We absolutely must leave room for doubt or there is no progress and no learning. There is no learning without posing a question. And a question requires doubt...Now the freedom of doubt, which is absolutely essential for the development of science, was born from a struggle with constituted authorities... FEYNMANN, 1964
SCOPE: The lectures are devoted to a rather informal overview of basic features of turbulent flows (TF). The main emphasis is on conceptual and problematic aspects, physical phenomena, observations, misconceptions and unresolved issues rather than on conventional formalistic aspects, models, etc.. The lectures are based on the book by the author *An informal introduction to turbulence, Kluwer, 2001* and its revision *A conceptual introduction to turbulence, Kluwer, 2007* and recent developments. PDF files of the lectures will be accessible from internet.

AUDIENCE AND PREREQUISITES: Ph.D students and higher. Fluid dynamics including turbulence. Knowledge of physics and mathematics at graduate level.

TIME: Wednesdays, 16-00-17-00 and/or 16-00-18-00.

Please, note that there will be no lectures at February 7 and 14.

PLACE: Seminar Room, Institute for Mathematical Sciences, Imperial College London, 53 Princes Gate, Exhibition Road, South Kensington, London SW7 2PG
This is a selection of lectures/topics rather than a course and contains as much questions and similar things (or even more) as answers, (hopefully) unbiased discussion of the unresolved issues, controversies major problems and misconceptions. In particular there are no lapses into brevity at difficult places. Naturally, this involves critical remarks, which are intended to be as constructive as possible. The specific sequence of themes depends on the audience response as the lectures will be made as interactive as possible.

It is much easier to present nice rational linear analysis than it is to wade into the morass that is our understanding of turbulence dynamics. With the analysis, professor and students feel more comfortable; even the reputation of turbulence may be improved, since the students will find it not as bad as they had expected. A discussion of turbulence dynamics would create only anxiety and a perception that the field is put together out of folklore and arm waving (Lumley, 1987).

In selecting the references I used the (genuinely small) parameter introduced by Saffman (1978), which he called information density, $I$, and defined as the ratio, $S/N$, in the literature, with $S =$ signal (understanding), and $N =$ noise (mountains of publications). In order to increase the value of $I$ I did all my best to concentrate on the numerator, $S$, and to reduce the denominator, $N$, to the best of my knowledge, ability and judgment/understanding (this includes scanning about 9000 references since the publication of my book at 2001). However, absence of references does not necessarily mean that - in my view - they belong to $N$, but is due to my ignorance or the lack of space needed to discuss them here.

I used extensively quotations which - to my view - are essential in any field of science.
SONNET TO TURBULENCE
by S. Corrsin
(For Hans Liepmann on the occasion of his 70th birthday, with apologies to Bill S. and Liz B.B.)

Shall we compare you to a laminar flow? You are more lovely and more sinuous.
Rough winter winds shake branches free of snow, And summer's plumes churn up in cumulus.
How do we perceive you? Let me count the ways.
A random vortex field--with strain entwined.
Fractal? Dig and small swirls in the maze
May give us paradigms of flows to, find.
Orthonormal forms non-linearly renew
Intricate flows with many free degrees
Or, in the latest fashion, merely few
As strange attractor. In fact, we need Cray 3's.
Experiment and theory, unforgiving
For serious searcher, fun ... and it's a living!
The following is an exemplifying list of some possible questions to be discussed and/or mentioned:

- Reynolds number dependence or where is the $\infty$? Is it (always) necessary to have large parameters to study the basic physics of turbulence? Is the nature of dissipation unimportant in the "inviscid" limit? Is the inertial range a conceptually well defined concept? Are its properties really independent of the nature of dissipation?

- Is "cascade" in genuine turbulence conceptually well defined notion or is it "mostly a pedagogical imagery"? Is there cascade in physical space? Is the Komlogorov 4/5 law an unequivocal evidence of such a cascade? How meaningful is "cascade" of passive objects as described by linear equations? Is "cascade" Eulerian, Lagrangian or both? Are decompositions aiding understanding or obscuring the physics of turbulence?

