Streamline-based dual-porosity simulation of reactive transport and flow in fractured reservoirs

Ginevra Di Donato and Martin J. Blunt
Department of Earth Science and Engineering, Imperial College London, London, UK

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[1] We present a streamline-based formulation to model two processes: transport of a tracer undergoing rate-limited sorption and two-phase (water/oil or air/water) transport in fractured systems, using a dual-porosity approach. We show that these two processes can be simulated using mathematically equivalent formulations. In both cases the system conceptually has two components: a flowing fraction connected to stagnant regions with transfer between the two domains. Streamlines capture movement through the flowing fraction. Fluid transfer between flowing and stagnant regions enters as a source/sink term in the one-dimensional transport equations along a streamline. To model flow and transport in fractured systems, we develop a new formulation for the transfer function that matches experimental imbibition data. Then we illustrate the streamline approach with synthetic reservoir problems. We use a finely gridded (over one million grid blocks) three-dimensional domain with a highly heterogeneous permeability field to study both fracture flow and tracer transport. We find breakthrough curves that are consistent with anomalous transport described by an exponent that characterizes the longtime tail of the transit time distribution. For fracture flow we demonstrate that the speed of fluid advance in the fractures is controlled by the imbibition rate. The run times for the simulations scale approximately linearly with system size, making the method appropriate for the simulation of large numerical models. INDEX TERMS: 5104 Physical Properties of Rocks: Fracture and flow; 1832 Hydrology: Groundwater transport; 5139 Physical Properties of Rocks: Transport properties; KEYWORDS: anomalous transport, dual porosity, fractured reservoirs, reactive tracer transport, streamline


1. Introduction

[2] Streamline-based simulation offers an attractive alternative to traditional grid-based methods since it naturally, accurately, and efficiently models advective-dominated transport in highly heterogeneous systems [Batycky et al., 1997; Crane and Blunt, 1999]. Crane and Blunt [1999] used this approach to model the transport of sorbing and decaying tracers and presented the method as an improvement over particle tracking techniques. Streamline simulation is recommended when advection is the principal transport process but is not applicable for circumstances where gravitational effects are dominant or for highly compressible systems [Batycky et al., 1997].

[3] There are two applications where streamline-based simulation may offer considerable advantages over other numerical methods: the modeling of field-scale anomalous transport and the prediction of multiphase flow in fractured reservoirs. Several tracer tests in heterogeneous aquifers [Adams and Gelhar, 1992; Sidle et al., 1998; Becker and Shapiro 2000; Haggerty et al., 2001] have indicated behavior that cannot be predicted assuming Gaussian spreading of the plume. Di Donato et al. [2003] demonstrated using streamline-based simulation that advective movement with particles experiencing a wide range of velocities could be the physical origin of anomalous transport at the large scale, confirming the theoretical and numerical studies of Berkowitz and Scher [1995, 1997, 1998, 2001]. We will pursue this concept further by including rate-limited sorption in the model.

[4] Multiphase flow in fractured systems with application to improved oil recovery in hydrocarbon reservoirs, non-aqueous phase contaminant transport, and nuclear waste containment offers a particular challenge to numerical modeling, since both the flow through the fracture network and transfer to a relatively stagnant matrix need to be modeled [Bear et al., 1993]. In this paper we show how to include fracture/matrix transfer simply and elegantly in a streamline-based approach by representing transfer through a source or sink term in the one-dimensional (1-D) transport equation along streamlines. The streamlines themselves model the flow through the fractures. There is no consensus in the literature on the appropriate form of the transfer function even for straightforward physical processes, such as capillary imbibition [de Swaan, 1978; Rossen and Shen, 1987; Civan, 1998; Terez and Firoozabadi, 1999]. Mathematically, rate-limited sorption and fracture/matrix transfer can be treated in a similar manner using a dual-porosity model [Barenblatt et al., 1960; Warren and Root, 1963; Gilman and Kazemi, 1983; Kazemi et al., 1992; Zimmerman et al., 1993]. In both cases, there is a flowing domain with
transfer to effectively stagnant regions (the matrix in the
case of fractured systems or the solid in the case of sorption).

[5] In our generalized dual-porosity model (Figure 1),
flow occurs along streamlines through the heterogeneous
high-permeability fraction of the system. There is transfer of fluid along the streamlines from the flowing to the stagnant regions, mediated principally by a retardation factor and a reaction rate for sorption or by capillary forces in the fracture flow model. There is no assumption that flow is only through the fractures or that the system is simply connected. Conceptually, the flowing regions simply represent the average single-phase flow field and may exclude some dead-end fractures, while including permeable fractions of the matrix.

[6] We will first develop the mathematical model to handle dual-porosity systems. Different forms of the transfer function to represent countercurrent imbibition from fracture to matrix will be discussed, and a new form will be proposed that matches experimental measurements on core samples. Then we present the results of the flow simulations obtained for a rate-limited sorbing tracer and for oil and water flow in a synthetic fractured reservoir.

2. Transport Models

2.1. Passive Tracer Transport Model

[7] The mass balance equation for single-phase, incompressible flow with no sources or sinks and without diffusion or dispersion is

\[
\phi \frac{\partial C}{\partial t} + \vec{u} \cdot \nabla C = 0
\]

(1)

where \(t\) is the time, \(\phi\) is the porosity of the rock, \(\vec{u}\) is the tracer velocity, and \(C\) is the tracer concentration. It is possible to include dispersion and diffusion (both longitudinal and transverse) in a streamline model using an operator splitting technique, where the dispersive portion of the conservation equation is solved on the underlying grid using conventional techniques [Di Donato et al., 2003]. The effects of ignoring dispersion on the results presented in this paper will be discussed later.

