Dynamic Pore-Scale Modelling of Two-Phase Flow

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Abstract

The architecture and geometry of the void space of a porous medium and its complementary grain matrix determine several macroscopic properties of the rock such as porosity, relative permeability, capillary pressure and resistivity index (Øren et al. (1998)). Therefore, it is important to study the microscopic structure of the reservoir pores and understand the physical fluid displacements through them. One approach to this problem is pore-scale modelling that requires a detailed understanding of the physical processes occurring on the pore scale and a complete description of the morphology of the pore space.

The pore scale network is a representation of the void space of the reservoir rock. Wide voids of the reservoir rocks are represented by the pore bodies which are interconnected by narrow spaces called pore throats. Flow through these networks is simulated using one of two types of model. The majority of existing pore network models are quasi-static models. At each stage in the displacement, a fixed capillary pressure is imposed on the network and the final static position of fluid-fluid interfaces is determined. However, there are several cases where the quasi-static displacement is not valid such as fracture flow (high flow rate), gas condensate reservoirs (low interfacial tension) and polymer injection (high viscosities). Therefore, studying dynamic fluid displacements through the reservoir pores becomes important. Dynamic network models that explicitly account for viscous forces are the second type of model, but are much less frequently used. Indeed, to date, no pore-scale model has been able to account properly for basic rate-dependent phenomena observed in micromodel experiments, such as the swelling of wetting layers and snap-off.

We present a dynamic pore-scale model for modelling drainage and imbibition processes, including reasonable assumptions and accurate formulas, which helps us to predict
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the events that are observed in micromodel experiments such as swelling of the wetting layers, snap-off and meniscus oscillations. We show that the contribution of disconnected oil flow to overall oil flow is insignificant in primary drainage and significant in imbibition even at typical reservoir flow rates. In addition, one of the key results of the network model is the ability to predict the trend in relative permeability and residual oil saturation with capillary number and to reproduce the displacement patterns observed in micromodel experiments.
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Chapter 1

Introduction: Multiphase flow and its importance to the oil industry

1.1 Impact of multiphase flow on oil production

The study of multiphase flow in porous media is of great industrial importance. Its applications include (Love et al. (2001)): aquifer purification, containment of toxic and nuclear waste, geological flows of magma, chemical reactions in catalysts, enhanced oil recovery and the study of blood flow through capillaries. The fluids may perform a variety of different functions. For example in oil field applications, fluids may be required not only to displace oil or gas and transport them to the surface, but also to act as coolants and lubricants for the drill bit. Another example of a multifunctional fluid is blood that transports salts and sugars in solution as well as white and red blood cells in colloidal suspension (Love et al. (2001)).

The importance of studying multiphase flow in oil field production rises from knowing that the reservoir rock contains two or more immiscible fluids in its pore space. In addition, the development of an oil field often involves flooding the reservoir rock with fluids that displace oil or gas. Important multiphase displacement processes in hydrocarbon reservoirs include (Marle (1981)):

1. Under the effect of production (i.e pressure drawdown), water from a neighbouring
aquifer may encroach into the oil reservoir.

2. Crude oil often contains dissolved gases which may be released into the reservoir rock when the pressure decreases.

3. Many development techniques use the injection of a fluid (typically water or gas) into some wells to displace the oil towards other wells.

In order to predict the behaviour in these and other cases, it is necessary to know the governing physical laws to describe multiphase flow. However, the literature shows that there is still no agreement on the governing equations. The lack of fundamental knowledge about multiphase flow is due to three reasons (Van Wachem and Almstedt (2003)):

- Multiphase flow is a very complex physical phenomenon, where many flow types can occur (e.g. gas/solid, gas/liquid and liquid/liquid) and with each flow type, several possible flow regimes can exist.

- The complex physical laws and mathematical treatment of phenomena occurring in the presence of the two phases (e.g. interface dynamics, coalescence and swelling of the wetting layers) are still largely undeveloped.

- The numerics for solving the governing equations and closure laws of multiphase flows are extremely complex. Very often multiphase flows show inherent oscillatory behaviour that requires costly transient solution algorithms.

Before explaining in detail equations conventionally used to model multiphase flow, it is worthwhile to define the different types of fluid displacement.

### 1.2 Fluid displacement types

Two types of fluid displacement are possible when two or more fluids in motion occupy a porous medium (Bear (1972)): 
Chapter 1. Introduction

1.2. Fluid displacement types

**Miscible displacement:** where the two fluids are completely soluble in each other. The interfacial tension between the two fluids is zero and the two fluids dissolve in each other. Therefore, in this type of displacement, there is no capillarity; instead there is mixing (i.e. dispersion) of the two fluids. This feature makes the miscible displacement a very efficient recovery procedure, where the elimination of capillary forces might lead to total recovery of the displaced phase (oil). In miscible displacement, when two fluids are in contact with each other, a transition zone due to hydrodynamic dispersion is immediately created. The compositions of the fluid varies from that of one fluid to that of the other fluid across the zone.

**Immiscible displacement:** where there is a simultaneous flow of two or more immiscible fluids or phases in the porous medium. The interfacial tension between the two fluids is non-zero and a distinct fluid-fluid interface separates the fluids within each pore. A capillary pressure difference exists at the interface at each point on it.

The flow of immiscible fluids in a porous medium can be conveniently subdivided into two types: steady-state, where all the macroscopic properties of the system are time independent at all points, or unsteady-state where the fluid and flow properties change with time.

In equilibrium steady-state flow of immiscible fluids the saturation of the medium with respect to all fluids contained in the system is constant at all points. Therefore, in steady-state flow there is no displacement of any fluid by any of the other fluids in the pores. This means each fluid is flowing through its own path without affecting on the flow of the other fluids (Figure 1.1). However, in unsteady-state flow, the saturation at a given point in the system is changing with time. Therefore, displacement phenomena (e.g. drainage and imbibition that will be described later) fall into this type of flow. In addition, the flow of two immiscible fluids can be classified according to its direction. Two types of flow can be classified (Dullien (1992)): co-current flow, when both phases flow in the same direction (Figure 1.1) and counter-current flow, when different phases are flowing in opposite directions. Co-current flow is seen when one phase displaces another through viscous forces, such as during water and gas injection. Counter-current
flow occurs when gravity and capillary forces are significant. For instance, imbibition of water into low permeability matrix blocks in fractured reservoirs is an example of a counter-current process: oil escapes by moving in the opposite direction to the water.

Figure 1.1: Two-dimensional representation of three-dimensional co-current steady two-phase flow in porous media. (a) Both fluids flow in separate channels. Fluid I wets the uniformly wet solid surface preferentially. b) Both fluids in the same channels. Fluid I wets the uniformly wet solid surface preferentially. c) Both fluids in the same channels. Each fluid wets preferentially different portions of the mixed-wet solid surface. After Dullien (1992).
1.3 Influence of medium and fluid properties on multiphase flow

Multiphase flow can be characterised by two parameters: residual saturations and relative permeabilities. These parameters are the most important parameters in reservoir engineering calculations, since they determine the rate of recovery and ultimate recovery of displacement processes.

Porosity is defined as the ratio between the void volume to the bulk volume of a certain material. In an oil reservoir, it represents the percentage of the total space that is available for occupancy by either liquids or gases. In a two-phase system, phase saturation can be defined as the percentage of the total volume available that the phase occupies. Residual phase saturation is a term used to indicate the phase saturation at which the phase becomes immobile.

Permeability is a term used to give an indication of fluid flow through pore spaces within reservoir rock. The tighter the rock the smaller the permeability is. It is defined more precisely via Darcy’s law. The absolute permeability stands for a porous material that is fully saturated with a single phase. In a two-phase system, where fluids flow simultaneously through the porous medium, the permeability of each fluid is dependent on its own saturation (Muskat (1981), Dullien (1992)). Each fluid will have its own effective phase permeability. Therefore the relative permeability term was introduced to relate the effective phase permeability to the absolute permeability of a porous medium. It is defined via Darcy’s law as follows:

\[ q_p = \frac{-k}{\mu_p} k_{rp} \nabla P_p - \rho_p g \]  \hspace{1cm} (1.1)

where the subscript \( p \) stands for phase type, \( q_p \) is the Darcy phase velocity, \( k \) is the absolute permeability, \( k_{rp} \) is the phase relative permeability, \( \nabla P_p \) is the phase pressure drop across the core sample, \( \rho_p \) is the phase density and \( g \) is the gravity acceleration.