- Is the physics of vortex stretching well understood? Is it a result of the kinematics of turbulence: are vortex lines on average stretched rather than compressed because two particles on average move apart from each other?
- Is the physics of vortex stretching well understood? Is it a result of the kinematics of turbulence: are vortex lines on average stretched rather than compressed because two particles on average move apart from each other? - Is enhanced dissipation in turbulence due to vortex stretching? Are vorticity and strain equal partners? - What is (are) the meaning(s) of non locality of turbulence? What are its manifestations? Is non locality important? Is there screening in turbulence? - How (much) statistical is turbulence? Is turbulence a part of statistical physics? Will non-equilibrium statistical mechanics play an increasingly important role in further progress of turbulence? Is turbulence ergodic? - How rigorous is turbulence modelling? Modelling versus physics of turbulence. - How analogous are quasi-two-dimensional flows? Diversity of strongly anisotropic flows (and the corresponding limiting states) as compared to pure two-dimensional ones. - How analogous are the genuine and passive turbulence? What can be learned about genuine turbulence from its signature on the evolution of passive objects? What is the importance (if any) of statistical conservation laws in genuine turbulence?
How prospective is Lagrangian description of turbulence? Is it different conceptually (not only technically) from the Eulerian one? Is it possible in this approach to separate the Lagrangian (kinematic) chaos from the genuinely dynamical intrinsic (Eulerian) stochasticity? Is this a conceptual difficulty? - What is universality of turbulence (if such exists)? What is the role of (the nature of) forcing and initial/upstream conditions in this and other issues? What are the situations and what are the properties which are (approximately) invariant of IC and BC? Is there qualitative universality? - What are the main reasons for slow progress in handling the physics of turbulence (which is the key for the progress in any aspect of the problem). Is it due to i) inadequate tools to handle both the problem and the phenomenon of turbulence, ii) lack of fresh ideas, which is directly related to the (in)ability/skill/art to ask the right and correctly posed questions, iii) insufficient conceptual progress and dominance of some misconceptions or ill defined conceptions?
Introductory remarks and premises. Main qualitative (universal) features. Examples of real turbulent flows. Definitions of turbulence and of the problem of turbulence? The nature of the problem, why turbulence is so impossibly difficult.
A BIT OF HISTORY

THE RISE AND FALL OF IDEAS IN TURBULENCE, LIEPMANN 1979

1500 Recognition of two states of fluid motion by Leonardo da Vinci and use of the term *la turbolenza*.
1839 ‘Rediscovery’ of two states of fluid motion by G. Hagen.
1883 Osborne Reynolds’ experiments on pipe flow. Concept of critical Reynolds number - transition from laminar to turbulent flow regime.
1887 Introduction of the term ‘turbulence’ by Lord Kelvin.
1895 Reynolds decomposition. Beginning of statistical approach.
1909 D. Riabuchinsky invents the constant-current hot-wire anemometer.
1912 J.T. Morris invents the constant-temperature hot-wire anemometer.
1921, 1935 Statistical approach by G.I. Taylor.
1922 L.F. Richardson’s hierarchy of eddies.
1924 L.V. Keller and A.A. Friedman formulate the hierarchy of moments.
1938 G.I. Taylor discovers the prevalence of vortex stretching.
1941 A.N. Kolmogorov local isotropy, 2/3 and 4/5 laws.
1943 S. Corrsin establishes the existence of the sharp laminar/turbulent interface in shear flows.
1949 Discovery of intrinsic intermittence by G. Batchelor and A. Townsend
1951 Turbulent spot of H.W. Emmons
1952 E. Hopf functional equation.
1967 Bursting phenomenon by S.J. Kline *et al.*
1976 Recapitulation of large scale coherent structures by A. Roshko.
I soon understood that there was little hope of developing a pure, closed theory, and because of the absence of such a theory the investigation must be based on hypotheses obtained in processing of experimental data... KOLMOGOROV 1985

In searching for a theory of turbulence, perhaps we are looking for a chimera... One of the things that I always found troubling in the study of the problem of turbulence is that I am not quite sure what the theoretical turbulence problem actually is. I just cannot think of anything where a genuine prediction for the dynamics of turbulent flow has been confirmed by an experiment. So we have a big vast empty field SAFFMAN 1978, 1991

Nothing can be proven in the theory of turbulence, BATCHelor, NOV 1998

... it is amazing how many different, nearly orthogonal, points of view there are about a phenomenon which is governed by Newton's innocent-looking, linear second law of motion, with a little help or hindrance from viscosity. BRADShAW, 2003

Every aspect of turbulence is controversial. SALMOn, 2003

Sometimes experiments provide us with so beautiful and clear results that it is a shame on theorists that they cannot interpret them.. YUDOVICH, 2003

... and so we can hold strong opinions either way FEYNMANn 1963.
Formal mathematical investigations have produced remarkably little value... A number of general procedures for calculation of various dynamical aspects of homogeneous turbulence have been devised, but none of them impresses me as being likely either to advance our understanding of turbulence or to achieve results on which we can place reliance... The universal similarity theory of the small-scale components of the motion stands out in this rather grey picture as a valuable contribution... 