2.2. Rate-Limited Sorption Model

[8] Now consider sorption of tracer that has a finite reaction rate. Physically, there is a flowing tracer in the water that transfers to a stationary sorbed phase and vice versa. In order to formulate a linear sorption model using streamlines, equation (1) is written for both the mobile and immobile zones, adding a source/sink term to account for mass transfer between the two regions:

\[
\phi \frac{\partial C_{sf}}{\partial t} + \vec{u} \cdot \nabla C_{sf} = -T
\]

(2)

\[
\phi \frac{\partial C_{ss}}{\partial t} = T
\]

(3)

where \(C_{sf}\) is the solute concentration in water (flowing fraction of the system) and \(C_{ss}\) is the concentration of the solute component sorbed to the rock (stagnant regions). The formulation of the transfer function \(T\) will be described in section 4.

2.3. Dual-Porosity Formulation for Fracture Flow

[9] In dual-porosity models, assuming incompressible flow of oil and water, the relevant mass conservation equations are

\[
\phi_f \frac{\partial S_{wf}}{\partial t} + \vec{u}_f \cdot \nabla F_{wf} + \nabla \cdot \vec{G}_w = -T
\]

(4)

\[
\phi_m \frac{\partial S_{wm}}{\partial t} = T.
\]

(5)

In equations (4) and (5), \(t\) is the time, \(\phi_f\) and \(\phi_m\) are the fracture and matrix porosity, respectively, and \(S_{\cdot}\) is the water saturation. The subscript \(f\) represents the fractures and \(m\) represents the matrix. \(T\) is the transfer function that describes the exchange of fluid between matrix and fractures. The fractional flow of fluid \(F_{\cdot}\) ignoring gravity is given by

\[
F_{\cdot} = \frac{k_{\cdot w} \mu_{\cdot w}}{k_{\cdot w} \mu_{\cdot w} + k_{\cdot m} \mu_{\cdot m}}
\]

(6)

and \(G_w\) is the water velocity due to gravity:

\[
\vec{G}_w = \frac{\vec{g}}{\mu_w} F_w (\rho_w - \rho_o) g \nabla h.
\]

(7)

The total velocity is

\[
\vec{u} = \vec{u}_w + \vec{u}_o
\]

(8)
where from Darcy’s law,
\[ \bar{u}_w = -\frac{k_{rw}}{\mu_w} \vec{g} \cdot \nabla (P_w - P_{gw}) \]
\[ \bar{u}_o = -\frac{k_{ro}}{\mu_o} \vec{g} \cdot \nabla (P_o - P_{go}), \]

where \( \mu_w \) and \( \mu_o \) are the water and oil viscosities, \( k_{rw} \) and \( k_{ro} \) are the water and oil relative permeabilities, respectively, \( K \) is the absolute permeability of the porous medium, \( P_w \) and \( P_o \) are the water and oil pressures, respectively, \( \rho_w \) and \( \rho_o \) are the water and oil densities, \( g \) is the gravity acceleration constant, and \( h \) is a depth below datum. If there is no capillary pressure \( (P_o = P_w = P) \) and no dispersion, the governing volume balance equation is
\[ \nabla \cdot \bar{u} = q \]

where \( q \) is a volumetric source/sink term representing the net addition of fluid to the porous medium per unit of total volume.

[10] Since the work of Warren and Root [1963], dual-porosity models incorrectly have often been thought of as representing the reservoir by an orthogonal array of matrix blocks surrounded by fractures; in fact, conceptually, the model simply represents flowing and stagnant domains with no assumptions as to their geological structure. Kazemi et al. [1976] extended the Warren-Root model to multiphase systems to account for capillary and gravity forces. All these methods assumed that the matrix/fracture flux in each block was directly proportional to the difference in pressure between the two systems, using a transfer coefficient introduced by Barenblatt et al. [1960] that was a function of the size and shape of the matrix blocks. Gilman and Kazemi [1983] added complexity to the dual-porosity models accounting for other transport processes such as tracer flow, but the basic approach remained the same. Notice that the conservation equations for rate-limited sorption and fracture flow are mathematically identical if in equations (4) and (5) we take \( S_{uf} \equiv C_{usf}, S_{wm} \equiv C_{usr}, F_{uf} \equiv C_{usf}, \) and \( \phi_f = \phi_m = \phi. \)

3. Streamline Method

[11] We first briefly review the streamline method for single porosity systems; further details are given by Crane and Blunt [1999] and Batycky et al. [1997].

[12] From equations (8)–(11) a pressure equation can be derived that is solved on a regular Cartesian grid using an algebraic multigrid method [Stueben, 2001]. Using Darcy’s law, the total velocity can be found at all cell edges. The streamlines are aligned along the total velocity at each point of the domain. They are traced from injectors to producers using a technique first developed by Pollock [1988]. To develop a transport equation, we define the time of flight \( \tau(s) \) as the time taken for a particle to move a distance \( s \) along a streamline [Pollock, 1988].

\[ \tau(s) = \int_0^s \frac{\phi}{|\bar{u}|} \, ds. \]

Notice that the saturation is transported in the direction of the streamlines, so equation (4) can be written as
\[ \phi_f \frac{\partial S_{uf}}{\partial t} + |\bar{u}_f| \frac{\partial F_{uf}}{\partial s} + \nabla \cdot \vec{G}_w = -T. \]

Using then the definition (12), we have
\[ \frac{\partial \tau}{\partial s} = \frac{\phi_f}{|\bar{u}_f|}, \]

so equation (13) becomes
\[ \frac{\partial S_{uf}}{\partial t} + \frac{\partial F_{uf}}{\partial \tau} + \frac{1}{\phi_f} \nabla \cdot \vec{G}_w = -\frac{T}{\phi_f}. \]

For the matrix we have
\[ \frac{\partial S_{wm}}{\partial t} = \frac{T}{\phi_m}. \]

In our model we do not account for capillary pressure in the flowing fraction (fractures), but we do account for capillary forces in the transfer of fluid from fracture to matrix, as described below.

4. Streamline-Based Model

4.1. Model Description

[13] The procedure for the simulation is as follows:

[14] 1. Flowing fraction saturation, permeability and porosity are defined on the underlying grid.

[15] 2. Using equations (8)–(11) for the flowing fraction of the system, the pressure field is computed on the grid.