The relative permeability characteristics depend on many factors such as saturation, saturation history, wettability, capillary pressure, initial water saturation, viscosity, pore geometry and interfacial tension. This section discusses the influence of some of these
1.3. Influence of fluid properties on multiphase flow properties on two-phase relative permeabilities.

1.3.1 Saturation and saturation history

Relative permeability a strong function of the phase saturation. Figure 1.2 shows relative permeability curves as obtained for different saturation histories. As the saturation of the wetting phase increases, the relative permeability of the wetting phase increases while that of the non-wetting phase decreases. The figure also indicates that at some saturations ($S_{wo}$ & $S_{nwo}$, i.e residual saturations), the relative permeabilities are zero for wetting and non-wetting phases respectively. This means at certain saturation, the phase becomes immobile and enhanced techniques are required to re-mobilise the fluid and reduce the saturation. The effect of saturation hysteresis on relative permeability shows that in a strongly wetting system, the wetting phase relative permeability is primarily a function of its own saturation which means the hysteresis of the wetting phase relative permeability curves is much smaller than that for the non-wetting phase (Figure 1.2).

![Figure 1.2: Effect of hysteresis on relative permeability. After Bear (1972).](image-url)
### 1.3.2 Wettability

Wettability may be defined as the rock/fluid interaction in which one fluid preferentially wets the surface of the solid. Wettability is a significant issue in multiphase flow, since it has influence on fluid distributions and consequently the fluid mobilities.

When two immiscible fluids say, $A$ and $B$, are in contact with a solid, one of the fluids, the wetting phase, may be more strongly attracted to the solid than the other. This is due to relative magnitudes of cohesive and adhesive forces (de Gennes (1985)). At equilibrium, one of two states will prevail: partial wetting (Figure 1.3a & b) or complete wetting (Figure 1.3c). The degree to which wetting occurs is measured by the contact angle $\theta$, where the contact angle is measured through the more dense fluid.

![Figure 1.3: Three possible equilibrium of two fluids (A & B) on a solid surface. (a-b) Partial wetting. (c) Complete wetting. After Bear (1972).](image)

Al-Futaisi and Patzek (2003) reported that wettability alteration has a profound effect on the type of imbibition displacement and hence on the two-phase flow characteristics. In a water-wet system, they found that water relative permeability remains about the same for contact angles less than 45° and it decreases very fast as the contact angle is further increased towards 90° (intermediate wettability). The oil relative permeability increases slowly. However, the water relative permeability in an oil-wet system changes very little with changing contact angle.

### 1.3.3 Capillary pressure

When two fluids are in contact with each other, an infinitesimally thin layer called an interface separates them. These two fluids are said to be immiscible and an existence surface or interfacial tension has been established between them. In drainage for example,
the non-wetting phase will displace the wetting phase if the pressure differential across the interface can be overcome. At equilibrium, this pressure differential can be given through the Young-Laplace equation (for a derivation see Adamson (1990)):

$$\Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

(1.2)

where $\Delta P$ is the pressure differential or capillary pressure, $\sigma$ is the interfacial tension between the two immiscible fluids and $R_1$ and $R_2$ represent the principal radii of the curvature normal to each other. The capillary pressure ($P_c$) can be related to the non-wetting phase pressure ($P_{nw}$), wetting phase pressure ($P_w$) and wetting phase saturation through the following expression:

$$\Delta P = P_c = P_{nw} - P_w = f(S_w)$$

(1.3)

The above capillary pressure equation is assumed to account for all effects and processes that influence the equilibrium distribution of fluids such as surface tension, presence of fluid-fluid interface, wettability, grain size distribution and microscale heterogeneities.

**Capillary pressure at microscale**

In the microscale, the capillary pressure plays an important role in fluid displacement through porous media. It is mainly a function of the fluid interfacial tension and pore geometries. Here, since the pore geometries or sizes differ from one pore to another, so will be the capillary pressure. Since the pore spaces of the reservoir rock are irregular, an approximation of the size is needed to compare the pores. The simplest way of determining the pore size is by assuming the pore radius is equivalent to the radius of the largest inscribed circle that can be enclosed within the pore. In this case, the capillary pressure can be written as follows:

$$P_c = \frac{2\sigma \cos \theta}{R}$$

(1.4)

where $R$ is the radius of the inscribed circle.

From this relation, it is clear that the capillary pressure is inversely proportional to the pore size. The local capillary pressure (pore capillary pressure) in many reservoir
simulators is used as a threshold pressure. For example, in primary drainage the oil does not invade a pore unless its pressure is higher than the local capillary pressure.

In this study, we use a pore capillary pressure equation which is similar to Equation 1.4, however, the capillary pressure is assumed to be a function of local fluid saturation. This is done by assuming that the pore size varies sinusoidally with the length (see chapter three).

**Capillary pressure at macroscale**

The standard definition of capillary pressure at the macroscale is derived from its definition at the microscale (Equation 1.3), where the macroscale capillary pressure \( P_c \) can be given through the following expression:

\[
P_c = \langle P_{nw} \rangle - \langle P_w \rangle
\]  

(1.5)

where \( \langle P_{nw} \rangle \) is average non-wetting phase pressure over the representative elementary volume of a porous medium and \( \langle P_w \rangle \) is average wetting phase pressure over the same volume.

Hassanizadeh and Gray (1993) reported that the capillary pressure as a function of pressure difference between non-wetting and wetting phases at equilibrium can be applied only on an interface and thus can be averaged only over that interface not over a volume as the case in Equation 1.5. They also reported that in the literature Equation 1.5 is not only used in steady-state equilibrium flow but also is assumed to be valid during all dynamic situations which is not true. They believe that relating the capillary pressure to the wetting phase saturation (Equation 1.3) is more accurate than relating it to the difference between the average phase pressures. They proposed a new approach for macroscale capillary pressure, they suggested that capillary pressure should be developed in the framework of a sound thermodynamic theory. They used the thermodynamic of two-phase flow in porous media that developed by Gray and Hassanizadeh (1991a,b) to define the capillary pressure. They concluded that the capillary pressure can be given by
the following equation:

\[ P_c = f(a_{nw}, S_w) \]  

(1.6)

where \( a_{nw} \) is the specific area of fluid-fluid interface per unit volume and \( S_w \) is the wetting phase saturation.

Hassanizadeh et al. (2002) extended the work developed by Hassanizadeh and Gray (1993) to study the dynamic effect of capillary pressure on unsaturated flow. In their study, the distinguished between two terms of capillary pressure: static capillary pressure (\( P_{c}^{\text{stat}} \)) which can be defined as the pressure difference between non-wetting and wetting phase pressures in an equilibrium or static flow at certain saturation. The second capillary pressure is the dynamic capillary pressure (\( P_{c}^{\text{dyn}} \)) and it is the pressure difference between the non-wetting and wetting phase pressures in dynamic fluid flow (unsteady-state or unsaturated flow). The dynamic capillary pressure can be related to the static capillary pressure through the following expression:

\[ P_{c}^{\text{dyn}} - P_{c}^{\text{stat}} = \tau \frac{\partial S}{\partial t} \]  

(1.7)

where \( \tau \) is the dynamic capillary coefficient.

In their study, they discussed a lot of experimental studies that attest the fact that the dynamic term presented in their theory (Equation 1.7) can actually be observed experimentally (see for instance Topp et al. (1967), Kalaydjian (1992), Hollenbeck and Jensen (1998)). They concluded that even if there is equilibrium at the pore scale, the upscaling (i.e. volume averaging) of the complex pore-scale processes will always introduce non-equilibrium effects. In addition, they believe that the presence of microheterogeneities also results in non-equilibrium effects. therefore, as they reported that the capillary pressure equation (Equation 1.7) that they proposed properly count for the dynamic effects.
1.4 **Motivation for studying two-phase flow in porous media**

The applications of multiphase flow to enhanced oil recovery have been proposed within the petroleum industry for some time. There is, however, a need for a deep understanding of how fluid properties (e.g. viscosity and interfacial tension) and medium properties (e.g. network topology and pore geometries) affect the micro-mechanisms of multiphase flow through the porous media.

