behaviour of $E_{d,\text{max}} = 0.69\%$. Different curves are given for deviatoric stress (black symbols) and excess pore water pressure (cross or hollow symbols). For strains $\epsilon_{ax} \geq 0.2\%$ the curves of both parameter sets are slightly below the laboratory data while the isotropic model over-predicts the development of deviatoric strain. The pore water pressure curves in this plot highlight the anisotropic behaviour of sets 1 and 2. High excess pore pressure are generated for set 2 due to the coupling of deviatoric and volumetric strain.

Figure 8.10 show the stiffness-strain curves for the two parameter sets together with the data for the original isotropic model. It can be seen that due to the change in $\alpha$ with increasing deviatoric strain the new model becomes isotropic as corresponding curves of $E_h$ and $E_v$ coincide for $E_d \geq E_{d,\text{max}}$ (Figure 8.10a). A similar behaviour can be found for the shear modulus were $G_{hh} = G_{hv}$ for large strains (Figure 8.10b).

The graphs show, that higher values of $\alpha$ lead to higher values of $E_h$ and $G_{hh}$. One would expect that $E_v$ and $G_{hv}$ reduce with $\alpha$ in order to maintain the same level of modified shear modulus $K^*$. However, Figure 8.10 shows a higher $E_v$-curve for set 1 ($\alpha = 1.265$) compared with results from the isotropic model. For $G_{hv}$ the parameter set 2 produces a higher curve
Chapter 8. Prediction of three-dimensional greenfield settlement  

Section 8.3

Figure 8.10: Stiffness-strain curves for isotropic (M1) and anisotropic (M2) soil models.
than set 1 which has a lower value of $\alpha$. This behaviour is due to the adjustment of $\tilde{A}$ and $\tilde{B}$ to give better predictions when applied to triaxial extension test analyses.

8.3.3.2 Results

This section presents results of two plane strain analyses carried out with the 2 anisotropic parameter sets and compares them with the isotropic calculation. Additionally a fully 3D analysis was performed adopting the anisotropic model with parameter set 2.

Figure 8.11 summarizes the 2D results. The graphs are similar to those shown in Figure 8.7 including field data from the St. James’s Park greenfield site (Nyren, 1998). Figure 8.11a plots the transverse settlement trough measured at the site when the tunnel face was just beneath it. The settlement curves obtained from plane strain FE analyses were chosen to give a comparable volume loss to $V_L = 1.6\%$ calculated from the field data. For the anisotropic FE study values of $V_L = 1.73\%$ and 1.65\% were obtained for parameter set 1 and 2, respectively (see footnote 1 on Page 224 about the calculation of $V_L$). Both results were taken from increment 7, equivalent to 47\% of unloading. This is one increment earlier than when a volume loss $V_L = 1.7\%$ was achieved in the isotropic plane strain analysis (M1) which is also included in the graph.

It can be seen that the use of parameter set 1 brings only little improvement to the wide
settlement trough obtained from the isotropic case. The increase in maximum settlement is less than 1mm and the curve shows the same settlement at the remote boundary at \( x = 80m \). The settlement curve calculated with parameter set 2 is slightly narrower but still shows settlement at 80m distance to the tunnel centre line.

The graph shown in Figure 8.11b presents similar results for field data at the end of the immediate settlement response. At this stage the distance between tunnel face and monitoring site was 41m. The FE results show a volume loss of \( V_L = 3.3\% \) and \( V_L = 3.6\% \) for parameter set 1 and 2, respectively, compared to \( V_L = 3.3\% \) calculated from the field data. These values were achieved at increment 10 (of 15 excavation increments, 67% of unloading) for parameter set 1 and increment 9 (60%) for parameter set 2 while in the isotropic analysis the volume loss of \( V_L = 3.3\% \) was reached in increment 12 (80%). The fact that similar levels of volume loss are achieved at earlier increments indicates that volume loss increases with anisotropy.

The results show the same trend as the previous graph. The high anisotropy of parameter set 2 leads to the deepest and narrowest of the three FE settlement curves. However, its maximum settlement is 5mm less than that of the field data. Parameter set 1 with its lower degree of anisotropy only improves slightly the wide settlement trough from the isotropic analysis. Compared with the field data, all FE curves are too wide, extending to the remote boundary at a distance of 80m from the tunnel centre line.

The trend is confirmed in Figure 8.12 which shows the development of volume loss (calculated over the whole mesh width) for the three plane strain analyses included in the previous figure. No tunnel lining was installed in these analyses.

The results show a slightly higher volume loss for parameter set 1 (\( V_L = 7.9\% \) for increment 15) compared with the isotropic analysis (5.3%) but reveals a vast increase for parameter set 2 leading to \( V_L = 18.5\% \) in the last increment. For plane strain analyses, where the results are chosen from an increment according to a specified volume loss, the different \( V_L \)-curves are of minor importance. However, the following paragraphs will present results from a fully 3D analysis where the influence of the high volume loss becomes more significant.

Figure 8.13 shows the settlement results from Figure 8.11b normalized against maximum settlement. The figure highlights the trend of narrower settlement troughs with increasing degree of soil anisotropy. The normalized settlement trough of parameter set 2 with its high anisotropy scale factor of \( \alpha = 2.5 \) coincides with the normalized field data within a
zone of approximately $x = 12m$ distance from the tunnel centre line. Outside this zone, the normalized settlement predictions obtained from parameter set 2 are higher than the field data leading to a settlement trough which is too wide, albeit narrower than the curves calculated for parameter set 1 ($\alpha = 1.265$) and for the isotropic case.

As parameter set 2 gave the best results of all plane strain analyses shown in Figures 8.11 and 8.13 it was adopted in a fully 3D greenfield analysis. The boundary conditions of the 3D mesh were outlined in Section 8.3.1. Figure 8.14 shows the longitudinal settlement profiles for different tunnel face positions which are indicated by the arrows. This graph is similar to Figure 8.4 which presented 3D results from the isotropic analysis. The vertical settlement calculated for the anisotropic model is nearly one order of magnitude higher than that of the isotropic case, indicating a significantly higher volume loss. Over the first increments a longitudinal settlement profile develops which has a similar shape as the cumulative error curve shown in Figure 2.4 on Page 31. It was found for the isotropic analysis that the settlement behaviour does not follow this expected trend for later increments (see Figure 8.4). Instead some hogging developed behind the tunnel face. This rear hogging zone is magnified in the anisotropic case. Its development starts approximately at a face position of $y = -50m$.

Previously it was shown that a high degree of anisotropy leads to a narrower transverse settlement trough. A similar trend can be expected for the longitudinal curve. With a
narrower and steeper curve, steady state conditions should develop earlier than for the wide trough obtained from isotropic analysis. Clearly, no steady state conditions are reached in Figure 8.14 but it can be assumed that the settlement profile is more advanced towards a steady state than its isotropic counterpart is.

It is therefore arguable that steady state condition will be reached somewhere between the face position and the rear hogging zone. This situation is shown in Figure 8.15 where a hypothetical settlement profile for a tunnel further advanced than modelled in the FE analysis is sketched. The increased settlement towards the \( y = 0 \)m boundary can be explained because it is the start position of the tunnel at the begin of the analysis. Due to the symmetry condition at this vertical plane, tunnel construction essentially commences simultaneously in both negative and positive \( y \)-directions (although only the negative \( y \)-part is modelled) leading to additional settlement at the beginning of the analysis. A similar longitudinal settlement profile, including the hogging zone, was presented by Vermeer et al. (2002) who performed fully 3D tunnel analysis for a \( K_0 = 0.65 \) situation (see Figure 2.17 on Page 51). In their ground conditions, steady state conditions were established approximately \( 5 \times D \)
behind the face after the tunnel was constructed over a length of approximately $10 \times D$ (with a tunnel diameter of $D = 8m$ and $z_0 = 16m$). Present analysis indicates that a much greater length of tunnel is required to reach steady state conditions. This problem will be addressed in more detail in the next chapter (Section 9.2.1).

Figure 8.16 normalizes the isotropic and anisotropic longitudinal settlement troughs for a tunnel face position of $y = -100m$ against their maximum value $S_{v,y=0m}$ at $y = 0m$. It can be seen that the settlement from the anisotropic calculation is steeper and shows relatively
less settlement on the remote boundary at $y = -155m$. The graph also shows field data from the St. James’s Park monitoring site. The data are plotted such that the face position corresponds to $y = -100m$ to compare them with the FE results. It can be seen that both FE curves are too wide and, furthermore, have a different shape compared with the field data.

The fact that the longitudinal anisotropic settlement curve is steeper and narrower than the isotropic one supports the above assumption that the anisotropic analysis is more advanced in establishing steady state conditions behind the tunnel face. The settlement at $y = 0m$ is exceptional high. It is therefore not reasonable to normalize settlement against this value. Instead, the results should be normalized against a value which can be expected to lie close to the steady state settlement proposed in Figure 8.15. The third curve in this graph shows the result of this normalization in which the anisotropic curve was normalized against its settlement $S_{v,y=-50m}$ at $y = -50m$. It has to be pointed out that this $y$-position is an arbitrary choice as clearly no steady state conditions are established. However, it comes close to the magnitude of settlement which can be expected in steady state conditions. This normalization is in much better agreement with the normalized field data than the previous two curves and therefore further supports the above assumption of the development of steady state conditions.

Figure 8.17 summarizes the transverse settlement profiles. As for the isotropic 3D analysis (results from which are also shown) the transverse monitoring section was at $y = -50m$ and results are taken for the last increment i.e. a face position of $y = -100m$. The field data are for the end of the immediate settlement response at St. James’s Park and where shown previously. The anisotropic plane strain result using parameter set 2 are also given. All results are normalized against maximum settlement as the volume loss varied between $V_L = 2.1\%$ for the isotropic 3D FE analysis and $V_L = 9.5\%$ for the anisotropic one.

It can be seen that the anisotropic 3D analysis is in reasonable agreement with the normalized field data. The curve is much steeper than the one obtained from the isotropic 3D analysis. Furthermore, the graph shows, that there is a slightly bigger difference between 2D and 3D results for the anisotropic cases than observed for the isotropic analysis (shown in Figure 8.8). This could be an effect of the significant difference in volume loss between the 2D and 3D analyses.

This study shows that including anisotropy improves the shape of the settlement trough.
as both transverse and longitudinal settlement profiles become steeper and narrower. However, as the results for parameter set 1 indicate, the effect is small for degrees of anisotropy measured in London Clay. A higher level of anisotropy improves the results further as seen for parameter set 2 which uses an anisotropic scale factor of $\alpha = 2.5$ which cannot be justified for London Clay. By increasing the horizontal soil stiffness while reducing the vertical stiffness leads to significantly higher volume losses. The results therefore have to be normalized in order to compare them with field measurements. No steady state was achieved in the fully 3D anisotropic analysis although the results indicate that the settlement behaviour in the last increment was closer to this situation than in the isotropic one.

### 8.4 Conclusions

A suite of 2D and 3D FE analyses was performed to investigate the influence of 3D effects on the tunnel induced surface settlement trough. The 3D excavation process was modelled by successive removal of elements in front of the tunnel while successively installing lining elements behind the tunnel face (step-by-step approach).

Tunnel work of the Jubilee Line Extension (JLE) beneath St. James’s Park, London, was modelled to study the performance of a fully 3D greenfield analyses. It was found that the longitudinal settlement trough did not develop steady state conditions. The curve was too wide and settlement was obtained on the vertical boundaries during the entire analysis although the total length of tunnel construction was chosen as $21.0 \times D$.
Chapter 8. Prediction of three-dimensional greenfield settlement

Section 8.4

The shape of the transverse settlement trough was similar to the curve obtained from equivalent plane strain analysis. Both results were too wide when compared with field data. It was concluded that 3D analysis does not improve the shape of transverse settlement troughs which are generally too wide in high $K_0$-regimes.

As suggested by several authors, soil anisotropy was included in the study to investigate if this additional soil characteristic, in combination with 3D effects, can improve results. For this purpose a transversely anisotropic soil model (proposed by Graham & Houlsby, 1983) was included into ICFEP and combined with the small strain stiffness formulation (Jardine et al., 1986). Plane strain results show little improvement in the transverse settlement trough when a level of anisotropy appropriate for London Clay was applied. When soil parameters were chosen to give an unrealistic high degree of anisotropy the settlement curve was further improved. However, the volume loss obtained in these analyses was unrealistically high.

A fully 3D analysis was carried out adopting this high level of anisotropy. As for the isotropic analysis no steady state conditions were established during the analysis. The longitudinal settlement trough is steeper and narrower than the isotropic curve. It is therefore arguable that in the last increment the anisotropic analysis is closer to such steady state conditions than the isotropic one. Towards the end of the analysis the longitudinal settlement developed hogging behind the tunnel face where normally sagging deformation is expected. This rear hogging zone can be explained from the boundary conditions at the start of tunnel construction. It has been concluded that for further tunnel progress steady state conditions might develop behind this rear hogging zone.

The transverse settlement profile of the anisotropic 3D analysis is narrower than all other corresponding results obtained within this study. However, the volume loss is unrealistically high.

Although the anisotropic 3D analysis improves the shape of the settlement curve it does not solve the problems observed in the isotropic analysis (no steady state, wide settlement trough compared with field data). Its volume loss is too high. Furthermore the degree of anisotropy adopted in the 3D analysis cannot be justified for London Clay. Instead the results from this analysis can be seen as an extreme example of anisotropy. When realistic parameters are chosen, the differences when compared to the isotropic results remain small.

This study showed that neither 3D effects nor soil anisotropy can be accounted for the
too wide settlement curves obtained from FE tunnel analysis in a high $K_0$-regime.
Chapter 9

The influence of step-by-step tunnel excavation

9.1 Introduction

This chapter investigates how 3D tunnel construction affects the deformation behaviour of an existing surface structure. For this purpose the tunnel construction was modelled in a step-by-step approach as outlined in the previous chapter. The tunnel diameter was $D = 4.146$ m with a tunnel depth of $z_0 = 20$ m. The building is modelled weightless and no relative movement between soil and structure was allowed. The building cases are therefore comparable with the plane strain situations analysed in Chapter 4. The soil models used to represent London Clay and the initial stress profile were described in Sections 3.3 and 3.4.1.

This chapter focuses on both the transverse and the longitudinal building behaviour. The transverse building deformation is compared with corresponding results of plane strain analyses. The development of strain and deflection in the longitudinal direction is studied and compared with similar results from greenfield conditions. Finally, twist deformation is investigated using case studies from the JLE construction and results from the FE analyses.

It has been shown in the previous Chapter that the predicted greenfield settlement obtained from fully 3D analysis is too wide both in the transverse and the longitudinal directions. The incorporation of soil anisotropy brought only little improvement when adopting soil parameters which are appropriate for London Clay. It was therefore decided to carry out all 3D...
analyses presented in this chapter with the isotropic soil model. This enables the results from
the following studies to be compared with the plane strain analyses presented in Chapters 4
to 6.

As a consequence of the settlement trough being too wide, no steady state conditions
were reached for the adopted mesh dimension. It was suggested that in order to achieve
steady state conditions the mesh could be extended further in the \( y \)-direction to increase
the total length of tunnel excavation. There are, however, limitations in computational time
and storage required for fully 3D analyses. Care must therefore be taken when choosing the
distance between the mesh boundaries in order minimize their influence on the results while
keeping reasonable calculation times.

A similar problem arises when specifying the excavation length \( L_{\text{exc}} \). This length should
be chosen so that a volume loss of approximately \( V_L = 1.5\% \) develops during the analysis.
A small excavation length reduces \( V_L \) but increases both the number of elements and the
number of increments over which the tunnel construction has to be simulated.

The following section will focus on these crucial parameters for FE analyses and will
discuss which values are adopted in the analyses presented subsequently.

\section*{9.2 Parameters for FE analysis}

\subsection*{9.2.1 Distances to mesh boundaries}

Three different greenfield meshes were analysed in order to investigate the influence of the lon-
gitudinal distance to the mesh boundaries. All meshes had the same cross section \( (z_0 = 20\text{m}) \)
and varied only in their longitudinal dimension. The excavation length was \( L_{\text{exc}} = 2.5\text{m} \).
The three meshes had the following geometries in the \( y \)-direction:

- Mesh 1: Total length: 50m, including tunnel excavation over 30m.
- Mesh 2: Total length: 105m, including tunnel excavation over 50m.
- Mesh 3: Total length: 135m, including tunnel excavation over 70m.

The following variables will be used:
Chapter 9. The influence of step-by-step tunnel excavation  
Section 9.2

Figure 9.1: Influence of mesh dimension, varying $L_{\text{soil}}$: (a) $L_{\text{tunnel}} = 30$ m; (b) $L_{\text{tunnel}} = 50$ m.

$L_{\text{tunnel}}$ describes the distance between the tunnel face and the $y = 0$ m boundary. This is the length over which the tunnel has been constructed since the beginning of the analysis. $L_{\text{soil}}$ is a measure of the longitudinal distance between the tunnel face and the remote vertical boundary. It is the length of remaining soil in front of the tunnel face.

Figure 9.1 compares results from the different meshes for situations of equal $L_{\text{tunnel}}$ but different $L_{\text{soil}}$ whereas Figure 9.2 compares results from analyses with the same $L_{\text{soil}}$ but different $L_{\text{tunnel}}$.

In Figure 9.1 the horizontal axis is expressed as distance to the tunnel face. Positive values indicate a position behind the face (where the tunnel is constructed) while negative values indicate a position in front of the face. Figure 9.1a shows a situation in which the
tunnel is constructed over $L_{\text{tunnel}} = 30\text{m}$. The longitudinal distance between tunnel face and remote vertical boundary for the three cases is $L_{\text{soil}} = 20\text{m}$ (Mesh 1), 75m (Mesh 2) and 105m (Mesh 3). It can be seen that the settlement curves for Mesh 2 and 3 coincide despite the difference in $L_{\text{soil}}$ of 30m. However, for a small mesh, such as Mesh 1, the short $L_{\text{soil}}$ affects the results over the whole mesh length.

Figure 9.1b presents similar results for a tunnel construction over $L_{\text{tunnel}} = 50\text{m}$. Only $L_{\text{soil}} = 55\text{m}$ and 85m of soil remain in front of the tunnel face for Mesh 2 and 3 respectively. Mesh 1 was too short for this comparison and is therefore not included in the graph. The curves for Mesh 2 and 3 are in good agreement. This demonstrates that $L_{\text{soil}} = 55\text{m}$ is a sufficient distance between tunnel face in the last excavation step and remote boundary and consequently this dimension was chosen for the parametric study presented in Section 9.3.
Chapter 9. The influence of step-by-step tunnel excavation  
Section 9.2

The next figure investigates the influence of $L_{\text{tunnel}}$. Figure 9.2a presents settlement results for Mesh 2 and 3 with $L_{\text{tunnel}} = 30\text{m}$ and $60\text{m}$, respectively. It can be seen that the settlement in front of the tunnel face is not much affected by the different length of constructed tunnel. Towards the $y = 0$ boundary, the settlement curves, however, diverge. A smaller $L_{\text{tunnel}}$ leads to less settlement. This difference becomes significant approximately 10m behind the tunnel face (i.e. over the first 20m of tunnel construction for Mesh 2).

If the tunnel is constructed for another 10m (as shown in Figure 9.2b) the settlement still diverges over those 20m at the beginning of Mesh 2. Within 20m behind the face the two profiles are, however, in relatively good agreement.

From the above study the longitudinal distance between tunnel face position at the end of the analysis and the remote boundary, $L_{\text{soil}}$ was chosen to 55m for all subsequent analyses. Furthermore, it was concluded that the longitudinal distance between a building and the start of tunnel construction should not be less than $L_{\text{tunnel}} = 30\text{m}$ in order to minimize boundary effects.

9.2.2 Length of tunnel construction

The previous section focused on the distances between the last position of the tunnel face and the remote boundary as well as between the building front and the $y = 0\text{m}$ mesh boundary. No conclusions were drawn about the total length of tunnel construction (i.e. the position of the tunnel face at the end of the analysis). It has been pointed out earlier that no steady state conditions develop during the 3D analyses presented in this thesis. Consequently, one could argue that the length of tunnel construction has to be increased leading to a larger number of elements and increments. For reasons of computational time and resources the number of elements cannot be increased beyond the mesh dimensions presented in Chapter 8. Furthermore, the dimensions adopted in mesh 3 in the previous section are already larger compared to those from recent studies by other authors, summarized in Table 2.1 (Page 48). Mesh 3 is over $32 \times D$ long with a total length of tunnel construction of over $16 \times D$. The longest tunnel length for step-by-step simulation listed in Table 2.1 is 80m ($10 \times D$) from a study by Vermeer et al. (2002). Modelling the tunnel construction in a low $K_0$-regime they showed that steady state conditions were reached approximately $5 \times D$ behind the tunnel face after the tunnel was constructed over a length of approximately $10 \times D$. 

246
Figure 9.3: Development of longitudinal settlement profile with tunnel progress for greenfield conditions with $z_0 = 20$ m and $K_0 = 1.5$.

Figure 9.4: Development of longitudinal settlement profile with tunnel progress for greenfield conditions with $z_0 = 20$ m and $K_0 = 0.5$.

For the $K_0 = 1.5$ situation modelled in this thesis such mesh dimensions are not sufficient as shown in Figure 9.3 which plots the longitudinal settlement profiles for different tunnel face positions for a greenfield analysis adopting mesh 3. The graph is similar to that shown in Figure 8.4 (Page 222). Clearly, no steady state conditions were achieved during the analysis as no horizontal settlement profile has developed behind the tunnel face. In addition, additional
settlement occur over the whole mesh length during the entire analysis.

Comparing these results with the results presented by Vermeer et al. (2002) indicates that the different values of lateral earth pressure coefficient at rest $K_0$ may be the reason for the different settlement behaviour. To investigate the role of $K_0$ the analysis presented in Figure 9.3 was repeated with a value of $K_0 = 0.5$ instead of 1.5. Figure 9.4 presents the longitudinal settlement profiles from this analysis. The curves in this graph show a steeper gradient than those in the previous plot, indicating narrower settlement troughs for the low $K_0$-regime compared with $K_0 = 1.5$. It can be seen, that for $K_0 = 0.5$ a horizontal plane of vertical settlement profile develops approximately 30m ($7 \times D$) behind the tunnel face from a face position of approximately $y = -50m$ ($12 \times D$). These distances are slightly higher than the values reported by Vermeer et al. (2002) who adopted a $K_0 = 0.66$ and modelled a larger tunnel diameter.