[16] 3. Using Darcy’s law and the known pressures, streamlines are traced across the grid. All grid blocks have at least one streamline going through them [Batycky et al., 1997].

[17] 4. Flowing and stagnant fractions saturations defined on the grid are mapped onto streamlines [Batycky et al., 1997].

[18] 5. Equations (15) and (16) are solved along streamlines, using an appropriate form for \( T \). The exact numerical schemes used are described later.

[19] 6. The flowing and stagnant fractions saturations are mapped onto the grid; how this is done is described by Crane and Blunt [1999] and Batycky et al. [1997]. In the dual-porosity model, fluid segregation due to gravity is simulated on the grid using an operator splitting technique [Batycky et al., 1997].

[20] 7. Whenever the flow field changes due to changing boundary conditions or because of the mobility contrast between the phases, streamlines are recomputed and the simulation returns to step 1.

4.2. Rate-Limited Sorbing Tracer

[21] In the case of sorption of tracer that has a finite reaction rate, the transfer function \( T \) is the sorption rate that can be written as [Zheng and Bennett, 1995]
\[ T = k_f C_{uf} - k_b C_{usf}. \]
Here $k_f$ is the forward reaction rate and $k_b$ is the backward reaction rate. There is a relation between the forward and backward rates, since in equilibrium $T = 0$ and

$$\frac{k_f}{k_b} = \frac{C_{swf}}{C_{swb}} = R - 1,$$  \hspace{1cm} (18)

where $R$ is a retardation coefficient. Then, using equation (18) we can write $T$ as

$$T = \phi k [(R - 1) C_{swf} - C_{swb}], \hspace{1cm} (19)$$

where $k$ is a finite reaction rate.

### 4.3. Fracture/Matrix Transfer Functions: Kazemi et al.’s [1992] Model

[22] In the 1980s, several authors [Dean and Lo, 1988; Rossen and Shen, 1987; Coats, 1989] used empirical transfer functions to describe matrix/fracture flow and scale laboratory results to the field, by defining “pseudo capillary pressure” functions. Kazemi et al. [1992] showed the advantages and the limitations of this approach. They derived an empirical transfer function when water imbibition is the dominant force for displacing oil from the matrix; in this case the cumulative oil recovery $R$ from a rock block surrounded by a water-saturated fracture matches an exponential form [Aronofsky et al., 1958]:

$$R = R_\infty (1 - e^{-\beta t}). \hspace{1cm} (20)$$

Here $\beta$ is a rate constant and $R_\infty$ is the total oil recoverable by imbibition:

$$R_\infty = (1 - S_{swm} - S_{num}) \phi_m, \hspace{1cm} (21)$$

where $S_{swm}$ is the initial matrix water saturation and $S_{num}$ is the remaining oil saturation in the matrix after imbibition. The recovery $R$ is given by

$$R = (S_{num} - S_{swm}) \phi_m, \hspace{1cm} (22)$$

so that when $S_{swf} = 1$, using equation (20), they found

$$(S_{num} - S_{swm}) = (1 - S_{swm} - S_{num})(1 - e^{-\beta t}). \hspace{1cm} (23)$$

Finally, using equation (5), they obtained a transfer function of the form

$$T = \phi_m \beta (1 - S_{swm} - S_{swf}). \hspace{1cm} (24)$$

They then derived $T$ when $S_{swf} \neq 1$ by using a convolution integral due to de Swaan [1978]:

$$T = \chi R_\infty \int_0^t e^{-\beta (t - \tau)} \frac{\partial S_{swf}}{\partial \tau} d\tau \hspace{1cm} (25)$$

from which was derived

$$T = \phi_m \beta [S_{swf}(1 - S_{swm} - S_{num}) - (S_{num} - S_{swm})]. \hspace{1cm} (26)$$

Equation (26) indicates that the transfer of fluids is a linear function of both water saturation in the fracture and water saturation in the matrix. Kazemi et al. [1992] derived an analytic 1-D solution for the transport equation (4) for $T$ given by equation (26) when $F_{swf} = S_{swf}$:

$$S_{swf}(x, t) = 0, \hspace{0.5cm} t < a$$

$$S_{swf}(x, t) = e^{-\alpha} e^{-\beta (t - a)} I_0 \left[ 2 \sqrt{\gamma (t - a)} \right]$$

$$+ \beta e^{-\beta (t - a)} \int_0^a e^{-\beta (t - \tau)} I_0 \left[ 2 \sqrt{\gamma (t - a)} \right] d\tau, \hspace{0.5cm} t \geq a \hspace{1cm} (28)$$

where $I_0$ is the zero-order Bessel function, $a = x/ut$, and $\gamma = a(R_\infty / \beta \phi_m)$.

### 4.4. New Linear Transfer Function

[23] We will formulate a similar transfer function chosen to match experimental imbibition data. Zhang et al. [1996] performed an extensive series of imbibition experiments in water-wet blocks surrounded by a water-saturated fracture. Gravity and viscous effects were negligible, and hence the recovery was controlled entirely by capillary pressure. The experimental data match the simple exponential form equation (20). Here $\beta$ is written as $\alpha t_D$, where $\alpha$ is a constant (with a value around 0.05 to match the data) and $t_D$ is a dimensionless time:

$$t_D = \int \frac{k_m \sigma}{\phi_m \sqrt{\mu_w \phi_o} L_c^2}. \hspace{1cm} (29)$$

where $k_m$ is the permeability of the matrix, $\sigma$ is the interfacial tension, $\mu_w$ and $\mu_o$ are the water and oil viscosities, respectively, and $L_c$ is an effective length:

$$L_c^2 = \frac{V}{\sum \frac{A_i}{l_i}} \hspace{1cm} (30)$$

where $V$ is the matrix block volume, $A_i$ is the area open to flow in the $i$th direction, and $l_i$ is the distance from the open surface to a no-flow boundary. Zhang et al. [1996] emphasized that the use of the geometric mean viscosity in equation (29) was a purely empirical fit to oil/water experimental data. Such a scaling does not reproduce results for air/water systems, where the viscosity of the nonwetting phase (equivalent to oil in this treatment) becomes negligible. It would be possible to pursue the same approach using other correlations, such as that derived by Zhou et al. [2002] that does reduce to an appropriate limit for low nonwetting phase viscosity.