This research will study the complex physical laws of the micro-flow phenomena that occur at the presence of the two phases (e.g. interface dynamics, meniscus oscillations, snap-off and swelling of the wetting layers). This can be done by modelling the wetting layer conductance as a function of fluid saturation and by allowing water to accumulate in the wetting layers. The main objectives of this study are:

- Study the influence of flowrate on fluid displacement patterns, fractional flow curve and residual saturation.
- Study the importance of oil ganglion movements.
- Study the influence of viscosity ratio on fluid movements and snap-off phenomenon.

Section 2.6 discusses these objectives in more details.
Chapter 2

Pore-Scale Network Models

2.1 Introduction

Fluid flow in porous media, as shown in the previous chapter, is of great interest in the petroleum industry. An accurate description of reservoir characteristics is therefore required on many length scales, ranging from $1 - 100 \mu m$ for pore-scale features, through $1 - 100 \text{ cm}$ for core samples to $1 - 100 \text{ km}$ for reservoir bodies. A full description of an oil reservoir cannot be gained except from combining the geological and geophysical data with petrophysical data that obtained from well tests, logs and cores (Dawe et al. (1987)).

To predict future production behaviour, engineers must be supplied with detailed and reliable information about the in situ properties of the reservoir such as the topology and geometry of the reservoir rocks. There are two approaches used by petroleum engineers to study the transport properties through porous media (Toubou et al. (1987)): the first is based on the continuum description of the porous medium associated with macroscopic laws (eg. Darcy’s law and relative permeabilities). The second is based on the microscopic description of the pore geometry and on the physical laws of flow and transport within the pores. The architecture and geometry of the void space of the rock and its complementary grain matrix determine several macroscopic properties of the rock such as porosity, relative permeability, capillary pressure and resistivity index (Øren et al. (1998)). The prediction of average or macroscopic transport properties from their microscopic origins requires two main steps (Øren and Bakke (2002)):
1. A quantitative geometric description of the complex microstructure of the porous medium.

2. Exact or approximate solutions of the equations of motion that govern the transport phenomena of interest.

One of the common tools that have been used to do this is pore-scale network modelling, which requires a detailed understanding of the physical processes occurring at the pore scale and a complete description of the morphology of the pore space. This research uses a pore-scale network model to study dynamic phenomena in two-phase flow. This chapter gives a brief review of pore-scale modelling.

### 2.2 Applications of network models

Network models are heavily used as investigative tools to study the nature of fluid flow from the pore (µm) to core (mm to cm) scale. Applications include chemical engineering, petroleum engineering, physics and hydrology. They are used to study diffusion and dispersion (eg. Hollewand and Gladden (1992), Burganos and Payatakes (1992), Sorbie and Clifford (1991), Koplik et al. (1988), de Arcangelis et al. (1986)), flow in fractures (eg. Fourar et al. (1993), Pyrak-nolte et al. (1992), Hughes and Blunt (2001)), pore-scale evaporation processes (eg. Nowicki et al. (1992)), foam flow (eg. Laidlaw et al. (1993)) and the interpretation of mercury porosimetry and associated characterization of pore size distributions (eg. Ioannidis and Chatzis (1993), Ioannidis et al. (1993), Tsakiroglou and Payatakes (1990), Chatzis and Dullien (1985)). In the petroleum industry and hydrology, their applications include prediction of relative permeability (eg. Blunt and King (1991), Rajaram et al. (1997), Fischer and Celia (1999)), the effect of pore structure on relative permeability and capillary pressure hysteresis in two-phase systems (eg Jerauld and Salter (1990)), prediction of permeability and hydraulic conductivity (eg. Bryant et al. (1993)), investigation of the functional relationship between capillary pressure, saturation and interfacial areas (eg. Reeves and Celia (1996)), drainage and imbibition (eg. Lowry and Miller (1995), Hilpert and Miller (2001), Singh and Mohanty (2003)), phase distributions, interfacial areas and mass transfer (eg. Dillard and Blunt (2000), Held and Celia
Chapter 2. Pore-Scale Network Models 2.3. Construction of the pore-scale network

(2001b)) and ganglion formation and mobilization (eg. Dias and Payatakes (1986b), Li and Wardlaw (1986a,b), Cohen et al. (1997)).

2.3 Construction of the pore-scale network

The pore-scale network can be simply defined as a representation of the void space of the reservoir rock. Wide voids are represented by the pores which are interconnected by narrower regions called throats. Constricting a realistic and fully characterized three-dimensional pore network is a very difficult and complicated process. Al-Raoush et al. (2003) classifies the methods used in generating pore-scale networks into two main types:

- Creating an equivalent network using distributions and correlations of basic morphologic parameters such as pore and throat size distributions, coordination number and spatial correlation within the system which includes size correlation between adjacent pores. Examples of this type include estimation of pore size from measured capillary pressure -saturation curves (eg. D’Hollander (1979), Mishra and Sharma (1988)), fitting generated pressure- saturation curves to the measured curves (eg. Fischer and Celia (1999)). Therefore, this approach can be simple defined as tuning a network model to match available experimental data which may involve removing throats to reduce the coordination number or adjusting the pore size distribution. The problem with this approach is the non-uniqueness of the solution obtained due to the dependency of macroscopic properties such as relative permeability and capillary pressure in both the topology of the pore space and the pore size distribution (Vogel (2000)).

- Direct mapping of a specific porous medium onto a network structure. This type requires a representation of the pore structure prior to the construction of the network. There are two main approaches to do this:

  1. Three-dimensional reconstruction of the pore space based on the measured porosity and correlation functions or serial cross sections (eg. Vogel and Roth (1997), Bakke and Øren (1997), Liang et al. (1999), Okabe and Blunt (2003)).
2. Direct non-destructive three-dimensional imaging using microtomography or magnetic resonance (eg. Baldwin et al. (1996), Rintoul et al. (1996)).

No attempt will be made to review all the work that has been done in the area of network construction. Instead in the remaining of this section, a brief description of the research of Bakke and Øren (1997) will be presented to point out the potential and difficulties of this approach. They developed a process-based reconstruction procedure which incorporates grain size distribution and other petrographical data obtained from two-dimensional thin sections to reconstruct three-dimensional sandstones. In their work, they specified three main stages for generating a pore-scale network.

The first stage is reconstructing the sandstone sample. This stage consists of three main modelling steps: Sedimentation, Compaction and Diagnosis. Modelling the sedimentation process commences with the generation of grain size distribution determined from image analysis of thin sections from the rock sample. In their model, the sand grains were picked randomly from the grain size distribution which helped to mimick the stochastic nature of the geological processes which determines the deposition process. The compaction step in the model is used to model the reduction in the bulk volume of the void space as a result of the vertical stresses from the overburden pressure. In their model, the compaction process was modelled as a linear process through Equation 2.1, where $Z_o$ is the Z-coordinate of the sand grain center before the compaction process is applied, $Z$ is the Z-coordinate after the compaction and $\lambda$ is the compaction factor which varies between zero and one.

$$Z = Z_o(1 - \lambda) \quad (2.1)$$

The diagenetic step is used to model the change in the heterogeneity of the system after the compaction process and other weathering process that might happen after deposition. In their model, they only modelled two kinds of diagenetic processes: quartz cement overgrowth and subsequent clay coating of the free surface. Figure 2.1a shows isosurface representation of the sandstone matrix.

The second stage of generating the pore network is reducing the sandstone model into its complimentary pore space network by finding the skeleton of the pore space network.
The final stage is representing the skeleton of the pore space as a network of pores interconnected by throats. The points where two or more throats meet determine the pores in the network. The connectivity or coordination number for each pore is the number of throats connected to that pore (Figure 2.1c).

