Answer THREE questions.
All questions carry equal marks.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.
1. (i) Describe briefly the simplifying assumptions made in the scalar wave approximation. Explain why the scalar wave approximation would be inappropriate for describing light propagation in Calcite. [2 marks]

(ii) Starting from Maxwell’s equations in vacuum (\( \mathbf{J} = 0, \rho = 0 \)) derive a wave equation describing a generalised electromagnetic wave. Why do we generally consider the \( \mathbf{E} \) rather than the \( \mathbf{B} \) component of the field when considering how an EM wave interacts with matter? Under what conditions might the \( \mathbf{B} \) term dominate over the \( \mathbf{E} \) term? [5 marks]

(iii) Write down an expression for the Poynting Vector \( \mathbf{S} \) and give brief descriptions of the physical quantities that \( \mathbf{S} \) and its time average over a single optical cycle \( \langle \mathbf{S} \rangle \) represent. [3 marks]

(iv) Light from a coherent, single wavelength 1kW point source expands freely into a uniform spherical volume. Explain with reference to conservation of energy why the electric field amplitude falls off with radial distance \( r \) as \( 1/r \) rather than \( 1/r^2 \). By considering the Poynting vector or otherwise find (a) the energy density and (b) the peak electric field strength at a distance of 1m from the point source. [5 marks]

(v) A device designed to lift mass into orbit, driven purely by photon momentum transfer is proposed. The “lifter” device comprises a horizontal 100% reflective 10\( m^2 \) mirror and payload with a total mass of \( 10^3 \) kg. The lifter mirror is illuminated from underneath by a ground based CW laser beam which makes a single bounce perpendicularly off the mirror. Estimate the minimum optical power required for the drive laser to just overcome the force of gravity. Comment briefly on the feasibility of constructing such a device. [5 marks]

[Total 20 marks]
2. (i) Define the terms “point spread function” and “far field pattern” and explain how the far field can be produced simply in the laboratory. A HeNe laser beam passes through a 1mm diameter circular aperture. At what distance from the aperture would a freely propagating beam be in the far field? 

(ii) A pair of parallel slits of width 2b are spaced a distance 2a apart to form an aperture in a screen, and uniformly illuminated by plane waves of wavelength \( \lambda_o \) and amplitude \( U_o \). Derive simple expressions for the amplitude and intensity of the Fraunhofer diffraction pattern produced perpendicular to the slits.

(iii) An instrument requires a single diffractive optical element of length 2L that when illuminated by a well collimated monochromatic beam of light produces a line of 5 equally spaced spots of light on a screen placed 1m down stream, with the central spot at the \( k_x = 0 \) point in the Fourier plane. The spacing between the spots on the screen must be 10mm, all spots must have equal amplitude and individual spots may be decorated by a sinc\(^2\) intensity profile. Assuming that the screen is in the far field, find a suitable transmission function \( T(\xi) \) for the device.

[Total 20 marks]
3. (i) When describing short duration optical pulses, what does the term “transform limited” refer to? Discuss briefly whether it possible for a real light pulse to be less than or more than transform limited. [3 marks]

(ii) A very narrow band transform limited cw laser beam passes through a moving “chopper wheel” which slices it into short sections. Explain why the chopped beam must contain new frequencies and explain briefly how these are generated. [3 marks]

(iii) By examining the Fourier transform of a wave train $E_0 \cos(\omega_0 t)$ of finite length ($\tau_c$) show that $\tau_c \approx \frac{1}{\Delta \nu}$ where $\Delta \nu$ is the bandwidth of the light source. [10 marks]

(iv) Estimate the bandwidth $\Delta \lambda$ in nm required to support a 5fs pulse centred at 800nm. Comment on this value with respect to the extent of the visible spectrum and describe the likely “colour” of such a light pulse. [4 marks]

[Total 20 marks]
4. (i) Explain briefly why Nd doped glass is preferable to Ruby as a gain medium for high energy flashlamp pumped laser systems. [2 marks]

(ii) What does the term “relaxation oscillation” refer to? Explain the benefits of Q-switching and describe how it could be implemented in a linear cavity solid-state laser system. Illustrate your answer with a simple sketch of a suitable cavity, taking care to identify all the necessary optical components. [4 marks]

(iii) A simple 0.5 m long laser cavity with mirror reflectivities $R_1 = 1.0$ and $R_2 = 0.98$ contains a high quality Nd:Glass rod with a length of 10cm and diameter of 1cm. Find the threshold gain $\alpha_T$ of the system and the duration of the output pulse $\tau_o$ when the system is Q-Switched. Estimate the output duration at the $2^{nd}$ harmonic if the laser was frequency doubled in a KDP crystal. [4 marks]

(iv) The laser rod of the system described in part (iii) is doped with $Nd^{3+}$ ions at a number density of $5 \times 10^{24} m^{-3}$ and pumped such that all of the lasant ions are promoted to the upper laser level.

(a) Determine the energy density available in the laser medium.
(b) Estimate the energy and peak power of the Q-switched pulse. [4 marks]

(v) High energy Nd:Glass laser systems such as the NIF are typically built using a series of amplifiers of increasing aperture. Briefly explain the reasoning behind this fact. Sketch a design for a system capable of amplifying a 10ns, 10mJ seed pulse to the 1 MJ ($10^6 J$) level assuming a single pass energy gain of x10 per stage and maximum area of 1$m^2$ for individual optical elements. You may ignore non-linear effects and should provide explanations for your choice of system geometry, number of gain stages and key limiting physical processes. [6 marks]

[Total 20 marks]
5. (i) What do we mean by the term “wavefront”. Describe briefly how linearly polarised light interacts with and propagates through a homogeneous, transparent medium at low intensities. [4 marks]

(ii) Give simple expressions for the group velocity $V_g$ and phase velocity $V_p$ and briefly explain what these two terms represent, with reference to a short pulse of light. What do the terms normal dispersion and anomalous dispersion refer to? [4 marks]

(iii) Write down an expression for the polarisation $P(t)$ of a medium illuminated by a light field as a power series expansion in terms of the applied electric field $E(t)$ and give an estimate of the ratios of the magnitudes of the coefficients $\chi_1: \chi_2: \chi_3$. [2 marks]

(iv) A medium with a large $\chi_3$ value is driven by an optical field with an amplitude $E(t) = E_0 \cos(\omega t)$. Find an expression for the component of the polarisation $P(t)$ arising from the $\chi_3$ term and briefly describe the two main physical processes that arise from this term. Explain why homogeneous materials typically limit the efficiency of frequency multiplication processes and describe a practical method to eliminate phase mismatch. [5 marks]

(v) The design of a white light continuum generation source requires that a well collimated 20 fs light pulse from a Ti:Sapphire laser undergo whole beam self-focusing in a slab of quartz 1 mm thick with an $n_2$ value of $4 \times 10^{-16} cm^2/W$. The laser pulse has a flat-top spatial profile of area $0.01 cm^2$. Use the B integral to determine the minimum pulse energy required for self-focusing. [5 marks]

[Total 20 marks]
6. Write notes on THREE of the following. Use sketches and mathematical expressions to illustrate your notes where appropriate. All section carry equal marks. [20 marks]

(i) Matrix methods for describing and modelling optical systems.
(ii) The nature of absolute phase in short laser pulses.
(iii) The properties of laser light and methods for pumping gain media.
(iv) Chirped pulse amplification.
(v) Selection and manipulation of the polarisation state of light.
(vi) Interference based measurement techniques.

[Total 20 marks]