[24] Following the derivation of equation (24) we find

$$T = \phi_m \beta (1 - S_{swm} - S_{num}) e^{-\beta t} \hspace{1cm} (31)$$

$$T = \phi_m \beta (1 - S_{swm} - S_{num}), \hspace{1cm} (32)$$

where

$$\beta = \frac{\alpha t_D}{t}. \hspace{1cm} (33)$$

For $S_{swf} \neq 1$ we will assume that the transfer function is given by equation (32); that is, that the transfer rate is
independent of fracture saturation as long as $S_{wf} > 0$. $T$ varies linearly with $S_{sw1}$; as a consequence we will call this the linear transfer function.

[25] Several authors have attempted to introduce a similar transfer function in the context of conventional finite difference dual-porosity simulators [de Swaan, 1978; Kazemi et al., 1992; Civan, 1998; Terez and Firoozabadi, 1999]. However, from the work of de Swaan [1978] onward, they have invariably used a convolution integral for the instantaneous transfer function, equation (25). This approach is valid if you consider the total transfer across a block that contains a front of saturation $S_{of} = 1$ passing through it. It is not appropriate, however, as a correct representation of a partial differential equation for the transfer, since it implies that the local transfer rate and ultimate recovery is proportional to the fracture saturation. Since the transfer occurs due to capillary forces, one would expect the transfer to cease when the capillary pressure in the fracture and matrix are equal. Since the capillary pressure in the fracture is typically negligible compared with the matrix, transfer will end when the matrix oil saturation is close to $S_{om}$, and is relatively insensitive to the fracture saturation. This is what is assumed in this model and is different from the formulations of other authors.

[26] Kazemi et al.’s transfer function equation (26) is the same as equation (32) for $S_{of} = 1$, but is smaller for $1 > S_{of} > 0$. Kazemi et al.’s transfer function is mathematically equivalent to rate limited sorption. Both models, equations (19) and (26), assume a dynamic equilibrium between flowing and stagnant regions and are equivalent if $S_{of} \equiv C_{of}$, $S_{sw} - S_{sw1} \equiv C_{sw}$, $1 - S_{om} - S_{sw1} \equiv R - 1$ and $\beta \equiv k$. Our transfer function, in contrast, models a nonreversible transfer of water from fracture to matrix.

4.5. Numerical Solutions

[27] We construct numerical solutions of equations (15) and (16), with $T$ given by equations (19), (26), and (32), along a streamline, by solving for the transport terms explicitly and the transfer terms implicitly. First, we compute the initial saturation $S_{int}^{n-1}$ corresponding to the flow in the flowing regions without any transfer into the stagnant regions, using single-point upstream weighting:

$$S_{int}^{n-1} = S_{of}^{n} - \frac{\Delta t}{\Delta x} \left( F_{of} \left( S_{of}^{n} \right) - F_{of} \left( S_{of}^{n-1} \right) \right).$$

where the superscript $n$ on $S$ labels the time step and the subscript $i$ labels the grid block.

[28] For the transfer function given by equation (19), we have ($C_{of} \equiv S_{of}$ and $F_{of} \equiv C_{of}$)

$$C_{of,i} = \frac{C_{of,i}^{n} + \frac{k \Delta t}{1 + k \Delta t} C_{sw,i}^{n}}{1 + \frac{k \Delta t (R - 1)}{1 + k \Delta t}}$$

(35)

$$C_{sw,i}^{n+1} = \frac{C_{sw,i}^{n} + k \Delta t (R - 1) C_{of,i}^{n+1}}{1 + k \Delta t}$$

(36)

For the Kazemi et al. transfer function given by equation (26) we find

$$S_{of,i}^{n+1} = S_{of,i}^{n} - \frac{\beta \Delta t}{1 + \beta \Delta t} \frac{\phi_m}{\phi_f} \left( S_{sw1,i}^{n+1} - S_{sw1,i}^{n} \right)$$

$$S_{sw1,i}^{n+1} = S_{sw1,i}^{n} - \frac{\beta \Delta t (1 - S_{om} - S_{sw1}^{n})}{1 + \beta \Delta t}$$

(37)

$$S_{of,i}^{n+1} = S_{of,i}^{n} - \frac{\beta \Delta t}{1 + \beta \Delta t} \frac{\phi_m}{\phi_f} \left( 1 - S_{om} - S_{sw1,i}^{n+1} \right).$$

(38)

For our linear transfer function given by equation (32), if the computed $S_{int}^{n+1}$ is equal to zero, then we set $T = 0$ and $S_{sw1,i}^{n+1} = 0$. If $S_{of,i}^{n+1} > 0$, we find

$$S_{of,i}^{n+1} = S_{of,i}^{n} - \frac{\beta \Delta t}{1 + \beta \Delta t} \frac{\phi_m}{\phi_f} \left( 1 - S_{om} - S_{sw1,i}^{n+1} \right).$$

(39)

If the water saturation in the fracture $S_{sw1,i}^{n+1}$ is negative, then we set $S_{sw1,i}^{n+1} = 0$ and find the matrix water saturation that is consistent with this and mass balance:

$$S_{sw1,i}^{n+1} = S_{sw1,i}^{n} + \frac{\phi_f}{\phi_m} S_{of,i}^{n+1}.$$ (40)

4.6. Analytical Solutions

[29] We have constructed 1-D analytical solutions for the water saturation valid at early times and a late time solution for the front location using the linear transfer function, equation (32). First, we derive the early time solution. Using the time of flight defined in equation (12) we assume that $S_{of} (t, \tau = 0) = 0$, $S_{of} (t, \tau = 0) = 1$, and $F_{of} = S_{of}$. We also assume that the leading edge of the water front moves at unit speed. This means that imbibition has not had sufficient time to reduce the fracture saturation to zero anywhere. Hence water first enters the fracture at time $t = \tau$, and at that moment water transfer from fracture to matrix starts with an exponentially decaying function $T$ equation (31):

$$T = \frac{\phi_m}{\phi_f} \left( 1 - S_{om} - S_{sw1} \right) e^{-\beta (t - \tau)} H(t - \tau).$$