### 2.4 Representation of the pore space

To use the pore network in fluid flow simulations, pores and throats of determined volume and size must be used as input parameters to the model. However, pictures of rocks in cross-section from thin sections show that the pore elements have very highly irregular geometries which is difficult if not impossible to represent precisely (Figure 2.2).

Therefore, it is necessary to replace this geometry by an equivalent idealized geometry which leads to mathematically tractable problems while at the same time reproducing the main features of the pore space pertinent to multiphase flow. Bryant and Blunt (1992) showed that if the structure of the pore space can be modelled accurately, then, network modelling can predict relative permeability from first principles. Some pore network models (see for example, Jerauld and Salter (1990), Heiba et al. (1992), Chaouche et al. (1993)) have assumed that the irregularly shape of the throats can be modelled by circular cylinders and the pores can be modelled by spheres. The drawback of this representation of pore space is that the pore element contains only one phase at a time. However, real systems allow two or even three phases to be simultaneously present in a single pore element. Micromodel experiments (Lenormand et al. (1983)) have shown that if two fluids coexist in the pore space, the non-wetting phase occupies the bulk of the pore and the wetting phase is held in the crevices near the pore walls. Therefore, to represent this, different types of pore shape have been proposed in the literature. These types are squares or rectangles (eg. Fenwick and Blunt (1998), Pereira et al. (1996)), triangles (eg. Bakke and Øren (1997), Hui and Blunt (2000)) and star-shapes (eg. Kovscek et al. (1993), Man and Jing (1999)). When two phases coexist in the pore space, the wetting phase is assumed to occupy the angular corners (see Figure 2.3). This feature of the pore shape
Figure 2.1: Construction of pore-scale network. a) Isosurface representation of a sandstone model. b) The skeleton of the pore network. c) pore space as a network of pores (spheres) connected by throats (tubes). After Bakke and Øren (1997)

allows both the wetting and the non-wetting phases to flow simultaneously through the same pore.
Figure 2.2: Schematic diagram showing the highly irregular geometry of the pore elements. After Dullien (1992).

Figure 2.3: Representation of fluid configuration in different types of pore space. a) Star-shaped pore. b) Square-shaped pore. c) Triangular pore.
2.5 Previously developed models

The earliest use of the pore scale modelling was by Fatt (1956a,b,c) who used a physical two-dimensional network of resistors, representing interconnected throats with various radius distributions to derive capillary pressure and relative permeability curves. He concluded that changing the throat radius distribution leads to a great change in the capillary pressure curves.

There are two main types of pore network model have been used to study fluid flow in porous media: quasi-static displacement models and dynamic displacement models. The quasi-static models are those in which the capillary force dominates and where the pores and throats change their configuration one at a time. However, in dynamic models, the effects of viscous forces are modelled in addition to the capillary effect through an explicit computation of the pressure field in the network.

This section is divided to three subsections. In the first subsection, a brief description and review of quasi-static models will be given. The second subsection highlights some of the circumstances where the approximation of quasi-static displacement is not valid. In the last subsection, a detailed description of the most important dynamic models will be given in which the advantages and disadvantages of each model will be clarified to distinguish them from the one proposed in this thesis.

2.5.1 Quasi-static models

In quasi-static models, capillary pressure is imposed on the network and the final, static position of all fluid-fluid interfaces is determined, ignoring the dynamic aspects of pressure propagation and interface dynamics. The pores and throats change their configuration one at a time. Quasi-static models can be considered as extensions of percolation models through the inclusion of more sophisticated displacement processes, especially in imbibition. In invasion percolation models, the invading fluid fills a pore or throat in order of its size. The pore element to be filled in these models it needs to be connected to an element that is filled with the invading fluid. Further details of the models and general overviews of their usefulness can be found in Heiba et al. (1992) and Berkowitz and Balberg (1993).
The majority of existing pore network models are quasi-static. Jerauld and Salter (1990) used this type of model to investigate the effects of various pore space morphologies on relative permeability and capillary pressure functions. They found that the pore/throat aspect ratio (ie the ratio of pore size to the throat size) is the parameter that affects the drainage/ imbibition hysteresis behaviour since it controls the degree of snap-off. They reported that increasing the amount of throat to throat size correlation leads to a change in the shape of the relative permeability curves by increasing the non-wetting phase relative permeability and reducing the wetting phase relative permeability.

Blunt et al. (1992) developed a pore-scale network model where pores were modelled as spheres and throats as cylindrical tubes. The film flow was modelled by allowing wetting phase to permeate the whole system as a thin film with thickness less than 1 µm. They verified percolation-theory results (Nickel and Wilkinson (1983)) for the effects of gravity forces on trapped saturation and they derived an expression for the correlation length in displacements perturbed by viscous forces. From their results they concluded that the measurements of relative permeabilities in quasi-static system at capillary equilibrium are meaningful in large-scale displacements where viscous forces predominate.

Blunt (1997a) studied the effects of long-range pore size correlation on imbibition relative permeability curves using a quasi-static model. He reported that spatial correlation is necessary to produce qualitatively realistic curves that captured measured hysteresis trends. He also found that changing the contact angle has a significant effect on both residual non-wetting saturation and relative permeability curves. Blunt extended this model (Blunt (1997b)) to incorporate wettability effects based on the work of Kovscek et al. (1993). Capillary pressure, relative permeability and residual saturation plots were presented for a variety of advancing and receding contact angles and for different oil-wet fractions of the pore space.

Øren et al. (1998) developed a quasi-static model that used a reconstructed sandstone pore network proposed by Bakke and Øren (1997) as input to simulate two-phase primary drainage and water injection for both water-wet and mixed-wet systems. The predicted capillary pressure and relative permeability curves agreed with measured results for water-wet system and they were in fairly good agreement with results for a mixed-wet system.
Man and Jing (1999) developed a quasi-static pore scale network model which simulated drainage and imbibition processes for water-wet systems. In later work (Man and Jing (2000, 2001)), they extended their model to mixed-wet systems. Their pore network was assumed to be made up of throats which have a star-shaped cross section and size varying sinusoidally along the length. With these features introduced in the model, they could model the fluid-fluid interface within a single pore. The purpose of their model was to study the effects of pore geometry and water saturation on the electrical resistivity and capillary pressure. We will use their model of a sinusoidally varying inscribed radius in our work.

Piri and Blunt (2002) used a quasi-static pore scale model to study three-phase flow in mixed-wet systems. They allowed any value for the advancing and receding oil/water, gas/water and gas/oil contact angle to be modelled. In their model, multiple phases can be present in each pore, in wetting and spreading layers as well as occupying the bulk of the pore space. The model predicted three-phase relative permeabilities measured by Oak (1990). They also used the model to study gas injection for different wettability systems, where the results were interpreted in terms of pore-scale displacement processes.

A more thorough review of recent developments in pore-scale modelling can be found in Blunt (2001) and Blunt et al. (2002).

2.5.2 Some of the circumstances where the approximation of quasi-static displacement is not valid

There are several circumstances where the approximation of quasi-static displacement is not valid. Examples include: fracture flow, where flow rates might be very large, often of the order of hundreds of meters a day; displacements with very low interfacial tension that substantially reduce capillary forces, such as near-miscible gas injection, gas condensate reservoirs and surfactant flooding; near well-bore flow; flow involving polymers, gels and foams where very large pressure gradients are found; and some cases where wetting layer flow and formation is significant, such as spontaneous wetting into a dry soil.

Experimental observations (Avraam and Patayakes (1995a)) suggest that the process
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2.5. Previously developed models

of mobilizing residual oil in a waterflood depends on a competition between viscous and capillary forces. This competition is specified by a dimensionless number, called the capillary number \( C_a \), which is interpreted as a ratio of viscous to capillary forces.

\[
C_a = \frac{\mu q}{\sigma}
\]

where \( \mu \) is the invading phase viscosity, \( q \) is the Darcy flow rate and \( \sigma \) is the interfacial tension.

