Imperial College London
BSc/MSci EXAMINATION May 2008

This paper is also taken for the relevant Examination for the Associateship

OPTICAL COMMUNICATIONS PHYSICS

For Third- and Fourth-Year Physics Students
Thursday, 22nd May 2008: 10:00 to 12:00

Answer ALL parts of Section A and TWO questions from Section B.
All questions carry equal marks.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.
SECTION A

1. (i) A data set consists of four symbols \{A, B, C, D\} occurring with probability \{0.5, 0.3, 0.15, 0.05\} respectively.

(a) How many bits per symbol would be required to transmit this data stream in a simple fixed length binary coded format? [1 mark]

(b) Calculate the entropy in the data set and hence the coding efficiency resulting from transmitting the data in this format. [2 marks]

A Huffman code can be derived for the data set that consists of the binary symbols \{0, 10, 110, 111\} respectively.

(c) What bit-rate is required to transmit the Huffman encoded symbols and what is the achieved coding efficiency? [2 marks]

(ii) A digital data-stream has a bit-rate of 1Gb/sec.

(a) Estimate the electrical bandwidth required to transmit the data and sketch the frequency content of the data-stream on a graph of electrical power versus frequency. [2 marks]

The digital signal is used to amplitude modulate an optical carrier at \(\lambda = 1.55\mu m\).

(b) On the same axes illustrate the spectrum of the resulting modulated optical signal. [1 mark]

(c) What are the central frequency and bandwidth of the transmitted optical signal? [2 marks]

(iii) A step index optical fibre is manufactured with a core index of \(n_{co} = 1.52\) and cladding index of \(n_{cl} = 1.50\).

(a) Draw a sketch showing the path of a paraxial ray travelling inside the fibre at the critical angle and calculate that critical angle. [2 marks]

(b) Draw the ray leaving the fibre at a normal cleaved end and calculate the angle at which the ray travels in air. [2 marks]

(c) What is the numerical aperture of the fibre? [1 mark]

(iv) A semiconductor DFB laser is fabricated with an active region of \(\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}\) with an energy gap \(E_g = 0.8eV\).

(a) Estimate the emission wavelength of the laser. [1 mark]

(b) The total efficiency of the laser is measured as 25%. Estimate the drive current required to give an output power of +10dBm. [You may assume a 0.8V forward bias voltage drop across the laser diode] [2 marks]

(c) The effective index of the laser mode is \(n_e = 3.4\). Estimate the pitch of the first order internal Bragg grating required to give single longitudinal mode operation. [2 marks]

[Total 20 marks]
2. The 1080p high definition television (HDTV) format prescribes the transmission of 1920x1080 pixel, full colour images at a rate of 60 frames per second.

(i) Calculate the uncompressed data rate required assuming a 256-level analog to digital encoder is used to digitise a red, a green and a blue image value at each pixel. [3 marks]

(ii) Explain the concepts of inter-symbol influence, conditional entropy and lossless compression. Describe briefly one technique for compressing image data without loss of information. How well would you expect this technique to perform with typical broadcast television data? [7 marks]

(iii) Explain the concepts behind perceptual coding techniques, in particular with reference to the way that JPEG coding can be used to compress single image data. Estimate the data rate you might expect from transmitting the video stream as JPEG images. [7 marks]

(iv) Explain how MPEG encoding uses both inter-symbol influence and perceptual coding to compress video streams. A typical digital HDTV signal is broadcast at a data rate of about 20Mb/s using MPEG-2 encoding. What is the achieved coding efficiency? [3 marks]

[Total 20 marks]
3. A single voice telephone signal can be carried by a standard 64 kb/s DS-0 digital data channel.

(i) Explain how the analogue voice signal is digitised. You should include in your description a discussion of:

(a) The analogue bandwidth of the voice signal.
(b) The digital sampling rate used.
(c) The number of discrete analogue levels converted in the digitisation process. [6 marks]

(ii) What analogue channel bandwidth and signal to noise ratio would be required to transmit the analogue voice signal with similar fidelity to the digital signal? [2 marks]

The Shannon-Hartley information theorem states that the channel capacity, \( C \), is:

\[
C = B \log_2(1 + S/N)
\]

where \( B \) is the analogue bandwidth of the channel and \( S/N \) is the power signal to noise ratio at the channel detector.

(iii) Calculate the capacity of the channel you described in part (ii) and compare this to the data-rate in the digitised DS-0 channel. [3 marks]

(iv) Describe how individual voice channels might be combined and transmitted over large distances through a network from London to Tokyo. [6 marks]

(v) Why is wavelength division multiplexing considered more practical for implementation in optical fibre systems than time division multiplexing? [3 marks]

[Total 20 marks]
4. (i) Describe the main causes of dispersion in:
   (a) Multi-mode optical fibres.
   (b) Mono-mode optical fibres. [5 marks]

An optical fibre link is constructed with a Fabry-Perot laser diode source of bandwidth $\Delta \lambda = 2\text{nm}$ operating at $\lambda = 1.55\mu\text{m}$ and a mono-mode optical fibre with dispersion $D_1 = +20\text{ps/nm/km}$.

(ii) What is the significance of the sign in the dispersion parameter $D_1$? [1 mark]

(iii) What is the maximum dispersion-limited bit-rate that can be transmitted over 50km using these components? [4 marks]

A second batch of fibre is available with dispersion of $D_2 = -60\text{ps/nm/km}$.

(iv) Describe how this second fibre could be used in conjunction with the first to construct a dispersion-free link over a distance of 100km. [4 marks]

As an alternative, a narrow linewidth DFB laser and external modulator are used as the source in the link.

(v) By considering the wavelength broadening that the modulation induces on the optical carrier, show that the dispersion-limited bit-rate $B$ of a link over a length $L$ of the original fibre is given by

$$B^2 < \frac{c}{D_1\lambda^2L}$$

and hence calculate the dispersion-limited bit-rate of a 100km link. [6 marks]

[Total 20 marks]
5. (i) Briefly describe the physical origin and characteristics of:

(a) shot noise
(b) thermal noise

in optical receiver circuits. [4 marks]

Consider an ideal shot-noise-limited detector operating in a communication link with an expected number $\mu$ of photons detected in each 1-bit and zero photons detected in each 0-bit.

(ii) Compare the characteristics of the noise at the receiver in 0-bits and in 1-bits. What is the optimal decision threshold? [2 marks]

(iii) What would the be the probability of a transmission error occurring in 0-bits and in 1-bits? [4 marks]

(iv) What value of $\mu$ is required to achieve a bit error rate, $\text{BER} < 10^{-9}$? [2 marks]

Consider a simple thermal-noise-limited p-i-n diode based receiver with a noise current level of $\sigma A_{rms}$ and an expected signal current level of 0A in each 0-bit and $\langle I \rangle A$ in each 1-bit.

(v) Compare the characteristics of the noise at the receiver in 0-bits and in 1-bits. What is the optimal decision threshold? [2 marks]

(vi) Show that the probability of an error occurring in a 0-bit is given by:

$$P(1/0) = \frac{1}{2} \text{erfc}\left(\frac{\langle I \rangle}{2\sqrt{2}\sigma}\right)$$

[6 marks]

[Total 20 marks]

You may assume:

The discrete Poisson probability distribution for a mean $\mu$:

$$P(n) = \frac{\mu^n \exp(-\mu)}{n!}$$

The continuous Gaussian probability distribution for a mean $\langle x \rangle$ and standard deviation $\sigma$:

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \langle x \rangle)^2}{2\sigma^2}\right)$$

The complementary error function:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt$$