(42)

Then, Laplace transforming the water mass conservation equation in the fracture system equation (15) and defining $\gamma = \frac{\phi_{sw1}}{\phi_f} \left( 1 - S_{om} - S_{sw1} \right)$, we obtain

$$s \bar{S}_{of} + \frac{\partial \bar{S}_{of}}{\partial \tau} = -\gamma \int_{0}^{\infty} e^{-s t} e^{-\beta (t - \tau)} H(t - \tau) dt;$$

(43)

solving the integral in equation (43), we obtain

$$s \bar{S}_{of} + \frac{\partial \bar{S}_{of}}{\partial \tau} = \frac{\gamma e^{-\beta t}}{s + \beta}.$$ (44)

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Once we invert equation (50) and use the definition of time of flight equation (12), we obtain an expression for the water front location with time:

$$
\tau(t) = \frac{1}{(\gamma + \beta)t + \gamma(1 - e^{-(\gamma + \beta)t})}. \tag{51}
$$

Equation (51) represents an analytical solution at all times. In the next section we compare the numerical solution given in the previous section with the analytical solutions equations (46) and (51); we also compare the numerical solution, obtained using Kazemi et al.’s transfer function with the analytical solutions equations (27) and (28).

5. Results

5.1. One-Dimensional Streamline Simulations

Table 1. Parameters and Values Used for Figures 2 and 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Figure 2</th>
<th>Figure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_f )</td>
<td>0.05 (Figure 2a); 0.02 (Figure 2b)</td>
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</tr>
<tr>
<td>( \phi_m )</td>
<td>0.2</td>
<td>0.16</td>
</tr>
<tr>
<td>( \nu_f )</td>
<td>1 m s(^{-1}) = 1.15 \times 10^{-5} m s(^{-1})</td>
<td>5.525 \times 10^{-7} m s(^{-1})</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>0.3 \times 10^{-3} Pa s</td>
<td>1.0 \times 10^{-3} Pa s</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>800 kg m(^{-3})</td>
<td>800 kg m(^{-3})</td>
</tr>
<tr>
<td>( k_{w,ref} )</td>
<td>0.0017 day(^{-1}) = 2 \times 10^{-8} s(^{-1})</td>
<td>0.1 day(^{-1}) = 1.16 \times 10^{-6} s(^{-1})</td>
</tr>
<tr>
<td>[\beta]</td>
<td>0.017 day(^{-1}) = 2 \times 10^{-8} s(^{-1}) (Figure 2a); 0.017 day(^{-1}) = 2 \times 10^{-8} s(^{-1}) (Figure 3a); 0.01 day(^{-1}) = 1.16 \times 10^{-6} s(^{-1}) (Figure 3b)</td>
<td></td>
</tr>
<tr>
<td>( K_f )</td>
<td>10^{-14} m(^2)</td>
<td>10^{-14} m(^2)</td>
</tr>
<tr>
<td>( S_{swi} )</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>( S_{swr} )</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>( \nu_r )</td>
<td>0.3 \times 10^{-3} Pa s</td>
<td>1.0 \times 10^{-3} Pa s</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>800 kg m(^{-3})</td>
<td>800 kg m(^{-3})</td>
</tr>
<tr>
<td>( k_{r,ref} )</td>
<td>1 - ( S_{swf} )</td>
<td>1 - ( S_{swf} )</td>
</tr>
</tbody>
</table>

The solution of this equation in the Laplace transform domain is

$$
\tilde{S}_{swf}(\tau, s) = -\frac{\gamma t e^{-\gamma t}}{s + \beta} + \frac{1}{s} e^{-\gamma t}, \tag{45}
$$

where we have accounted for the condition \( \tilde{S}_{swf}(0, s) = 1/s \).

Inverting (45), we find the time-dependent solution:

$$
S_{swf}(\tau, t) = \left(1 - \gamma t e^{-\gamma (t - \tau)}\right) H(t - \tau). \tag{46}
$$

Equation (46) describes how the water flows in the fracture at early times.

At late times, when the amount of imbibition into the matrix exceeds the pore volume of the fracture, another analytical approach is required. We assume that to a good approximation \( S_{swf} = 1 \) for some distance \( x = L(t) \) and \( S_{swf} = 0 \) for \( x > L(t) \). We find an expression for \( L(t) \) by considering the global volume balance in the system using an approach similar to that of Rangel-German and Kovscek [2002]. If \( u_t \) is the total injection rate into the fracture per unit area of the system, then it has to be equal to

$$
\dot{u}_f = T + \dot{u}_f, \tag{47}
$$

where \( T \) is the total transfer rate from fracture to matrix and \( \dot{u}_f \) is the flow rate in the fracture per unit area. Notice that \( T \) is not an explicit function of the position \( x \), but of the time \( t - \dot{t} \), where \( t - \dot{t} \) is the time of first contact of the matrix by water in the fracture. Thus we can rewrite equation (47) as

$$
\dot{u}_f = \int_0^t T(t - \dot{t}) \frac{dL}{dt} dt + \dot{u}_f \frac{dL}{dt}. \tag{48}
$$

If we now take the Laplace transform of equation (48) and use equation (42), we have

$$
\frac{u_t}{s} = \phi_f \frac{\gamma}{\beta + s} \tilde{L}(s) + \phi_f \tilde{L}(s), \tag{49}
$$

so, solving for \( \tilde{L}(s) \), we find

$$
\tilde{L}(s) = \frac{u_t(\beta + s)}{s^2 \phi_f (\beta + \gamma + s)}. \tag{50}
$$