The capillary number is important in determining the microscopic displacement efficiency and the shape of fractional flow-saturation curves as will be seen in chapters four and five. This subsection will describe the effects of interfacial tension, viscosity ratio and flow rate on fluid displacements.

**Interfacial tension**

The interfacial tension between two fluids or substances can be defined as the amount of work that must be performed in order to separate a unit area of one substance from the other (Bear (1972)). There are some factors which have influence on the oil-water interfacial tension in oil reservoirs such as temperature, pressure, dissolved gas, viscosity and specific gravity. An increase in the temperature or pressure lowers the interfacial tension. However, an increase in viscosity or specific gravity leads to an increase in the interfacial tension. In systems with low interfacial tension (< 1mN/m) such as gas condensate and volatile oil reservoirs, the fluid flow characteristics are different to those of normal oil or gas reservoirs (Marle (1981)).

In oil reservoirs, the interfacial tension has a great influence on the residual saturation. At low interfacial tension, the fluids flow together and this leads to a reduction in the residual saturation and an increase in the relative permeability. Figure 2.4 shows the relative permeability as a function of interfacial tension. From the figure it is clear that at an interfacial tension of 10 mN/m, a typical relative permeability curve for a water-wet system is shown. Then as the interfacial tension decreases, the residual saturation decreases and the relative permeability increases. At zero interfacial tension, the curves become straight.
diagonal lines which cross at 50% saturation and 0.5 relative permeability of each phase. Therefore, the residual oil saturation becomes zero. Here, there is no capillary pressure difference between the two fluids which means the fluid/fluid interactions during the flow are such that the fluids behave in the same way as a single fluid.

Figure 2.4: Relative permeability curves as a function of interfacial tension. After Bear (1972).

In the gas-condensate research area, experimental work published by Boom et al. (1995), studied the effect of capillary number and interfacial tension on the mobility of gas condensate. From their work, they deduced that the key parameter that controls the mobility of the gas condensate is the Bond number, which is the ratio of gravitational to capillary forces ($\frac{\Delta \rho g k}{\sigma}$), and not the interfacial tension alone (see Figures 2.5). Figure 2.5a shows the relationship between wetting phase permeability and the wetting phase saturation for different Bond numbers (corresponding to different interfacial tensions). An increase of up to nearly three orders of magnitude in the mobility of the wetting phase is observed when the Bond number increased from $3.8 \times 10^{-4}$ to $3.8 \times 10^{-2}$. However, Figure 2.5b shows the same relation for systems of constant interfacial tension ($\sigma = 0.55$ mN/m) and different Bond numbers (different centrifugal acceleration) where a reduction in the residual saturation was observed with increasing the Bond number which proves that the interfacial tension is not the only parameter controlling the gas condensate mobility. Clearly, in this work, the behaviour cannot be predicted by considering capillary forces alone.
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Figure 2.5: Wetting-phase relative-permeability curves for six core samples from a gas/condensate reservoir in Europe. a) Wetting-phase relative-permeability as a function of Bond number \( \left( \frac{\Delta \rho g k}{\sigma} \right) \) and interfacial tension. b) Wetting-phase relative-permeability as a function of Bond number. After Boom et al. (1995).
Viscosity ratio

Fluid viscosity can be defined as the resistance of flow due to internal friction when one layer of fluid is caused to move in relationship to another layer. Fluids are called Newtonian fluids if the ratio of shear stress to the rate of shear between the layers is constant. In a system of two Newtonian fluids, the viscosity ratio can be defined as the ratio between the two viscosities.

Viscosity ratio has a great influence on the fluid movements and relative permeability curves. Many experimental and theoretical studies have been performed to investigate the influence of viscosity ratio on the relative permeability curves and fluid distributions, see for instance Lefebvre du Prey (1973), Vizika et al. (1994), Aker et al. (1998) and Al-Shuraiqi et al. (2003). The conclusions that can be drawn from these studies are as follows: in the case of low viscosity ratio (defending fluid viscosity/invasion fluid viscosity), the displacement is piston-like with late breakthrough, however, in the case of high viscosity ratio, the displacing fluid fingers through the displaced fluid and the breakthrough happens earlier. Figure 2.6 shows experimental results for drainage published by Lefebvre du Prey (1973). He reported that the higher the viscosity of one of the liquids is, the lower is the relative permeability of the other liquid.

Flow rate

Injection rate has a very important effect on fluid distributions and consequently on the fractional flow and relative permeability curves. Experimental work by Richardson (1952) was performed to study the effect of displacement rate on the residual saturation. They observed in the case of wetting phase displacing non-wetting phase, only regions near the inlet and outlet of the core were filled with wetting phase, resulting in a very high overall non-wetting phase residual saturation. Increasing the displacement rate resulted in a reduction of residual saturation. From this observation, they deduced that the relative permeability and residual saturation are strong functions of the displacement rate. The sensitivity of relative permeability to flow rate has been confirmed subsequently by many authors (see, for instance, Lake (1989)).

If the displacing phase is strongly wetting, the capillary number is low and the vis-
Figure 2.6: Comparative effect of viscosity ratio ($M = \frac{\mu_w}{\mu_{nw}}$) and wettability on relative permeability. a) $M = 1/36.5$. b) $M = 1$. c) $M = 26.5$. After Lefebvre du Prey (1973).

Cosities are not very high, the non-wetting phase will be trapped in form of disconnected "blobs" or "ganglia" of various size and shape (Dullien (1992)). This trapped non-wetting phase can be displaced by overcoming the capillary forces that hold it in the pore space, such as by decreasing the interfacial tension or increasing the flow rate. Displacing the non-wetting phase under a high capillary number reduces the residual non-wetting phase saturation (Dullien (1992)). Figure 2.7 shows a classic graph of residual non-wetting and wetting phase saturations as a function of capillary number ($C_a$) (Lake (1989)). It is clear
from the graph that the residual saturation is constant at low capillary numbers until a critical capillary number is reached where a knee in the curves occurs and the residual saturation begins to decrease.

![Schematic capillary desaturation curve](image)

Figure 2.7: Schematic capillary desaturation curve. After Lake (1989).

To explain how exactly the flow rate affects the relative permeabilities and residual saturations, consider an imbibition process for a pore-scale model, where the pores and throats have corners and roughness in cross-section. There are three types of invasion process in imbibition (Lenormand and Zarcone (1984)).

- The first type is piston-like or connected advance in throats, in which the fluid advances in a connected front occupying the centre of the pore space.

- The second type of water invasion is called pore body filling in which the threshold capillary pressure is controlled by the largest radius of curvature required to invade the pore body. This radius depends on the size of the pore body and the number of connecting throats filled with wetting phase. For example, if co-ordination number is $Z$, there will be $(Z - 2)$ pore body filling mechanisms. Here, as the number of connecting throats that are filled with oil increases, the pressure required for filling the pore body decreases (i.e $P_c(2) > P_c(3) > P_c(4) > ...$).
• The third type of water invasion is called snap-off. As the capillary pressure decreases, the radius of the curvature of the fluid interface increases and the wetting layers in the crevices start to swell. The swelling of the wetting layers continues until a point where further filling of the crevices causes the interfacial curvature to decrease which leads to fluid instability and the centre of the pore space spontaneously fills with wetting fluid.

_The question that rises at this point is what governs the sequence of these three types of water invasion?_

Many authors (see, for instance, Blunt (1997a), Øren et al. (1998) and Patzek (2001)) have used the contact angle and the capillary pressure to determine a sequence of water invasion events, ignoring rate effects. However, Lenormand and Zarcone (1984) reported that the dominant factor in determining the displacement sequence is the flow of the wetting phase in corners and roughness. They observed three types of wetting phase flow: flow by roughness of the surface, flow along edges or corners and flow through the bulk of the pore or throat. They reported that each type of flow is a function of capillary number (or flow rate). They classified their observations of wetting phase flow under three types of flow rate:

**Very low flow rate:** The wetting fluid can reach all the parts of the network by flowing along the surface roughness and the dominant mechanism is snap-off.