Figure 9.4 also shows additional settlement at the $y = 0$m boundary for the last 10m of tunnel excavation. Strictly spoken no steady-state conditions are therefore achieved. However, the additional settlement obtained at $y = 0$m during the tunnel advance from $y = -60$m to -70m is 0.1mm and lower than the additional settlement of 0.16mm obtained in the $K_0 = 1.5$ study over the same increments. Considering that the $K_0 = 0.5$ situation yields larger settlement along the $x = 0$m profile (noting the different vertical scales between Figures 9.3 and 9.4) this change becomes even less significant: for the $K_0 = 0.5$ case the additional settlement over the last 10m of tunnel excavation is only 0.6% of the settlement at the $y = 0$m boundary at the end of the analysis. With a $K_0 = 1.5$ this increase was 2.3%.

This study shows that for low values of $K_0$ a horizontal settlement profile develops behind the tunnel face and steady-state conditions are nearly achieved. This is, however, not the case for the $K_0 = 1.5$ scenario adopted within this thesis. The total length of tunnel construction should therefore be as long as possible depending on the limitation imposed by computational resources. How much of these are required depends not only on the tunnel length but also on the excavation length $L_{exc}$. Its influence on the tunnel-soil-structure interaction will be studied in the following section.


### Table 9.1: Mesh details and calculation time for analyses with different $L_{exc}$.

<table>
<thead>
<tr>
<th>$L_{exc}$</th>
<th>Increments</th>
<th>Nodes</th>
<th>Elements</th>
<th>Calculation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0m</td>
<td>40</td>
<td>45947</td>
<td>10212</td>
<td>291.3h</td>
</tr>
<tr>
<td>2.5m</td>
<td>32</td>
<td>38083</td>
<td>8436</td>
<td>194.9h</td>
</tr>
</tbody>
</table>

**9.2.3 Excavation length**

After having specified the longitudinal distances to the mesh boundaries, the excavation length $L_{exc}$ is another crucial parameter to be chosen. Two analyses were performed with the meshes having identical dimensions (with the tunnel being constructed over 50m and 40m of soil remaining at the end of the analyses). Mesh 4 had an excavation length of $L_{exc} = 2.0m$ while in Mesh 5 the tunnel was excavated over $L_{exc} = 2.5m$ per increment. A 1-storey building was included in the analysis. Its plan dimension was 100m $\times$ 20m in the transverse and longitudinal directions respectively. It had no eccentricity towards the tunnel and therefore only half of this symmetrical problem was modelled. In longitudinal direction the building was located between $y = -30m$ and $-50m$.

Table 9.1 summarizes details of the meshes and calculation times for these analyses. In both meshes the tunnel had a total length of 80m. Consequently the construction was simulated over 40 increments for Mesh 4 in contrast to only 32 increments required for Mesh 5. The different excavation lengths have significant implications on the mesh size. Mesh 4 was modelled with 10212 solid elements. This number reduced with increasing $L_{exc}$ and only 8436 solid elements were required to model the same geometry for Mesh 5. Both the lower number of elements and increments lead to reduced calculation times. Tunnel construction using $L_{exc} = 2.0m$ was calculated over 291.3h while it took only 194.9h to simulate the same problem with the increased excavation length – a difference of approximately 4 days.

The next figures compare the development of volume loss, deflection ratio and horizontal strain for both analyses. All results were taken for the transverse centre line of the building at $y = -40m$. Figure 9.5 shows the increase of volume loss with tunnel advance. Both $L_{exc}$-cases give a similar response until the tunnel face reaches the monitoring section ($y = -40m$). From this point the $L_{exc} = 2.5m$ analysis shows higher volume loss leading to $V_L = 2.29\%$ when the tunnel face reaches $y = -80m$. The equivalent value obtained for the $L_{exc} = 2.0m$ excavation
is \( V_L = 2.07\% \).

Similar behaviour can be found in Figure 9.6 which plots (a) \( DR_{sag} \) and (b) \( DR_{hog} \) against position of the tunnel face. Both excavation lengths give a similar response until the tunnel construction reaches the position of the monitoring section but then diverge. When plotting the same results against volume loss, as shown in Figure 9.7a and b, both pairs of results are in better agreement. Only for the last data points does the \( L_{exc} = 2.0m \) case show slightly smaller (absolute) values of deflection ratio.
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.2

For maximum horizontal compressive strain, $\epsilon_{hc}$, plotted against tunnel advance (Figure 9.8) the curves have a similar shape as those in the corresponding $DR_{sag}$ and $DR_{hog}$ graphs. When $\epsilon_{hc}$ is plotted against $V_L$, the results of both $L_{exc}$-cases are in good agreement.

This study shows that a variation in excavation length has little influence on the deformation behaviour of a transverse profile until the tunnel face reaches this section. With the tunnel construction moving on behind this location a smaller $L_{exc}$ leads to lower values of $V_L$. For $L_{exc} = 2.0$ the volume loss reached at the end of the analysis was $V_L = 2.07\%$ which is higher than $V_L = 1.5\%$ which was applied in the plane strain analyses. Because of the large computation time involved it was not possible to reduce $L_{exc}$ further in order to decrease $V_L$ to its plane strain value. However, when comparing results obtained from fully 3D analyses with 2D data their volume loss should be of approximately the same size.

To achieve this, results from the transverse profiles will be adjusted to a volume loss of $V_L = 1.5\%$. Deformation criteria will be adjusted by linearly interpolating volume losses from the two results either side $V_L = 1.5\%$. Figure 9.10 illustrates this principle for $DR_{sag}$. The figure shows the two result-curves for $L_{exc} = 2.0m$ and $2.5m$. As pointed out before, both curves are in good agreement until a volume loss of approximately $V_L = 1.8\%$. It is therefore possible to interpolate a value of $DR_{sag}$ for a volume loss of $V_L = 1.5\%$. This point is marked by the solid diamond.

This figure also extends both ICFEP curves to an assumed final value of volume loss. This situation would be achieved at the end of the immediate settlement response. As pointed out earlier such steady state conditions were not established during the analyses. A hypothetical curve for a smaller excavation length leading to a final volume loss of $V_L = 1.5\%$ is also
sketched. This is the situation which should ideally be modelled in order to be consistent with the plane strain results presented in earlier chapters of this thesis. This graph demonstrates that the interpolation of deformation criteria to a common volume loss is likely to over-predict the results and is, therefore, conservative. This effect is less significant for the $\epsilon_{hc}$-curves which do not diverge for different excavation lengths.

When comparing settlement troughs from different increments with corresponding plane strain results a simplified adjustment method was chosen by linearly scaling these data from their $V_L$ to $V_L = 1.5\%$. For results close to $V_L = 1.5\%$ the difference between interpolation and scaling was found to be negligible while for early increments with small volume losses scaling can under-estimate results. This method was therefore only used to compare shapes of settlement profiles with each other.

In order to achieve reasonable calculation times the excavation length for all subsequent analyses was fixed at $L_{exc} = 2.5m$. In Section 9.2.1 a distance between the building and the tunnel start boundary of 30m was proposed. Building lengths of $L = 20m$ and 30m are included in the following parametric study. A further 40m of excavation was performed behind the building leading to a total tunnel length of either 90m or 100m and each analysis is modelled over 36 or 40 increments (for the $L = 20m$ and 30m case respectively).

![Figure 9.10: Interpolation of deformation criteria to a common volume loss.](image-url)
9.3 Parametric study

A parametric study was performed in order to investigate building behaviour when subjected to a step-by-step tunnel construction. Different building geometries were included in this study. The buildings were $20m \times 100m$, $30m \times 100m$ and $20m \times 66m$ in plan with the longer dimension being perpendicular to the tunnel axis. For the first building geometry tunnel depths of $z_0 = 20m$ and $34m$ were analysed while for the other building geometries only a $20m$ deep tunnel was modelled. The stiffness of the buildings was varied and values equivalent to 1, 3 and 5 storeys were investigated. In addition for the $20m \times 100m$ building geometry a 10-storey case was also included.

The mesh for an analysis with a $30m \times 100m$ structure is shown in Figure 9.11. The structure is indicated by the dotted pattern with its corner (or mid-side) nodes denoted to $a$, $b$, $c$ and $d$. Only half of the problem is modelled. The boundary conditions with vertical planes being planes of symmetry was described previously (see Section 7.2).

The mesh in the $x$-$y$ plane is a simplified version of the 2D mesh shown in Figure 3.8. To compare results obtained from the simplified mesh with data from the original analyses a plane strain study was performed on both meshes analysing a $100m$ wide structure with a range of different stiffnesses.

When comparing the deformation criteria between the two meshes the maximum difference was found to be 15% (for $D\rho_{hog}$ of a 5-storey structure). For the corresponding modification factors the difference was smaller and was below 10% for all cases.

In the mesh shown in Figure 9.11 the length of final tunnel construction is 100m. Over this length the $y$-dimension of all elements is $L_{exc}$. The only exception are the elements beneath the front and rear edge of the surface structure where the mesh was refined. The $y$-dimension of these element slices is $L_{exc}/2$, hence two slices of elements within the tunnel boundary had to be excavated in the corresponding increments.

Greenfield analyses were performed for tunnel depths of $z_0 = 20m$ and $34m$. Greenfield values for the 3 building geometries modelled above the $20m$ deep tunnel were all taken from the same analysis.

The following subsections will present the results of this parametric study. First the behaviour of deflection ratio and horizontal strain is studied along profiles on the transverse
centre line of the building. These data can be compared with results from corresponding plane strain analyses carried out with the plane strain version of the 3D mesh. Secondly the deformation behaviour along the longitudinal centre line of the building (above the tunnel centre line) is investigated. Finally fully 3D building deformation will be presented by discussing twist which develops within the structure during tunnel excavation.

**Figure 9.11:** Finite element mesh for fully 3D analyses of tunnel-soil-building interaction.
Figure 9.12: Transverse (a) settlement and (b) horizontal displacement for different tunnel face position on transverse centre line of 3-storey building (20m × 100m, $z_0 = 20$ m). Results are scaled to $V_L = 1.5\%$.

### 9.3.1 Transverse behaviour

Figure 9.12a and b show the development of the transverse settlement ($S_v$) trough and horizontal $S_{hrr}$-displacement, respectively, for a 20m × 100m 3-storey structure. The tunnel depth was $z_0 = 20$ m. All results are scaled to a volume loss of $V_L = 1.5\%$. Different curves are given for different tunnel face positions. For vertical settlement (Figure 9.12a) the relative shape of the trough does not change significantly after the tunnel face has reached $y = -30$ m. The graph also includes results from a corresponding plane strain analysis, marked with cross symbols. It can be seen that these results are in good agreement with the data from the fully 3D calculation.
A similar trend can be found for the development of transverse horizontal displacement, shown in Figure 9.12b. As in the previous graph, results are scaled to a common volume loss of $V_L = 1.5\%$. The graph shows that early increments result in relative high horizontal displacement. However, after the tunnel face reaches a position of approximately -40m, the relative shape of the horizontal displacement curves does not change significantly. Again, corresponding plane strain results are included which lie near to the results obtained from the 3D analysis.

These results demonstrate that from a certain tunnel face position (and therefore volume loss) the relative shapes of settlement and horizontal displacement along a transverse section do not change significantly. This is further illustrated in Figure 9.13 where $i$, the position of the point of inflection, is plotted against face position. The value of $i$ was calculated from the nodal displacement values, as explained in Section 3.4.5. Results for greenfield conditions and for buildings with different number of storeys for the building geometry of the previous figure are included in this plot. The arrows indicate plane strain results obtained from corresponding analyses. During the first excavation increments, $i$ reduces for all building stiffness cases indicating that the settlement trough becomes narrower. For buildings, $i$ remains constant after the tunnel face reaches $y = -30m$. This is in agreement with the results presented in Figure 9.12a which showed similar adjusted settlement troughs from this

![Figure 9.13](image1.png)  ![Figure 9.14](image2.png)

**Figure 9.13:** Development of $i$ with tunnel progress. Results are for the 20m × 100m geometry, $z_0 = 20m$.  

**Figure 9.14:** Development of $V_L$ with tunnel progress. Results are for the 20m × 100m geometry, $z_0 = 20m$. 

256
Chapter 9. The influence of step-by-step tunnel excavation

9.3

Tunnel face position. The greenfield case shows initially the biggest decrease, reaching its minimum at a face position of \( y = -40 \text{m} \). It then increases slightly but remains constant beyond \(-60 \text{m}\). Figure 9.12 shows that, at the end of the analysis, 3D settlement curves (adjusted to \( V_L = 1.5\% \)) and plane strain results are in good agreement. Figure 9.13 presents a more accurate comparison in plotting \( i \) from corresponding plane strain analyses as arrows on the right hand side of the graph. All arrows are slightly below their corresponding 3D results. This shows that the 3D building settlement troughs are wider than similar curves obtained from plane strain analysis.

Figure 9.14 shows the volume loss for the same building and greenfield cases. The graph demonstrates that a higher building stiffness leads to lower values of volume loss. At the end of analysis (face position of \( y = -90 \text{m} \)) the 10-storey structure gives a volume loss of \( V_L = 2.11\% \) which is 87.9% of the corresponding volume loss obtained from greenfield analysis.

The following subsections will investigate how the deformation criteria \( DR_{sag} \), \( DR_{hog} \) and \( \epsilon_{hc} \) develop with tunnel progress and how they compare with corresponding plane strain results. All data presented subsequently are taken from a profile on the transverse centre line of the surface structure.

9.3.1.1 Deflection ratio

In Figure 9.15a and b the increase of \( DR_{sag} \) and \( DR_{hog} \) with \( V_L \) is plotted respectively. The solid line denotes the greenfield case while dotted lines with hollow symbols indicate 3D building analyses. The curves for the different number of storeys have a similar shape as those discussed in Figure 9.7 (Page 250). Their gradient decrease for high values of \( V_L \). As found for 2D analyses, higher building stiffness leads to lower deflection ratios. This trend is more distinct for the hogging case.

The graphs also include the corresponding 2D results marked by a corresponding solid symbol (a star for the greenfield case). These results are adjusted to a volume loss of \( V_L = 1.5\% \) although the variation in \( V_L \) is small within the 2D analyses. It can be seen that the 2D and 3D results are in good agreement.

It is possible to calculate modification factors for each data point obtained from the 3D study. Therefore the results for the building analyses were adjusted to the greenfield volume loss obtained in the same increment (i.e. position of the tunnel face) of the analysis.
The adjusted values were then divided by the corresponding greenfield deformation criterion. Figure 9.16a and b shows these plots for sagging and hogging respectively. $M^{DR_{sag}}$ for all stiffness cases initially reduces as $V_L$ increases. However, from approximately $V_L = 1\%$ all curves indicate little change in $M^{DR_{sag}}$. The 2D modification factors included in the graph (solid symbols) are in good agreement with the 3D curves.

For hogging (Figure 9.16b) there is only a small decrease in $M^{DR_{hog}}$ as $V_L$ increases from 0 to 1%. Above this value the curves remain on a relatively constant level apart from the 1-storey curve which shows some further reduction followed by a slight increase. For the other stiffness cases the agreement between 2D and 3D data is good.

The ratios between 3D and 2D sagging deflection ratios and modification factors for all cases of this parametric study are summarized in Table 9.2a and b respectively. For this table the 3D $DR_{sag}$ was adjusted to 1.5% (in contrast to the results shown in Figure 9.16, which

**Figure 9.15:** Development of (a) $DR_{sag}$ and (b) $DR_{hog}$ with volume loss and comparison with plane strain results. 3D geometry: 20m × 100m, $z_0 = 20$m.

**Figure 9.16:** Development of (a) $M^{DR_{sag}}$ and (b) $M^{DR_{hog}}$ with volume loss and comparison with plane strain results. 3D geometry: 20m × 100m, $z_0 = 20$m.
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

Table 9.2: Summary of (a) $DR_{sag}$ and (b) $M^{DR_{sag}}$ of all 3D analyses.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$DR_{sag,3D} / DR_{sag,2D}$</th>
<th>sagging</th>
<th>$M^{DR_{sag}} / M^{DR_{sag,2D}}$</th>
<th>sagging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of storeys</td>
<td>GF</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>100×20</td>
<td>1.00</td>
<td>0.99</td>
<td>1.04</td>
<td>1.06</td>
</tr>
<tr>
<td>100×30</td>
<td>1.00</td>
<td>0.94</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>66×20</td>
<td>0.90</td>
<td>0.97</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>100×20, $z_0=34m$</td>
<td>1.10</td>
<td>1.02</td>
<td>1.06</td>
<td>1.08</td>
</tr>
</tbody>
</table>

(a)

Table 9.3: Summary of (a) $DR_{hog}$ and (b) $M^{DR_{hog}}$ of all 3D analyses.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$DR_{hog,3D} / DR_{hog,2D}$</th>
<th>hogging</th>
<th>$M^{DR_{hog}} / M^{DR_{hog,2D}}$</th>
<th>hogging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of storeys</td>
<td>GF</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>100×20</td>
<td>1.07</td>
<td>0.94</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td>100×30</td>
<td>1.07</td>
<td>0.86</td>
<td>0.86</td>
<td>0.98</td>
</tr>
<tr>
<td>66×20</td>
<td>0.85</td>
<td>0.51</td>
<td>0.64</td>
<td>0.73</td>
</tr>
<tr>
<td>100×20, $z_0=34m$</td>
<td>1.06</td>
<td>0.96</td>
<td>0.90</td>
<td>0.80</td>
</tr>
</tbody>
</table>

(a)

were scaled to the volume loss obtained in the same increment of the greenfield analysis) and then compared with the (also adjusted) 2D value. The modification factors were also calculated from these adjusted data. The tables show that all 3D $DR_{sag}$ lie within 10% of the corresponding 2D results. Apart from one exception (the 5-storey 66m × 20m building) the same margin can be found for the sagging modification factors.

Table 9.3 gives a similar overview for the hogging parameters. The differences found are slightly larger than those for sagging but most 3D results lie within 20% of the corresponding 2D data. It has to be pointed out that the hogging zone in the 66m wide structure is small. The hogging results of this structure are therefore subject to a relatively high scatter as discussed in Section 3.4.5. However, apart from 3 cases all ratios are smaller than unity indicating that 2D analyses overestimate hogging (i.e. are conservative).
9.3.1.2 Strain

Figure 9.17 shows the development of maximum horizontal compression within the building\(^1\). The results are for the same 20m \(\times\) 100m geometry as those in the previous section. The greenfield case is not included in this graph as its horizontal strain is one order of magnitude higher. The curves of increasing \(\epsilon_{hc}\) with \(V_L\) are similar to those displayed in Figure 9.9. The 2D results marked by the solid symbols are in very good agreement with the curves obtained from the 3D analyses.

The change of corresponding modification factors with \(V_L\) is shown in Figure 9.18. Up to approximately \(V_L = 1.5\%\) all \(M_{\epsilon_{hc}}\) curves reduce, they then remain constant as \(V_L\) further increases. The 2D results coincide well with the 3D curves. A summary of the ratio between 3D and 2D \(\epsilon_{hc}\) and \(M_{\epsilon_{hc}}\) values for all cases of this parametric study is provided in Table 9.4a and b respectively. Apart from the results of the 30m wide geometry all 3D data lie within 20\% of the corresponding 2D results. The ratios calculated for the 30m \(\times\) 100m cases are, however, significantly lower with a minimum value of \(\epsilon_{hc,3D}/\epsilon_{hc,2D} = 0.52\) for the 5-storey building stiffness.

This effect can also be seen in Figure 9.19a and b in which settlement (\(S_v\)) and horizontal displacement (\(S_{hx}\)), respectively, are plotted for the 30m \(\times\) 100m and 20m \(\times\) 100m geometry together with corresponding plane strain results. The building stiffness was equivalent to

\(^1\)No tension was obtained in any of the building cases included into this parametric study.
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

Table 9.4: Summary of (a) $\epsilon_{hc}$ and (b) $M_{hc}^c$ of all 3D analyses.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>No. of storeys</th>
<th>$\epsilon_{hc,3D}/\epsilon_{hc,2D}$</th>
<th>$M_{hc}^c/ M_{hc}^{2D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100×20</td>
<td>1</td>
<td>0.83</td>
<td>1.17</td>
</tr>
<tr>
<td>100×30</td>
<td>3</td>
<td>0.83</td>
<td>0.69</td>
</tr>
<tr>
<td>66×20</td>
<td>5</td>
<td>0.82</td>
<td>1.16</td>
</tr>
<tr>
<td>100×20, $z_0=34$m</td>
<td>10</td>
<td>1.07</td>
<td>1.11</td>
</tr>
</tbody>
</table>

3 storeys. The 3D results are taken for a face position of $y = -40m$ and all results are adjusted to a common volume loss of $V_L = 1.5\%$. It can be seen that the variation in settlement between the different geometries is small and the plane strain results lie within the range of the 3D data. The horizontal displacement (Figure 9.19b), in contrast, shows a significant difference between the $L = 30m$ and $20m$ cases. The $30m$ long structure exhibits much smaller horizontal $x$-displacements than obtained from the plane strain analysis while the $20m$ long structure is in good agreement with the plane strain results (as already shown in Figure 9.12b). The smaller horizontal displacement leads to lower compressive strains developing within the structure.

This effect is similar to the results obtained in Chapter 7. There, it was shown that an increase in the longitudinal building dimension $L$ reduces the transverse horizontal displac-

![Figure 9.19: Comparison between $L = 20m$ and $30m$ geometry: (a) settlement trough, (b) horizontal displacement. Building stiffness: 3-storeys.](image-url)
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

Figure 9.20: Longitudinal settlement profiles for different tunnel face positions. Results for 3-storey building, 20m × 100m, z₀ = 20m.

ment and strain. This effect explains why the $L = 30m$ building gives lower $\epsilon_{hc,3D}/\epsilon_{hc,2D}$ ratios than the $L = 20m$ structure (note that both cases are compared with the same plane strain results). However, this effect is stronger than expected from the results obtained in Chapter 7 where the 3D surface structure was subjected to plane strain tunnel construction.

9.3.2 Longitudinal behaviour

This section presents the behaviour of the surface structure in the longitudinal direction, parallel to the tunnel axis. All results in the following subsections are taken along a profile on the longitudinal centre line of the building. Figure 9.20 shows settlement profiles on this line for different tunnel face positions marked by arrows. The results are for a 20m × 100m structure with a stiffness of 3 storeys.

The graph demonstrates that the centre line of the structure tilts towards the tunnel face as it is approached by the tunnel2. The building centre line shows its highest inclination when

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2 As the graph only shows results on the centre line no information is given whether the whole building tilts. This problem will be addressed in Section 9.3.3 where the 3D deformation of the structure will be studied.
the tunnel is just excavated beneath the building. The inclination then reduces as tunnel construction moves on. At the end of the analysis the centre line still shows some tilt. Apart from tilting, Figure 9.20 shows some bending (hogging) of the structure, most notably for the curves associated with the tunnel face at $y = -70m$ and beyond.

