Securitizing and Tranching Longevity Exposures

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AGENDA

1. Motivation
2. The model
3. Optimal securitization
4. Optimal tranching
5. Aggregation of exposures
6. Conclusions
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MOTIVATION

Objectives

★ provide theoretical framework guiding the development of a securitization market in longevity risk
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  ○ general equilibrium perspective
  ○ asymmetric information in longevity trends

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★ provide theoretical framework guiding the development of a securitization market in longevity risk
  ○ general equilibrium perspective
  ○ asymmetric information in longevity trends

Answers to the following questions

★ how to optimally securitize longevity exposures?
★ how to optimally design longevity-linked securities?
★ what happens when longevity exposures are aggregated?
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THE MODEL

Transactions:
THE MODEL

Transactions:

informed holders of exposures
THE MODEL

Transactions:

informed holders of exposures → uninformed investors
THE MODEL

Transactions:

informed holders of exposures → uninformed investors

informed intermediary → uninformed investors
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ASYMMETRIC INFORMATION

Stylized longevity exposure

★ death rate $D$
ASYMMETRIC INFORMATION

Stylized longevity exposure

- death rate $D$

Representation

$$D = q(X) + \varepsilon$$
ASYMMETRIC INFORMATION

Stylized longevity exposure
  ★ death rate $D$

Representation

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Information
  ★ uninformed agents have no access to $X$
  ★ informed agents have access to $X$
ASYMMETRIC INFORMATION

Stylized longevity exposure
  ★ death rate $D$

Representation

$$D = q(X) + \varepsilon$$

Information
  ★ uninformed agents have no access to $X$
  ★ informed agents have access to $X$

$$q(X) \in [q_{\text{min}}, q_{\text{max}}]$$
LONGEVITY TRENDS: CMI scenarios
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YEAR
DEATH PROBABILITY FORECAST
0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02 0.022 0.024
0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02 0.022 0.024

q_max
q_min

age 75
age 70
age 65
LONGEVITY TRENDS: CMI scenarios

![Graph showing death probability forecasts for ages 65, 70, and 75 from 2008 to 2030.]

- **Motivation**: The model
- **Optimal securitization**
- **Optimal tranching**
- **Aggregation of exposures**
- **Conclusions**
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TRANSACTION TIMELINES

Risk-neutral agents, interest rates normalized to zero

- \( \delta \in (0, 1) \) discount factor of the holder of the exposure
- capital requirements, incentive to trade
- retention of part of the exposure is a credible signal
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Given \( X \), the issuer computes \( q(X) \)
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given $X$, the issuer computes $q(X)$
given $q(X)$, the issuer transfers $\gamma \in [0, 1]$
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\[ \text{given } X, \text{ the issuer computes } q(X) \]
\[ \text{given } q(X), \text{ the issuer transfers } \gamma \in [0, 1] \]

\[ \text{given } \gamma, \text{ investors try to infer } q(X) \text{ and offer } P(\gamma) \]
TRANSACTION TIMELINES

Risk-neutral agents, interest rates normalized to zero

★ \( \delta \in (0, 1) \) discount factor of the holder of the exposure

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Resulting equilibrium

★ optimal securitization fraction \( \gamma^* \): depends on \( q(X)/q_{\text{min}}, \delta \)

★ equilibrium market demand \( P^*(\gamma) \): depends on \( q_{\text{min}}, \delta \)
OPTIMAL SECURITIZATION I

Supplying longevity exposures to the market

PRIVATE VALUATION $q(X)$

SECURITIZATION PAYOFF

SECURITIZATION FRACTION

$0.00037$ $0.00086$ $0.00132$ $0.00176$ $0.00233$

$0$ $0.023$ $0.024$ $0.025$ $0.026$ $0.027$ $0.028$ $0.029$ $0.03$ $0.031$
OPTIMAL SECURITIZATION I

Supplying longevity exposures to the market
OPTIMAL SECURITIZATION

Supplying longevity exposures to the market

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SECURITIZATION PAYOFF

SECURITIZATION FRACTION

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0.00086
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Market demand for longevity exposures

\[ P(\gamma) \]
OPTIMAL SECURITIZATION II

Market demand for longevity exposures

\[ P(\gamma) \]

\[ P_1 \]

\[ \gamma_1 \]
OPTIMAL SECURITIZATION II

Market demand for longevity exposures
OPTIMAL SECURITIZATION III

The role of $q(X)/q^\text{min}$ and $\delta$
OPTIMAL SECURITIZATION III

The role of $q(X)/q^{\min}$ and $\delta$

- The lower $\delta$, the higher the retention costs
- Retention is a more credible signal