**Low flow rate:** In which, the wetting fluid is flowing along the corners of the pores and the dominant invasion mechanism is pore body filling.

**High flow rate:** In which the wetting fluid is only flowing through the bulk of the pores and the dominant mechanism is frontal drive without fingers or trapping of the non-wetting phase.

_Now, how can this be modelled mathematically and how does it differ from quasi-static models that ignore rate effects?_
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2.5. Previously developed models

2.5.3 Dynamic models

In dynamic models, a specified inflow rate for one of the fluids is imposed and the subsequent transient pressure response and the associated interface positions are calculated. In these models, a given volume of invading fluid is injected during a time step and Poiseuille flow is assumed in the throats. At each time step, the element pressures are computed and the displacement decisions are taken based on pressure difference rules. The dynamic pore network models can be divided into two main types:

1. Models that focus on the throats, where no volume is assigned to the pores and all the volume is assumed to be distributed among the throats (see Man and Jing (1999)).

2. Models that assumed all the volume is distributed among the pores and the throats are treated as volumeless connections between the pores (see Blunt and King (1991)).

The first rigorous of study of imbibition process at the pore scale model was by Lenormand and Zarcone (1984). Because micromodel experiments showed that both fluids (wetting & non-wetting) can be simultaneously present and flow in the same throat or duct, they studied the different flow mechanisms in a straight duct with a square cross-section. They assumed that the wetting fluid flowed in the corners and surface roughness of the ducts while the non-wetting fluid occupied the center of the duct. The purpose of their study was to describe and study the physical mechanisms during the imbibition process at different flow rates. They concluded that for very small capillary number (ie. very low flow rate), the wetting phase is flowing through the surface roughness and connected throughout the network. The dominant displacement mechanism of this flow type is snap-off which results in considerable trapping of the non-wetting fluid (Figure 2.8a). At higher capillary numbers, they observed that the wetting fluid is flowing through corners and roughness. However, the roughness flow is negligible and the dominant displacement mechanism is pore body filling with some trapping of non-wetting phase (Figure 2.8b). Piston type displacement mechanisms dominate at high capillary number, where the wetting phase is flowing in the bulk of the duct displacing the non-wetting phase without any trapping or fingering (Figure 2.8c).
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Mohanty and Salter (1983) published a study of the effect of capillary number on the waterflood residual oil saturation for water-wet and mixed-wet systems using the three-dimensional pore network model developed by Mohanty and Salter (1982a,b). However, in their study, they did not consider wetting layer flow or snap-off. From their model, they noticed that in strongly water-wet media, residual oil saturation decreased sharply above some critical capillary number and decreased to zero at a capillary number 100 times higher than the critical value. However, in mixed-wet media, the reduction of the residual oil saturation with increase in the capillary number was more gradual. In addition, they noticed that at high capillary number, the oil is not only recovered from the oil initially mobilized, but also from the trapped oil that broke-up into smaller blobs that became mobile.

Two years later, a study of dynamic flow of two phases in a random network was published by Koplik and Lasseter (1985). In this study, they assumed circular pores of variable radii centered on a regular lattice, connected randomly by straight throats of variable radii. They only considered the flow through the bulk of the throats, neglecting wetting layer flow. They showed how to calculate the pressure drop along a throat for the case where one or more menisci are present in the pores or the throat connecting them.

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They assumed that the throat capillary pressure can be given by its equilibrium value $P_c = \frac{\sigma}{r_m}$, where $\sigma$ is the surface tension and $r_m$ is the meniscus radius of curvature. They used the oil pressure ($P_o$) in formulating the flow equation by modelling the oil pressure as the actual pressure if oil is present at pore centre space and as the actual pressure plus the pore capillary pressure if water occupies the pore centre space. Figure 2.9 illustrates these ideas. They reported that given the saturation of the corresponding throat and pore, the interface saturation (which is a factor of one or zero used in the pressure drop equation to make the equation only a function of oil pressure (see Figure 2.9)) can be defined by the following rules:

1. If one saturation is single-phase or both saturations are the same single-phase, then the interface saturation is the single-phase saturation.

2. If one saturation is all water and the other is all oil, then the interface saturation is the upstream saturation meaning that of the throat or pore from which the liquid exits.

3. If both saturations are two-phase, the interface saturation is the upstream saturation.

Toubou et al. (1987) used a simplification of the model of Koplik to simulate the dynamics of drainage on a 100x100 two-dimensional network. They noticed that due to capillary pressure, the system of equations used to solve for pressure at each node becomes non-linear. Therefore, they chose to approximate directly the solution to the non-linear problem by using a relaxation technique by replacing the non-linear problem with a sequence of linear problems. The purpose of the study was to study the effect of mobility ratio and capillary number on displacement mechanisms. The results were in good agreement with micromodel experimental results.

Blunt and King (1991) developed a dynamic two-phase model of drainage and they invoked a relaxation technique to calculate the pore pressures. Their model can be classified under the second type of dynamic pore network models where all the fluid is considered to be contained in the pores. The main purpose of their study was to investigate the effect of viscosity ratio and capillary pressure on pore-scale displacement in both two and three dimensions, as well as studying how the microscopic physics of flow affected...
averaged properties on the centimeter scale. In their model, they used up to 80,000 circular or spherical pores that are connected by non-intersecting straight throats. They ran their model assuming the capillary pressure difference across an interface between the two fluids at the entrance of the throat is inversely proportional to the throat radius and this pressure drop is neglected in the pores due to their wider radius. In addition, they assumed that the throats are totally filled with one fluid. However, the pores may contain both fluids. Their simulations showed that drainage relative permeability was a function of both viscosity ratio and capillary number.

Dias and Payatakes (1986b,a) introduced a pore-scale network model to study water flooding for a range of capillary number and viscosity ratio. Their network consisted of unit cells of converging-diverging geometry, where the pore walls between the pore centres were assumed to vary sinusoidally. The patterns and rate of the displacement were obtained by assuming quasi-static flow. They ignored wetting layer flow in their simulations. From their results they concluded that the microdisplacement is a function of capillary number and viscosity ratio. At high capillary number and favourable viscosity ratios (oil is less viscous than water), the displacement is piston-like and the residual
saturation is low. However, at low capillary number and unfavourable viscosity ratios the displacement is ramified and the residual saturation is high.

*Vizika et al.* (1994) published a theoretical and experimental study that investigated the effect of the viscosity ratio over a wide range of values of capillary number on immiscible displacement. In their theoretical study, the model that they used was akin to the one developed by *Dias and Payatakes* (1986b,a). However, the network in their study had been extended to two layers in order to account for the non-planarity of real porous rocks. They modelled the pore space of the permeable medium as a network of unit cells of constricted tube type which was depicted as a network of spherical pores connected through a long cylindrical throats. The constriction of the unit cell is made by assuming the wall profile of the pores is sinusoidal as shown in Figure 2.10. In their study, they performed a series of imbibition simulations, keeping all parameters constant and changing only the network size. They observed that the residual oil saturation increases with increasing the network size as illustrated in Figure 2.11. In the investigation of the effect of the viscosity ratio on residual saturation, they used a network of size $12 \times 5 \times 2$. They reported that at high capillary number, the role of viscosity ratio is very important. In the case of viscosity ratio (defending fluid viscosity/invasion fluid viscosity) greater than one, there are long microfingers of the wetting fluid forced into the nonwetting fluid and

![Figure 2.10: Typical unit cell of the network used. After *Vizika et al.* (1994).](image-url)
the displacement is highly irregular which led to large parts of nonwetting fluid becoming trapped. However with this high capillary number, many oil ganglia became mobilized and they kept moving for more than two or three units cells before they became stranded or broke up into smaller ganglia.