The graph highlights, again, the fact that no steady state conditions have established over the mesh. Figure 9.20, therefore, does not give any indication whether the remaining building inclination and/or deformation in the last increment of the analysis is only a result of the building still lying within the developing longitudinal settlement trough or whether building deformation remains after the building is under steady state conditions. Such a situation could arise from different values of volume loss developing in front of the building (i.e. the direction where the tunnel construction comes from) and at the rear of the structure. To investigate this behaviour the volume loss was calculated for different transverse profiles during the analysis. Figure 9.21 shows these values of $V_{L}$ plotted against their longitudinal position. Each data point of one curve represents the volume loss calculated from a transverse surface settlement trough. The graph includes results from two analyses which modelled a greenfield situation and a 3-storey building of which results were presented in the previous graph. For each analysis, three different curves are given corresponding to a tunnel face positions of $y = -30m$ (front edge of the building), $y = -50m$ (rear edge) and $y = -90m$ (end
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

Figure 9.22: Longitudinal profile of volume loss calculated from soil movement at tunnel boundary for different tunnel face position of greenfield and building analysis.

The graph shows that along transverse settlement profiles which include the building (i.e. between $y = -30m$ and $-50m$), a lower value of $V_L$ is achieved than in the corresponding greenfield situation. This is in agreement with the results shown in Figure 7.12 (Page 202) where a similar plot was presented for a 3D building subjected to plane strain tunnel construction. Between building and the $y = 0m$ boundary the building analysis shown in Figure 9.21 exhibits slightly higher volume losses than found under greenfield conditions. Behind the building, in contrast, the volume losses obtained from both building and greenfield situations coincide regardless of the position of the tunnel face.

The small difference found between the building case and the greenfield situation between $y = 0m$ and $-30m$ might cause some permanent deformation within the structure. However, it is too small to account for the remaining inclination shown in Figure 9.20 at the end of the analysis. The fact that the presence of the (weightless) surface structure has only a limited influence on the volume loss is further illustrated in Figure 9.22 which plots the volume loss calculated from the displacement at the circumferential tunnel boundary at different tunnel cross sections against their longitudinal coordinate. It was found that mid-side nodes along the tunnel boundary show significantly higher soil movement towards the tunnel. This can be explained as the existing tunnel lining and the soil elements at the tunnel face support the
corner nodes of solid elements representing the soil around the tunnel while nodes at $L_{exc}/2$ are not supported. This leads to a zigzag curve of soil displacement on the tunnel boundary. A similar effect was shown by Shin (2000) when performing 3D tunnel analysis.

For clarity Figure 9.22 does not plot this zigzag curve which also would be obtained when calculating $V_L$ from the soil displacements at all nodes along the longitudinal direction. Instead, two different data sets are presented, one for corner nodes and one for mid-side nodes. The mid-side nodes show approximately twice the value of $V_L$ calculated from the corner nodes. The only exception are the corner nodes of the mesh refinement below the building edge (in building case only) and the corner node at $y = 0m$. At these locations two slices of elements were excavated during one excavation step. The cross section between these two slices (at $L_{exc}/2$) shows a similar high value of $V_L$ as the mid-side nodes.

All curves show a constant level of $V_L$ over the length the tunnel is constructed except close to the $y = 0m$ boundary where the volume loss changes within a zone of less than 10m (for the corner nodes) and 30m (for the mid-side nodes). In front of the tunnel face $V_L$ reduces drastically reaching $V_L = 0\%$ within approximately 10m.

Apart from some scatter around the mesh refinement adopted in the building analysis, all curves (regardless of face position and greenfield/building case) practically coincide over the length the tunnel is constructed. This indicates that the presence of the (weightless) surface structure has no influence on the volume loss generated around the tunnel boundary.

The previous two figures indicate that building inclination and/or deformation that remains at the end of the analyses is a consequence of the fact that no steady-state conditions develop in the FE analyses. There might be some permanent deformation due to differences in volume loss on either side of the building. Figure 7.12, however, showed that this difference remains small and it is therefore not possible to separate this effect from the larger inclination and/or deformation caused by the developing longitudinal settlement profile.

Figure 9.20 only showed the settlement along a longitudinal profile with $x = 0m$. Visual inspection of this graph indicates that the building shows some hogging behaviour in the longitudinal direction at the end of the analysis. The following subsection will investigate this deformation in more detail. This will then be followed by results focusing on the horizontal strain in the longitudinal direction which develops within the building during tunnel construction. The results are not adjusted to a common volume loss as no comparison with
plane strain analyses is possible.

9.3.2.1 Deflection ratio

A greenfield section experiences hogging and subsequently sagging when it is approached by the longitudinal settlement trough caused by tunnel construction. To study this behaviour, a spread sheet calculation was performed in which a 20m long greenfield section was subjected to an approaching longitudinal settlement profile calculated from the cumulative error function (Equation 2.7 on Page 30). Different distances between tunnel face and greenfield section were considered and for each case the hogging and sagging deflection ratios were calculated. The point of inflection was set to $i = 10m$, $z_0 = 20m$ and the volume loss was $V_L = 1.5\%$. The solid lines in Figure 9.23a and b show the results from this calculation for sagging and hogging, respectively. The greenfield section was located between $y = -30m$ and $-50m$ to be consistent with the dimensions used in the 3D FE analyses. It can be seen that $DR_{hog}$ increases (in terms of absolute value) as the tunnel face approaches. It reaches its maximum when the tunnel face is just beneath the front edge of the section ($y = -30m$). Figure 2.4 (Page 31) showed that the point of inflection of the longitudinal settlement trough is above the tunnel face. The maximum hogging is therefore reached just when the point of inflection enters the greenfield section. From this face position on, $DR_{sag}$ starts to develop. It can be seen that considered together, the curves for sagging and hogging are symmetrical with respect to $y = -40m$, the centre of the greenfield section, which is a consequence of the symmetrical cumulative error settlement trough. $DR_{sag}$, therefore, reaches its maximum at $y = -50m$. At this position hogging reduces to $DR_{hog} = 0$.

The graphs also show results from a 3D FE greenfield analysis of the same geometry. Hogging develops from the beginning of the analysis and a maximum value is reached at a face position of approximately $y = -30m$ which is in good agreement with the spread sheet calculation. However the magnitude of $DR_{hog}$ obtained from the FE is approximately half of that from the cumulative error trough. This is a consequence of the relative wide FE settlement curve. The FE results would be even smaller in magnitude if they were not compensated for by the higher volume loss compared to $V_L = 1.5\%$ specified for the spread sheet calculation. Figure 9.14 showed that for the greenfield FE analysis a volume loss of approximately 1.5% developed at a face position of $y = -40m$. For the spread sheet calculation
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

Figure 9.23: Development of greenfield longitudinal (a) $D_{\text{R}_{\text{sag}}}$ and (b) $D_{R_{\text{hog}}}$ with tunnel progress. Comparison between spread sheet calculation and FE results.

Figure 9.24: Development of longitudinal $D_{R_{\text{hog}}}$ in 20m $\times$ 100m buildings.

the preset $V_L = 1.5\%$ is, in contrast, the final volume loss. Above the tunnel face the volume of the transverse trough is only half this final value.

The FE $D_{R_{\text{hog}}}$-curve reduces until the tunnel face reaches $y = -55m$ after which very low values of $D_{R_{\text{hog}}}$ remain (which is a result of the hogging developing behind the tunnel face, as shown in Figures 8.4 and 9.3 and discussed on Page 235). Consequently, the curve is wider than the corresponding distribution from the spread sheet calculation.

The $D_{R_{\text{sag}}}$ results calculated from the FE analysis show a similar trend as the $D_{R_{\text{hog}}}$ data. The curve is wider than the one computed from the cumulative error curve and its maximum value is approximately half of the spread sheet results. Both trends are a consequence from the fully 3D FE settlement trough being too wide.

Figure 9.24 shows how the behaviour studied for greenfield conditions changes when a building is included in the analysis. In this graph only $D_{R_{\text{hog}}}$ is plotted against tunnel face position as no longitudinal sagging was obtained on the centre line of the structure. Results for 1, 3, 5 and 10 storeys are presented. The curve for 1-storey shows an increase in $D_{R_{\text{hog}}}$
until the tunnel face reaches the front edge of the building where a first maximum is reached. This behaviour is similar to that found for greenfield conditions. Over approximately the next 10m of tunnel construction $DR_{\text{hog}}$ reduces but increases again as the tunnel moves behind the building. Towards the end of the analysis the gradient of the curve reduces but does not become horizontal which would be expected for steady state conditions. A similar behaviour can be found for the other stiffness cases although their shapes are less distinct. The longitudinal settlement profile for the 3-storey building was shown in Figure 9.20 and it was pointed out previously that the surface structure bends towards the tunnel face as the tunnel moves away from the building. This explains the increase in $DR_{\text{hog}}$ during the last increments of the analysis.

The magnitude of the longitudinal $DR_{\text{hog}}$, however, remains small when compared with values obtained for transverse building sections. For the 1-storey building a longitudinal $DR_{\text{hog}} = 5.98 \times 10^{-6}$ develops at the end of the analysis (tunnel face at $y = -90$ m). This is only 25% of the transverse hogging deflection ratio reached at the same increment ($DR_{\text{hog}} = 2.43 \times 10^{-5}$).

The geometries included in this parametric study have a small length (in the longitudinal direction) compared to the transverse building width. The building behaves therefore relatively stiffly in the longitudinal direction. It has to be pointed out that longitudinal hogging would become more critical if the buildings were longer in the tunnel direction. However, it was not possible to include such geometries into the parametric study due to the excessive computational resources required.

9.3.2.2 Strain

The previous section compared settlement data from a cumulative curve with results from a 3D FE analysis before the behaviour of buildings was studied. A similar approach is chosen in this section to investigate the development of longitudinal horizontal strain $\epsilon_{\text{h}y}$.

For the spread sheet calculation, the strain was calculated using Equation 2.10 (Page 32) derived from the cumulative error function. The parameters used in this spread sheet calculation were the same as those used in the previous section. It was noted in Section 2.2.1.2 that the horizontal strain (in the $y$-direction) changes from tension to compression above the tunnel face. Tension develops ahead of the tunnel face while compression can be found behind
Chapter 9. The influence of step-by-step tunnel excavation  

Section 9.3

Figure 9.25: Development of greenfield longitudinal horizontal strain with tunnel progress. Results from spreadsheet calculation.

Figure 9.26: Development of greenfield longitudinal horizontal strain with tunnel progress. Results from FE analysis.

It. This pattern can be seen in Figure 9.25 which plots $\epsilon_{hy}$ along a 20m long greenfield section above the tunnel centre line. Different curves refer to different tunnel face positions. Tensile strain develops as the tunnel approaches the site with maximum tension occurring 10m ahead of the tunnel face. When the tunnel face is below the front edge of the section ($y = -30m$)
the strain at this surface point reduces to $\epsilon_{hy} = 0$. With the tunnel construction moving on, compressive strain is generated. Its maximum (in terms of absolute value) can be found approximately 10m behind the face. As the tunnel is constructed further the compression reduces.

Figure 9.26 shows a similar graph based on the results of a 3D greenfield analysis. Tensile longitudinal horizontal strain develops at the beginning of the analysis. This tension reaches its maximum value as the tunnel construction approaches the greenfield section (tunnel face at -30m). From there on compressive strains develop. At a face position of $y = -40m$ the strain shows a similar distribution to that seen in the previous plot: Tensile strain ahead of the face and compression behind. Compression can be found over the whole section as the tunnel face moves beyond the greenfield site. From there on compression reduces.

The FE results and the spread sheet calculation show similar magnitudes of $\epsilon_{hy}$. However, the volume loss developed during the FE study is higher than the value specified for the spread sheet calculation, as discussed in the previous section. If the FE study was performed with a lower volume loss (by reducing $L_{exc}$) lower strain values would be obtained.

The development of $\epsilon_{hy}$ within a building is shown in Figure 9.27. The results are for a 20m × 100m 5-storey building. It can be seen that tension increases within the structure until

![Development of longitudinal horizontal strain with tunnel progress in 5-storey building.](image)

**Figure 9.27**: Development of longitudinal horizontal strain with tunnel progress in 5-storey building.
the tunnel face reaches the building edge \( (y = -30\text{m}) \). The building, however, already shows compression with the tunnel face at this position. The distribution of strain changes little while the tunnel underpasses the structure. Tension and compression only reduce slightly as the tunnel construction moves away from the building.

It has been pointed out earlier that the building remains within the influence zone of the tunnel excavation during the whole FE analysis. It is therefore not possible to investigate whether longitudinal strain remains in the building when the tunnel construction is completed (i.e. steady state conditions are established). The magnitude of \( \epsilon_{h_y} \) is, however, much smaller than corresponding transverse strain values. For the above example a maximum compressive strain of \(-1.12 \times 10^{-6}\) developed. The highest value of transverse compressive strain during the same analysis was \(-7.68 \times 10^{-6}\). However, tensile strain is more critical than compression. Tensile strain values calculated in this analysis reach similar magnitudes as found for plane strain analyses of eccentric buildings (see for example Figure 4.29, Page 130) although the higher volume loss of the 3D analysis amplifies the development of strain.

### 9.3.3 Twist

The previous sections studied the tunnel induced building deformation along the transverse and longitudinal centre lines of the surface structure. These results were of a two-dimensional nature although the buildings were subjected to three-dimensional tunnel excavation. This section will focus on the twisting deformation of the building which is a fully 3D deformation mode.

#### 9.3.3.1 Definition

There is only a limited number of publications on twisting of buildings due to ground subsidence. The only literature (the author could find) which addresses this topic are by Standing & Selman (2001) and Cooper & Chapman (2000). Both presented field measurements from existing tunnels which were subjected to ground subsidence caused by the construction of new tunnels. Standing & Selman (2001) studied the behaviour of the Northern Line, London, influenced by the construction of the JLE. They introduced the term *relative twist rotation* to quantify the twist measured in the tunnels. The twist was calculated from the difference in settlement between a pair of levelling points installed opposite to each other on the cross
section of the tunnel. This difference divided by the distance between the two points leads to an angle (precisely the sine of this angle which is the angle itself for small values) and was converted to the unit of arc minutes.

Cooper & Chapman (2000) present a case study from the construction of the Heathrow Express tunnel (London) which run beneath the existing tunnel of the Piccadilly Line. They expressed rotation by the differential settlement of a pair of opposite measurement points across the tunnel cross section (noting that the distance between the points was constant). Such pairs were installed over a certain length of the existing tunnel to study the longitudinal distribution of ‘rotation’.

In these definitions the longitudinal change of differential settlement was not considered. The authors pointed out that this definition therefore includes tunnel rotation caused by overall tunnel settlement and distortion of the tunnel circumference. For a long structure (such as an existing tunnel) the above approach is applicable. When buildings of different geometry are subjected to tunnel induced settlement, both dimensions, building width and length have to be considered.

The twist expression commonly used in structural engineering incorporates both geometric parameters. The definition is based on the torsion of a constant circular cross section of a rod (Timoshenko, 1955) as shown in Figure 9.28. A section of this circular rod with a diameter

![Figure 9.28: (a) Torsion of a single rod; (b) definition of angle of twist per unit length.](image)
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

$d$ and a height of $dx$ shows a rotation $d\varphi$ of its upper cross section with respect to its base. If the shaft shown in this figure is only twisted by a torque at its end the quantity $\frac{d\varphi}{dx}$ is constant over $x$. This ratio is the *angle of twist per unit length* and will be denoted with the symbol

$$\Theta = \frac{d\varphi}{dx} \quad (9.1)$$

From the above definition it follows that $\Theta$ is not dimensionless but has the unit $[1/\text{length}]$.

This definition can be extended to more general geometries such as shell and plates. Timoshenko & Woinowsky-Krieger (1959) show that the twist of a shell, shown in Figure 9.29 can be expressed as

$$\Theta_{xy} = \frac{1}{r_{xy}} = \frac{\partial^2 w(x, y)}{\partial x \partial y} \quad (9.2)$$

where $w(x, y)$ is the displacement normal to the shell, $r_{xy}$ is the curvature of the surface with respect to the $x$ and $y$ axis defined in the figure and $\Theta_{xy}$ is the twist with respect to these axes. With this definition the twist is the rate of change of slope in the $x$-direction as one moves in the $y$-direction. It has to be pointed out that this expression is, strictly speaking, only applicable to thin shells with small deflections which fulfill the following assumptions:

1. The shell is thin.
2. The deflection of the shell is small.
3. Normals to the middle surface of the shell remain normal to it and undergo no change in length during deformation.
4. Changes in direct stresses normal to the middle surface are negligible.

The shell elements representing the building are not necessarily fulfilling the first assumption. The thickness of shells representing a 1-storey building is 5.9m which increases to 20.1m for the 5-storey structure (see Table 3.4, Page 97). Although their deflection remains small (Kraus (1967) suggest that the thickness of a thin shell is at all points less than one tenth of its curvature which is the case for all building situations presented here) such shells cannot be considered to be thin. In addition the requirement of the preservation of normals to the middle surface and of no change in stresses normal to the middle surface are not necessarily
Chapter 9. The influence of step-by-step tunnel excavation  

Section 9.3

The implementation of shell elements into ICFEP is based on the formulation presented by Kraus (1967) which takes account of additional effects which arise from abandoning the above assumptions (Schroeder, 2003).

However, when presenting twist results from the FE parametric study the definition of twist will be based on Equation 9.2. In this equation the displacement $w(x, y)$ depends on the coordinates $x$ and $y$. This function is normally not known when analysing field measurements.

Figure 9.29: Twist of a thin shell caused by moments $m_{xy}$.

Figure 9.30: Deformation of shell and definition of displacement of corner nodes.
Instead measurements are available for discrete points. Figure 9.30 shows this situation where
the displacement is measured only at the 4 corner nodes. If a linearly varying displacement
is assumed on the edges of the rectangle and on any lines parallel to these edges then the
twist can be calculated from the differential settlement of the corner nodes as
\[ \Theta_{BL} = \frac{S_a - S_b}{L} - \frac{S_c - S_d}{L} \]
where \( S_a, S_b, S_c, S_d \) are the settlement of the four corners, \( B \) and \( L \) are the width and the
length of the building, respectively. The geometry, the coordinate system and the indexes
denoting the corners in this figure are chosen to be consistent with the building cases used
in the FE analyses presented in this chapter (compare with Figure 9.11 on Page 254). In
this case two of the nodes are on the longitudinal edge of the building while the other two lie
on the symmetry axis of the longitudinal building centre line. The index ‘\( BL \)’ indicates that
the twist is calculated along axes parallel to the building width \( B \) and length \( L \). It will be
omitted in the following paragraphs.

It has been stated earlier that the above definitions of twist are not dimensionless in
contrast to other building deformation criteria such as deflection ratio or horizontal strain
which have no dimension. The dimension \([1/\text{length}]\) is a consequence from twist being a one
dimensional deformation of a two dimensional structure. The definition given in Equation 9.3
will be applied in the following sections when presenting results from both the FE study and
field measurements.

9.3.3.2 Case studies

As pointed out above there is only a limited number of case studies available which address
the problem of twisting. Standing & Selman (2001) present precise levelling data along the
tunnels of the Northern Line (London Underground) which were subjected to movement
cau\( se \)d by the construction of the Jubilee Line Extension (JLE) near Waterloo station, Lon-
don. The route of the JLE tunnel was approximately perpendicular to the existing tunnels
of the Northern Line.

Twist was calculated from the differential settlement of pairs of levelling points across the
tunnel. As described earlier the longitudinal distance between different pairs of points was
not included into the calculation of twist which was referred to as \textit{relative twist rotation}. It
was expressed as an angle rather than having the dimension [1/length].

Standing & Selman (2001) showed that relative twist rotation occurred when the JLE tunnel construction passed beneath the existing tunnels. However, their results demonstrate that this relative twist rotation was only temporarily and some counter twist was observed as the construction of the tunnel advanced beyond the existing tunnels.

The study demonstrated that structures which are perpendicular to the route of the new tunnel construction experience twist mainly during the construction of the tunnel and twist deformation reduces or even vanishes when the tunnel face moves away from the structure. The parametric FE study presented in this chapter models a building geometry which is perpendicular to the tunnel. This trend will be verified by the results from the FE analyses presented later although only surface structures were included in these analyses in contrast to the subsurface structures reported by the field study.

Not all existing structures are, however, perpendicular to the tunnel to be constructed. Cooper & Chapman (2000) present measurements from the running tunnel of the Piccadilly Line in Heathrow, London, which was affected by the construction of the Heathrow Express tunnel. The centre lines of the tunnels crossed in a skew angle of approximately $70^\circ$. As Standing & Selman (2001), Cooper & Chapman (2000) report rotation of the existing tunnel towards the new tunnel as its tunnel face approached. As tunnel construction passed beyond the existing tunnel, rotation reduced. As the new tunnel was subsequently enlarged (from the same direction as the initial tunnel construction) the rotation in the existing tunnel increased again followed by a reduction in rotation as the excavation of the enlargement passed beyond the existing tunnel. However, some rotation remained in the Piccadilly Line tunnel and Cooper & Chapman (2000) concluded that this behaviour is due to the skew angle between the route of the existing and the new tunnel.

To demonstrate the effect of twisting on surface structures whose geometry is inclined to the axis of the new tunnel, field data from Elizabeth House will be presented in the following paragraphs. This building was under passed by the construction of the JLE. The building centre line was inclined with respect to the route of the JLE as shown in Figure 9.31. The building was extensively monitored during the tunnel construction process. The instrumentation included precise levelling and tape extensometers in the basement, photogrammetry for the facade monitoring and extensometers and inclinometers to monitor ground movement.
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

beneath the building. A full presentation of these measurements and a detailed description of the building and the tunnelling works can be found in Standing (2001). The measurements from the precise levelling will be used to demonstrate how the building twisted during the construction of the JLE tunnel. There was only negligible damage to the building as a consequence of the construction of the JLE tunnels.

Elizabeth House is a reinforced concrete structure which consists of a 10-storey and a 7-storey block which are linked by an expansion joint. Only the 10-storey structure (the southern part, Figure 9.31) will be considered here. This building part is relatively narrow in plan view. It is 110m long and its width varies between approximately 18 and 24m. Its north-west side will be referred to as York Road side while the opposite site will be termed Waterloo side.

The building has two basement levels. Its foundation consists of a 1.4m thick monolithic reinforced concrete slab founded on Terrace Gravels which overlay the London Clay in which the JLE tunnels were constructed. The tunnel axis level was approximately 23m below the top of the foundation slab. The tunnels are 5.6m in diameter. The tunnels were constructed using SCL (NATM) techniques except for a short length of the westbound tunnel for which a segment lining was installed.

The tunnel geometry beneath the building is shown in Figures 9.31 and 9.32. Work started on an access adit which was completed by the end of July 1994. It nearly reached below the building on the York Road side, shown in Figure 9.31. Precise levelling started after the construction of this access adit and before westbound tunnel excavation started towards Elizabeth House from this position in January 1995 using first segment lining and then SCL technique. Consequently the measurements do not document the effect of the approaching westbound tunnel on the structure as it was nearly beneath it when precise levelling started.