... that sense of frustration that afflicted Batchelor (and many others) from 1960 onward... These frustrations came to the surface at the now legendary meeting held in Marseille (1961) to mark the opening of the former Institut de M’ecanique Statistique de la Turbulence (Favre 1962). 

MOFFATT (2002)
The story started before...

My overall impression of the Symposium is that no really new and important ideas have been presented... I think we must admit that little new theory has been put before us.

THREE MAJOR DIFFICULTIES

- Inadequate tools to handle both the problem and the phenomenon of turbulence are not developed enough.
- Lack of fresh ideas, which is directly related to the (in)ability/skill/art to ask the right and correctly posed questions.
- Insufficient conceptual progress and dominance of some misconceptions or ill defined conceptions.

*The Rise and Fall of Ideas in Turbulence*, LIEPMANN 1979
The entire experience with the subject indicates that the purely analytical approach is beset with difficulties, which at this moment are still prohibitive....our intuitive relationship to the subject is still too loose – not having succeeded at anything like deep mathematical penetration in any part of the subject, we are still quite disoriented as to the relevant factors, and as to the proper analytical machinery to be used, VON NEUMANN, 1949.
THEORY: TOO MANY “THEORIES” (MOSTLY PRETTY USELESS, BUT ALL AGREEING WELL WITH EXPERIMENT)

- I just cannot think of anything where a genuine prediction for the dynamics of turbulent flow has been confirmed by an experiment. So we have a big vast empty field. SAFFMAN, 1991

- Nothing can be proven in the theory of turbulence. BATCHELOR, NOV. 1998

There is no way and tools so far, if ever, to treat turbulence analytically -- turbulence is beyond analytics (TBA). Unfortunately, this is true of other theoretical approaches such as attempts to construct statistical and/or other theories.

EXPERIMENTS: PHYSICAL AND NUMERICAL

The essential mathematical complications of the subject were only disclosed by actual experience with the physical counterparts of these equations, VON NEUMANN, 1949

REMAIN A MAJOR EXPLORATORY TOOL BOTH IN

- Elucidating the properties of turbulence as a physical phenomenon
- Host of applications
THESE ARE THE REASONS WHY THE DISCUSSION OF
conceptual and problematic aspects, physical
phenomena, observations, misconceptions and
unresolved issues (rather than conventional
formalistic aspects, models and similar)

IS SO IMPORTANT
Turbulence is a phenomenon which sets in in a viscous fluid for small values of the viscosity coefficient \( \nu \) ... hence its purest, limiting form may be interpreted as the asymptotic, limiting behavior of a viscous fluid for \( \nu \to 0 \) \( \text{NEUMANN, 1949} \).

Turbulence can be defined by a statement of impotence reminiscent of the second law of thermodynamics: flow at a sufficiently high Reynolds number cannot be decelerated to rest in a steady fashion. The deceleration always produces vorticity, and the resulting vortex interactions are apparently so sensitive to the initial conditions that the resulting flow pattern changes in time and usually in stochastic fashion. \( \text{LIEPMANN 1979} \), The rise and fall of ideas in turbulence, American Scientist, 67. 221.

See Appendix A in my book for a collection of attempts to give a definition and snags.
Are attempts to give a definition of what is turbulence conceptually correct?

In mathematics the definition of the main object of study precedes the results.

In physics it is vice versa. Usually it happens when one studies a new phenomenon and only at a later stage, after understanding (!) it sufficiently (!), classifying it, etc. a most reasonable definition is found. Turbulence in not a new field but this time has yet to come.

Can one define what is mathematics? One has to learn it first.
IN LIEU OF DEFINITION: MAJOR QUALITATIVE UNIVERSAL FEATURES

- **Intrinsic spatio-temporal stochasticity**, both E- and L-turbulent, in contrast to pure Lagrangian chaos, which is E-laminar but L-turbulent. **Loss of predictability**

- **Huge range of strongly interacting ‘scales’/degrees of freedom.**

- **Highly dissipative**

- **Three-dimensional and rotational.**

- **Strongly diffusive**

Generally, a single property (or part) as above is not synonymous to turbulence. For example, randomness alone as was believed for a long time. Today too there is a tendency not to make a clear distinction between genuine turbulence (the big T-problem) and a set of phenomena in evolution of passive objects (PS, PV, diffusion/dispersion) in random velocity fields (small t-problems) prescribed a priori (more in lectures on analogies). As a latest example see Falkovich, G. and Sreenivasan, K.R. (2006) Lessons from Hydrodynamic Turbulence, *Physics Today*, 59(4), 43-49. More in lectures on analogies.