In the case of high capillary number and viscosity ratio less than one, the microfingers almost disappeared and high microdisplacement efficiency was obtained due to the relatively smooth displacement front. They showed a remarkable influence of the viscosity ratio on the fluid movement and the residual oil saturation at low capillary number. For viscosity ratios greater than one, viscous microfingers appeared promptly and grew into
the oil leading to high values of residual oil saturation (see Figure 2.12). In addition, the phenomenon of ganglion mobilization was again observed with these capillary numbers. However, the mobilized ganglia kept moving for a short time and then usually disintegrated into smaller ganglia that became stranded which did not lead to any improvement of the sweep efficiency. In the case of viscosity ratios less than unity, the residual oil saturation was quite high due to the capillary microfingers caused by capillary forces. However, it was well below the corresponding value obtained for viscosity ratios greater than one (see Figure 2.12).

![Figure 2.12](image.png)

Figure 2.12: Effect of the viscosity ratio ($M = \frac{\mu_o}{\mu_w}$) on the residual non-wetting saturation for various values of the capillary number ($C_a$), using the theoretical simulator. After Vizika et al. (1994).
In their experimental study, they used a 2D square lattice network constructed from copper coated glass plates with unit cells 1221 μm long. The network consisted of 11,300 pores and 22,600 throats. According to their observations, in the case of low capillary number and viscosity ratio of 1.45, extensive microfingers were observed which led to the formation of numerous ganglia that immediately become trapped. The residual oil saturation tended to be constant for capillary numbers less than $10^{-6}$ and decreased as the capillary number increased (Figure 2.13). Increasing the viscosity ratio to 3.35 caused the microfingering phenomenon to be excessive and the disturbance at the front became amplified. This made the wetting fluid advance faster at the forward interfaces, forming new microfingers that developed in a treelike fashion.

Figure 2.13: Effect of the viscosity ratio ($M$) on the residual non-wetting saturation for a range of capillary number, from the experimental study. After Vizika et al. (1994).
In the case of high capillary number and viscosity ratio more than unity, the microfin-
gering phenomenon was remarkable, although the menisci were moving with high dis-
placement velocity. In the case of viscosity ratio less than unity, the extent of microfin-
gering is substantially reduced and a relatively smooth displacing front gave relatively
good microdisplacement efficiency. The comparison between the simulation results (Fig-
ure 2.12) and the experimental ones (Figure 2.13) shows that the influence of viscosity
eratio on residual oil saturation observed in the experimental results is stronger than that
Predicted by the simulator especially in the region of low capillary number. This might
be due to the wetting layer flow that was ignored in the simulation.

The generation of oil ganglia that had been observed in their experimental study en-
couraged them to perform more experimental work to study the influence of ganglion
movements on fractional and relative permeability curves (see Avraam et al. (1994),
Avraam and Patayakes (1995a,b), Tzimas et al. (1997)). The conclusion that they drew
was that the principal pore-scale flow mechanism during steady-state two-phase flow is
ganglion dynamics. Four main flow regimes were observed: large-ganglion dynamics
(LGD), small-ganglion dynamics (SGD), drop-traffic flow (DTF) and connected path-
way flow (CPF). These observations encouraged Valavanides et al. (1998) to extended
the pore-scale model used by Vizika et al. (1994) to account for the ganglion dynamics
flow regimes. The approach of their model was based on the ganglion population bal-
ance equation in combination with a microflow network simulator. In their method, the
fundamental information of two-phase flow at a scale of a few hundred pores was ex-
pressed through so-called system factors, which are functions of the system parameters
obtained using the microflow simulator. Then these system factors were utilized by the
population balance equation to predict the macroscopic behavior of the flow process (Fig-
ure 2.14). From Figure 2.14, it is clear that their method can be divided to three stages.
In the first stage, they used a pore-scale network model to obtain the system parameters.
a pore-scale network of size of $30 \times 20 \times 5$ nodes was used to obtain capillary number
($C_a$), viscosity ratio, wettability, coalescence factor ($C_o$) –probability of coalescence of
two colliding ganglia– and water saturation. In the second stage, they used these system
parameters as an input to a ganglion dynamics simulator that was used to obtain system
factors for each individual ganglion member of the interacting ganglia population. These factors are the velocity of ganglia, coefficients of stranding and breakup, breakup mode, the probability of stranding of a newly formed ganglion and the maximum length of the ganglion projected in the direction of the macroscopic flow. The ganglion dynamics simulator that they used was only capable of modelling the LGD and SGD mechanisms. The last stage in their method was using the system factors as an input to ganglion population balance equations model to predict the macroscopic behavior of the flow process. Sample calculations were performed for steady-state fully developed and steady-state non-fully developed conditions, where relative permeability and fractional flow curves were studied as function of the system parameters. Three years later, the model was extended by Valavanides and Payatakes (2001) to include the drop-traffic flow (DTF) and connected pathway flow (CPF) mechanisms. They suggested that even at small capillary numbers, the main transport process for the non-wetting phase is the flow of disconnected ganglia. This conceptual model of multiphase flow is very different from the traditional view illustrated in Figure 1.1, where each phase flows through its own connected sub-network. The authors neglected wetting layers in their simulations and used uniform pore shapes in their micromodel experiments which limited layer flow. Furthermore, they only studied two-dimensional or quasi two-dimensional (only two layers) systems where it is difficult for two phases to span the model through fully-occupied elements. This might lead them to over-emphasise the impact of the ganglion dynamics. In this thesis we will test how significant disconnected flow is in a model where wetting layers are accounted for.

Van der Marck et al. (1997) developed a dynamic model to simulate two-phase drainage flow. The fluid flow in their model was subject to three forces: capillary, viscous and gravity. In the network simulator, they introduced a geometry that resembles the shape of an etched channels (throats) used in the micromodel experiments (Figure 2.15). They allowed up to two interfaces to be present between the pore centres. They compared their model results with micromodel experiments where they measured both the saturation and the pressure drop across the micromodel. The simulations agreed with experimental results except for a high rate, high viscosity ratio case.

A study investigating some of the dynamic phenomena of microflow (e.g capillary
Chapter 2. Pore-Scale Network Models

2.5. Previously developed models

<table>
<thead>
<tr>
<th>Scale</th>
<th>Microscopic</th>
<th>Mesoscopic</th>
<th>Macroscopic</th>
</tr>
</thead>
<tbody>
<tr>
<td># pores</td>
<td>1</td>
<td>$10^3$</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>

Fingering and viscous fingering) was published by Aker et al. (1998). In this study they simulated primary drainage in a two-dimensional lattice of throats and neglected film flow. To account for the dynamics aspects of the flow, they let the local capillary pressure change as the menisci move through the throats.

Figure 2.14: Range of number of pores over which the various models are applicable and scale-up of the model. Modified after Valavanides et al. (1998).

Figure 2.15: Schematic diagram of the throat geometry used in the model. Modified after Van der Marck et al. (1997).
The porous medium in their study was modelled by a regular lattice. Different radii were assigned to the throats at random. The throats were considered to hold all the network volume and the pores were treated as volumeless joining points. In simulating fluid flow in the network, they used a modified equation for the local capillary pressure:

\[ P_c = \frac{2\sigma}{r}(1 - \cos(2\pi x)) \]  

(2.3)

where \( x \) represents the position of the interface within the throat, i.e. \( 0 \leq x \leq 1 \). The volume flux equation that they used was:

\[ q_{ij} = \frac{\pi r_{ij}^2}{8\mu_{\text{eff}} d}(\Delta P_{ij} - \bar{P}_c) \]  

(2.4)

where \( q_{ij} \) is the volume flux through a throat connecting pore \( i \) to pore \( j \), \( r_{ij} \) is the radius of the throat, \( d \) is the length of the throat, and \( \Delta P_{ij} \) is the pressure drop between the pores. The \( \bar{P}_c \) term in the above equation stands for the summation of the capillary pressures in a single throat, which is given by the following expression:

\[ \bar{P}_c = \sum_{i=1}^{n} P_{ci} \]  

(2.5)

where \( n \) is the number of the interfaces in a single throat, and \( P_{ci} \) is the capillary pressure at each interface given in Equation 2.3. The \( \mu_{\text{eff}} \) in Equation 2.4 is the effective viscosity and it is the sum of the amount of each fluid multiplied by their respective viscosities. Using the effective viscosity term in the flowrate equation simplifies the problem and it becomes no more complex than solving a single-phase flow problem. However, this is a very rough approximation, and the expression in the front of the bracket in Equation 2.4 may not be an accurate expression for the equivalent hydraulic conductance of two fluids flowing simultaneously in a cylindrical tube.