The westbound tunnel face was beyond the Waterloo side on 6 February 1995 when construction of the eastbound tunnel started on the York Road side. This tunnel passed beneath the Waterloo side of the building on 24 February 1995. The data presented subsequently are from the time between 23 December 1994 (before construction of the westbound tunnel) and 6 March 1995 (when both tunnels are beyond the Waterloo side of the building). The cross-over tunnel which was constructed later is not considered.

Precise levelling was carried out to monitor the settlement in the basement of the structure
Figure 9.31: Plan view of Elizabeth House with position of JLE tunnel.

Figure 9.32: Tunnelling sequence beneath Elizabeth House.
both along and across the building. All measurements were referenced to the deepest anchor of
the extensometer with the largest distance to the tunnels. A full description of the surveying
programme can be found in Standing (2001). In the following paragraphs measurements from
4 cross sections of the building, referred to as grid 9, 12, 14 and 16 (shown in Figure 9.31)
will be presented.

Figure 9.33 shows the development of settlement along each of the four cross sections.
The measurements are taken from Standing (2001). Different curves refer to different dates
(see Figure 9.32 for the position of the tunnel faces on these dates). The curve of 6 February
indicates that the construction of the westbound tunnel causes some settlement along grid
line 9 and 12 but its influence on the settlement along grid line 14 and 16 is negligible. There
is no significant differential settlement along the cross sections up to this stage of tunnel
construction. As the eastbound tunnel is constructed, however, the grid lines start to tilt
towards the tunnel. For grid line 9 this results in a rotation towards the York Road side
while a rotation towards the Waterloo side can be seen for grid line 12. Grid line 14 shows a
steeper slope towards the Waterloo side (compared with grid line 12), albeit its settlements
are smaller. Grid line 16 which is furthest away from the eastbound tunnel shows the smallest
settlement but the tilt towards the Waterloo side can clearly be seen.

The different slopes of the settlement curves for the different grid lines cause twist along

\[ \text{Figure 9.33: Settlement across Elizabeth House during construction of westbound and eastbound tunnels.} \]
the structure. Equation 9.3 was applied to quantify this twist. Only the end points of each settlement curve shown in Figure 9.33 were considered and divided by the distance between them (leading to the slope of the settlement along one grid line). The difference in slope between adjacent grid lines was then divided by the distance between these lines leading to the twist Θ.

The sign convention adopted for this calculation is that, looking northwards, a positive twist results in an increase in slope towards the York Road side, this is the side where the tunnel construction approaches from.

Figure 9.34 shows the results from this calculation. There are three result points for each date, plotted between the position of the grid lines, which are also indicated in the graph. The horizontal axis marks the distance from the southern building end.

The graph shows no significant development of twist as the westbound tunnel passes beneath the building (6 February). This can be explained as the precise levelling started after construction of the access adit. When the westbound tunnel excavation started from this adit, it was already very close to the building. However, as the westbound tunnel moves beyond the Waterloo side, negative twist develops between grid lines 9 and 12 (10 February). At the same time the eastbound tunnel construction approaches the building causing initially positive twist between grid lines 12 and 16. As the construction of the eastbound tunnel passes

![Figure 9.34: Development of twist along Elizabeth House during construction of westbound and eastbound tunnels.](image-url)
beneath the building negative twist develops between grid line 12 and 14.

The figure shows that twist increases (in terms of absolute value) until the end of the monitoring period. This observation is in contrast to the field study by Standing & Selman (2001) who reported only temporary development of relative twist during the construction of tunnels. This indicates that the twist observed in Elizabeth House is mainly a result of the skew angle between building centre line and tunnel axis. This inclination causes negative twist to develop on the southern part of the building while positive twist occurs north of the tunnels. The position where twist changes its sign marks the point of maximum slope along the grid lines. It can be seen that this point moves northwards as tunnel construction progresses northwards.

The trend of increasing twist with progressing tunnel construction is further illustrated in Figure 9.35 which plots the results from the previous figure against time. The three curves refer to the three sets of results between the grid lines. The graph confirms that no significant twist develops until the westbound tunnel moves beyond the building and the eastbound tunnel approaches it (6 February). Negative twist develops between grid line 9 and 12 from then on and slightly later also between 12 and 14. The twist between grid line 14 and 16 is (in terms of absolute value) smaller and has a positive sign. There is a clear trend of increasing twist with advancing tunnel construction. To get a view over a longer time scale a similar graph is given in Figure 9.36 presenting twist up to the start of the cross-over passage construction. The graph shows that the twist increases approximately until April 1995 (this is one month more than shown in the previous figures) but then remains stable. This graph demonstrates that there is no reduction of twist as the tunnels move away from the building. Such a behaviour would have been expected for a structure perpendicular to the tunnel axis.

These case studies demonstrate that twist deformation can be caused temporarily as the tunnel passes beneath an existing structure. If the building is perpendicular to the tunnel axis no further twist develops and twist reduces or even vanishes as the tunnel moves away from the building. If the building axis, however, is inclined with respect to the tunnel axis a permanent twist deformation develops.

The next paragraphs present results from the parametric FE study including different building geometries and stiffnesses. For reasons of computation time only perpendicular geometries were included. However, the results give a general overview of how building
9.3.3.3 Results

The settlement results from the parametric study presented in earlier sections were used to calculate twist for the different building cases. Twist was calculated adopting Equation 9.3 using the settlements of the 2 corners of the building (denoted as $c$ and $d$ in Figure 9.11) and the two nodes above the tunnel centre line ($a$ and $b$). The sign convention was similar to that used for Elizabeth house. Looking in transverse direction towards the route of the tunnel characteristics, such as geometry and stiffness, influence twist deformation.

Figure 9.35: Development of twist of Elizabeth House with time during construction of westbound and eastbound tunnel.

Figure 9.36: Development of twist of Elizabeth House with time between westbound tunnel construction and start of cross-over excavation.
The influence of building stiffness on twist is illustrated in Figure 9.37 for a 20 m × 100 m geometry. Twist is plotted against position of the tunnel face. Different curves are given for greenfield and different building cases. For the greenfield scenario twist increases as the tunnel approaches the site (between $y = -30$ m and -50 m). This shows that the $x = 0$ m centre line of the geometry tilts towards the tunnel face while its edge shows less rotation. The maximum twist is reached at a face position of $y = -40$ m i.e. beneath the transverse centre line of the site. The twist reduces but then remains approximately constant for a tunnel face position beyond $y = 70$ m. Consequently the high values of twist are only of temporary nature. Whether the remaining twist is only a consequence of the longitudinal settlement trough being too wide or/and a result of different values of volume loss in front and behind the building cannot be determined from the analysis as discussed in Section 9.3.2, Page 264.

Analyses including a building give a similar picture with a peak twist reached when the tunnel face is beneath the building. The influence of the structure’s stiffness can clearly be seen in this graph when comparing the different curves: buildings with a higher stiffness show less twist deformation. From these result it is possible to calculate a twist modification factor $M_{\Theta}$. It is defined as the ratio between the peak twist obtained from any building case divided by the peak value of the corresponding greenfield scenario. With this definition it is
Chapter 9. The influence of step-by-step tunnel excavation

Section 9.3

possible that corresponding peaks occur at different positions of the tunnel face.

The 1-storey building gives a modification factor of \( M^\Theta = 0.77 \). This value reduces to 0.30 for the 3-storey building. The 10-storey case only shows \( M^\Theta = 0.047 \). With increasing stiffness, the tunnel face position where the peak twist occurs moves slightly towards the direction from where the tunnel construction approaches. In all building cases twist deformation remains constant for a tunnel face position beyond approximately \( y = -70 \text{m} \). For the 1-storey structure the remaining twist is higher than the corresponding values obtained from the greenfield analysis. However, it is only 31% of the 1-storey peak value and this ratio reduces to 21% for the 10-storey case.

The next two graphs investigate the influence of building geometry on the twist behaviour. Figure 9.38a and b plot twist against tunnel face position for 20m \( \times \) 100m, 30m \( \times \) 100m and 20m \( \times \) 66m geometries. Figure 9.38a presents results from greenfield analyses. It can be seen that the largest geometry (30m \( \times \) 100m) develops the smallest twist deformation. The curve for the 20m \( \times \) 100m geometry is slightly narrower leading to a higher peak value. The peak value increases further as the greenfield site width decreases from \( B = 100 \text{m} \) to 66m. These three curves demonstrate that for greenfield conditions the twist deformation is concentrated around the tunnel centre line. A narrower greenfield geometry (i.e. smaller \( B \)) therefore experiences a higher degree of twist than a wider one.

When comparing the curves for the two 100m wide geometries, the \( L = 30 \text{m} \) case shows initially less twist than the 20m long greenfield site. This can be explained as the 30m long structure extends further away from the tunnel face and therefore shows less twist deformation. As the tunnel construction moves beyond the building the opposite is the case and the 30m structure is nearer to the actual tunnel face than the 20m long building and, consequently, the twist deformation is higher for the 30m long site. The difference between the two curves is, however, smaller than that between the 20m \( \times \) 100m and the 20m \( \times \) 66m geometries.

It should be noted that the different curves for different geometries are a consequence of the application of Equation 9.3 using only the settlement of the 4 corner nodes of half of the greenfield geometry. If the twist was calculated locally over each geometry (for example for every element on the ground surface) the peak twist found within each geometry would have been the same leading to identical curves for all three cases. However, this is not the case for
building scenarios where the twist deformation is influenced by both geometry and building stiffness.

Figure 9.38b shows results from 3-storey buildings with three different geometries. The graph reveals the opposite trend to the previous graph. The 30m \( \times \) 100m shows the highest twist values while the 20m \( \times \) 66m structure develops the smallest twist deformation. The curves for the two 20m long structures lie closer together than the results of both 100m wide buildings, the opposite trend was observed in the greenfield case. This trend of smaller
structures leading to smaller twist deformation shows that a smaller building geometry yields a stiffer response. However, this stiffer response leads to an increased rigid-body motion of the whole structure as shown in Figure 9.39. This figure plots the rotation of the longitudinal building edge (at \( x = 50 \text{m or } 33 \text{m} \)) against tunnel face position. Positive rotation is defined as a higher settlement on the \( y = -30 \text{m} \) building side (where the tunnel construction comes from). It can be seen that the \( 20 \text{m} \times 66 \text{m} \) structure shows the highest values of rotation along the building edge. Together with the relatively low values of twist this implies a high degree of rigid-body rotation. The \( 30 \text{m} \) long building, in contrast, shows only small rotation along the edge, changing its sign as the tunnel construction progresses.

Decreasing twist for reducing building size on the one hand, and increasing corresponding greenfield values on the other hand lead to a reduction in \( M^\Theta \). The 3-storey \( 30 \text{m} \times 100 \text{m} \) case shows \( M^\Theta = 0.57 \) which reduces to 0.30 for the \( 20 \text{m} \times 100 \text{m} \) building and further to 0.15 for the \( 20 \text{m} \times 66 \text{m} \) geometry.

Figure 9.40 shows how the tunnel depth \( z_0 \) influences the development of twist. A greenfield situation and a \( 20 \text{m} \times 100 \text{m} \) in plan building of 3 storeys was analysed for a \( 34 \text{m} \) deep tunnel and results are compared with the \( z_0 = 20 \text{m} \) case. The length of the tunnel and all horizontal distances to the vertical mesh boundaries were the same as for the corresponding \( z_0 = 20 \text{m} \) mesh.

The results of the two greenfield cases are shown Figure 9.40a. It can be seen that the deeper tunnel causes less twist over the \( 20 \text{m} \times 100 \text{m} \) greenfield site; its peak value is only 40% of the corresponding \( z_0 = 20 \text{m} \) result. A similar picture can be found for the building case, shown in Figure 9.40b. The ratio between the \( z_0 = 34 \text{m} \) and \( 20 \text{m} \) cases is 51% which is slightly higher than for greenfield conditions. The modification factors for the 3-storey building above the \( 34 \text{m} \) deep tunnel is \( M^\Theta = 0.38 \) compared with 0.30 for the corresponding \( z_0 = 20 \text{m} \) case. This shows that \( M^\Theta \) increases with increasing tunnel depth while the twist itself reduces.

Figures 9.37 to 9.40 demonstrate that twist deformation of a structure compared to the corresponding greenfield scenario is in agreement with the relative stiffness formulation, proposed by Potts & Addenbrooke (1997). An increase in building stiffness (due to an higher number of storeys) reduces \( M^\Theta \) while increasing building width and/or length increases it. The twist modification factor also increases with increasing \( z_0 \). Similar trends were observed.
when the influence of building stiffness, building width and tunnel depth on the plane strain factors $M_{DR}$ and $M_{eh}$ were studied in Chapter 4. Because of the limited number of analyses included into this case study, it is not possible to develop any design charts for twist. Furthermore it has been pointed out that the geometry of a building being perpendicular to the tunnel is a special case and that permanent twist increases as the skew angle between building and tunnel increases. However, this case study provides a general overview on the influence of a building’s characteristics on its twist deformation and it can be concluded that it is in good agreement with the relative stiffness framework postulated by Potts & Addenbrooke (1997).

### 9.4 Conclusions

This chapter studied how an existing surface structure is affected by approaching tunnel construction. A number of fully 3D analyses were presented in which step-by-step tunnel construction was modelled by successively removing elements in front of the tunnel.

Before carrying out a parametric study the dimensions of the 3D mesh were carefully chosen. A set of analyses were performed to determine the longitudinal distance between the vertical boundaries. The total length of tunnel construction in the analysis was set to a maximum value of 100m (over $24 \times D$). However, no steady state conditions were achieved.
behind the tunnel face at the end of the analyses. It was shown that for stress conditions with \( K_0 = 0.5 \) (instead of 1.5 used for all other analyses in this thesis) such conditions develop at approximately \( 7 \times D \) behind the face for the 20m deep tunnel geometry analysed in this chapter.

Another study compared results from different excavation lengths (\( L_{\text{exc}} = 2.0\text{m} \) and \( 2.5\text{m} \)). The adoption of \( L_{\text{exc}} = 2.5\text{m} \) in the following parametric study led to higher volume losses than specified in corresponding plane strain analyses. It was shown that it is possible to adjust results to a common volume loss in order to compare them with plane strain data.

As for greenfield conditions it was found that 3D transverse building settlement is very similar to corresponding plane strain results. This became clearer when comparing deformation criteria obtained from 3D and 2D analyses. Only a few 3D results gave higher values (with a maximum increase of 10%) while most deformation criteria showed lower values when calculated from 3D analyses. For the corresponding modification factors, few results showed increases above 10%. However, for \( M_{\text{DR\_hog}} \) all 3D modification factors were lower than calculated from 2D analyses.

This comparison showed that plane strain analysis tends to over-predict deformation criteria and subsequently modification factors. Showing that most 2D results are conservative when compared with 3D analysis provides further confidence in the relative stiffness approach, proposed by Potts & Addenbrooke (1997) which was based on a suite of plane strain FE analyses.

In the longitudinal direction, the building showed hogging deformation which increased over the last increments of the analyses as the tunnel moved away from the building. The magnitude of \( DR_{\text{hog}} \) in all analyses was smaller than \( DR_{\text{hog}} \) calculated for the transverse profile. As no steady state conditions developed during the analysis it is not possible to say to which degree longitudinal deformation remains within the structure after tunnel construction is completed.

In addition to hogging, tensile strain developed within the building in the longitudinal direction and remained until the end of the analysis. The magnitude of this tension was similar to the tensile strain levels obtained in 2D analysis of eccentric geometries. However, the volume loss obtained in the 3D analyses was approximately twice the value calculated from plane strain analyses. Consequently, a lower level of tensile strain can be expected if
3D analyses were performed for a similar volume loss as specified for plane strain conditions. In addition to the deformation criteria, deflection ratio and horizontal strain, twist deformation was calculated. The definition of twist was derived from expressions used in structural engineering. By considering both building dimensions, $B$ and $L$ this expression is not dimension-less but has the unit $[1/\text{length}]$.

By presenting two case studies it was shown that twist can occur temporarily during tunnel construction in the vicinity of the building. However, this temporary deformation only applies to structures which are perpendicular to the route of the new tunnel. In case where there is a skew angle between building and tunnel, permanent twist develops as a consequence of tunnel construction. Such a situation was found for Elizabeth House, which was subjected to the construction of the JLE.

The parametric FE study, however, only included perpendicular buildings and, consequently, twist deformation reduced towards the end of the analyses although it did not vanish. The parametric study demonstrated that the influence of building geometry and stiffness on twist is similar to their influence on the other deformation criteria. Twist modification factors were defined which decrease with increasing building stiffness, increase with increasing building geometry and with increasing tunnel depth. Similar relationships are described by the Potts & Addenbrooke (1997) relative stiffness method.

Because of the limited number of analyses included in the parametric study it was not possible to develop similar design charts for twist as provided by Potts & Addenbrooke (1997) for $M^{DR}$ and $M^{\text{th}}$. In addition, further research is required to investigate the relation between twist deformation and strain developing in the structure leading to potential cracking.

This chapter demonstrated that 3D analysis has only limited influence when analysing transverse surface settlement. For this purpose the use of plane strain analysis is a less time-consuming alternative and it was shown that results obtained from 2D analyses tend to be slightly conservative. Settlement profiles for both 2D and 3D analyses are too wide when comparing with field data. As a consequence, no steady state settlement was obtained when simulating step-by-step tunnel construction. More research has to be carried out to improve these settlement predictions.
Chapter 10

Design Charts

10.1 Introduction

The previous chapters investigated the influence of various building characteristics, such as the geometry, the building weight and the nature of the contact between the soil and the foundation on tunnel induced building deformation. In addition, the 3D effect of the process of tunnel construction was studied. It was shown how these features affect the mechanisms of the tunnel-soil-building interaction problem. To quantify their influence, deformation criteria deflection ratio ($DR_{sag}$ and $DR_{hog}$) and horizontal strain ($\epsilon_{hc}$ and $\epsilon_{ht}$) were evaluated.

According to the relative stiffness approach of Potts & Addenbrooke (1997) these deformation criteria were related to corresponding greenfield situations by calculating modification factors $M_{DR_{sag}}$, $M_{DR_{hog}}$ (for deflection ratio) and $M_{\epsilon_{hc}}$, $M_{\epsilon_{ht}}$ (for horizontal strain) as defined in Equations 2.30 and 2.31.

Potts & Addenbrooke (1997) introduced the following relative stiffness expressions

$$\rho^* = \frac{EI}{E_s \left( \frac{B}{2} \right)^4}; \quad \alpha^* = \frac{EA}{E_s \left( \frac{B}{2} \right)}$$

(see Equation 2.29, Page 72)

which relate the bending and axial building stiffness $EI$ and $EA$, respectively, to the soil stiffness. Their study included over 100 plane strain FE analyses with varying building stiffness and geometry. Consequently, the above stiffness expressions were adopted for plane strain situations leading to the relative bending stiffness $\rho^*$ having the dimension [1/length] while the relative axial stiffness $\alpha^*$ is dimension less. When plotting the modification factors
obtained from their study against relative stiffness, upper bound curves were fitted to the data giving the design curves shown in Figure 2.27 on Page 77.

The relative stiffness method has been described as the most promising development of recent years for taking account of relative stiffness between soil and structure (Burland et al., 2001) and was applied for settlement predictions in London (Mair & Taylor, 2001). This thesis, however, has highlighted the following shortcomings in the relative stiffness formulation:

- In their work, Potts & Addenbrooke (1997) applied the relative stiffness to plane strain situations. Consequently, $\rho^*$ had the dimensions $[1/\text{length}]$ while $\alpha^*$ was dimensionless. In Chapter 7 of this thesis the above relative stiffness formulation was used in a 3D context in which $\rho^*$ was dimension-less while $\alpha^*$ was expressed in $[\text{length}]$. It would be more consistent if both $\rho^*$ and $\alpha^*$ were dimension-less regardless of whether they are applied in 2D or in 3D conditions.

- Chapter 4 presented a large number of plane strain analyses in which buildings were modelled in a similar way as in the study of Potts & Addenbrooke (1997). Including a wider variation of building geometries and tunnel depths revealed more scatter than observed by Potts & Addenbrooke (1997) when plotting modification factors against the corresponding relative stiffness values. Furthermore, it was found that changing the initial stress conditions can lead to strain modification factors exceeding the design curves.

- Potts & Addenbrooke (1997) modelled the building in plane strain, having no self-weight and no relative movement between soil and foundation was allowed. This thesis studied the influence of these building characteristics and additionally modelled the 3D progress of tunnel construction. It is therefore possible to adjust the relative stiffness approach to these conditions.

In the previous chapters it was shown how the relative stiffness formulation could be modified to account for the above situations. These proposals will be summarized in the next section before combining them to introduce new design charts in Section 10.3.
10.2 Summary of previous chapters

10.2.1 Transverse geometry

In Chapter 4 the geometric parameters building width $B$, eccentricity $e$ and tunnel depth $z_0$ were varied separately and their influence on tunnel induced building subsidence was quantified. It was found that the current definition of relative bending stiffness over-estimates the influence of $B$ by incorporating it raised to a power of 4 in the denominator of $\rho^*$. The results also indicated that, in contrast, the tunnel depth is not sufficiently represented in the relative stiffness expression. The tunnel depth is included in $\rho^*$ only via the soil stiffness $E_S$ which is taken from half tunnel depth. Consequently Chapter 4 proposed to incorporate $B$ and $z_0$ into the relative bending stiffness by changing $\rho^*$ to

\[
\rho_{m1}^* = \frac{EI}{z_0^3 E_S B^2} \tag{see Equation 4.1, Page 143}
\]

By plotting $M^{DR}$ results against both the original definition of $\rho^*$ and against $\rho_{m1}^*$ it was shown that the above formulation reduces the scatter for $M^{DR}_{sag}$ but increases it slightly for $M^{DR}_{hog}$ (see Figures 4.42 to 4.45, Pages 144 and 145).

A similar formulation for relative axial stiffness was introduced as

\[
\alpha_{m1}^* = \frac{EA}{z_0 E_S B^2} \tag{see Equation 4.2, Page 146}
\]

It was shown, however, that the original expression produced less scatter than $\alpha_{m1}^*$ when plotting $M^{\epsilon_h}$ against relative axial stiffness (see Figures 4.46 and 4.47, Page 146). It was therefore concluded that $\alpha^*$ adequately incorporates the geometry of the tunnel-soil-building interaction problem while $\rho^*$ should be changed in order to more accurately account for the influence of $B$ and $z_0$.

10.2.2 Building weight

Chapter 5 studied the influence of the building’s self-weight on tunnel induced building deformation by performing analyses considering a wide range of building stiffness-load combinations. For both $M^{DR}$ and $M^{\epsilon_h}$ it was found that deformation criteria increased with increasing building load. This effect was, however, compensated by the vast decrease of
modification factors with increasing building stiffness, when realistic building stiffness-load combinations were modelled.