Mount GALUNGGUNG, west Java in August 1982, M.-A. del Marmol
Flow patterns of a square jet of cold flow (nitrogen, top) and combusting gas (propane, bottom) exhibiting strong dependence on forcing (4 Hz, left column) at the jet exit by piezoelectric actuators. Courtesy A. Glezer (1992), Phys. Fluids, A4, p.1877.
Where turbulence is present and where it is essential for flow regimes (e.g. stenotic vessels, aneurisms; artificial devices), transport processes (PO) and on micro-scales (hemolysis, thrombosis...)?
Partly Turbulent Flows

Flow past a 4 cm FLAT PLATE $Re \sim 1000$

Oil slick past a WRECKED TANKER $Re \sim 100$ million

A turbulent boundary layer flow

A turbulent jet from testing a Lockheed rocket engine in the Los Angeles hills

It seems more meaningful to look at a turbulent flow (TF) as a whole and ‘indecomposable’: separate components are not so meaningful, e.g. it is impossible to say that the flow is completely laminar or not at smallest scales. It is a matter of principle whether one looks at TF as a whole or via various decompositions. The latter is pretty problematic (more later), just like the following statements:


- at the small scales it becomes more difficult to argue for fundamental differences between these two types of flows. (Soutterland, K.B., Frederiksen, R.D. and Dahm, W.J.A., 1994, Comparisons of mixing in chaotic and turbulent flows, Chaos, Solitons and Fractals, 4(6), 1057-1089. see p 1065.)
TURBULENCE IS ‘RANDOM’, but there are so many random phenomena having nothing to do with turbulence.

W. Feller (1964) An introduction to probability theory and its applications, vol I, p. 82, fig. 5: The record of 10,000 tosses of an ideal coin.
In lieu of definition of turbulence:

**Definition of Turbulence Problem**

It has been realized since the beginning that the problem of turbulence is a statistical problem; that is a problem in which we study instead of the motion of a given system, the distribution of motions in a family of systems...It has not, however, been adequately realized just what has to be assumed in a statistical theory of turbulence, **Wiener, 1939**.

There is probably no such thing as a most favored or most relevant, turbulent solution. Instead, the turbulent solutions represent an ensemble of statistical properties, which they share, and which alone constitute the essential and physically reproducible traits of turbulence. **Von Neumann, 1949**.
Juts like in statistical physics, the statistical approach should be adopted in turbulence theories from the outset/start due to the extreme complexity of turbulence phenomenon. In both cases certain statistical hypotheses are made. But the former was quite successful in making a number of important predictions, whereas the latter, with few exceptions, such as the Kolmogorov four fifths law (Kolmogorov, 1941b), was unable to produce genuine predictions based on the first principles. All the rest -- in the words of P.G. Saffman -- are postdictions.

Apart from the above-mentioned reasons for such a failure it should be mentioned that, unlike statistical physics, in turbulence neither `simple objects' (such that a collection of these objects would adequately represent turbulent flows) `to do statistical mechanics' with them, nor `right' statistical hypotheses have so far been found. The question about the very existence of both remains open.

Two differences: BG SM is non-dissipative and discrete i.e. ODEs. T is strongly dissipative and continuous, i.e. PDEs. Hence, problematic relevance of dynamical systems methods in T.
let us give the present-day formulation of the general problem of the statistical description of turbulent flows (or, in short the “problem of turbulence”). For simplicity, we shall confine ourselves to the case of an incompressible fluid. In this case the flow the velocity field with the aid of the equations of motion). The problem of turbulence is reduced here to finding the probability distribution $P(d\omega)$ in the phase space of turbulent flow $\Omega = \{\omega\}$, the points $\omega$ of which are all possible solenoidal vector fields $u(x, t)$ which satisfy the equations of fluid mechanics and the boundary conditions imposed at the boundaries of the flow.

