Imperial College London
BSc/MSci EXAMINATION May 2008

This paper is also taken for the relevant Examination for the Associateship

PLASMA PHYSICS

For Third- and Fourth-Year Physics Students
Wednesday, 28th May 2008: 10:00 to 12:00

Answer THREE questions.
All questions carry equal marks.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of
each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corre-
sponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, ac-
curacy and clarity of expression.
1. (i) For the fusion reaction: \( ^1_2 D + ^3_1 T \rightarrow ^4_2 He + n \)
   Calculate the kinetic energy of each of the reaction products in MeV ? [3 marks]

   (ii) Derive the Lawson criterion and evaluate it for this reaction at 8keV assuming an efficiency of 1/3. [5 marks]

   (iii) For the ITER Tokamak the proposed parameters are a volume of approximately 800 m\(^3\), an ion number density of 10\(^{20}\) m\(^{-3}\), and a temperature of 8keV. What is anticipated output power from the neutron flux ? [3 marks]

   (iv) If the anticipated energy confinement time for ITER is 3.7 seconds, calculate whether the alpha particle heating will be sufficient to maintain a constant temperature. [4 marks]

   (v) Briefly describe three different processes that will be used, during the initial stages of the pulse on ITER, to heat the plasma up to the point where fusion reactions start to occur. [5 marks]

[Total 20 marks]

Charge on the electron = \(-1.6 \times 10^{-19}\) Coulomb

Rest masses in a.m.u. (1 a.m.u. = 1.66 \times 10^{-27} kg)
\( ^4_2 He, 4.0015, n, 1.0087, ^2_1 D, 2.0136, ^3_1 T, 3.0155 \)

\(< \sigma v > \) for \( ^2_1 D + ^3_1 T \rightarrow ^4_2 He + n \) at 8keV = \( 1.1 \times 10^{-22} \) m\(^3\)s\(^{-1}\)
2. (i) The drift velocity of charged particles resulting from a uniform magnetic field and an arbitrary force \( \mathbf{F} \) is given by:

\[
\mathbf{v}_D = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}
\]

Use this formula to derive an expression for the drift velocity due to magnetic field curvature, stating any assumptions made. [3 marks]

(ii) A Z-pinch is formed from a plasma cylinder of length 1cm and radius 1mm. A voltage of 1MV is used to drive a current of 1MA flowing between an anode and cathode. Calculate the magnitude and direction of the \( \mathbf{E} \times \mathbf{B} \) and magnetic curvature drifts for deuterons with energy 100eV, at the plasma surface. Assume \( v_\parallel = v_\perp \). [7 marks]

(iii) An electron in the Van-Allen radiation belts, starts at the equator with an energy of 5 keV and with equal \( v_\perp \) and \( v_\parallel \), but then moves towards one of the Poles and experiences an increasing magnetic field strength. Which two important properties of the electron’s motion are conserved? When the magnetic field reaches twice the value at the equator what are the magnitudes of the electron’s parallel and perpendicular velocities. What is happening to the electron? [6 marks]

(iv) If \( v_\parallel \) and \( v_\perp \) are the velocity components of the electron, parallel and perpendicular to the magnetic field, at the equator, derive a condition for the maximum \( v_\parallel \) that the electron can have and still remain confined in the Van-Allen belts, in terms of \( v_\perp \), \( B_E \) and \( B_M \) (where \( B_E \) is the magnetic field near the equator and \( B_M \) is the maximum magnetic field experienced by the electron). What observable phenomenon do electrons with very large \( v_\parallel \) contribute to? [4 marks]

\[
\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}
\]
Electron Charge = \(-1.6 \times 10^{-19} \) Coulomb
mass of electron = \( 9.11 \times 10^{-31} \) kg

[Total 20 marks]
3. (i) The mean free path for electrons undergoing Rutherford scattering by ions is given by:
\[
\lambda_e = \frac{\pi}{32n_i b_0^2 \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)}
\]
Describe the physical significance of \(b_0\), \(b_{\text{min}}\), \(b_{\text{max}}\), and \(\lambda_e\). [4 marks]

(ii) If the average time between collisions is given by:
\[
\tau_e = 3.44 \times 10^{11} T^{3/2} \frac{T^{3/2}}{Z^2 n_i \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)}
\]
show that the resistivity of the plasma is given by
\[
\eta = 1.03 \times 10^{-4} Z T^{3/2} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \Omega m
\]
[4 marks]