When plotting $M^{DR}$ against $\rho^*$ it was shown that all modification factors for realistic load cases lay within the boundaries described by the Potts & Addenbrooke (1997) design curves (see Figure 5.10, Page 159). For $M^{ch}$, however, some data points lay outside the design curves (see Figure 5.14, Page 163). It was noted, however, that the fact that some results lay outside the curves was not only due to building load but also because of different initial stress conditions adopted between the current study and the analyses performed by Potts & Addenbrooke (1997).

### 10.2.3 Soil-structure interface

Interface elements were included in the plane strain analyses in Chapter 6 to vary the nature of the contact between soil and building foundation. When separation between building and soil was allowed, it was shown that no gap develops between foundation and soil as long as building weight is considered in the analysis. When horizontal relative movement between soil and building was modelled, the horizontal strain in the building nearly vanished leading to strain modification factors being less than 1% in magnitude of those obtained from corresponding no-interface analyses. Compared to the vast reduction of $M^{ch}$ the deflection ratio modification factors were less affected showing a reduction with decreasing interface shear stiffness (see Figure 6.22, Page 187).

### 10.2.4 Longitudinal geometry

In Chapter 7 three-dimensional FE analysis was used to model buildings with a length $L$ in a direction parallel to the tunnel axis in addition to a width $B$ in the transverse direction. The tunnel was constructed simultaneously over the whole mesh length simulating effectively a plane strain excavation beneath a 3D structure. The study revealed that modification factors increase as $L$ reduces to small values. For all deformation criteria this trend was smallest for sagging while large increases were found for hogging (although there was a large scatter in the result). For compressive strain the trend of increasing $M^{ch}$ with decreasing $L$ was uniform for all building stiffness cases included in the study. As only buildings with no eccentricity were analysed, no tension was obtained in any building case.
For deflection ratio it was found that the reduction of of $M^{DR}$ with increasing building stiffness outweighed the increase due to reducing $L$. When the deflection ratio modification factors were plotted against their plane strain relative bending stiffness (i.e. not considering the building length $L$ in the second moment of area $I$ of the building) most factors remained below the Potts & Addenbrooke (1997) design curves which were developed from results of a plane strain FE study (Figure 7.24, Page 211).

It was therefore concluded that it is justified to use plane strain relative stiffness expressions to describe the deflection ratio of 3D structures. However, it was noted that the dimension of $\rho^*$ changes between 2D and 3D conditions because the numerator of $\rho^*$ contains the out of plane dimension ($L$ is included in $I$, see Page 96) while the denominator does not. To achieve the same dimensions for 2D and 3D conditions no out of plane dimension should be incorporated into the definition of relative stiffness or, alternatively, such a dimension must be present in both numerator and denominator. The latter approach was introduced in Chapter 7. Rearranging the relative bending stiffness to

$$\rho_{m2}^* = \frac{EI}{E_S \left(\frac{B}{2}\right)^4 L}$$

(see Equation 7.1, Page 213)

gives a relative bending stiffness which always keeps its dimension $[1/\text{length}]$ regardless of whether it is used in 2D or 3D analysis. Note that in 3D conditions $I$ has the dimension $[\text{length}^4]$ while $[\text{length}^3]$ is adopted under plane strain conditions in which the length is expressed as $L = 1\text{m/m}$.

Similarly the relative axial stiffness $\alpha^*$ can be rearranged to

$$\alpha_{m2}^* = \frac{EA}{E_S \left(\frac{B}{2}\right) L}$$

(see Equation 7.2, Page 214)

which is always dimension-less whether used in 2D or in 3D.

10.2.5 Tunnel construction process

Chapters 8 and 9 presented a suite of FE analyses in which the tunnel construction process was modelled fully 3D by adopting a step-by-step construction sequence. Chapter 8 focused on greenfield conditions while buildings were considered in Chapter 9. Buildings of different geometry and building stiffness were included in the study. All buildings were perpendicular to the tunnel axis with $B$ (being the longer dimension) in the transverse and $L$ (the shorter
dimension) in the longitudinal direction. As the buildings had no eccentricity no tensile strain was obtained in any of the structures. Building deformation in the transverse direction to the tunnel was compared to the results from earlier chapter of this thesis.

Generally it was found that deformation criteria and corresponding modification factors obtained from 3D analyses were in good agreement with results from corresponding plane strain analyses when situations involving a similar volume loss were compared. Tables 9.2b to 9.4b (Page 259) summarize $M_{\text{DR}_{\text{sa}}}$, $M_{\text{DR}_{\text{ho}}}$ and $M_{\text{fc}}$ for all 3D analyses by normalizing their values against the corresponding plane strain results. The tables show that most ratios are below unity, indicating that the 3D analyses tend to give lower modification factors than the plane strain ones.

10.3 An alternative formulation for dimension less modification factors

The conclusions drawn in the previous chapters (summarized above) can be used to modify the relative stiffness expressions $\rho^*$ and $\alpha^*$ to take into account the change in modification factors due to additional building characteristics.

10.3.1 Deflection ratio

In Chapter 4 it was shown that the influence of building width $B$ in the original definition of $\rho^*$ (Equation 2.29a) was overestimated while the tunnel depth $z_0$ was not considered sufficiently. Equation 4.1 introduced a modified relative bending stiffness $\rho_{\text{m1}}^*$ (shown above) which reduced the scatter for $M_{\text{DR}_{\text{sa}}}$ but increased it for $M_{\text{DR}_{\text{ho}}}$. Defining an alternative modified relative bending stiffness as

$$\rho_{\text{mod}}^* = \frac{EI}{E_S z_0 B^2 L} \tag{10.1}$$

can be seen as a compromise between the original Potts & Addenbrooke (1997) relative bending stiffness and the modified expression $\rho_{\text{m1}}^*$. It also includes the approach outlined in Chapter 7 by including $L$. As noticed above, $I$ includes the out of plane geometry (hence has the dimension [length$^4$]). Dividing by $L$, however, reduces it to its plane strain expression. It can be seen that the above equation is then dimension less.
The use of the building width $B$ rather than the half width of the building $\frac{B}{2}$ as adopted by Potts & Addenbrooke (1997) has no implication on the relative position of the results to each other when plotted against a log-scale of relative stiffness. The use of $B$ is consistent with expressing the degree of eccentricity as $e/B$.

The data presented in Section 4.5.1 are used in Figures 10.1 and 10.2 to assess this new definition. In the 2D analyses presented in Chapter 4 buildings were modelled weightless and the foundation was assumed to be perfectly rough. Figure 10.1 reprints the graphs previously shown in Figure 4.42. It plots the modification factors $M_{DR}$ for a wide range of buildings with different geometries against the original definition of $\rho^*$. Only geometries with no eccentricity are plotted in the figure.

The graph shows a relatively wide scatter for $M_{DR_{sag}}$. It was shown in Chapter 4 that this arises as the influence of $B$ on $M_{DR_{sag}}$ is overestimated while the effect of $z_0$ is underestimated.

![Figure 10.1](image1.png)  
**Figure 10.1:** $M_{DR}$ for cases with no eccentricity plotted against $\rho^*$.

![Figure 10.2](image2.png)  
**Figure 10.2:** $M_{DR}$ for cases with no eccentricity plotted against modified relative bending stiffness $\rho^*_\text{mod}$. 

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Chapter 10. Design Charts

Section 10.3

The scatter for $M_{DR_{hog}}$ (Figure 10.1b) is less. Figure 10.2 replots these data adopting the relative bending stiffness $\rho_{\text{mod}}$ defined above.

The new definition reduces the scatter when plotting $M_{DR_{sag}}$ (Figure 10.2a) against relative stiffness. Through changing the relative stiffness expression all data points are shifted towards the right hand side in this plot compared to the corresponding graph in Figure 10.1 (note that the limits on the x-axis have changed between both figures), although the use of $B$ instead of $\frac{B}{h}$ reduces this shift. A new upper bound curve has been incorporated in this graph.

For $M_{DR_{hog}}$ (shown in Figure 10.2b) the scatter increased slightly compared with the original definition of $\rho^*$, shown in Figure 10.1b. It is, however, still possible to fit a new upper bound curve to the data.

Figure 10.2 only includes the building cases which adopted the same simplifications as used by Potts & Addenbrooke (1997). Chapters 5 to 7 and 9 showed how additional building characteristics can be incorporated into the numerical model. Figure 10.3 plots $M_{DR}$ from analyses presented in these chapters against $\rho_{\text{mod}}^*$. Only results from 100m wide structures

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10_3.png}
\caption{$M_{DR}$ for cases with no eccentricity but additional building features plotted against $\rho_{\text{mod}}^*$.}
\end{figure}
with no eccentricity but varying building stiffness and tunnel depth are presented. Different symbols refer to different sets of analyses focusing on certain building features: ‘Weight’ and ‘IF’ refer to analyses modelling realistic building weight and the soil-structure interface, respectively. The 3D structures with an out of plane dimension of $L = 1\text{m}$ and $4\text{m}$, subjected to plane strain tunnel construction, are assigned by the labels ‘$L = 1$’ and ‘$L = 4$’; respectively, while ‘$3D, 20$’ and ‘$3D, 30$’ represents data from analyses modelling fully 3D tunnel construction with a building length of $L = 20\text{m}$ and $30\text{m}$ respectively. Buildings with no additional features but with the same geometry (i.e. which were included in the previous plot) are referred to as ‘standard’. The upper bound curves fitted to the data in Figure 10.2 are also shown in the graphs of Figure 10.3.

When comparing results with the same value of $\rho_{\text{mod}}^*$ (i.e. with the same transverse geometry) the graph shows increases in $M_{\text{DR}}$ (compared to the ‘standard’ analyses) for analyses including weight$^1$ and a small value of building length $L$. Most of the fully 3D analyses show a moderate reduction in $M_{\text{DR}}$ while modelling an extremely low interface shear stiffness causes a significant decrease in $M_{\text{DR}}$. All results lie inside the upper bound curves despite the wide variation of different building parameters.

The previous figures only focused on building geometries with no eccentricity with respect to the tunnel. Deflection ratio modification factors for buildings with eccentricity are presented in Figures 10.4 and 10.5. As previously $M_{\text{DR}}$ is first plotted against the original relative bending stiffness $\rho^*$ in Figure 10.4 (whose data were previously presented in Figure 4.44) before re-plotting the data against the modified expression $\rho_{\text{mod}}^*$ in Figure 10.5.

Comparing Figure 10.4a with Figure 10.5a shows that data points of different $e/B$ can be better distinguished when $\rho_{\text{mod}}^*$ is adopted as relative bending stiffness instead of $\rho^*$. As in the previous figures, due to the use of $\rho_{\text{mod}}^*$ all results are shifted to the right hand side and new upper bound curves had to be fitted to the data in Figure 10.5. For $M_{\text{DR}}^\text{avg}$ (Figures 10.4b and 10.5b) the scatter is similar when using $\rho^*$ and $\rho_{\text{mod}}^*$. New upper bound curves were fitted to the data shown in Figure 10.5b.

Figure 10.6 shows the upper bound curves for $e/B = 0.2$ and compares them with results of building cases with this eccentricity which additionally considered building weight.

$^1$The increase of $E_S$ due to building load was not considered in this and the following graphs as it was shown in Figure 5.10 (Page 159) that the influence of building load on $E_S$ taken at half tunnel depth is small.
and relative movement at the soil-structure interface. As pointed out in earlier chapters no eccentric geometries were studied in 3D analyses as they would have required a substantial increase in computer resources. Each building geometry included in this graph has therefore three data points with equal $\rho^\text{mod}$. The graph shows that the influence of weight on $M^\text{DR}$ is small for eccentric cases. The reduction of shear stiffness in the soil-structure interface, in contrast, has a more significant influence as it reduces $M^\text{DR}$ and therefore moves the results further away from the upper bounds. Therefore, all results lie within the boundaries fitted to the data shown in Figure 10.5.

### 10.3.2 Horizontal strain

It was concluded in Chapter 4 that the transverse geometry was adequately represented in the original formulation of relative axial stiffness $\alpha^*$. Even when wide ranges of building widths,
eccentricities and tunnel depths were considered, $M^{\text{eh}}$ when plotted against the original relative axial stiffness $\alpha^*$ followed the pattern of the Potts & Addenbrooke (1997) design curves, although some results exceeded the limits imposed by the curves. This situation is illustrated in Figure 10.7 which plots strain modification factors against $\alpha^*$ (the same graphs were shown in Figure 4.46). It was shown in Chapter 4 that changes in the initial stress profile between analyses presented in this thesis and the study by Potts & Addenbrooke (1997) accounts for data points lying outside the design curves.

It was shown that including the tunnel depth $z_0$ into the relative axial stiffness did not improve the distribution of $M^{\text{eh}}$ but lead to a less distinct pattern between data points of different eccentricity (see Figures 4.46 and 4.47, Page 146).

It was highlighted before, that the dimension of $\alpha^*$ is [length] when analysing 3D structures compared to a dimension-less expression in plane strain situations. To overcome this problem the building length $L$ was included into the relative axial stiffness expression proposed in Chapter 7. This leads to a relative axial stiffness which is always dimension-less, whether used in plane strain or in 3D. Based on this expression, the relative axial stiffness can be
modified to

\[ \alpha_{\text{mod}}^{*} = \frac{EA}{E_{S}BL} \]  

(10.2)

where the half building width \( \frac{B}{2} \) in the denominator was replaced by \( B \) resulting in the results being shifted to the left when plotted against a horizontal log-scale axis.

This can be seen when comparing Figures 10.7 and 10.8 in which \( M_{\text{th}}^{*} \) for both eccentric and concentric building cases are plotted against \( \alpha^{*} \) and \( \alpha_{\text{mod}}^{*} \), respectively. New upper bound curves are fitted to the data plotted against \( \alpha_{\text{mod}}^{*} \). As with the original curves the values of \( M_{\text{th}}^{*} \) have a small magnitude compared with the corresponding upper bound curves of the \( M_{\text{DR}}^{*} \) graphs.

The results of analyses in which additional building features were considered are shown in Figure 10.9. For the 3D studies (labelled as ‘L = 1’, ‘L = 4’, ‘3D, 20’, ‘3D, 30’) only non-eccentric building cases were analysed. No tension was obtained in these analyses. Data
Figure 10.8: $M_{th}^{th}$ plotted against modified relative axial stiffness $\alpha^*_{mod}$.

points for analyses considering building weight also include eccentric building scenarios ($e = 20\,m$, $B = 100\,m$, $z_0 = 20\,m$ and $34\,m$) in which tensile strain developed in the building during tunnel construction. Analyses with interface elements are not represented in this figure as the tunnel induced strain in the building was so small that the corresponding modification factors were more than 2 orders of magnitude smaller than $M_{th}^{th}$ obtained from the corresponding no-interface analyses.

It can be seen that building weight slightly increases the modification factors for both tension and compression. Fully 3D tunnel construction (‘3D, 20’ and ‘3D, 30’) also has a small influence with 20m long buildings tending to increase $M_{th}^{th}$ while 30m long structures reduce its value. The largest increases, however, are due to small values in building length $L$, as discussed in Chapter 7. The graph shows that data points for buildings with $L = 1m$ lie outside the upper bounds which were fitted to the data shown in Figure 10.8. However, a
length of $L = 1\text{m}$ was chosen as an extreme case and is an unrealistic value when the stiffness of entire buildings is considered. Results for $L = 4\text{m}$, which is still a small value for building length, are also included in the graph. Results for $L = 8\text{m}$ were smaller than those for $L = 4\text{m}$ and are not plotted in the graph. It can be seen, that the changes in $M_{thc}$, imposed by these building cases are much smaller than for $L = 1\text{m}$. $M_{thc}$ for 4m long buildings lie inside the upper bound curves.

**Figure 10.9:** $M_{th}$ for cases with additional building features plotted against $\alpha^*_\text{mod}$. 
10.4 Conclusions

This chapter summarizes the modification factors obtained from a wide range of different building scenarios and correlates this data against relative building stiffness. This approach is based on the relative stiffness method, proposed by Potts & Addenbrooke (1997).

In their original study Potts & Addenbrooke (1997) included a wide range of different geometries including two tunnel depths. This thesis presented a larger number of analyses in which additional building features, such as building weight, low friction between the soil and the foundation and the longitudinal dimension in the direction of the tunnel axis were modelled. Addition tunnel depths were also included. These analyses showed that their design concept is still applicable when data with these additional building characteristics are considered.

It was, however, shown that redefining the relative bending stiffness expressions $\rho^*$ can reduce the scatter observed when plotting modification factors against $\rho^*$. A new definition of relative bending stiffness, $\rho_{\text{mod}}^*$, was presented, which is dimension-less when applied to both 2D and 3D situations in contrast to the original $\rho^*$ which had the dimension $[1/\text{length}]$ in plane strain situations. New upper bound curves, shown in Figure 10.10 were fitted to the deflection modification factors when plotted against $\rho_{\text{mod}}^*$.

For axial strain it was found that the original definition of relative axial stiffness $\alpha^*$ gives a good correlation with $M^{\epsilon_h}$ for all cases analysed. The expression was, however, rearranged to be dimension-less when applied to both 2D and 3D conditions. As for the relative bending stiffness the half building width used by Potts & Addenbrooke (1997) was replaced by $B$ to be more consistent with the degree of eccentricity which is expressed as $e/B$. For this modified relative axial stiffness expression new upper bound curves, shown in Figure 10.11 were fitted to the data. These curves can then be incorporated into a building damage assessment as outlined by Potts & Addenbrooke (1997) (see Section 2.4.6.2).

This Chapter demonstrated that the relative stiffness approach for predicting tunnel induced building subsidence can be adopted for a wide range of building scenarios. Incorporating the building length into the relative stiffness formulations yields dimension-less expressions. The new upper bound curves presented here can be used as design curves which allow the relative stiffness method to be used for design purposes.
Figure 10.10: Proposed design curves for $M^{DR}$ adopting the modified relative bending stiffness $\rho_{\text{mod}}$. 

$$\rho_{\text{mod}} = \frac{EI}{E_S z_0 B^2 L}$$
Figure 10.11: Proposed design curves for $M^{th}$ adopting the modified relative bending stiffness $\alpha_{\text{mod}}$. 
Chapter 11

Conclusions and recommendations

11.1 Introduction

The objective of this research was to investigate the mechanisms of tunnel induced ground movement and to assess methods of including for soil-structure interaction effects in the evaluation of building deformation and potential damage caused by tunnel construction. This thesis presented parametric studies using both two and three dimensional FE analysis to investigate the influence of certain building characteristics on ground and building deformation. The process of tunnel excavation was modelled three dimensionally to study the implication of such a construction process on existing surface structures. Following the relative stiffness method proposed by Potts & Addenbrooke (1997) the building deformation was quantified by calculating modification factors for deflection ratio and horizontal strain, which correlate to relative stiffness expressions. This thesis showed that these relative stiffness terms can be redefined in order to become dimension-less when used in both two and three dimensional situations.

This chapter summarizes the conclusion drawn in this thesis. Firstly the mechanisms which control the tunnel-soil-building interaction problem are discussed in Section 11.2. Secondly, the implications of these mechanisms on building deformation are shown in Section 11.3. The lessons learned from performing three dimensional (3D) FE analyses are presented in Section 11.4. Finally recommendations for future research are given.
11.2 Mechanisms of ground movement

It has been pointed out by Potts & Addenbrooke (1997) that tunnel induced building deformation is an interactive problem. Not only does tunnel construction deform the surface structure but the presence of this structure also affects the ground movement around the tunnel. With the results presented in this thesis it is possible to draw a more detailed picture of this interaction problem.

The following building characteristics were addressed in this research project:

- Building stiffness, expressed in number of storeys
- Geometry in the transverse direction to the tunnel axis (Building width $B$ and eccentricity $e$) and in the longitudinal direction (building length $L$)
- Building load
- Soil - building interface

The following features of the tunnel were investigated

- Tunnel depth
- Progress of tunnel excavation

The tunnel diameter was $D = 4.146m$ which is a common value for running tunnels on the London Underground system. In plane strain analyses tunnel construction was modelled by controlling the volume loss. A value of $V_L = 1.5\%$ was adopted.

The tunnel is deformed by the surrounding soil which then transmits this movement to the surface where, consequently, the building is deformed. In the analyses the soil was represented by a non-linear elasto-plastic model with the soil stiffness depending on strain level and on mean effective stress. During tunnel construction the ground was modelled fully undrained. The following characteristics of soil and stress regime were found to be particular influential:

- Soil stiffness $E_S$. As it depends on mean effective stress it initially increases with depth. The soil stiffness at tunnel axis level and near to the ground surface are of particular interest.
Chapter 11. Conclusions and recommendations

Section 11.2

- Ratio of lateral effective stress to vertical effective stress $\sigma'_h / \sigma'_v$. Initially this ratio is described by the lateral earth pressure coefficient at rest $K_0$ which was set to a value of 1.5 in this thesis. However, the application of building load can change this ratio.

Chapter 4 studied the influence of the transverse building geometry on ground and building deformation. It was shown that the geometry of the building has little influence on the tunnel induced ground movement in the vicinity of the tunnel. This influence reduces even further as the tunnel depth $z_0$ increases. Tunnel construction was modelled over a number of increments to achieve a volume loss of approximately 1.5%. It was found that with increasing building stiffness the volume loss at a certain excavation increment (i.e. a certain percentage of unloading) decreased slightly. This effect had no influence on the deformation criteria presented in this thesis as they were adjusted to a volume loss of $V_L = 1.5%$. This variation in volume loss, however, had implications on the ground movement when 3D analyses were performed. In a first step Chapter 7 modelled tunnel construction simultaneously over the whole mesh length, essentially representing plane strain tunnel excavation beneath a 3D surface structure. When calculating volume loss from the surface settlement trough the variation of $V_L$ found in plane strain analyses was confirmed: Settlement troughs which included the building gave lower values of $V_L$ than those in greenfield conditions obtained at large distances from the building. However, this variation was not found at tunnel depth when $V_L$ was calculated from the tunnel displacement. Consequently, lateral soil movement in the longitudinal direction parallel to the tunnel took place towards the building in order to compensate for the variation of $V_L$ caused at ground surface level by the presence of the structure. Similar behaviour was found when modelling the 3D tunnel construction process in Chapter 9. Generally it was found that soil movement in the vicinity of the tunnel was little affected by varying the geometry of the surface structure.

The fact that tunnel induced ground deformation depends on the stress regime and the soil properties at tunnel depth was highlighted in Chapter 2 were a number of FE studies performed by other authors were summarized. Different soil models (linear pre-yield/non-linear pre-yield and isotropic/anisotropic) and stress regimes (high or low $K_0$-situations) were included in these studies. It was shown by several authors that $K_0$ has an essential influence on tunnel induced ground deformation. In this thesis a value of $K_0 = 1.5$ was adopted. However, application of building load (modelled as fully drained) reduces the ratio of $\sigma'_h / \sigma'_v$. 
prior to tunnel construction. It also increased the level of mean effective stress $p'$.

