SYNCHRONIZED PHASOR MEASUREMENT TECHNIQUES

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Lecture outline:

Evolution of PMUs
  • Development of Phasor Measurement Units
  • Phasor Estimation
  • Off-nominal frequency phasors

Standards
  • Comtrade
  • Synchrophasor

Applications of PMUs
  • Power system state estimation
  • Control with feed-back
  • Adaptive relaying
  • Remedial Action Schemes
History of Wide-Area Measurements

- Wide-area measurements in power systems have been used in EMS functions for a long time. Economic Dispatch, tie line bias control etc. all require wide area measurements.

- However, the birth of modern wide-area measurement systems can be traced back to a very significant event which took place in 1965.

- The state estimators as we know them today were developed following the technical assessments of the causes of the failures in 1965.
• **The Birth of the PMUs**

  • Computer Relaying developments in 1960-70s.
  • Symmetrical Component Distance Relay Development.
  • Significance of positive sequence measurements.
  • Importance of synchronized measurements.

  • Development of first PMUs at Virginia Tech ~ 1982-1992
  • Development funded by AEP, DOE, BPA, and later NYPA
  • First prototype units assembled at Va Tech and installed on the BPA, AEP, NYPA systems.
PHASOR ESTIMATION
• Introduction to phasors

- The starting time defines the phase angle of the phasor.
- This is arbitrary.
- However, differences between phase angles are independent of the starting time.
• Sampling process, Fourier filter for phasors

Input signal

Data samples

\( x_n \)

\( x_{n-1} \)

\( \cdot \)

\( x_1 \)

Cosines

Sines

Sin and cos functions

Phasor

\[ X = \frac{\sqrt{2}}{N} \sum_{k} x_k (\cos k\theta - j \sin k\theta) \]
• Sampling process, Fourier filter for phasors

Fourier filters can also be described as:

• Least-squares on a period
• Cross-correlation with sine and cosine
• Kalman filters (under many circumstances)

• Phasors from fractional cycle:

High speed relaying

\[ X_c - jX_s = \frac{\sqrt{2}}{N} \sum x_k (\cos k\theta - j \sin k\theta) \]

Phasor \( X = (AX_c + BX_s) + j(CX_c + DX_s) \)
Non-recursive phasor calculations

The non-recursive phasor rotates in the forward direction, one sample angle per sample.
Recursive phasor calculations

The recursive phasor remains fixed if the input waveform is constant.
• **Effect of noise on phasor calculations**

  • Harmonics eliminated correctly if Nyquist criterion is satisfied.

  • **Non-harmonic components**

  • Random Noise

![Diagram showing the effect of noise on phasor calculations](image)

- **Circle of uncertainty**
- **Size of circle of uncertainty**
- **True Phasor**
- **Measurement data window**
Motivation for synchronization

By synchronizing the sampling processes for different signals - which may be hundreds of miles apart, it is possible to put their phasors on the same phasor diagram.
• Sources for Synchronization

  • Pulses
  • Radio
  • GOES
  • GPS
• A phasor measurement unit

Except for synchronization, the hardware is the same as that of a digital fault recorder or a digital relay.
Sampling process, Fourier filter for phasors

Input signal

Data samples

\[ x_n \]

\[ x_{n-1} \]

\[ \cdot \]

\[ x_1 \]

\[ \text{cosines} \]

\[ \text{sines} \]

\[ \sin \text{ and cos functions} \]

Sampling clock based on nominal frequency

\[ \text{Phasor } X = \frac{\sqrt{2}}{N} \sum_{k} x_k (\cos k\theta - j \sin k\theta) \]
• Fixed clocks, DFT at off-nominal frequency

• Consider frequency excursions of ± 5 Hz

• The definition of phasor is independent of frequency

\[ X = \left( \frac{X_m}{\sqrt{2}} \right) e^{j\phi} \]
Input signal at off-nominal frequency:

\[ \hat{X} = \frac{\sqrt{2}}{N} \sum_{k} x_k (\cos k\theta - j \sin k\theta) \]
• Fixed clocks, DFT at off-nominal frequency

Using the normal phasor estimation formula with ‘\(x_r\)’ being the first sample, the estimated phasor is:

\[
\hat{X}_r = PX_\varepsilon \, j^{r(\omega - \omega_0)\Delta t} + QX^*\varepsilon \, -j^{r(\omega + \omega_0)\Delta t}
\]

where \(\Delta t\) is the sampling interval, \(\omega\) is the actual signal frequency, and \(\omega_0\) is the nominal frequency. \(P\) and \(Q\) are independent of ‘\(r\)’, and are given below:

\[
P = \frac{\sin \frac{N(\omega - \omega_0)\Delta t}{2}}{N \sin \frac{(\omega - \omega_0)\Delta t}{2}} \varepsilon^{j(N-1)\frac{N(\omega - \omega_0)\Delta t}{2}}
\]

\[
Q = \frac{\sin \frac{N(\omega + \omega_0)\Delta t}{2}}{N \sin \frac{(\omega + \omega_0)\Delta t}{2}} \varepsilon^{-j(N-1)\frac{N(\omega - \omega_0)\Delta t}{2}}
\]
• Fixed clocks, DFT at off-nominal frequency

• At off-nominal frequency constant input, the phasor estimate is no longer constant, but depends upon sample number ‘r’.

• The principal effect is summarized in the ‘P’ term. It shows that the estimated phasor turns at the difference frequency.

• The ‘Q’ term is a minor effect, and has a rotation at the sum frequency.

• For normal frequency excursions, P is almost equal to 1, and Q is almost equal to 0.
• Fixed clocks, DFT at off-nominal frequency

For small deviations in frequency, P is almost 1 and Q is almost 0.