As a result, in the formulation under discussion, the problem of turbulence is a problem of the evolution of the probability measure in a functional space with given initial conditions.
Consequently, the fundamental problem of the theory of turbulence (e.g., for the case of an incompressible fluid) may be formulated as follows. Given the probability distribution of the values of the three velocity components at different points of space at the instant \( t = t_0 \), concentrated on a set of doubly differentiable solenoidal vector fields, it is required to determine the probability distribution of the values of the velocity and pressure fields at all subsequent times (including distributions for values at several different times).
The approach discussed in Sect. 3.2, which treats the fields of hydrodynamic variables of a turbulent flow as random fields, was initiated by the works of Kolmogorov and his school [see, e.g., Millionshchikov (1939)] and the work of Kampé de Fériet (1939). At present, this approach is generally accepted in all investigations on the theory of turbulence [see, e.g., the special survey articles of Kampé de Fériet (1953), and Obukhov (1954) and the monographs by Hinze (1959) and Lumley and Panofsky (1964)]. Adopting the assumption of the existence of probability distributions for all fluid dynamic fields, we may further make wide use of the mathematical techniques of modern probability theory; the operation of averaging is then defined uniquely and has all the properties naturally required of it.
The key words here are treats the fields of hydrodynamic variables of a turbulent flow as random fields and adopting the existence of probability distributions. This provokes the question on how ‘statistical’ is turbulence or (the dichotomy of) determinitic versus random/stochastic. Related to issues on ergodicity, the meaning of average, decompositions and similar.

The theory of turbulence by its very nature cannot be other than statistical, i.e., an individual description of the fields of velocity, pressure, temperature and other characteristics of turbulent flow is in principle impossible. Moreover, such description would not be useful even if possible, since the extremely complicated and irregular nature of all the fields eliminates the possibility of using exact values of them in any practical problems. MONIN AND YAGLOM 1971
The real problem is that turbulence (being studied by all kinds of statistical methods of description) cannot be considered as just a problem of statistical physics/mechanics only (*more later*). This, for example is admitted by Kraichnan (the papers by Kraichnan, R.H. and Chen, S., 1989, Is there a statistical mechanics of turbulence?, *Physica* D 37, 160—172 and Goto and Kraichnan, R.H. 2004, Turbulence and Tsallis statistics, *Physica* D 193, 231–244). The problem is even broader: there is no effective satisfactory theoretical framework (statistical or whatever) to handle turbulence (nothing new: this was said by von Neumann in 1949 and by many later), though I do not mind that turbulence can be seen also (but not only (!)) as a problem of nonequilibrium statistical physics or whatever.
Nevertheless there are many ‘theories’ (all statistical in some sense) none of which ‘solves’ the problem (and which all agree with some experimental data) and all of which belong to the category (using the wording of Saffman) of postdictions rather than predictions.

It seems that progress in the understanding of (the physics of) turbulence is needed for some real theoretical progress to be possible (if at all) and not the other way round.
So far not much is known about the physics of turbulence. There is a major lack of knowledge about the basic physical processes of turbulence and its generation and origin, and poor understanding of the processes which are already known. This is a difficulty of a more general nature than described above which are mostly of a formal/technical nature. For example, the underlying mechanisms of predominant vortex stretching, that is why in turbulent flows vorticity is stretched more than compressed, are (at best) poorly understood and essentially not known, though there is a widespread belief in the opposite - a theme to be discussed along with other misconceptions.
Another matter are statistical methods of describing turbulent flows (including studying and hunting its structure(s)) which should not be confused with statistical theories of turbulent flows. It seems that this is the only way. Indeed, due to very large dimension and complicated (stochastic?) structure of the underlying attractors, assumed to be in existence, was invoked in the justification of the unavoidable necessity of statistical methods of description (and `theories') of turbulent flows: one may never be able to realistically determine the fine-scale structure and dynamical details of attractors of even moderate dimension. ... The theoretical tools that characterize attractors of moderate or large dimensions in terms of the modest amounts of information gleaned from trajectories [i.e. particular solutions]... do not exist ...they are more likely to be probabilistic than geometric in nature (GUCKENHEIMER, 1986)
Very large dimension and complicated (stochastic?) structure of turbulent flows poses a problem with the attempts of the so called reduced description (by a low dimensional system). So it is quite plausible that any fluid flow which is adequately represented by a low dimensional system is not turbulent -- a kind of definition of 'non-turbulence'. The immediate examples are low dimensional chaotic fluid flows.

The question whether adequate low dimensional description of turbulent flows is possible depends on the meaning of the term 'adequate'. In the strict sense, i.e. from the basic point of view, it seems that there does not exist such a description, though as a (semi) empirical tool it may be more than satisfactory.
Statistical **description** sounds pretty modest and much less ambitious (as compared to ‘theories’), but the **art** of this description is far from being trivial as it is a consequence of the art/ability/skill to ask the right and correctly posed questions. This in turn dictates what kind of statistics one needs.