Chapter 5 studied the implications of these stress changes. The study focused on two distinct zones: Either adjacent to the tunnel and within the uppermost soil layer in the vicinity to the surface structure. It was found that application of building load reduces horizontal soil movement at tunnel axis level. It was shown that this reduction was a consequence of two effects.

- The increase of soil stiffness $E_S$ (due to the increase in $p'$) reduces the volume loss. It was shown that this behaviour does not influence results of ground and building deformation when adjusted to a common value of $V_L$.
- The application of building load reduces $\sigma_h'/\sigma_v'$ at tunnel axis level which leads to a decrease of horizontal soil movement at this level. The volume loss is only little affected by this change of stress regime.

Thus, building weight influences the deformation of the tunnel and the surrounding soil as it changes the stress regime in this zone.

Building load also increases the mean effective stress and, consequently, the soil stiffness close to the ground surface. Furthermore the increase in this zone is more significant as the magnitude of overburden stress is low. It was shown in Chapter 5 that this increased soil stiffness on the one hand reduces horizontal surface soil movement in the vicinity to the building. On the other hand it means that the soil is more able to transfer this movement to the structure. The net result of this interaction is an increase in horizontal strain in the building.

The nature of the interface between building and soil and its consequences on the horizontal soil movement were studied in Chapter 6. Interface elements were included in the 2D FE analyses to vary the nature of the contact between soil and building foundation. The study focused on situations of frictionless soil-structure interaction. Relative movement between the soil and the building was allowed by adopting a low interface shear stiffness while opening of a gap was restricted due to the use of a high interface normal stiffness.

This extreme situation enabled the investigation of the influence of the horizontal boundary condition at ground surface independently from the vertical one. For a given building stiffness the vertical building settlement was little affected by changes of the interface shear
stiffness. The width of the building settlement trough at ground surface level was not affected by the change in interface properties while it led to narrower subsurface settlement troughs. The horizontal ground movement close to the surface structure increased when a low interface shear stiffness was modelled reaching similar magnitudes to those obtained under greenfield conditions while the horizontal building deformation reduced drastically in these cases. This can lead to relative horizontal movement between soil and foundation which might affect service connections with the building.

**Figure 11.1:** Relation between different elements of the tunnel-soil-building interaction problem.

The detailed picture of the tunnel-soil-structure interaction problem outlined above is illustrated in Figure 11.1. In general it was found that the presence of the building has only a small effect on ground displacement in the vicinity of the tunnel. The only factor which was found to influence the stress state around the tunnel was the building load (i.e. self weight). However, the effect was small compared to the influence that building load imposes on the soil stiffness close to the surface structure. The tunnel is the source of the ground movement...
while the building acts as a boundary condition. Its influence is limited close to the ground surface. However, in this zone the building interacts with the soil as the presence of building load alters the stiffness of the ground which then affects how the soil transmits the tunnel induced ground movement to the building.

11.3 Building deformation

Building deformation was quantified by using deformation criteria deflection ratio ($DR_{\text{sag}}$ and $DR_{\text{hog}}$ for sagging and hogging, respectively) and maximum horizontal strain ($\epsilon_{\text{hc}}$ and $\epsilon_{\text{ht}}$ for compression and tension, respectively). Following the relative stiffness method proposed by Potts & Addenbrooke (1997) modification factors were calculated by dividing each of the deformation criteria by the corresponding greenfield deformation criterion. These modification factors correlate with the relative bending or axial stiffness, $\rho^*$ and $\alpha^*$, respectively. These relative stiffness expressions, introduced by Potts & Addenbrooke (1997), relate the building stiffness and its geometry to the stiffness of the soil.

In their work Potts & Addenbrooke (1997) included over 100 plane strain FE analyses for which upper bound curves were fitted when modification factors were plotted against relative stiffness. This thesis presented similar analyses (Chapter 4) but also included additional building features such as building weight (Chapter 5), frictionless soil-building interface (Chapter 6) and building dimension in both the transverse and the longitudinal direction (Chapters 7 and 9). The 3D nature of the tunnel construction process was also studied (Chapter 9).

**Geometry:** The following geometric parameters were varied in this study: Building width $B$, eccentricity $e$, building length $L$ and tunnel depth $z_0$. The relative stiffness approach by Potts & Addenbrooke (1997) predicts that the building’s deformation criteria increase with increasing $B$. This trend was generally confirmed by this study. However, it was found that from a certain value of $B$ the deformation criteria do not increase further. This trend was most distinct for $DR_{\text{sag}}$. It led to a significant scatter when plotting modification factors against relative stiffness showing that the influence of $B$ in the original formulation of Potts & Addenbrooke (1997) was over-estimated.

When eccentricity $e$ was varied it was shown that $DR_{\text{sag}}$ and $\epsilon_{\text{hc}}$ reduce with increasing
While $\Delta R_{\text{hog}}$ and $\epsilon_{\text{ht}}$ increased. For hogging and tension, maximum deformation criteria were obtained for degrees of eccentricity of approximately $e/B = 0.6$. The trend for the corresponding modification factors was similar and is in good agreement with the design charts proposed by Potts & Addenbrooke (1997). Despite the trend of reducing $\Delta R_{\text{sag}}$ and $\epsilon_{\text{hc}}$ with increasing $e/B$ the study showed that high degrees of eccentricity can exhibit high values in the corresponding modification factors. It was shown that this increase is a result of the fact that the positions of the building’s points of inflection change with eccentricity. This can lead to situations in which a small value of building $\Delta R_{\text{sag}}$ is divided by an even smaller value of greenfield $\Delta R_{\text{sag}}$ leading to unreasonable high modification factors.

Varying the tunnel depth $z_0$ showed that all deformation criteria decrease with increasing $z_0$. In contrast, the corresponding modification factors increased with increasing $z_0$. For deflection ratio modification factors it was found that this trend was not appropriately predicted by the relative stiffness method when low relative bending stiffnesses were considered. It was therefore argued, to include the tunnel depth directly into the formulation of relative bending stiffness, instead of the indirect representation due to the soil stiffness being taken at half tunnel depth.

3D FE analyses were applied to study the influence of the building dimension $L$ in the longitudinal direction to the tunnel axis. By constructing the tunnel simultaneously over the whole length (instead of step-by-step excavation) it was shown that building deformation does not significantly vary in the longitudinal direction. When $L$ was varied it was found that the building settlement was not greatly affected while the horizontal displacement of the building increased significantly with reducing $L$. It was concluded that this behaviour is due to the fact that buildings considered in this study behave much more stiffly in the axial direction than in bending. When deformation criteria were plotted against $L$ the graphs revealed, for all criteria, an increase with reducing $L$.

Even with this increase most deflection ratio modification factors remained below the Potts & Addenbrooke (1997) design curves when plotting $M_{\text{DR}}$ against the plane strain definition of $\rho^*$. It was therefore concluded that the plane strain relative bending stiffness formulation is applicable to 3D problems. Furthermore it was demonstrated that including $L$ into the definition of relative bending stiffness can lead to consistent dimensions of relative stiffness whether used in 2D or in 3D. For the horizontal strain modification factors a similar
approach was applied although modification factors lay outside the Potts & Addenbrooke (1997) design curves.

**Building weight:** It was shown that generally building deformation criteria increase as the load imposed by building weight increases. When focusing in realistic building stiffness-load combinations it was shown that the increase of $M^{DR}$ with building load was outweighed by the decrease of $M^{DR}$ with increasing building stiffness. When plotted against relative bending stiffness $\rho^*$ the results of both no-load and load cases were close together and lay inside the Potts & Addenbrooke (1997) design curves. A more severe increase with load was found for $M^{th}$ causing modification factors to lie outside the corresponding design curves although they still remained small.

**Soil-structure interface:** Soil-structure interface elements were included in the analyses to study the influence of the contact between soil and the building foundation. When modelling the building weightless and assigning a Mohr-Coulomb material model to the interface it was shown that during tunnel construction a gap develops between soil and building. The settlement trough of the building became flatter and wider in such a case. However, such a separation was not found when building weight was applied.

In another scenario low interface shear stiffness allowing free relative horizontal movement between soil and building while adopting a high normal interface stiffness restricting any relative normal movement were used. In such cases the building showed very small horizontal movement and hardly any horizontal building strain. The settlement behaviour was less affected with the deflection ratio reducing leading to lower $M^{DR}$ values.

**Tunnel construction progress:** The three dimensional process of tunnel excavation was modelled by adopting a step-by-step approach. All buildings in these analyses were in plan perpendicular to the tunnel axis with the longer side being the width, transverse to the tunnel axis. In the 3D study higher volume losses were obtained than in the corresponding plane strain analyses. In order to compare the results from these analyses with the plane strain ones, deflection ratios and horizontal strain were adjusted or interpolated to a volume loss of $V_L = 1.5\%$. Comparing the modification factors obtained from these analyses with corresponding factors from plane strain analyses revealed that the 3D analyses tend to give
lower modification factors.

Modelling tunnel construction as a step-by-step excavation caused a progressing settlement trough in the longitudinal direction. Consequently, building deformation develops in this direction. When comparing the magnitude of longitudinal $DR_{hog}$ with the values of transverse $DR_{hog}$ obtained in the same analyses it was concluded that the transverse deflection ratio shows more critical values. Similar conclusions were drawn for tensile horizontal building strain developing in the longitudinal direction during the tunnel face advance.

In addition to deflection ratio and horizontal strain in transverse and longitudinal direction, building twist deformation was calculated from the 3D analyses. It was shown that both building width and length should be included in the definition of twist $\Theta$, leading to a formulation which is not dimension-less (as the other deformation criteria) but is expressed in $[1/\text{length}]$. A case study presented by other authors indicated that twist deformation is mainly of a temporary nature when the route of the tunnel is perpendicular (in plan view) to the existing structure. Another case study showed that if there is an skew angle between building and tunnel, twist deformation remains after completion of tunnel construction. However, in this thesis only buildings perpendicular to the tunnel axis were modelled. The trend of the peak in twist deformation occurring when the tunnel face is beneath the building was confirmed by the analyses. With further tunnel face advance the twist reduced but a small value remained when the tunnel had advanced a considerable distance from the building. It was concluded that this could be due to different magnitudes of volume loss on either side of the building. However, the analyses indicated that this effect might have been amplified due to the fact that no steady-state settlement conditions were established during the analyses (which will be discussed in more detail in the next section). Nevertheless the parametric study showed some important trends when twist modification factors $M^\Theta$ were calculated: An increase in building stiffness reduces $M^\Theta$ while increasing building width and/or length increases it. $M^\Theta$ also increases with increasing $z_0$ (although the magnitude of twist itself reduces with increasing $z_0$). These trends are in agreement with the trends predicted by the relative stiffness method for transverse building deformation modification factors.

**New design charts:** The above conclusions for transverse building deformation were used to modify the relative stiffness expressions proposed by Potts & Addenbrooke (1997). It was
Chapter 11. Conclusions and recommendations

Section 11.4

shown that including $z_0$ and reducing the influence of $B$ in the relative bending stiffness reduces the scatter when plotting $M^{DR}$ against relative bending stiffness. By incorporating $L$ in the formulation it was shown that the stiffness expression keeps its dimension regardless whether used in 2D or 3D situations. A modified relative bending stiffness expression was proposed:

$$\rho^*_{\text{mod}} = \frac{EI}{E S z_0 B^2 L}$$  \hspace{1cm} (see Equation 10.1)

where $B$ and $L$ are the building dimensions in the transverse and longitudinal direction, respectively. $EI$ is the bending stiffness of the building. If applied in 3D conditions $L$ is included in the definition of the second moment of area, $I$, which then has the dimension $[\text{length}^4]$. However, if plane strain conditions apply, $I$ has the dimension $[\text{length}^4/\text{length}]$ while $L$ has $[\text{length}/\text{length}]$. It can be seen that the above expression always remains dimension-less.

For the relative axial stiffness a similar approach was adopted by redefining the relative axial stiffness to

$$\alpha^*_{\text{mod}} = \frac{EA}{E S B L}$$  \hspace{1cm} (see Equation 10.2)

where the cross sectional area $A$ includes $L$. As shown for $\rho^*_{\text{mod}}$, $\alpha^*_{\text{mod}}$ remains dimension-less for both 2D and 3D scenarios.

When plotting the modification factors of the various parametric studies presented in this thesis against these new relative stiffness expressions, upper bound curves were fitted to the data to provide design charts similar to those introduced by Potts & Addenbrooke (1997). These design charts (shown in Figures 10.10 and 10.11, pages 305 and 306, respectively) can be used to assess potential building damage caused by tunnel construction.

11.4 Three-dimensional analysis

Three dimensional FE analyses were performed to investigate the 3D nature of the tunnel construction process and to assess its influence on building deformation. Tunnel excavation was modelled by a step-by-step approach: Successive removal of elements in front of the tunnel while successively installing lining elements behind the tunnel. The excavation length $L_{\text{exc}}$ over which soil was excavated in one increment was set to 2.5m in most analyses. It was shown that this dimension controls the volume loss and higher values of $V_L$ than in
plane strain analyses were obtained when the above value of $L_{exc}$ was adopted. It was shown that reducing $L_{exc}$ but keeping the same mesh dimensions would require substantially more computational resources.

As a first step only greenfield situations were considered (Chapter 8). For this purpose construction of the westbound tunnel of the Jubilee Line Extension beneath St. James’s Park, London was modelled and the results were compared with those from a corresponding 2D analysis. For plane strain analyses of tunnel construction in a high $K_0$-regime ($K_0 = 1.5$ was adopted throughout the thesis to model London Clay) it was found that the surface settlement trough is too wide compared to field data. It was suggested that this is due to the fact that plane strain analyses do not account for 3D effects ahead of the tunnel face. However, it was found that both 3D and 2D analyses give similar transverse settlement troughs which were too wide when compared with the field measurements reported for St. James’s Park. Furthermore, the 3D analysis exhibited a longitudinal surface settlement profile (above the centre line of the tunnel) which was also too wide. As a consequence, no steady state settlement conditions were achieved behind the tunnel face. One could argue that this behaviour indicates that the dimensions of the FE mesh were chosen to be too small. However, the total length of tunnel construction was 100m which is more than $21.0 \times D$ (with a tunnel diameter of $D = 4.75m$) with another 55m ($11.5 \times D$) of soil ahead of the last tunnel face position. These dimensions are significantly larger than those of the 3D meshes used by other authors summarized in Chapter 2.

It was stated by other authors that soil anisotropy accounts for the discrepancy between FE results and field measurements. A transversely anisotropic soil model was therefore included in the analyses to study whether this constitutive relation in combination with 3D modelling is capable of improving the shape of surface settlement troughs. The transversely anisotropic formulation proposed by Graham & Houlsby (1983) was combined with the small-strain-stiffness behaviour used in the non-linear pre-yield model of the isotropic analyses. When applied with soil parameters appropriate for London Clay the analyses (both 2D and 3D) showed little improvement compared with the isotropic study. An unrealistic high degree of anisotropy was also applied and gave narrower settlement troughs in both transverse and longitudinal directions. It was concluded that this 3D analysis was, in the last increment, closer to steady state conditions than the isotropic one. The volume loss, developed during
Chapter 11. Conclusions and recommendations

Section 11.4

this 3D analysis, however, was unrealistic high.

The literature review of 3D FE analyses presented several publications in which it was stated that steady-state settlement conditions were established at the end of the analyses. In contrast, the results presented in this thesis did not agree with these conclusions. It was noted, that different values of $K_0$ could be the source for this discrepancy. A 3D analysis was performed for both $K_0 = 1.5$ and 0.5. The longitudinal settlement profile of the $K_0 = 0.5$ case became horizontal at a distance of approximately 30m ($7 \times D$) behind the tunnel face. The analysis revealed, however, that during the last excavation step incremental settlement (albeit of small value) still developed over the whole mesh length. This indicated that despite a horizontal profile behind the face no steady-state conditions were reached. This study highlighted that presenting a longitudinal settlement profile from the end of an analysis is not sufficient to demonstrate that the settlement process has developed steady-state conditions. Settlement profiles from consecutive increments should also be given. It is interesting to note that only one publication discussed in the literature review provided this information.

After investigating the influence of 3D tunnel construction on greenfield surface settlement, buildings were included in the 3D model. Only buildings with no eccentricity with respect to the tunnel were analysed and the longer side of all buildings was perpendicular to the tunnel axis.

During the 3D analyses higher volume losses were obtained than adopted in the plane strain studies. By reducing the excavation length $L_{exc}$ lower values of $V_L$ developed. However, it was found that analyses with different values of $L_{exc}$ exhibited a similar development of transverse deformation criteria with increasing volume loss during the analyses. It was concluded that $L_{exc}$ has only a small influence on these deformation criteria when values are taken for a certain volume loss. It was therefore possible to adjust or interpolate results of deflection ratio and horizontal building strain to a common volume loss of $V_L = 1.5\%$ and to compare them with corresponding 2D analyses. The building deformation in the transverse direction was similar (with a tendency to be lower) to results obtained from 2D models - a similar conclusion as drawn from the greenfield analyses.

The fact that no steady-state settlement conditions were achieved during the analyses had implications on the building’s deformation behaviour. As the tunnel approached the building, the longitudinal $DR_{hog}$ increased. For buildings of low stiffness (1 storey) $DR_{hog}$ reached a
local maximum as the tunnel face was beneath the building front and then reduced as the face moved towards the rear of the structure. From then on, $DR_{h_{og}}$ increased again until the end of the analysis. For buildings of higher stiffness there was no reduction but $DR_{h_{og}}$ increased during the whole analysis reaching its maximum at the end. Greenfield analysis, in contrast, predicted $DR_{h_{og}}$ to vanish after the tunnel face reached a certain distance from the greenfield section. Closer inspection of the development of volume loss in the longitudinal direction showed little difference between greenfield and building cases. Only between the building and the start boundary of the FE mesh did the analyses develop a slightly higher amount of $V_{L}$ compared to greenfield situations. Such a difference could cause building deformation to remain after the tunnel face has moved away from the building. However, the small difference in volume loss detected between the two scenarios is unlikely to cause such final values of longitudinal $DR_{h_{og}}$ as observed in the analyses. It was therefore concluded that there might be some remaining deformation in the building which is magnified by the fact that settlement has not reached steady-state conditions.

Similar behaviour was found for twist deformation although a maximum twist developed in all analyses (with varying building stiffness, geometry and tunnel depth) when the tunnel face was beneath the building. Twist then reduced until it reached a level at which it remained constant until the end of the analysis. It is not possible to say whether the twist found at the end of the analysis remains in the building or whether it is purely a result of the ongoing settlement process as no steady-state settlement conditions developed during the analysis.

### 11.5 Recommendations for future research

The work presented in this thesis has helped to provide a better understanding of the mechanisms which control the tunnel-soil-building interaction problem. There were, however, limitations when modelling the interaction problem.

- The structure was represented by an elastic beam or in 3D analyses by elastic shell elements. This approach is consistent with the evaluation of limiting strain proposed by Burland & Wroth (1974). However, it does not account for the non-linear behaviour that masonry structures exhibit when cracking occurs. Similar parametric studies should be undertaken to investigate the influence of such a material on both building and ground
deformation.

- In the FE analyses tensile strain was only obtained in buildings with eccentricity. However, such a building geometry were not modelled in 3D due to the substantially higher computational resources required for asymmetric geometries compared to symmetric ones. The influence of 3D tunnel construction on tensile strain was therefore not investigated. Eccentric building cases would also give a better picture of how hogging develops when step-by-step tunnel excavation is modelled.

- In the 3D analyses all buildings were perpendicular to the tunnel axis in plan. This configuration allowed the mesh to be generated by expanding a plane strain mesh into the longitudinal direction. Recently, fully 3D mesh generation has become possible in ICFEP. With this tool it is possible to include buildings with a skew angle with respect to the tunnel centre line. Such analyses would enable the study of more general forms of building twist. Such situations, however, are asymmetric with respect to the tunnel centre line and, therefore, require a large amount of computer resources.

- During construction of the Jubilee Line Extension twist deformation was observed in different structures. Although no damage was recorded in these particular cases the research highlighted the need to incorporate twist deformation into the damage assessment by extending the deep beam model to three dimensions.

- This thesis focused on short term effects of tunnel construction. In all analyses presented in this thesis tunnel construction was modelled undrained. Coupled consolidation analysis should be incorporated into the 3D model to study the long-term effects tunnel construction has on buildings.

- This thesis assessed the relative stiffness method by incorporating additional building features into the FE analyses. For twist, the general behaviour found in the FE analyses was compared with data from case studies. However, further comparisons of the predictions obtained from the relative stiffness approach with field data is essential. Unfortunately it was found that most case studies obtained from the JLE do not provide an appropriate comparison as additional construction events such as twin tunnel construction or compensation grouting influenced the building’s deformation behaviour.
• It was shown that 3D analysis did not reach steady-state settlement conditions at the end of the analyses although large mesh geometries were applied. It was shown that for a high $K_0$-regime the FE 3D settlement trough is too wide in both the longitudinal and the transverse direction. Furthermore, comparison of 3D with corresponding 2D analyses showed that 3D modelling does not substantially improve transverse settlement predictions obtained from 2D analyses. Improving greenfield settlement predictions for high $K_0$-situations should be the focus of future research.
Appendix A

The Finite Element Method

A.1 Basic formulation

This appendix summarizes the basic equations of the finite element method. A more detailed description can be found in Potts & Zdravković (1999) and Zienkiewicz & Taylor (1989). The finite element formulation as used in the geotechnical context uses displacement as the primary variable. When simulating consolidation the pore water pressure is an additional primary variable. The following equations include three dimensional displacement but no pore pressure as coupled analyses were not used in this work. For simplification, only two dimensional geometries are considered. The equations are derived following an incremental formulation.

**Displacement:** The 3D displacement at any point, defined by its location $\mathbf{x}_G$, is given by

$$\Delta \mathbf{d}(\mathbf{x}_G) = \begin{bmatrix} \Delta u(\mathbf{x}_G) \\ \Delta v(\mathbf{x}_G) \\ \Delta w(\mathbf{x}_G) \end{bmatrix}$$

where $\mathbf{x}_G$ is a vector containing the global coordinates. The displacement is described by dividing the problem domain into small regions so called *finite elements*. In each element the displacement is approximated using *shape functions* which derive the displacement of any point within the element from the displacement at discrete nodes. Figure A.1 shows a single element with four corners and eight nodes. The following equations are derived for the parent
Appendix A. The Finite Element Method

Section A.1

The element shown in Figure A.1 has 8 nodes leading to shape functions being 2nd order polynomial. Elements with only 4 nodes use linear shape functions while shape functions of higher order than 2 can be described by introducing more nodes. If a shape function for a corner node of the 8-node element is given as

\[ N_2(x_N) = \frac{1}{4} (1 + S) (1 - T) - \frac{1}{4} (1 - S^2) (1 - T) - \frac{1}{4} (1 + S) (1 - T^2) \]  

(A.2)

where \( S \) and \( T \) are the components of the natural coordinate system than it can be seen that this function gives unity at the corner node with \( S = 1 \) and \( T = -1 \) (which is denoted as node 2 in Figure A.1) and zero at all other nodes. Figure A.2 shows this shape function.