Figure 2. Comparisons of analytical (lines) and numerical (points) 1-D solutions for the fracture saturation using the linear transfer function. The parameters used are given in Table 1. (a) For small values of \( \beta t \), where \( \beta \) is the transfer rate and \( t \) is the time, the advancing front in the fracture moves at the same speed as flow without transfer, but the fracture saturation behind the front is lower due to imbibition into the matrix. (b) For larger values of \( \beta t \), the front location moves slower, as the leading edge of the front is completely drained by transfer into the matrix.
used a 1-D grid with 200 blocks of 10 m length each and we injected water from one edge at a constant rate (see Table 1 for the parameters used). We also fixed the transfer rate $\beta$ to a constant value of $2 \times 10^{-9}$ s$^{-1}$. The matrix saturation changes significantly in a time $t$ corresponding to $\beta t \approx 1$, that is about 600 days; for $t \ll t$ the flow in the fracture is described by the analytical solution at early times equation (46) that in Figure 2a is compared to the finite difference solution equations (39) and (40); clearly there is a very good agreement between them. At later times a front with $S_{wfr} = 1$ propagates through the system with a lower speed, with a transition zone to $S_{wfr} = 0$. Figure 2b shows the all times solution equation (51), compared to the numerical solution equations (39) and (40). The front location is defined as where $S_{wfr} = 0.5$. In this case we used a $\beta$ value of $2 \times 10^{-7}$ s$^{-1}$, so that $t$ is of the order of 60 days. There is a transition from fast flow of water in the fracture at small times to a slower flow associated with increasing water imbibition into the matrix at later times.

We have also checked that our finite difference solution equation (37), when using Kazemi et al.’s transfer function, was correct by comparing it with the analytical solution given by equations (27) and (28). The numerical solution is for a 1-D system of 50 blocks and time steps of 150 days (see also Table 1); we used all the same parameters that were needed to compare analytical and numerical solutions of Kazemi et al. [1992]. Figures 3a and 3b show fracture saturation profiles at different time steps for two rate constants ($\beta = 0.1$ days$^{-1} = 1.16 \times 10^{-6}$ s$^{-1}$ and $\beta = 0.01$ days$^{-1} = 1.16 \times 10^{-7}$ s$^{-1}$, respectively). The agreement between the analytical and numerical solutions is good, especially for $\beta = 0.01$ days$^{-1}$.

We then compared the numerical solutions obtained by using the two different transfer functions. We used 200 grid blocks and we show the 1-D displacements in the fractures and in the matrix at early time in Figure 4 and at late time in Figure 5. We used $\beta = 5 \times 10^{-9}$ s$^{-1}$ (see also Table 2). One pore volume injected corresponds to about 9000 days as it is in the 3-D flow simulations presented in the next section. As mentioned previously, the Kazemi et al. model predicts less transfer into the matrix for $S_{wfr} < 1$, as evident in Figures 4b and 5b. Since less water transfers to the matrix, more is retained in the fracture. The Kazemi et al. model predicts a diffuse water profile in both matrix and fracture, while our model gives a more frontal advance with a relatively narrow transition region from high $S_{wfr}$ to $S_{wfr} = 0$; see Figure 5.

5.2. Three-Dimensional Streamline Simulations

Simulations were carried out for the two different transport processes described above. We used a model based

![Figure 4. Comparisons of 1-D displacements in the (a) fracture and (b) matrix for the two transfer functions implemented in the streamline code at early time. For both the linear model, equation (32), and the Kazemi et al. model, equation (26), we use $\beta = 5 \times 10^{-5}$ s$^{-1}$.](image_url)
on a North Sea oil field with high-permeability meandering sand channels surrounded by low-permeability shale [Christie and Blunt, 2001]. The model is described on a regular Cartesian grid of dimensions $366 \times 670 \times 52$ m; the number of grid cells used for the following simulations was $1,122,000$, given by $60 \times 220 \times 85$ in the $x$, $y$, and $z$ directions, respectively. The highly heterogeneous permeability field of the model had a standard deviation $\sigma_{\ln K_x} = 5.79$ and $\sigma_{\ln K_y} = 3.5$. We assume that initially the system is in equilibrium so that the flowing and sorbed phases coexist in the domain at $t = 0$. Specifically the sorbed phase concentration is $C_{\text{sw}} = (R - 1)C_{\text{inf}}, \quad t = 0$ (52)

where $R$ is the retardation factor defined in section 4.2. In the simulations we have considered two different values of $R$: 2 and 10. We considered reaction with a finite rate, $10^{-7}$ s$^{-1}$, and equilibrium sorption (corresponding to the limit $k \to \infty$).

Figure 7 shows the breakthrough curves obtained for the different tracers. Notice the huge range of timescales; while breakthrough can occur in the order 10 days ($8.64 \times 10^5$ s), significant amounts of tracer still remain in the system after 10,000 days ($8.64 \times 10^8$ s). This anomalous behavior has been explained and predicted by Berkowitz and Scher [1995, 1997, 1998, 2001], using a continuous time random walk (CTRW) formulation. Define $\psi(s, t)$ as the probability per unit time for tracer movement between sites separated by a distance $s$ in a time $t$. If, at large times, $\psi(s, t) \sim t^{-(1 - \lambda)}$ for $1 > \lambda > 0$, then at late times the concentration at a production well, $C(t) \sim t^{-(1 - \lambda)}$ [Berkowitz et al., 2000]. For a passive tracer Di Donato et al. [2003] associated $\psi(s, t)$ with the time-of-flight distribution through each grid block and demonstrated power law
computed C scaling at late times with an exponent \( \lambda \) consistent with the computed \( C(t) \).

[38] Each breakthrough curve in Figure 7 has a long tail at late time with a log-log slope 1 + \( \lambda \) of approximately 1.43. Since a source of tracer is initially present in the porous medium occupying a region with both high and low permeabilities, the tracer flows with highly variable velocities once water injection begins [Di Donato et al., 2003]. The first arrivals of the tracer depend on how high the permeability values are, while the behavior of the breakthrough curves at late time is controlled by the presence of the tracer in the low-permeability regions. This scenario is similar to that observed at the Columbus Air Force Base [Adams and Gelhar, 1992].