Graph with lines $\delta=0.90$ and $\delta=0.95$ showing the relationship between $q(x)/q^{\min}$ and securitization fraction.
OPTIMAL SECURITIZATION III

The role of $q(X)/q_{\text{min}}$ and $\delta$

$q(x)$ high with respect to $q_{\text{min}}$ ⇒ transfer less to the market
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OPTIMAL TRANCING

 Manufacture derivative written on $D$

 ★ contract $C = \phi(D)$, with $\phi(D) \leq D$

 ★ find $C^*$ maximizing the payoff to the holder of the exposure from issuing the contract
OPTIMAL TRANCHEING

Manufacture derivative written on $D$

- contract $C = \phi(D)$, with $\phi(D) \leq D$
- find $C^*$ maximizing the payoff to the holder of the exposure from issuing the contract

In some interesting cases, the optimal contract is an option

$$C^* = \min(q^*, D) = q^* - \max(0, q^* - D)$$
OPTIMAL TRANCHEING

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There is a payoff maximizing floor $q^*$ to the issuer’s exposure
OPTIMAL TRANCHING

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In some interesting cases, the optimal contract is an option

$$C^* = \min(q^*, D) = q^* - \max(0, q^* - D)$$

There is a payoff maximizing floor $q^*$ to the issuer’s exposure

- tranching level, strike level
- at level $q^*$, the two parties are indifferent to obtaining/offering additional protection
THE OPTIMAL DESIGN PROBLEM
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![Graph showing the expected profit to the issuer as a function of the strike level. The graph has a peak at a certain strike level, indicated by q*. The x-axis represents the strike level, ranging from 0.005 to 0.025. The y-axis represents the expected profit to the issuer, ranging from 0 to 1.8 x 10^-3. The peak of the curve is at a strike level q*.](image-url)
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AGGREGATION OF EXPOSURES

Aggregation (pooling) of multiple exposures

* aggregation of exposures destroys info on longevity trends
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★ can diversification benefits outweigh information losses?
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Yes, provided the aggregated exposures are suitably tranching
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Aggregation (pooling) of multiple exposures

- aggregation of exposures destroys info on longevity trends
- can diversification benefits outweigh information losses?

Yes, provided the aggregated exposures are suitably tranched

So long as the contract is optimally designed

- the expected payoff to the issuer is maximized
- the strike increases, because investors are more confident about diversification benefits
INFORMATION LOSS vs. DIVERSIFICATION BENEFITS

DEATH RATE

YEAR

2008 2009 2010 2011 2012 2013

0 0.01 0.02 0.03 0.04

$q_{max}$ $q_{min}$
INFORMATION LOSS vs. DIVERSIFICATION BENEFITS
INFORMATION LOSS vs. DIVERSIFICATION BENEFITS
INFORMATION LOSS vs. DIVERSIFICATION BENEFITS
TRANCHE WITH & WITHOUT POOLING

![Graph showing expected payoff to issuer against strike level, comparing pool and tranche, and tranche separately.]
TRANCHE WITH & WITHOUT POOLING

![Graph showing expected payoff to issuer vs. strike level for pool and tranche separately and for pool and tranche combined. The graph illustrates the payoff differences between pooling and tranche separately approaches.]
DIVERSIFICATION BENEFITS: IMPACT OF POOL SIZE
DIVERSIFICATION BENEFITS: IMPACT OF POOL SIZE
OPTIMAL POOLING OF EXPOSURES

Pooling and tranching works best when

★ info on longevity trends positively correlated across exposures
★ residual risk poorly correlated across exposures
Optimal Pooling of Exposures

Pooling and tranching works best when
- info on longevity trends positively correlated across exposures
- residual risk poorly correlated across exposures
- examples
  - different ages, same population (‘age-bucketing’) 
  - different cohorts, same underwriting characteristics (geographic area, social class, etc.)
POSITIVELY CORRELATED PRIVATE INFORMATION
POSITIVELY CORRELATED PRIVATE INFORMATION
POORLY CORRELATED PRIVATE INFORMATION
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CONCLUSIONS

informed holders of exposures

retain $1 - \gamma_1^*$
transfer $\gamma_1^*$

tranche at $q^*$

uninformed investors

informed intermediary

retain $1 - \gamma_2^*$
transfer $\gamma_2^*$

uninformed holders of exposures

pool $n$ exposures, tranche at $q_n^*$

pool and transfer
CONCLUSIONS

We have shed some light into how supply and demand may equilibrate when longevity exposures are exchanged, emphasizing the role played by

★ retention costs
★ trend uncertainty
★ asymmetric information
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The model is flexible, but could be improved

- by extending it to a dynamic setting
- by introducing learning
- by letting $q_{\text{min}}$ and $\delta$ arise endogenously
THANKS FOR YOUR ATTENTION