As it is naturally not just to contemplate statistical outcomes of the data many people wondered what they mean. In many cases the latter (i.e. wondering about the **meaning**) was reduced to the question on what sort of **individual** events account for the **statistical** trends. This question – though very popular - seems to be conceptually ill posed as very different **individual** events may account for the **same statistical** trends (**more later**).
.. even wrong theories may help in designing machines.

**Feynman, 1996**

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**Examples on Conceptual Aspects**

Dealing with conceptual aspects one encounters the issue of misconceptions.

- **Leonardo-Reynolds, Decomposition** - conceptually correct
- **Boussinesq, Eddy Viscosity**/diffusivity and mixing length - conceptually incorrect, though useful as an empirical tool.
- **Richardson-Kolmogorov-Onsager, Cascade**/spectral transfer - ill defined
- **Taylor, Vorticity amplification** is a result of the kinematics of turbulence: vortex lines are on average stretched rather than compressed because two particles on average move apart from each other — RRWR. Is dissipation due to vortex stretching?
- **Batchelor, For the very smallest eddies the motion is entirely laminar.**
WHY TURBULENCE IS SO IMPOSSIBLY DIFFICULT?

THE THREE BIG N's

- **NONLINEARITY**
- **NONINTEGRABILITY**
  - NOT SOLVABLE 'BY HANDS'. Chaotic nature. Self-randomization/Intrinsic stochasticity. PO, Burgers, KdV.
- **NONLOCALITY**
  - DIRECT AND BIDIRECTIONAL INTERACTION OF LARGE AND SMALL SCALES. High dimension, intermittency

SOME 'SMALL' n's

- non-gaussianity, non-lognormality, non-markovianity
  - Intermittency
- no low-dimensional description
- no small parameters and no theory (frustration of a theoretician)

Information density, $\varepsilon$, defined as the ratio, $S/N$, in the literature, with $S = $ signal (understanding), and $N = $ noise (mountains of publications). **Saffman, 1978.**
The most familiar difficulty.

Just to mention one of distinctly conceptual nature

Coupled with a decomposition (of whatever form) - which is a good tool for linear problems - results in interaction between its components (‘cascade’). That is ‘cascade’ is not independent on the nature/form of the decomposition and, therefore, is not a good means for describing a physical process, since the latter cannot be decomposition (which is ours - not Natures’) dependent.
In 1788 LAGRANGE wrote: One owes to EULER the first general formulas for fluid motion ... presented in the simple and luminous notation of partial differences... By this discovery, all fluid mechanics was reduced to a single point analysis, and if the equations involved were integrable, one could determine completely, in all cases the motion of a fluid moved by any forces.
The if in the above citation is crucial: the Navier-Stokes equations are not integrable. Integrable systems, such as those having a solution ‘in closed form’ exhibit regular organized behaviour, even those having (formally) an infinite number of strongly coupled degrees of freedom. A prominent example is provided by the solitons in the systems described the Korteweg de Vries and Schrödinger equations. Another example is the Burgers equation, which is an integrable equation, and exhibits random behaviour only under random forcing, otherwise its solutions are not random*. That is, these examples represent the response of nonlinear systems to random forcing and which otherwise are not random, and should be distinguished from problems involving genuine turbulence. Navier-Stokes equations at sufficiently large Reynolds number have the property of intrinsic stochasticity in the sense that they possess mechanisms of self-randomization (most probably at all scales) which are poorly understood (more in lectures on origins of turbulence and analogies).

* there is no consensus on the meaning of the term integrability, but it is agreed mostly that integrable systems behave nicely and are globally ‘regular’, whereas the nonintegrable systems are not ‘solvable exactly’ and exhibit chaotic behaviour
NONLOCALITY

‘KINEMATIC’ AND DYNAMIC

A SET OF PHENOMENA/MANIFESTAIONS UNDER A COMMON NAME ‘DIRECT AND BIDIRECTIONAL INTERACTION OF LARGE AND SMALL SCALES’. TO BE DISCUSSED IN A SEPARATE LECTURE.
COLLECTION OF ATTEMPTS OF DEFINITION OF TURBULENCE

* Turbulence is the name given to imperfectly understood class of chaotic solutions to the Navier-Stokes equation in which many degrees of freedom are excited. Aref 1999

* It is a well-known fact that under suitable conditions, which normally amount to a requirement that the kinematic viscosity $\nu$ be sufficiently small, some of these motions are such that the velocity at any given time and position in the fluid is not found to be the same when it is measured several times under seemingly identical conditions. In these motions the velocity takes random values which are not determined by the ostensible, or controllable, or 'macroscopic', data of the flow, although we believe that the average properties of the motion are determined uniquely by the data. Fluctuating motions of this kind are said to be turbulent. Batchelor 1953