[39] When the tracer is undergoing sorption to the rock, the breakthrough time increases as the retardation factor increases (see the dashed and dotted lines in Figure 7). If adsorption is rate-limited, breakthrough occurs early as in the case of the passive tracer, since some tracer has been in the system for an insufficient time to sorb. However, for \( kt > 1 \) (greater than approximately 100 days), the tracer has reached equilibrium and the breakthrough curves resemble those of a corresponding system with instantaneous reaction. All the breakthrough curves of the different tracers undergoing sorption behave as a power law at late time and have approximately the same double-log slope shown by the breakthrough curve obtained by the purely advective transport model (solid curve in Figure 7). This is a key result, and it confirms that it is the advective flow through heterogeneous media that causes the breakthrough curves to behave as a power law at late time. This contrasts with the work of other authors [Haggerty and Gorelick, 1995; Haggerty et al., 2000; Harvey and Gorelick, 2000] who have invoked models of rate-limited transfer to explain anomalous transport.

[40] This model does not account for dispersion. Including physical amounts of dispersion has no qualitative effect on the results, and power law tails with the same exponent are observed for passive tracer [Di Donato et al., 2003]. Di Donato et al. also showed that the purely advective model is able to yield a power law behavior at late time with different slopes of the breakthrough curves as the degree of heterogeneity varies in the field and as different test designs are considered.

### 5.2.2. Fracture Flow Simulations

[41] We constructed a synthetic fractured reservoir based on the model presented above. We assumed a fixed matrix permeability of 1 mdarcy \((10^{-15} \text{ m}^2)\). The permeability field from the model was taken as the fracture permeability. The only difference is that the minimum allowed permeability value was 1 mdarcy and the matrix permeability was smaller than or equal to the fracture permeability in every grid block. The matrix porosity \( \phi_m \) had a constant value of 0.2 and the fracture porosity \( \phi_f \) was 0.02. While the permeability and porosity values are typical of fractured reservoirs [Bear et al., 1993], the geological model is, however, based on an unfractured sandstone. There was one injection well (with a constant flow rate of 300 m\(^3\) d\(^{-1}\)) at the center of the reservoir, and there were four extraction wells at the four corners (each with a constant pressure of 27,578.8 kPa). There was no flow across the model boundaries. Additional parameters for the simulations are given in Table 2.

[42] Before showing the simulation results, we will consider typical values of the matrix/fracture transfer rate \( \beta \). Considering a matrix permeability of 1 mdarcy and an interfacial tension of 0.05 N m\(^{-1}\), equations (29) and (33) give \( \beta = 3.8 \times 10^{-6} \alpha/L_c^2 \text{ s}^{-1} \). Typical fracture spacings \( L_c \) at the field scale are around 1 m. Setting \( L_c \geq 1 \) m and for a water-wet system (\( \alpha = 0.05 \)), \( \beta = 1.9 \times 10^{-7} \text{ s}^{-1} \). However, most oil reservoirs are not strongly water-wet and the fracture spacing can be larger. Furthermore matrix permeabilities of 0.1 mdarcy or lower are not unusual. For mixed-wet systems imbibition is much slower and \( \alpha \) may be in the range \( 10^{-5} \text{ to } 10^{-4} \) [Zhou et al., 2000]. Thus a realistic range of \( \beta \) is \( 10^{-11} \text{ to } 10^{-7} \text{ s}^{-1} \). In our simulations one pore volume of water is injected in 8 \( \times \) 10\(^8\) s (9000 days). The \( \beta t \) represents a dimensionless measure of transfer; when \( \beta t \approx 1 \) there will be appreciable imbibition where the fracture has been flooded with water.

[43] Figure 8 shows the water saturation maps obtained using our linear transfer function, at three different time steps; the left column shows the saturation maps, while the right column displays the saturation in the matrix. A transfer rate of 5 \( \times \) 10\(^{-8}\) s\(^{-1}\) was chosen to reproduce a relatively rapid matrix/fracture fluid exchange versus time. After one pore volume of water is injected, \( \beta t = 40 \), indicating significant transfer. The effect of changing the transfer rate is discussed later. As shown in Figure 8, water channels through the high-permeability regions; the areas with no flow are related to very small values of fracture permeability, and hence no transfer of water to the matrix is possible here (see the white regions in Figure 8, left column, and the corresponding lightly shaded ones in the right column). Figure 9 shows the oil recovery curves. The high transfer rate chosen for the current flow simulation gives fast imbibition of water and displacement of the oil out of the matrix. Notice the almost linear relationship between the amount of water imbibed and time at late time. This behavior has been observed for experiments of water infiltration in fractured systems and is called the “filling-fracture” regime [Rangel-German and Kovscek, 2002]. In addition, Rangel-German and Kovscek [2002] derived this scaling analytically using an approach similar to that used to obtain equation (51).
A different fracture flow regime is observed when the matrix/fracture transfer rate is only $5 \times 10^{-8} \text{ s}^{-1}$. In this case, the water advance is primarily through fractures. Water propagates quickly, moving toward the producers at the four corners of the domain, causing an early breakthrough. Little water imbibes into the matrix at any time, so the corresponding oil recovery (Figure 9) is low. The behavior of this second regime is similar to that observed experimentally and has been called the “instantly filled fracture” regime [Rangel-German and Kovscek, 2002] because the time to fill the fractures is much less than the imbibition time.

Simulation results using Kazemi et al.’s transfer function are shown in Figure 10, using the same parameters as before. The volume of water imbibed into the matrix is always less than or equal to the mass of water transferred into the matrix when using the linear transfer function, as mentioned in section 4.4. However, the more diffusive water front (see the 1-D simulations in Figures 4 and 5) mitigates the mobility contrast between oil and water, leading to a greater sweep of the reservoir. This effect, evident by comparing Figures 8 and 10, will tend to increase recovery. The corresponding oil recovery curve (see the dashed line in Figure 9) is below the line obtained using the linear transfer function, and the difference between the recoveries increases with time as water imbibition into the matrix becomes the only process responsible for the oil recovery. When $\beta$ is $5 \times 10^{-10} \text{ s}^{-1}$, the two dotted curves are superimposed since there is almost no transfer from fractures to matrix in both cases.