A shape function for a mid side node is defined as

\[ N_5(x_N) = \frac{1}{2} (1 - S^2) (1 - T) \]  

(A.3)

Figure A.1: Coordinate systems of (a) parent element and (b) global element (after Potts & Zdravković (1999)).
Appendix A. The Finite Element Method

Similar shape functions can be defined for the other nodes. The displacement at any point within the element can then be expressed by

\[
\Delta u = N_1 \Delta u_1 + N_2 \Delta u_2 + \ldots + N_7 \Delta u_7 + N_8 \Delta u_8 \quad (A.4)
\]

\[
\Delta v = N_1 \Delta v_1 + N_2 \Delta v_2 + \ldots + N_7 \Delta v_7 + N_8 \Delta v_8 \quad (A.5)
\]

\[
\Delta w = N_1 \Delta w_1 + N_2 \Delta w_2 + \ldots + N_7 \Delta w_7 + N_8 \Delta w_4 \quad (A.6)
\]

or in a more general form

\[
\Delta d = N \Delta d_E \quad (A.7)
\]

where \( d_E \) contains the displacement for each node of the element and \( N \) contains the shape functions. The coordinates in the above equations are omitted for simplicity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{shape_function.png}
\caption{Shape function for corner node of 8 node 2D element.}
\end{figure}

It follows from the definition of the shape functions that displacement values on an element boundary only depend on the nodal values along this border. The shape functions above therefore satisfy continuity between two elements connected to each other.

The geometry of the global element can be derived from the parent element using the same shape functions. In this case the element is termed isoparametric. With the parabolic shape functions shown above it is possible to describe curved element boundaries which can be found in many geotechnical problems.
Appendix A. The Finite Element Method

Section A.1

Strain: Strains are derived from the displacements as follows
\[
\begin{align*}
\Delta \epsilon_x &= - \frac{\partial \Delta u}{\partial x} \\
\Delta \epsilon_y &= - \frac{\partial \Delta v}{\partial y} \\
\Delta \epsilon_z &= - \frac{\partial \Delta w}{\partial z} \\
\Delta \gamma_{xy} &= - \frac{\partial \Delta u}{\partial y} - \frac{\partial \Delta v}{\partial x} \\
\Delta \gamma_{yz} &= - \frac{\partial \Delta v}{\partial z} - \frac{\partial \Delta w}{\partial y} \\
\Delta \gamma_{zx} &= - \frac{\partial \Delta w}{\partial x} - \frac{\partial \Delta u}{\partial z}
\end{align*}
\] (A.8)
or in matrix formulation
\[
\Delta \epsilon = S \Delta d
\] (A.9)

with
\[
S = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 \\
0 & \frac{\partial}{\partial y} & 0 \\
0 & 0 & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\
0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\
\end{bmatrix}
\quad \text{and} \quad
\Delta \epsilon = \begin{bmatrix}
\Delta \epsilon_x \\
\Delta \epsilon_y \\
\Delta \epsilon_z \\
\Delta \gamma_{xy} \\
\Delta \gamma_{yz} \\
\Delta \gamma_{zx}
\end{bmatrix}
\] (A.10)

and introducing nodal displacement and shape functions:
\[
\Delta \epsilon = S \Delta d = S \Delta \hat{d} = B \Delta d_E
\] (A.11)

The matrix B contains the derivatives of \( N_i \) with respect to the global coordinate system.

Applying the chain rule leads to
\[
\begin{bmatrix}
\frac{\partial N_i}{\partial S} \\
\frac{\partial N_i}{\partial T}
\end{bmatrix} = \mathbf{J} \begin{bmatrix}
\frac{\partial N_i}{\partial x} \\
\frac{\partial N_i}{\partial y}
\end{bmatrix}
\] with the Jacobian matrix \(
\mathbf{J} = \begin{bmatrix}
\frac{\partial x}{\partial S} & \frac{\partial y}{\partial S} \\
\frac{\partial x}{\partial T} & \frac{\partial y}{\partial T}
\end{bmatrix}
\) (A.12)

where \( x \) and \( y \) are the components of the global coordinate system \( \mathbf{x}_G \) while the coordinates \( S \) and \( T \) form the natural system \( \mathbf{x}_N \).

Constitutive model: The material behaviour is described by the constitutive model
\[
\Delta \sigma = D \Delta \epsilon
\] (A.13)

with \( \Delta \sigma^T = [\Delta \sigma_x \Delta \sigma_y \Delta \sigma_z \Delta \gamma_{xy} \Delta \gamma_{yz} \Delta \gamma_{zx}] \) and \( D \) being the constitutive matrix. As the strain and the stress vectors have 6 components \( D \) must be a 6 \( \times \) 6 matrix. For symmetrical reasons it only has 21 independent components. For an isotropic linear elastic material the number of independent material parameter reduces to 2.
Appendix A. The Finite Element Method

Section A.1

**Equilibrium:** The equilibrium is invoked using the principle of minimum potential energy which may be stated as follows (Huebner & Thornton, 1982):

The displacement which satisfies the differential equations of equilibrium as well as the boundary conditions yields a smaller value for the potential energy than any other displacement which satisfies the same boundary conditions.

The potential energy (in incremental form) is defined as

\[
\Delta E = \Delta W - \Delta L
\]  
(A.14)

with the strain energy \(\Delta W\)

\[
\Delta W = \frac{1}{2} \int Vegaso \Delta \epsilon^T \sigma dVol = \frac{1}{2} \int Vegaso \Delta \epsilon^T D \Delta \epsilon dVol
\]  
(A.15)

and the work done by applied loads:

\[
\Delta L = \int Vegaso \Delta d^T T dVol + \int Srf \Delta d^T T dSrf
\]  
(A.16)

\(\Delta F\) are body forces and \(\Delta T\) are surface tractions acting on the surface of the domain.

**Element equations:** The element equation are based on the principle of minimum potential energy. Minimizing Equation A.14 leads to

\[
\delta \Delta E = 0
\]

\[
= \frac{1}{2} \int Vegaso 2 \delta \epsilon^T D \Delta \epsilon dVol - \int Vegaso \delta d^T T F dVol - \int Srf \delta d^T T T dSrf
\]  
(A.17)

and after discretising the problem domain into elements and invoking shape functions to approximate the displacement:

\[
\approx \sum_{E} \left( \int Vegaso \delta d_E^T B^T D B \Delta d_E dVol 
\right.
\]

\[
- \int Vegaso \delta d_E^T N^T \Delta F dVol - \int Srf \delta d_E^T N^T \Delta T dSrf
\]  
(A.18)

\[
= 0
\]
Appendix A. The Finite Element Method

Section A.2

where $n_E$ is the number of elements and $\sum n_E$ is the sum over all elements. Defining the element stiffness matrix:

$$K_E = \int_{Vol} B^T DB \, dVol$$  \hspace{1cm} (A.19)

and the right hand side load:

$$\Delta R_E = \int_{Vol} N^T \Delta F \, dVol + \int_{Srf} N^T \Delta T \, dSrf$$  \hspace{1cm} (A.20)

leads to the element equilibrium equation:

$$K_E \Delta d_E = \Delta R_E$$  \hspace{1cm} (A.21)

The integral in the element stiffness matrix $K_E$ has to be calculated in terms of the global coordinate system. This is done by invoking the Jacobian matrix $J$ (Equation A.12) as $dVol = t \, dx \, dy = t|J|dSdT$ with the thickness $t$ being unity in plane strain problems.

Global equation: The global equation is formed by assembling the separate element equations. This is done by replacing each element’s parent node numbers by the numbers of the global system and then summing up

$$K_G = \sum_{E} K_E; \quad \Delta d_G = \sum_{E} \Delta d_E; \quad \Delta R_G = \sum_{E} \Delta R_E$$  \hspace{1cm} (A.22)

where $K_G$ is the global stiffness matrix. The global equilibrium equation is then given by

$$K_G \Delta d_G = \Delta R_G$$  \hspace{1cm} (A.23)

The global stiffness matrix $K_G$ normally shows a band structure with the band width depending on the global numbering. This band width can be minimized in order to reduce the computer storage.

To solve Equation A.23 for $\Delta d_G$ (and therefore invert $K_G$) several approaches both direct and iterative exist. Algorithms based on the Gauss elimination are most commonly used (Potts & Zdravković, 1999). If, however, the global stiffness matrix is non-linear (due to the use of a non-linear material model) then the evaluation of the unknown displacement is not straight forward and special non-linear approximation schemes have to be applied.

A.2 Non linear solution method

As mentioned in Section 3.2.3 the incorporation of non linear material behaviour leads to a constitutive matrix $D$ which is not constant but depends on stress and/or strain level. Special
Appendix A. The Finite Element Method

Section A.2

solution strategies have to be applied to solve the non linear global equilibrium equation (Equation A.23). The modified Newton-Raphson (MNR) method in combination with the substepping stress point algorithm was applied within this work. Both methods are explained in the following sections.

A.2.1 Modified Newton-Raphson method

The MNR method is an iterative solution technique applied to each increment of the analysis. It assumes the global stiffness matrix $K_G$ to be constant over each increment. As $K_G$ is non linear an error arises expressed as the residual $\Psi$ shown in Figure A.3. The iterative approximation has the following form:

$$K^i_G \Delta d^{i,j}_G = \Psi^{j-1}$$

where the index $i$ and $j$ stand for the increment and iteration number respectively. For $j = 1$, $\Psi^0 = \Delta R^i_G$ as shown in Figure A.3. The iteration is repeated until the norm of the residual load $\Psi$ is smaller than a certain tolerance. It has to be pointed out that the modified Newton-Raphson method uses a constant global stiffness matrix $K^i_G$ over the whole increment $i$ while the (original) Newton-Raphson scheme recalculates (and inverts) $K^{i,j}_G$ for each iteration $j$. Although the recalculation of $K^{i,j}_G$ normally results in less iterations to approximate the true solution it requires significantly more computation time to evaluate the stiffness matrix in each iteration. To reduce the amounts of iteration required with the MNR method a acceleration technique is used.

The convergence criteria for this method are expressed by the norm of the iterative displacement $\| \Delta d^{i,j}_G \|$ and of the residual loads $\| \Psi^j \|$. The convergence criteria are set as a certain percentage of the iterative displacement compared with the incremental and with the accumulated displacement. In this work these criteria were set to 2%. Similar criteria can be defined for the iterative residual load by comparing it with the incremental and accumulated right hand side load vectors.

A key step in the MNR approach is the evaluation of the residual load vector $\Psi$. The strain change at the end of each iteration $j$ is calculated from the displacement in the iteration $\Delta d^{i,j}$. The corresponding stress change must then be estimated by integrating the non linear constitutive model along the strain path. This integration can be performed using a so called
stress point algorithm. For the analyses in this thesis a *substepping stress point algorithm* was used.

### A.2.2 Substepping stress point algorithm

The determination of the stress change associated with the strain change in each iteration is not straightforward. Initially it is assumed that the material behaves linearly elastic and the incremental stress can be estimated by integrating the elastic constitutive matrix $D$ along the incremental strains $\Delta \epsilon$. Problems arise when the stress state (in an integration point) is elastic at the begin of an iteration but becomes plastic at the end. If $\sigma_0$ is the stress state at the begin and $\Delta \sigma^j$ the stress change during one iteration giving the stress state at the end of the iteration $\sigma^j = \sigma_0 + \Delta \sigma^j$ than it has to be checked if this stress state is on or below the yield surface $F$. This is the case if $F(\sigma, k) \leq 0$ where $k$ are the hardening parameters. The index $j$ is omitted for simplification. If in contrast $F(\sigma, k) > 0$ the stress state becomes plastic at the end of the iteration. In this case the elastic portion of the stress increment has to be determined by evaluating a scalar $\alpha$ so that $F(\sigma_0 + \alpha \Delta \sigma, k) = 0$. The value of $\alpha$ can be determined using an iterative method such as a secant iteration approach which is used in ICFEP (Potts & Zdravković, 1999).
Having determined $\alpha$ the elastic portion of the stress increment can be calculated as

$$\Delta\sigma^e = \alpha \Delta\sigma \quad \text{and} \quad \Delta\epsilon^e = \alpha \Delta\epsilon$$  \hfill (A.25)

where the index ‘e’ denotes the elastic part. The plastic strain portion can be expressed by $(1 - \alpha)\Delta\epsilon$. To calculate the plastic stress the elasto-plastic constitutive matrix $D^{ep}$ has to be integrated along the remaining portion of the strain path. For this integration the plastic portion has to be subdivided into smaller increments. The integration is then performed using an approximative scheme such as the modified Euler integration algorithm which is described in the following section.

**A.2.2.1 Modified Euler integration**

This approach integrates the elasto-plastic constitutive matrix $D^{ep}$ along the stress path $\Delta\epsilon_s = (1 - \alpha)\Delta\epsilon$. Therefore $\Delta\epsilon_s$ is subdivided into smaller substeps $\Delta\epsilon_{ss} = \Delta T \Delta\epsilon_s$ with $0 < \Delta T \leq 1$. As a first approximation the stress state at the end of the substep is calculated using $D^{ep}$ evaluated at the begin of the substep. This gives the stress state $\Delta\sigma_1$ as shown in Figure A.4. The plastic strain along this stress increase is also evaluated as well as the change in hardening parameter. With these values a second estimate for the changes in stress are calculated with $D^{ep}$ calculated from the stress state and hardening parameter at the end of the substep. A more accurate estimate of stress increase, plastic strain and of the hardening parameter can then be found by taking the average of the first and of the second estimate, e.g.:

$$\Delta\sigma = \frac{1}{2} (\Delta\sigma_1 + \Delta\sigma_2); \quad \Delta\epsilon^p = \frac{1}{2} (\Delta\epsilon_1^p + \Delta\epsilon_2^p); \quad \Delta k = \frac{1}{2} (\Delta k_1 + \Delta k_2)$$  \hfill (A.26)

There is of course an error in this estimation which can be estimated to be $E \approx 0.5(\Delta\sigma_2 - \Delta\sigma_1)$ and can be related to the stress state by

$$R = \frac{\|E\|}{\|\sigma + \Delta\sigma\|}$$  \hfill (A.27)

This relative error has to be smaller than a user defined error tolerance $SSTOL$. It has been shown that the error associated with the above estimation is of order $O(\Delta T^2)$ and that the substep has to be decreased by a factor $\beta$ which can be obtained from

$$\beta = \left(\frac{SSTOL}{R}\right)^{\frac{1}{2}}$$  \hfill (A.28)
so that the substep is reduced to $\Delta T_{\text{new}} = \beta \Delta T$ (Potts & Zdravković, 1999). As the estimate of $\beta$ is only an approximation the new substep should be further reduced. ICFEP uses a factor of 0.8 for this additional reduction.

For this new substep the stress increase and the change $\Delta \epsilon^p$ and $\Delta k$ are calculated and the error $R$ again compared with $SSTOL$. When the error is smaller then the tolerance the stress state, plastic strain and hardening parameter are then updated in order to consider the next substep.

It is, however, possible that the updated stress state may not satisfy the updated yield surface, e.g. $|F(\sigma, k)| > YTOL$ where $YTOL$ is a user defined tolerance. Whether the new stress state violates the yield condition depends on the choice of $SSTOL$ and $YTOL$. This error is called \textit{yield surface drift} and can lead to cumulative errors when not corrected. There are different approaches to deal with this phenomenon. A comparison can be found at Potts & Gens (1984) and the approach used by ICFEP is described by Potts & Zdravković (1999).
Appendix B

A non-linear transversely anisotropic soil model

B.1 Introduction

Most of the results presented in this thesis are from analyses using the isotropic non-linear elasto-plastic soil model, described in Section 3.3. It has been noted in this thesis that the surface settlement trough predicted by these FE analyses for a high $K_0$ regime is too wide compared to field data. Similar results have been reported by other authors (Addenbrooke et al., 1997; Negro & de Queiroz, 2000; Tang et al., 2000) as summarized in Section 2.3.1. Greenfield analyses carried out by Simpson et al. (1996), however, showed that incorporating soil anisotropy into the soil model improves the calculated surface settlement curve. Similar conclusions were drawn by Lee & Rowe (1989) who included anisotropy into a linear elasto-plastic model for analysing tunnelling in a low $K_0$-regime. However, Addenbrooke et al. (1997) demonstrated that applying anisotropic soil parameters which are appropriate for London Clay gave little improvement to the results. Gunn (1993) stated that including soil anisotropy did not improve settlement predictions. However, he did not provide any further details of his analyses.

In order to get a clearer picture about the influence of soil anisotropy on tunnel induced surface settlement troughs, a cross anisotropic material model, based on the formulation by Graham & Houlsby (1983) was implemented into ICFEP and both 2D and 3D analyses were
Appendix B. A non-linear transversely anisotropic soil model

performed. This appendix gives details of this soil model and presents results from analyses to evaluate the new model. Its application to 2D and 3D tunnel simulation is discussed in Chapter 8.

B.2 Original model formulation

For the general description of an anisotropic linear elastic material 21 constants are required. If a material has the same material properties in any horizontal direction but different properties in the vertical direction the material is called transversely or cross anisotropic. Such a situation is shown in Figure B.1. Soil is likely to show such behaviour as it is normally deposited onto a horizontal plane. The direction of deposition (i.e. the vertical direction) is an axis of symmetry. Only 5 material parameter are necessary to describe such a material (Pickering, 1970), which are:

1. $E_v$ - Young’s modulus in the vertical direction
2. $E_h$ - Young’s modulus in the horizontal direction
3. $\nu_{vh}$ - Poisson’s ratio for horizontal strain due to vertical strain
4. $\nu_{hh}$ - Poisson’s ratio for horizontal strain due to horizontal strain in the orthogonal direction
5. $G_{hv}$ - shear modulus in the vertical plane (also written as $G_{vh}$)

Further material constants can be derived from these parameters. The Poisson’s ratio for vertical strain due to horizontal strain $\nu_{hv}$ can be calculated from

$$\frac{\nu_{hv}}{E_h} = \frac{\nu_{vh}}{E_v}$$  \hspace{1cm} (B.1)

The horizontal plane is a plane of isotropy. The shear modulus in this plane can therefore be expressed as

$$G_{hh} = \frac{E_h}{2(1 + \nu_{hh})}$$  \hspace{1cm} (B.2)
Appendix B. A non-linear transversely anisotropic soil model

Figure B.1: Horizontal layers of transversely isotropic material.

The constitutive behaviour can then be expressed by

$$\begin{bmatrix}
\Delta \epsilon_{xx} \\
\Delta \epsilon_{yy} \\
\Delta \epsilon_{zz} \\
\Delta \epsilon_{yz} \\
\Delta \epsilon_{xz} \\
\Delta \epsilon_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_h} & -\frac{\nu_{hh}}{E_h} & -\frac{\nu_{ch}}{E_v} \\
-\frac{\nu_{hh}}{E_h} & \frac{1}{E_h} & -\frac{\nu_{ch}}{E_v} \\
-\frac{\nu_{ch}}{E_v} & -\frac{\nu_{ch}}{E_v} & \frac{1}{E_v} \\
\frac{1}{G_{hv}} & & \\
& \frac{1}{G_{hv}} & \\
& & \frac{2(1+\nu_{hh})}{E_h}
\end{bmatrix}
\begin{bmatrix}
\Delta \sigma'_{xx} \\
\Delta \sigma'_{yy} \\
\Delta \sigma'_{zz} \\
\Delta \sigma'_{yz} \\
\Delta \sigma'_{xz} \\
\Delta \sigma'_{xy}
\end{bmatrix}$$

where the vertical direction is described by the $z$-coordinate.

Graham & Houlsby (1983) showed that it is not possible to recover more than three elastic constants from a triaxial test as no shear stresses are applied to the sample. They showed that by introducing an anisotropic scale factor $\alpha$ it is possible to derive a special form of transverse anisotropy. They proposed a three-parameter formulation adopting a Young’s modulus $E^*$, a Poisson’s ratio $\nu^*$ and an anisotropy factor $\alpha$. The five independent material
constants above become coupled and can be expressed as

\[ E_v = E^* \]  \hspace{1cm} (B.4a)
\[ E_h = \alpha^2 E^* \]  \hspace{1cm} (B.4b)
\[ \nu_{vh} = \frac{\nu^*}{\alpha} \]  \hspace{1cm} (B.4c)
\[ \nu_{hh} = \nu^* \]  \hspace{1cm} (B.4d)
\[ G_{hv} = \frac{\alpha E^*}{2(1 + \nu^*)} \]  \hspace{1cm} (B.4e)

and the shear modulus in the horizontal plane can then be calculated as

\[ G_{hh} = \frac{\alpha^2 E^*}{2(1 + \nu^*)} \]  \hspace{1cm} (B.5)

It can be seen that in this formulation the ratios of Young’s modulus, Poisson’s ratio and shear modulus are linked to each other by

\[ \alpha = \sqrt{\frac{E_h}{E^*}} = \frac{\nu_{hh}}{\nu_{vh}} = \frac{G_{hh}}{G_{hv}} \]  \hspace{1cm} (B.6)

This coupling is the main simplification in the Graham & Houlsby (1983) model. However, Lings et al. (2000) point out that there is no a priori reason why these ratios should be the same as the five parameters of the original formulation are independent.

Incorporating Equations B.4 into Equation B.3 leads to

\[
\begin{bmatrix}
\Delta \epsilon_{xx} \\
\Delta \epsilon_{yy} \\
\Delta \epsilon_{zz} \\
\Delta \epsilon_{yz} \\
\Delta \epsilon_{xz} \\
\Delta \epsilon_{xy}
\end{bmatrix}
= \frac{1}{E^*} \begin{bmatrix}
\frac{1}{\alpha^2} & -\frac{\nu^*}{\alpha^2} & -\frac{\nu^*}{\alpha} \\
-\frac{\nu^*}{\alpha^2} & \frac{1}{\alpha^2} & -\frac{\nu^*}{\alpha} \\
-\frac{\nu^*}{\alpha} & -\frac{\nu^*}{\alpha} & 1 \\
\frac{2(1+\nu^*)}{\alpha} & \frac{2(1+\nu^*)}{\alpha} & \frac{2(1+\nu^*)}{\alpha^2}
\end{bmatrix}
\begin{bmatrix}
\Delta \sigma'_{xx} \\
\Delta \sigma'_{yy} \\
\Delta \sigma'_{zz} \\
\Delta \sigma'_{yz} \\
\Delta \sigma'_{xz} \\
\Delta \sigma'_{xy}
\end{bmatrix}
\]  \hspace{1cm} (B.7)

which when inverted leads to the constitutive matrix.
Appendix B. A non-linear transversely anisotropic soil model

Section B.3

\[ D = \frac{E^*}{(1 + \nu^*)(1 - 2\nu^*)} \begin{bmatrix} \alpha^2(1 - \nu^*) & \alpha^2\nu^* & \alpha\nu^* \\ \alpha^2\nu^* & \alpha^2(1 - \nu^*) & \alpha\nu^* \\ \alpha\nu^* & \alpha\nu^* & (1 - \nu^*) \end{bmatrix} \frac{\alpha(1 - 2\nu^*)}{2} \frac{\alpha(1 - 2\nu^*)}{2} \frac{\alpha^2(1 - 2\nu^*)}{2} \] (B.8)

For \( \alpha = 1 \) the material model becomes isotropic.