* Turbulence is a three-dimensional time-dependent motion in which vortex stretching causes velocity fluctuations to spread to all wavelengths between a minimum determined by viscous forces and a maximum determined by the boundary conditions of the flow. It is the usual state of fluid motion except at low Reynolds numbers. Bradshaw 1972

* The only short but satisfactory answer to the question "what is turbulence?" is that it is the general-solution of the Navier-Stokes equation. Bradshaw 1972

* The distinguishing feature of turbulent flow is that its velocity field appears to be random and varies unpredictably. The flow does, however, satisfy a set of differential equations, the Navier-Stokes equations, which are not random. This contrast is the source of much of what is interesting in turbulence theory. Chorin 1975.

* Creation of small scale activity and dissipation, is the principle of turbulence. Classical fluid dynamical instabilities play a role of the fuel, vortex stretching is the engine, and viscous dissipation is the breaks. Constantin 1994.
* - The next great era of awakening of human intellect may well produce a method of understanding the qualitative content of equations. Today we cannot. Today we cannot see that the water flow equations contain such thing as the barber pole structure of turbulence that one sees between rotating cylinders. Today, we cannot see whether Schrödinger's equation contains frogs, musical composers, or morality - or whether it does not. We cannot say whether something beyond it like God is needed, or not. And we can all hold strong opinions either way. FEYNMAN 1963

* - Turbulence with its limit of self-excitation, with the characteristic hysteresis in its appearance and disappearance as the velocity of flow producing it is increased or reduced and the primary role of nonlinearity in its developed (stationary) state, is, in fact, a self-oscillation. Its specific features are determined by the fact that it is self-oscillation of a continuous medium, i.e., a system with an infinite number of degrees of freedom. GORELIK, 1956


* - The following definition of turbulence can thus be tentatively proposed and may contribute to avoiding the somewhat semantic discussion on this matter:  
  a) Firstly, a turbulent flow must be unpredictable, in the sense that a small uncertainty as to its knowledge at a given initial time will amplify so as to render impossible a precise deterministic prediction of it evolution; 
  b) Secondly, it has to satisfy the increased mixing property defined above; 
  c) Thirdly, it must involve a wide range of spatial wave lengths. LESIEUR, 1997.
Turbulence can be defined by a statement of impotence reminiscent of the second law of thermodynamics: flow at a sufficiently high Reynolds number cannot be decelerated to rest in a steady fashion. The deceleration always produces vorticity, and the resulting vortex interactions are apparently so sensitive to the initial conditions that the resulting flow pattern changes in time and usually in stochastic fashion. LIEPMANN 1979

A body of fluid is a mechanical system with an infinite number of degrees of freedom. It may therefore be expected to execute a rather random motion comparable to that of the molecules in a gas. If one regards such a chaotic motion as analyzed into harmonic components of various scales, one recognizes that frictional forces tend to dissipate the small scale oscillations and keep the motion more or less regular. Thus, when viscous forces are sufficiently strong, i.e. at sufficiently low Reynolds numbers, the motion will become laminar. On the other hand, at sufficiently high Reynolds numbers the motion will tend to become random fluctuating, even when external conditions are steady. LIN AND REID 1963

Perhaps a satisfactory definition would be an ensemble of nonperiodic solutions of the Navier-Stokes equations. Ensembles of solutions of simplified or otherwise modified forms of the Navier-Stokes equations will not qualify as turbulence; we shall instead regard them as models of turbulence. LORENZ 1972

We have therefore defined turbulence as random fluctuations of the thermodynamic characteristics of vortex flows, thereby distinguishing it at the outset from any kind of whatever random irrotational i.e., potential flows, ... MONIN 1978
- Definition of Randomness  

- Of special interest to us here are the strange attractors, on which phase trajectories display the following properties of randomness:

1. An extremely sensitive dependence on initial conditions, due to exponential divergence of trajectories which are initially close together (and leading to their unpredictability for initial conditions which are given with arbitrarily high (but finite) precision).
2. The everywhere-denseness at the attractor of almost all trajectories, i.e., their arbitrarily close approach to any of the attractor's points (which implies that they return infinitely often to the attractor), and the property that any initial nonequilibrium probability distribution (measure) over the phase space (or, more precisely, over the region of attraction of the strange attractor) reduces to some limiting equilibrium distribution at the attractor (an invariant measure).
3. The mixing property: For any (measurable) subsets A and B of the attractor, the probability after emerging from A of arrival at B is proportional after a long time of measure of B:

\[ \lim_{t \to \infty} P\{F^tA \cap B\} = P(A)P(B) \]

where the symbol \( \cap \) denotes set intersection. A consequence of the mixing property is the fact that the time-averaged value \( \langle \Phi[u(t)] \rangle \) of any function \( \Phi(u) \) defined on the strange attractor is independent of the initial conditions \( u_0 \) (for almost all \( u_0 \)) and that this average value coincides with the average \( \Phi(u) \) over the invariant measure (ergodicity):

\[ \langle \Phi \rangle \equiv \lim_{T \to \infty} \frac{1}{T} \int_0^T \Phi[u(t)]dt = \int \Phi(u)Pdu \equiv \Phi \]

A characteristic of the mixing property is a rather rapid decay of the correlation functions as \( \tau \to \infty \):

\[ B^j(t) = \langle [u^j](t) - \langle u^j \rangle \rangle \]

which is to say continuity of their Fourier transforms with respect to \( \tau \), i.e., their spectral functions. It appears expedient to have the term turbulence refer to the random evolution [in the sense of (1) - (3) above] of the flow of a (viscous) fluid which possesses vorticity. Stochastic potential flows of a fluid are by preference referred to as random wave fields, while for nonhydrodynamic systems one should preferably restrict oneself, where necessary, to the adjective stochastic.  

MONIN, 1978
Turbulence is a phenomenon which sets in in a viscous fluid for small values of the viscosity coefficient \( \nu \) (reckoning \( \nu \) in significant units, that is, as the reciprocal Reynolds' number \( 1/Re \)), hence its purest, limiting form may be interpreted as the asymptotic, limiting behavior of a viscous fluid for \( \nu \to 0 \). The circumstances described above made it very plausible that turbulence is a phenomenon of instability. A complete theory of the general solutions of the Navier-Stokes equations are called for. Nothing less than a thorough understanding of the system of all their solutions would seem to be adequate to elucidate the phenomenon of turbulence. Turbulence proper is tied to 3-dimensionality. VON NEUMANN, 1949.

One of the best definitions of turbulence is that it is a field of random chaotic vorticity. SAFFMAN 1981.

The turbulence syndrome includes the following symptoms: The velocity field is such a complicated function of space and time that a statistical description is easier than a detailed description; it is essentially three-dimensional, in the sense that the dynamical mechanism responsible for it (the stretching of vorticity by velocity gradients) can only take place in three dimensions; it is essentially nonlinear and rotational, for the same reasons; a system of partial differential equations exists, relating the instantaneous velocity field to itself at every time and place. STEWART, 1960.

Turbulence is an irregular motion which, in general, makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another. TAYLOR AND KARMAN 1937.
In many cases, it is allowable to deal with a system of finite dimensions as a model of a continuous fluid and this is particularly the case at the stage of generation of "turbulence", at which only a limited number of degrees of freedoms of motion have been excited. The approximation of a fluid in terms of a model system of finite dimensions provides us with a powerful means of analysis and it is by this reason that the recent progress in the theory of "chaos" has enabled us to look straight at the fundamental mechanism of "turbulence". On the other hand, it is generally recognized that "turbulence" in its fully developed state has a singular structure in space and time and that the singularity is closely connected with the peculiar property of "turbulence" such as the nonzero viscous dissipation in the limit of vanishing viscosity. Such a singular behaviour of the fluid cannot be described correctly by means of a model system of finite dimensions which remains regular in the inviscid limit. Thus, in this restricted area, "chaos in fluids" covers only a part of "turbulent" phenomena. TATSUMI 1984.

Before 1970, I would not have dreamt of putting the words turbulence and predictability side by side, as in the title of this summer course. To me, turbulence was unpredictable by definition. Turbulence was the chaos that arises in fluids because of the innumerable instabilities associated with vortex stretching. These days, I tend to think of turbulent flow as flow in which deterministic calculations become useless in a finite time interval. TENNEKES 1985.

Everyone who, at one time or another, has observed the efflux from a smokestack has some idea about the nature of turbulent flow. However, it is very difficult to give a precise definition of turbulence. All one can do is list some of the characteristics of turbulence flows: Irregularity ... Diffusivity ... Large Reynolds numbers ... Three-dimensional vorticity fluctuations ... Dissipation ... Continuum ... Turbulent flows are flows ... TENNEKES AND LUMLEY 1972.
It is proposed to read the following short papers and to discuss the questions, comments… arised some time during this lecture series.


You may wish to have a look (through) the papers by