Simulation results using Kazemi et al.’s transfer function are shown in Figure 10, using the same parameters as before. The volume of water imbibed into the matrix is always less than or equal to the mass of water transferred into the matrix when using the linear transfer function, as mentioned in section 4.4. However, the more diffusive water front (see the 1-D simulations in Figures 4 and 5) mitigates the mobility contrast between oil and water, leading to a greater sweep of the reservoir. This effect, evident by comparing Figures 8 and 10, will tend to increase recovery. The corresponding oil recovery curve (see the dashed line in Figure 9) is below the line obtained using the linear transfer function, and the difference between the recoveries increases with time as water imbibition into the matrix becomes the only process responsible for the oil recovery. This shows that the reduced transfer rate dominates the recovery. When $\beta$ is $5 \times 10^{-10} \text{ s}^{-1}$, the two dotted curves are superimposed since there is almost no transfer from fractures to matrix in both cases.

To investigate the computational efficiency of our approach, we first performed single-porosity simulations on the same reservoir model with all properties the same as the 10th SPE Comparative Solution project [Christie and Blunt, 2001]. As well as the fine grid model, we generated a series of coarser grids: $20 \times 55 \times 17$, $20 \times 55 \times 85$, $60 \times 55 \times 85$. 

Figure 8. Fracture (left column) and matrix (right column) saturations at the 53rd layer of the reservoir obtained using our linear transfer function with a transfer rate $\beta = 5 \times 10^{-8} \text{ s}^{-1}$ after (a) 1000 days, (b) 4500 days, and (c) 9000 days. Notice that the water rapidly saturates the high-permeability fractured regions, while the saturation in the matrix is controlled by the transfer rate.

Figure 9. Comparison of oil recoveries for different values of the transfer rate $\beta$ for the two transfer functions studied. For low values of $\beta$, essentially only water in the fractures is recovered and recovery is low. For higher values of $\beta$ there is rapid matrix/fracture transfer and good recoveries are achieved. The Kazemi et al. model gives slightly poorer recovery than the linear model for the higher transfer rate. Despite the improved sweep efficiency (see Figure 10), breakthrough occurs earlier using the Kazemi et al. model due to an advancing water front of low saturation; see Figure 4. Furthermore, the imbibition rate into the matrix is slower than for the linear model.

Figure 10. Fracture (left column) and matrix (right column) saturations at the 53rd layer of the reservoir obtained using the Kazemi et al.’s transfer function with a transfer rate $\beta = 5 \times 10^{-8} \text{ s}^{-1}$ after (a) 1000 days, (b) 4500 days, and (c) 9000 days. Notice that the higher sweep efficiency of the Kazemi et al. model compared with the linear model (Figure 8), while the water saturation in the matrix is lower at each time step since the transfer rate is slower.
Permeability was upsealved using geometric averaging. We also generated a finer model $120 \times 440 \times 85$ (4,488,000 grid blocks) by downscaling the permeability [Huang et al., 2004]. We will not show the results here, as they were identical to those obtained in the 10th SPE Comparative solution project. We then ran the streamline-based dual-porosity model for the different grids using the well rates as before. For both the single- and dual-porosity streamline runs we used approximately 20 time steps to the end of the simulation. Figure 11 shows comparisons of the run times for the various simulations. All the times are for a 3 GB RAM, 2.4 GHz Pentium Xeon workstation. For both single- and dual-porosity models the run times scale approximately linearly with the number of grid blocks, indicating optimal computational efficiency. The dual-porosity model is slower because of the more complex 1-D transport solver.

6. Conclusions

[47] We have presented the extension of streamline-based simulation to two new areas: contaminant transport with rate-limited sorption and multiphase flow in fractures using a dual-porosity formulation. We have demonstrated the mathematical similarity between the formulations of the two different transport processes. Transfer between flowing and stagnant regions is accommodated through source/sink terms in the 1-D transport equations along a streamline.

[48] We tested the simulator on a highly heterogeneous million-grid-block model of a North Sea reservoir. We showed that the simulation time scales approximately linearly with the number of grid blocks used, allowing transport to be computed in very finely gridded domains.

[49] We analyzed the non-Gaussian behavior of field-scale tracer transport. At late time the breakthrough curves showed a power law tail consistent with anomalous transport. The same power law exponent was observed for simulations of a passive tracer and sorbing tracer with and without a finite sorption rate. We suggest that using a streamline-based approach, a purely advective transport model allows efficient analysis of the contaminant flow in heterogeneous media at the large scale, which may not be possible using classical grid-based advection-dispersion models.

[50] We developed a simple expression for the matrix/fracture transfer function based on experimental measurements of imbibition in water-wet matrix blocks. The transfer rate is a linear function of matrix saturation and matches core-scale experiments. We compared our model with the empirical transfer function obtained by Kazemi et al. [1992] and implemented both formulations in our streamline simulator.

[51] For high transfer rates, typical of high matrix permeability, dense fractures, and water-wet media, simulation results show that the amount of water imbibed scales almost linearly with time and high recoveries can be achieved. This behavior has been recognized as the “filling-fracture” regime identified in air-water experiments on fractured systems [Rangel-German and Kosscek, 2002].

[52] For low transfer rates, typical of mixed-wet systems, the water front moves faster toward the producers and a smaller quantity of water is transferred into the matrix. The amount of oil recovered is very low at all times. This corresponds to the “instantly filled fracture” regime [Rangel-German and Kosscek, 2002]. Kazemi et al.’s transfer function gave poorer recoveries than our linear model and rapid breakthrough of water in the fractures, because it predicts a lower transfer rate.

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References


M. J. Blunt and G. Di Donato, Department of Earth Science and Engineering, Imperial College London, Prince Consort Road, London SW7 2AZ, UK. (g.didonato@imperial.ac.uk)