(iii) In a deuterium Z-pinch plasma with a \(\ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)\) value of 10, the dominant radiation loss is due to bremsstahlung with an output of \(1.69 \times 10^{-38} Z^2 n_e n_i T^{1/2}\) Watts per m\(^3\). Derive an expression for the Pease-Braginskii current and evaluate it for this plasma. Explain what can happen to a Z-pinch plasma if the current is much larger than the Pease-Braginskii value. [7 marks]

(iv) Briefly describe the physical processes involved in bremsstahlung, recombination radiation and line emission from plasmas. Describe how the emission spectrum from a plasma changes with increasing opacity. [5 marks]

[Total 20 marks]

\[\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}\]
Electron Charge = \(-1.6 \times 10^{-19}\) Coulomb
mass of electron = \(9.11 \times 10^{-31}\) kg
Note that the Bennett relation is \(\mu_0 I^2 = 8\pi(Z + 1)NkT\)
4. Write short notes on THREE of the following topics (using equations and diagrams to illustrate). All parts carry equal marks.

(i) The magnetic Reynolds number

(ii) Debye Shielding

(iii) The $\rho R$ condition for inertial confinement fusion

(iv) Dusty plasmas and plasma crystals

(v) Tokamak divertors

(vi) Moments of the distribution function

[Total 0 marks]
5. (i) Show how the $j \wedge B$ force used in the MHD momentum equation can also be written in terms of magnetic pressure and tension forces. [3 marks]

(ii) Briefly describe how magnetic pressure can drive a longitudinal magneto-sonic wave. [3 marks]

(iii) Describe two different modes of MHD instability in a Z-pinch plasma and show how these instabilities can be stabilised by the introduction of a further magnetic field component. [5 marks]

(iv) For the ITER Tokamak the proposed parameters are a major radius of 6.2 m, a minor radius of 2.0 m, an ion number density of $10^{20} \text{ m}^{-3}$ and a temperature of 8 keV. What is the approximate total thermal energy of the plasma?. [2 marks]

(v) What is meant by the plasma Beta? If Beta = 0.025 for ITER, estimate the total energy of the magnetic fields. [2 marks]

(vi) What is meant by the safety factor for a Tokamak? If the safety factor = 1.3 for ITER, estimate the magnitude of the poloidal and toroidal magnetic fields and the toroidal current. [5 marks]

[Total 20 marks]

$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$

Electron Charge = $-1.6 \times 10^{-19}$ Coulomb

Integration by parts formula $\int u \, dv = uv - \int v \, du$

Note that $\nabla (a \cdot b) = a \wedge (\nabla \wedge b) + b \wedge (\nabla \wedge a) + (a \cdot \nabla)b + (b \cdot \nabla)a$
6. (i) If the electrons and ions in a plasma become separated by a distance $x$, an electric field $E$ results. Describe the subsequent motion of the electrons and derive an expression for the plasma frequency $\omega_p$. [3 marks]

(ii) A full derivation of the motion based upon the distribution function $f$ for the electrons, yields an equation of the form:

$$\omega = \omega_p + \frac{\omega_p^2}{\omega^2} \frac{3kT}{m} K^2 - \frac{\omega_p^2}{K^2n} \omega \frac{\partial f_0}{\partial v} \bigg|_{v=\omega/k}$$

where $K$ is the wave number. Briefly explain the physical significance of the three terms on the right hand side of this equation. [3 marks]

(iii) Describe how electrons can be accelerated by a Langmuir wave (or electron plasma wave). What determines the range of velocities of the electrons which experience acceleration? Describe how a Maxwellian electron distribution function is changed by this effect. Why is this process self-limiting? [6 marks]

(iv) The dispersion relation for electro-magnetic waves propagating through plasma has the form $\omega^2 = \omega_p^2 + K^2 c^2$. Draw a graph of $\omega$ versus $K$. Use this to illustrate what is happening to the group velocity and the phase velocity of the wave as $\omega \rightarrow \omega_p$. What is the refractive index of the plasma in this limit? [4 marks]

(v) Use simple diagrams to show what happens to an intense laser pulse as it strikes a plasma formed from the ablation of a solid target, at oblique incidence. If the laser has a wavelength of 532nm, estimate the electron density where most of the laser energy will be absorbed. [4 marks]

[Total 20 marks]