The function P
• Fixed clocks, DFT at off-nominal frequency

For small deviations in frequency, $P$ is almost 1 and $Q$ is almost 0.

The function $Q$
Fixed clocks, DFT at off-nominal frequency

A graphical representation of $X_r$:

Errors have been exaggerated for illustration.

In reality, $Q$ is very small.
**Fixed clocks, DFT at off-nominal frequency**

If a cycle by cycle phasor is estimated at off-nominal frequency, the magnitude and angle will show a ripple at \((\omega + \omega_0)\), and the average angle will show a constant slope corresponding to \((\omega - \omega_0)\).
• Fixed clocks, Symmetrical Components at off-nominal frequency

If the off-nominal frequency input is unbalanced, and has symmetrical components of $X_0$, $X_1$, and $X_2$, the estimated symmetrical components are given by

$$\begin{bmatrix}
\hat{X}_{r0} \\
\hat{X}_{r1} \\
\hat{X}_{r2}
\end{bmatrix} = P \varepsilon \, j\tau(\omega-\omega_0)\Delta t \begin{bmatrix}
X_0 \\
X_1 \\
X_2
\end{bmatrix} + Q \varepsilon \, -j\tau(\omega+\omega_0)\Delta t \begin{bmatrix}
X^*_0 \\
X^*_1 \\
X^*_2
\end{bmatrix}$$

Note that positive sequence creates a ripple in the negative sequence estimate, and vice versa. The zero sequence is not affected by the other components.

Also, more importantly, if the input has no negative sequence then the positive sequence estimate is without the corrupting ripple.
• Fixed clocks, Symmetrical Components at off-nominal frequency

The ripple components of the three phase voltages are equal and 120° apart, and thus cancel in the positive sequence estimate.
Error in positive sequence estimate as a function of per unit negative sequence and frequency deviation.
• **Summary of fixed clock DFT estimation of phasors:**

  • For small frequency deviations, a single phase input with constant magnitude and phase will lead to an estimate having minor error terms.

  • The principal effect is the rotation of the phasor estimate at difference frequency \((\omega - \omega_0)\), and a small ripple component at the sum frequency \((\omega + \omega_0)\).

  • A pure positive sequence input at off-nominal frequency produces a pure positive sequence estimate without the ripple. The positive sequence estimate rotates at the difference frequency.
APPLICATIONS FOR MONITORING, PROTECTION AND CONTROL
Frequency measurement with phasors

Positive sequence voltage at $\omega$

3-phase voltages at $\omega$

$\omega - \omega_0$

d$\theta$/dt

$\theta$/dt
Present practice

Measurements are primarily $P, Q, |E| = [z]$

State is the vector of positive sequence voltages at all network buses $[E]$

Phasor measurement based state estimation offers many advantages as will be seen later.
Since the currents and voltages are linearly related to the state vector, the estimator equations are linear, and no iterations are required. The weighted least square solution is obtained with a constant gain matrix.

\[ [Z] = [A] [E] \]
Incomplete observability estimators:

One of the disadvantages of traditional state estimators is that at the very minimum complete tree of the network must be monitored in order to obtain a state estimate. The phasor based estimators have the advantage that each measurement can stand on its own, and a relatively small number of measurements can be used directly if the application requirements could be met.

For example consider the problem of controlling oscillations between two systems separated by great distance.

In this case, only two measurements would be sufficient to provide a useful feed-back signal.
Incomplete observability estimators:

How many PMUs must be installed?

For complete observability, about 1/3 the number of buses (along with the currents in all the connected lines) in the system need to be monitored.
Incomplete observability estimators:

PMU placement for *incomplete observability* and interpolation of unobserved states:

The unobserved set can be approximated by interpolation from the observed set

\[
[E_{\text{un-observed}}] = [B][E_{\text{neighbors}}]
\]
State estimation with phasor measurements:

Summary:

• Linear estimator
• True simultaneous measurements
• Dynamic monitoring possible

• Complete observability requires PMUs at 1/3 buses
• Incomplete observability possible
• Few measurements become useful for control
ADVANCED CONTROL FUNCTIONS
Present system: model based controls

Controller

Measurements

Controlled Device
ADVANCED CONTROL FUNCTIONS
Phasor based: Feedback based control

Control with feedback

Controller

Measurements

Controlled Device
Example of control with phasor feed-back

Power demand Controller

System A  System B

\[ \delta_A - \delta_B \]

Desired performance

Performance with constant power control law

\text{time}
Example 1: HVDC Controller

- 680 MVAR
- 590 MVAR
- (200+j20) MVA
- 820 MW
- 1640 MW

3 phase fault cleared in 3 cycles
Example 1: HVDC Controller

Control law 1: Constant current, constant voltage on HVDC

Control law 2: Optimal controller

\[ \delta_1(t \text{ (seconds)}) \]

\[ \delta_2(t \text{ (seconds)}) \]
Adaptive Relaying

Definition

Adaptive protection is a protection philosophy which permits and seeks to make adjustments in various protection functions automatically in order to make them more attuned to prevailing power system conditions.
• Adaptive out-of-step relaying
• Adaptive out-of-step relaying

![Diagram showing adaptive out-of-step relaying with PMUs and variables P, δ, t, observe, predict.](image-url)
Adaptive Relaying

Controlled Security & Dependability

System State

Logic Arbitration

Protection No 1
Protection No 2
Protection No 3

Or
And
Vote

To Circuit Breakers
FUTURE PROSPECTS

• Applications a very active area of investigation
• Intense industry interest in installations of PMUs
• New revised standard a step forward
• System post-mortem analysis the first application
• State estimation is an ideal application
• Control and adaptive relaying applications will follow