Graham & Houlsby (1983) showed that for the interpretation of triaxial tests the constitutive equation can be expressed as

\[
\begin{bmatrix} \Delta p' \\ \Delta J \end{bmatrix} = \begin{bmatrix} K^* & M \\ M & G^* \end{bmatrix} \begin{bmatrix} \Delta \epsilon_v \\ \Delta E_d \end{bmatrix}
\] (B.9)

where the mean effective stress \( p' \), the deviatoric stress \( J \), the volumetric strain \( \epsilon_v \) and the deviatoric strain \( E_d \) were defined on page 90 (Equations 3.9 to 3.13). The entries of the constitutive matrix are \( K^* \), the modified bulk modulus; \( G^* \), the modified shear modulus; and \( M \), which describes the cross-coupling behaviour between volumetric and deviatoric behaviour\(^1\). For isotropic behaviour \( M = 0 \) and volumetric and deviatoric behaviour becomes uncoupled. From the previous equations, Wood (1990) derived expressions which link \( K^* \), \( G^* \) and \( M \) with \( E^* \), \( \nu^* \) and \( \alpha \):

\[
K^* = \frac{E^*(1 - \nu^* + 4\alpha\nu^* + 2\alpha^2)}{9(1 + \nu^*)(1 - 2\nu^*)} \] (B.10)

\[
G^* = \frac{E^*(2 - 2\nu^* - 4\alpha\nu^* + \alpha^2)}{6(1 + \nu^*)(1 - 2\nu^*)} \] (B.11)

\[
M = \frac{E^*(1 - \nu^* + \alpha\nu^* - \alpha^2)}{3(1 + \nu^*)(1 - 2\nu^*)} \] (B.12)

Equations linking \( K^* \), \( G^* \) and \( M \) directly with \( E_h \), \( E_v \), \( \nu_{vh} \) and \( \nu_{hh} \) are given by Lings et al. (2000)

\(^1\)In their formulation Graham & Houlsby (1983) used \( J \) for the cross-coupling term. In this thesis \( J \) is defined as the deviatoric stress and therefore \( M \) is used instead.
B.3 Model development

The above equations describe an anisotropic linear elastic material. As described in Section 3.3.1 soil behaves highly non-linear for small strains. In the analyses presented in this thesis the non linear response is modelled applying tangent stiffness-strain expressions which were developed from a stiffness-strain formulation proposed by Jardine et al. (1986), see Equations 3.14 and 3.15 (Page 91). To model both the anisotropic and the non linear behaviour the anisotropic three parameter model described in the previous section was combined with the small strain model.

In the original small strain stiffness formulation the tangent shear modulus \( G \) (normalized against mean effective stress) depends on deviatoric strain \( E_d \) (Equation 3.14) while the bulk modulus \( K \) (normalized against mean effective stress) is linked to volumetric strain \( \epsilon_v \) (Equation 3.15). The isotropic Young’s modulus \( E \) and Poisson’s ratio \( \nu \) can then be calculated from \( K \) and \( G \).

In order to include anisotropic features into this soil model the vertical Young’s modulus was expressed by

\[
\frac{E_v}{p'} = \tilde{A} + \tilde{B}\cos(\beta X\gamma) - \frac{\tilde{B}\beta\gamma X^{\gamma-1}}{2.303}\sin(\beta X\gamma) \quad \text{with} \quad X = \log_{10}\left(\frac{E_d\sqrt{3}}{C}\right) \quad (B.13)
\]

which is derived from Equation 3.14. The model parameters \( \tilde{A}, \tilde{B}, C, \beta \) and \( \gamma \) describe the reduction of \( E_v \) with increasing deviatoric strain\(^2\). As for the original small strain stiffness model minimum and maximum deviatoric strains \( E_{d,\text{min}} \) and \( E_{d,\text{max}} \) are defined below or above which \( E_v \) only depends on \( p' \). Further input parameters are the Poisson’s ratio \( \nu_{\text{hh}} \) and the anisotropic factor \( \alpha \). Using Equations B.4 the other anisotropic parameters can be

\(^2\)The tilde indicates that \( \tilde{A} \) and \( \tilde{B} \) are different from \( A \) and \( B \), respectively, used in the original small strain model. For the other parameters the same values were adopted in both models.
Appendix B. A non-linear transversely anisotropic soil model

Section B.4

calculated and the constitutive matrix is then evaluated from

\[
D = \begin{bmatrix}
(E_h - E_v \nu_{hv}^2)Y & (E_h \nu_{hh} + E_v \nu_{hv}^2)Y & E_v \nu_{hv}(1 + \nu_{hh})Y \\
(E_h \nu_{hh} + E_v \nu_{hv}^2)Y & (E_h - E_v \nu_{hv}^2)Y & E_v \nu_{hv}(1 + \nu_{hh})Y \\
E_v \nu_{hv}(1 + \nu_{hh})Y & E_v \nu_{hv}(1 + \nu_{hh})Y & E_v(1 - \nu_{hv}^2)Y \\
\end{bmatrix}
\]

with

\[
Y = \frac{1}{(1 + \nu_{hh})(1 - \nu_{hh} - 2\nu_{hv}^2 \frac{E_v}{E_h})}
\]

which is equivalent to the constitutive matrix shown in Equation B.8.

In addition to the variation of \(E^*\) with deviatoric strain an option to vary \(\alpha\) with \(E_d\) was implemented such that \(\alpha\) reduces linearly from its initial value to 1.0 between \(E_{d,\text{min}}\) and \(E_{d,\text{max}}\), respectively. Such a reduction of anisotropy with strain has been observed in test results. Jovičić & Coop (1998) present laboratory test data of reconstituted clay which indicate that the ratio of \(G_{hh}/G_{hv}\) reduces gradually with increasing volumetric strain. They concluded, however, that very large strains would be necessary to obtain isotropy. The variation of \(\alpha\) implemented into ICFEP might exaggerate this trend. However, it can be used as an extreme to investigate the effect of a changing level of anisotropy with strain.

B.4 Model evaluation

B.4.1 Constant anisotropic scale factor

As a first step the anisotropic scale factor \(\alpha\) in the new model was set to \(\alpha = 1.0\) to compare results of this model with analyses performed with the original isotropic small strain stiffness model. The input parameter \(\tilde{A}\) and \(\tilde{B}\) for the new model were derived from \(A\) and \(B\) used in the original small strain formulation (listed in Table B.1) from

\[
G = \frac{E_{\alpha=1}^*}{2(1 + \nu^*)}
\]

where \(E_{\alpha=1}^*\) denotes the (isotropic) Young’s modulus calculated from the new model while \(G\) is the shear modulus calculated from the original isotropic model. The Poisson’s ratio \(\nu^*\) of

338
the new model is isotropic in this specific case. The above equation leads to

\[
\tilde{A} = 2A(1 + \nu^*) \quad \text{and} \quad \tilde{B} = 2B(1 + \nu^*)
\]  

(B.16)

The other corresponding parameters were left identical for both material models. A value of \(\nu^* = 0.2\) was adopted in all following analyses.

Figure B.2 shows the development of shear modulus \(G\) and Young’s modulus \(E\) with deviatoric strain \(E_d\) for both the original small strain model (labelled as ‘M1’) and the anisotropic model (‘M2’) which in this figure has isotropic parameters. For M1 the shear modulus is calculated from Equation 3.14, Page 91. For the calculation of \(E\) in this model it was assumed that \(\varepsilon_v < \varepsilon_{v,\text{min}}\) as this model will be applied for undrained tunnel excavation and that therefore the bulk modulus remained constant. The Young’s modulus \(E\) and the Poisson’s ratio \(\nu\) can be calculated from \(G\) and \(K\). As a consequence \(\nu\) changes with strain level in M1 while it is a constant input parameter for M2.

The figure shows that the choice of the above parameters results in identical curves for \(G\) with varying \(E_d\). The variation of \(E\) with deviatoric strain, however, differs between both models. This is because of the different Poisson’s ratios.

![Figure B.2: Strain-stiffness curves for isotropic material behaviour.](image)

![Figure B.3: Settlement trough for isotropic soil behaviour \((z_0 = 20m)\).](image)
Appendix B. A non-linear transversely anisotropic soil model

Section B.4

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$ [%]</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$E_{d,\text{min}}$ [%]</th>
<th>$E_{d,\text{max}}$ [%]</th>
<th>$G_{\text{min}}$ [kPa]</th>
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<table>
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<th>$S$</th>
<th>$T$ [%]</th>
<th>$\delta$</th>
<th>$\mu$</th>
<th>$\epsilon_{v,\text{min}}$ [%]</th>
<th>$\epsilon_{v,\text{max}}$ [%]</th>
<th>$K_{\text{min}}$ [kPa]</th>
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<td>506.0</td>
<td>$1.00 \times 10^{-3}$</td>
<td>2.069</td>
<td>0.42</td>
<td>$5.00 \times 10^{-3}$</td>
<td>0.15</td>
<td>3000.0</td>
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</table>

Table B.1: Material parameters used for non-linear elastic soil model M1 (identical to Table 3.2).

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\tilde{A}$</th>
<th>$\tilde{B}$</th>
<th>$C$ [%]</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$E_{d,\text{min}}$ [%]</th>
<th>$E_{d,\text{max}}$ [%]</th>
<th>$E^*_{\text{min}}$ [kPa]</th>
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<td>812.8</td>
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<td>1.335</td>
<td>0.617</td>
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<tr>
<td>2.0</td>
<td>403.2</td>
<td>365.8</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

For all parameter sets: $\nu^* = 0.2$

Table B.2: Input parameter for anisotropic soil model and different cases of $\alpha$.

Both models were used to model the excavation of a 20m deep tunnel with a diameter of $D = 4.146$ m in a plane strain FE analysis. Figure B.3 presents the greenfield settlement trough which was taken at a volume loss of approximately $V_L = 1.5\%$ (see Section 3.4.4). It can be seen that both models with parameters chosen to give identical shear stiffness, give the same settlement trough.

In this previous example M2 modelled isotropic conditions. When $\alpha$ is increased from $\alpha = 1.0$ the model becomes anisotropic. Anisotropy ratios are often used to describe a soil. Simpson et al. (1996) reports a ratio of $G_{hh}/G_{hv} = 1.5$ for London clay obtained from bender element tests under an isotropic stress state. Joviˇ ci´ c & Coop (1998) concluded a similar ratio. Ratios of $E_h/E_v = 4.0$ and $G_{hh}/G_{hv} = 2.25$ were measured by Lings et al. (2000) with samples of Gault clay using small strain measurements in drained triaxial tests.

If $\tilde{A}$ and $\tilde{B}$ were kept constant while increasing $\alpha$ the overall response of the material would become stiffer as the stiffness $E_h$ and $G_{hh}$ would increase (see Equations B.4b and B.5). In order to maintain a comparable stiffness response $E_v$ has to be reduced as $\alpha$ increases. New input parameters $\tilde{A}$ and $\tilde{B}$ were calculated by keeping the modified shear modulus $G^*$ (Equation B.11) constant when increasing $\alpha$. This calculation was carried out for small strains (i.e. $E_d < E_{d,\text{min}}$). The input parameters used in M1 and in M2 for $\alpha = 1.0, 1.5$
and 2.0 are summarized in Tables B.1 and B.2. Figure B.4 shows stiffness-strain curves for M1 and M2 with different values of $\alpha$. All curves have a similar pattern. This is because the input parameter $C$, $\beta$ and $\gamma$ used in Equation B.13 are the same as those used for the original small strain definition. Figure B.4a demonstrates how the increase in $E_h$ is compensated for by a reduced $E_v$. Figure B.4b shows higher $G_{hh}$ curves with increasing $\alpha$ while the $G_{hv}$ curves remain approximately on the same level.

Ideal undrained triaxial extension tests were modelled with these input parameters using a single axisymmetric finite element. The initial stress within the sample was equivalent to an isotropic consolidation of $p^\prime = 750$ kPa. Strain controlled tests were then performed by prescribing vertical displacements at the top of the element. The failure criterion was described by a Mohr-Coulomb model as outlined in Section 3.3.2. The soil mechanics sign convention with compression defined positive is adopted when presenting results in the following figures.

Figure B.4: Strain-stiffness curves for isotropic (M1) and anisotropic (M2) material.

Figure B.5: Stress path for isotropic (M1) and anisotropic (M2) material in triaxial extension.
Figure B.5 shows the stress paths followed by the test in $p'-q$ stress space where $q$ is the deviatoric stress defined in triaxial stress space as

$$q = \sigma_{ax} - \sigma_r$$

with $\sigma_{ax}$ and $\sigma_r$ being the axial and radial stress in the sample respectively. For the isotropic model M1 the graph shows a constant $p'$ until the stress path reaches the Mohr-Coulomb failure criterion. The stress paths of the anisotropic model M2 do not follow this vertical stress path. This is because volumetric and deviatoric behaviour are coupled in Equation B.9 by the cross coupling term $M$. The pre-failure behaviour is illustrated by Figure B.6 which shows the development of both, $q$ and excess pore water pressure $u$, with axial strain $\epsilon_{ax}$. Figure B.6a plots results for small axial strains up to $\epsilon_{ax} = 0.01\%$. The data in Figure B.6b are for strains up to $\epsilon_{ax} = 0.4\%$ which is equivalent to the upper limit of non-linear behaviour defined by $E_{d,\text{max}} = 0.693\%$. Laboratory results of an equivalent test reported by Addenbrooke et al. (1997) are also presented.

For small strains the first graph shows that all three cases give the same deviatoric stress response. For larger strains an increase in anisotropy gives slightly higher deviatoric stress curves. The anisotropic nature of the $\alpha = 1.5$ and 2.0 analyses can be seen from the pore water response. The development of excess pore water pressure with strain increases drastically with
an increase in degree of anisotropy.

The above parameters were used to analyse the same 2D tunnel excavation presented previously. Figure B.7 presents the results of this study. The top graph shows the settlement trough for M1 (isotropic) and M2 with $\alpha = 1.5$ and 2.0. All results were taken from the 7th of 15 excavation increments. The volume loss for the three curves is $V_L = 1.51\%$, 1.39\% and 1.47\% for M1, M2 with $\alpha = 1.5$ and M2 with $\alpha = 2.0$ respectively. The curve for M1 was already presented in Figure B.3. The case of $\alpha = 1.5$ shows the same maximum settlement, the settlement trough, however, is narrower. This trend continues for $\alpha = 2.0$ which shows a deeper and even narrower settlement trough.

The trend of narrower settlement troughs with increasing anisotropy is further illustrated in the lower graph which presents the above settlement curves normalized against maximum settlement. The curve of M2 with $\alpha = 2.0$ is the narrowest of the three settlement curves. The three curves coincide once the distance from the tunnel centre line exceeds 70m. This

Figure B.7: Settlement troughs for isotropic (M1) and anisotropic (M2) soil behaviour ($z_0 = 20m$).
Appendix B. A non-linear transversely anisotropic soil model

Section B.4

shows that even in a highly anisotropic environment the settlement trough is too wide. This becomes clear when comparing the results with the Gaussian settlement trough calculated for the tunnel diameter adopted in this analyses (with \( V_L = 1.5\% \) and \( i = 0.5z_0 \)). Only within a zone of approximately 12m from the tunnel centre line are the \( \alpha = 2.0 \) results in reasonable agreement with the Gaussian settlement trough. In general, all FE results whether isotropic or anisotropic, are too wide when compared with the Gaussian curve.

B.4.2 Variable anisotropic scale factor

The anisotropic analyses presented in the previous section used a constant value of \( \alpha \). However, the soil model as implemented into ICFEP provides the possibility to vary \( \alpha \) with strain level. Figures B.8 and B.9 show strain-stiffness curves for the case of \( \alpha \) varying between 2.0 at \( E_{d,\text{min}} \) and 1.0 at \( E_{d,\text{max}} \). Figure B.8a and b illustrates the reduction of tangent Young’s modulus with deviatoric strain. The first graph shows the development over a strain range between \( E_d = 10^{-5}\% \) and 1% while the second graph provides a close up for strains larger than \( E_d = 10^{-2}\% \). The graphs include the corresponding curves for calculations with constant \( \alpha = 1.0 \) and 2.0 which were already shown in Figures B.2 and B.4 respectively. The curves of \( E_v \) for variable and constant \( \alpha = 2.0 \) are identical (as \( E_v = E^* \) is an input parameter). For these two cases the same values of \( E_h \) are obtained for \( E_d < E_{d,\text{min}} \) for which \( \alpha \) remains constant. As \( E_h \) starts to reduce with increasing deviatoric strain both curves still coincide. It has to be noted that \( \alpha \) decreases linearly with strain while the graphs in this figure are plotted against a log-scale. From approximately \( E_d = 0.01\% \) both curves of \( E_h \) can be distinguished with the variable \( \alpha \) case decreasing more. It finally approaches the \( E_v \) curve of \( \alpha = 2.0 \) and, hence, becomes isotropic. In this situation the Young’s modulus of the variable \( \alpha = 2.0 \) is lower than that of the isotropic \( \alpha = 1.0 \) set.

The same pattern can be found for the variation of shear modulus with deviatoric strain presented in Figures B.9a and b. For small strains \( G_{hv} \) and \( G_{hh} \) for both variable and constant \( \alpha = 2.0 \) coincide. With increasing strain the variable \( \alpha \) case becomes more isotropic until it reaches \( G_{hv} = G_{hh} \) as shown in the close up. In this situation the shear modulus is lower than the corresponding value obtained for a constant \( \alpha = 1.0 \). Consequently, the material with variable \( \alpha \) behaves for deviatoric strains \( E_d > E_{d,\text{max}} \) softer than the isotropic soil.

This change from anisotropic to isotropic behaviour is further illustrated in Figure B.10.
Figure B.8: Development of Young’s moduli with strain for isotropic and anisotropic (constant and variable) material.

Figure B.9: Development of shear moduli with strain for isotropic and anisotropic (constant and variable) material.

which plots the mean effective stress $p'$ against the triaxial deviatoric stress $q$. In addition to the variable $\alpha = 2.0$ stress path the figure also includes the constant $\alpha = 2.0$ and 1.0 cases. At the begin the variable $\alpha = 2.0$ case follows the constant $\alpha = 2.0$ stress path. However, as $\alpha$ reduces to 1.0 its stress path becomes vertical and runs parallel to the constant $\alpha = 1.0$ path until it reaches the failure surface.

The tunnel excavation previously studied for constant $\alpha$ was analysed applying a variable $\alpha$. The surface settlement profile obtained from this 2D FE analysis is shown in Figure B.11a
Appendix B. A non-linear transversely anisotropic soil model  Section B.4

Figure B.10: Stress paths for isotropic and anisotropic (constant and variable) material. Figure B.11: Settlement troughs for isotropic and anisotropic (constant and variable) material behaviour.

together with the troughs for constant $\alpha = 2.0$ and for the isotropic Model M1. The variation of $\alpha$ leads to a slightly deeper settlement trough than calculated for constant $\alpha = 2.0$ which was already deeper and narrower than the trough from the original small strain model M1. The volume loss for the variable $\alpha$ case was significantly higher than for the other analyses. The curve presented in this graph is therefore taken from increment 6 (of 15 excavation increments). The volume loss for this increment is $V_L = 1.5\%$ and increases to $2.16\%$ in increment 7. The other results presented in this figure are taken from the 7th excavation increment.

The deeper settlement above the tunnel crown and the higher volume loss can be explained by the soft behaviour, a material with variable $\alpha$ exhibits when $\alpha$ is reduced to 1.0. The largest strains occur around the tunnel leading to a relatively soft response of the soil compared with the constant $\alpha = 2.0$ and the isotropic model. This behaviour explains the high volume loss.
and the deeper settlement curve obtained from the variable $\alpha$ analysis.

When normalizing the settlement troughs against maximum settlement (Figure B.11b) the variable $\alpha$ curve shows little improvement compared to the constant $\alpha = 2.0$ case. Both curves are much narrower than the curve of the isotropic small strain model. However, compared to the Gaussian trough, also shown in this figure, they are still too wide.

### B.5 Summary

A pre-yield soil model describing non-linear transversely anisotropic behaviour was developed and implemented into ICFEP. The model was evaluated by analysing both single element tests and a boundary value problem.

- The anisotropic formulation by Graham & Houlsby (1983) requires three input parameters, one of them being an anisotropic scale parameter $\alpha$ which describes the ratio between horizontal and vertical stiffness. Further material parameters are required to describe the reduction of stiffness with deviatoric strain. This formulation based on the isotropic soil model by Jardine et al. (1986).

- The Graham & Houlsby (1983) and Jardine et al. (1986) models were combined to a new non-linear transversely pre-yield model. The new model was tested by choosing isotropic input parameters which result in the same shear stiffness reduction as calculated from the isotropic small strain stiffness model. When applied to a tunnel excavation analysis both models gave the same surface settlement trough.

- The anisotropic scale factor was then increased leading to higher values of $E_h$ and $G_{hh}$. In order to maintain a comparable overall stiffness, $E_v$ had to be reduced. This was achieved by keeping the modified shear modulus $G^*$ at the same level. Material parameter sets with an anisotropic scale factors of $\alpha = 1.5$ and 2.0 were chosen and compared with results from the isotropic analyses.

- When analysing an undrained triaxial extension test with the different parameter sets, similar stress-strain curves were obtained which were close to laboratory data. The development of excess pore water pressure increased with anisotropy as a result of the coupling between volumetric and deviatoric behaviour. In addition a plane strain
tunnel excavation was analysed. The results showed that the surface settlement trough becomes deeper and narrower with an increasing level of anisotropy. However, all curves were still too wide when compared to the Gaussian curve.

- The soil model provides the opportunity to reduce $\alpha$ with increasing strain so that the material becomes isotropic for large strains. Analyses were performed with this option with an initial $\alpha = 2.0$. This resulted in a deeper and narrower settlement trough than the equivalent analysis with a constant $\alpha$. The reason for this behaviour is that the soil becomes softer as $\alpha$ reduces. For large strains $E_h$ and $G_{hh}$ become equal to the lower values of $E_v$ and $G_{hv}$, respectively and the material behaves softer than the corresponding isotropic case.