In this study, they simulated primary drainage by injecting oil with constant flowrate into a system that initially is completely filled with water. To achieve the desired injection rate, they assumed that the injection rate varies linearly with the pressure drop across the
system:

\[ Q = A \Delta P + B \]  \hspace{1cm} (2.6)

where \( \Delta P \) is the pressure across the system and \( A \) & \( B \) are parameters depending on the geometry of the medium and the current fluid configurations. The first part of the above equation is the Darcy’s law for single-phase flow through the porous medium. The second part (i.e \( B \)) results from the capillary pressure between the two phases. Therefore, the unknowns \( (A \) & \( B) \) in the above equation are calculated by solving the pore pressures for two different pressure drops across the system, and then, the corresponding injection rates are calculated. The calculated pressure drops and injection rates are inserted in the above equation to give a set of two equations, from which \( A \) & \( B \) are found. Then the above equation is reused to find the pressure drop across the system for the desired injection rate. Since \( A \) & \( B \) are functions of the fluid configurations, the above procedure should be repeated every time step.

They assumed that the simultaneous flow could be represented by a finite number of small bubbles of each fluid, placed next to each other inside the throat. Then, they sorted the bubbles of the same type of fluid and replaced them by one or two menisci as shown in Figure 2.16. When the meniscus in the bottom throat reaches the end of the throat, it is moved a distance \( \delta \) into the neighbouring throats as shown in Figures 2.16a & b. Then, due to the opposite flow-direction in the right throat, the created meniscus in this throat is moved back to the pore (Figure 2.16c) and then pushed into the neighbouring throats as shown in Figure 2.16d. When the meniscus in the bottom throat approaches the pore again from below, it creates a configuration with three menisci in the top and left throats Figure 2.16f. Here, to avoid having more than two menisci inside a single throat, they reduced the three menisci in the top and left throats to one placing the wetting fluid on the top of the non-wetting fluid Figure 2.16g.

They performed three types of simulation, where the behaviour of the global capillary pressure and the pressure across the system are discussed qualitatively for three regimes of viscosity ratio: viscous fingering, stable displacement and capillary fingering. They observed viscous fingering when the viscosity of the invading fluid was less than the viscosity of the defending fluid. The fingers were driven by viscous forces when capillary forces
forces are less dominant. In the case of the stable displacement regime (i.e. the viscosity of the invading fluid is higher than the viscosity of the defending fluid), the fluid movements are dominated by the viscous forces in the invading fluid. The pressure gradient in the invading phase was found to stabilize the displacement and a compact pattern with an almost flat front between the fluids was generated. In the case of capillary fingering (i.e. unit viscosity ratio), a rough front between the fluids was observed with many trapped
clusters of the defending fluid of different size.

Later, the model was extended to study the viscous stabilization of the invasion front (Aker et al. (2000)) and the relation between pressure, saturation and fractional flow (Knudsen et al. (2001), Knudsen and Hansen (2002)).

Dahle and Celia (1999) developed a dynamic pore-scale model that is similar to the one developed by Aker et al. (1998). However, the capillary pressure in their model was given by Young-Laplace equation for a cylindrical tube. This means if there are two interfaces in a throat, the net capillary force be zero. In addition, they allowed ganglion movements although only one interface was allowed to move in a single throat at any time. The $P_c - S$ curves for drainage were studied based on both the pressure drop over the network and on the internal average phase pressures. They found that these curves were in good agreement as long as most of the wetting fluid is connected to the outlet. However, at low wetting phase saturation the wetting phase was represented in the network as ganglia. Therefore, the $P_c - S$ curves defined by reservoir pressure and phase pressure were not the same. The model was extended to study interfacial area and its relation to capillary pressure (Held and Celia (2001a)) and interfacial velocity (Nordhaug et al. (2003)).

**Dynamic models that included wetting layer flow**

All the above models only allow a single phase to be present in any cross-section through a throat, which means they ignore contributions to flow through wetting layers that occupy the roughness and crevices of the pore space even when the centre of the pore or throat is filled by non-wetting phase. This is a significant approximation for imbibition where snap-off is important.

Blunt and Scher (1995) extended the model introduced by Blunt et al. (1992), where pores and throats were modelled as cubic or square shapes to account for wetting layer flow. They assumed a fixed conductance in wetting layers. They introduced a dimensionless crevice resistance factor $\alpha$ (the ratio of bulk flow in an element to the crevice flow). This parameter in combination with the capillary number was found to have a major influence in the wetting behaviour. They showed that an increase in flow rate led to decrease in the trapped oil saturation. The flow rates at which this effect is significant were when
Hughes and Blunt (2000) used a pore network to study the effects of flow rate, contact angle and initial wetting phase saturation on imbibition relative permeability curves. They used an irregular shape (square and equilateral triangular) for the pores to account for wetting layer flow. They assumed a fixed conductance to the wetting layers. They developed an approximate method where pores and throats were filled one at a time and there was no simultaneous filling which would occur in reality due to the swelling of the wetting layers and snap-off. The filling sequence in their method was a function of what they called sorting pressure. In this method, they first solved for the wetting phase pressure \( P_{i}^w \) for each element. Then, for each element in the network they calculated the wetting phase pressure drop between the inlet and the element \( \Delta P^i \). The sorting pressure \( P_{sort} \) then was calculated as follows:

\[
P_{sort} = \Delta P^i - P_{i}^c
\]  

(2.7)

where \( P_{i}^c \) is the local displacement capillary pressure. They assumed that the non-wetting phase pressure gradient was negligible and so the sorting pressure could be used as the inlet pressure necessary to fill the element. Since the sorting pressure was a function of fluid configurations, it was periodically recomputed during the displacement. Their model was essentially quasi-static with a perturbative rule to include rate effects. While they were able to reproduce qualitatively the micromodel results of Lenormand and Zarcone (1984) as illustrated in Figure 2.17, the model was only valid for relatively low flow rates.

Mogensen and Stenby (1998) presented a dynamic pore-scale mode of imbibition in which the wetting layer flow was included. They used the model to investigated the effect of viscosity ratio, contact angle, aspect ratio and capillary number on residual oil saturation. They used a 3-D network of \( 15 \times 15 \times 15 \) pores considering different types of pore geometry. Snap-off was assumed to happen if \( \frac{r_{throat}}{r_{pore}} < \frac{1-\tan(\theta) \tan(\alpha)}{2f} \), where \( \alpha \) is a corner half angle and \( f \) is geometry factor. They obtained the radius of interface curvature for each pore or throat as a function of the wetting and non-wetting phase pressures. This was done by providing an iteration loop for calculating fluid pressures before the
Chapter 2. Pore-Scale Network Models  

2.5. Previously developed models

Figure 2.17: Comparison of simulated results (left) and experimental result (right) using information from Lenormand and Zarcone (1984). Simulation on a $128 \times 128$ grid. $\theta = 0$. Capillary numbers are a) $C_a = 6 \times 10^{-7}$, b) $C_a = 1.4 \times 10^{-5}$, c) $C_a = 3 \times 10^{-4}$. The same number of elements have been filled in each simulation. $\alpha = 2150$. After Hughes and Blunt (2000).
displacement simulation. Once they obtained the required radius of curvature for each element, they assumed it was constant which means that they assumed that in the wetting layers the flow rate was constant throughout the simulation. Then the time step was chosen as the minimum time required for snap-off or piston like mechanisms to occur in a single pore or throat. The invasion process was simulated in a series of time steps. By knowing the flow rates, the minimum filling time could be found among those pores and throats that contain an interface. In the case of a piston-like mechanism, the filling time was:

\[ t_{piston-like} = \min \left( \frac{S_{nw,i} V_i}{q_i} \right) \quad (2.8) \]

where the minimum is with respect to all pore/throats \( i \) containing a moving interface. In a throat where the snap-off criterion is satisfied, the time step is defined as the time it takes for the crevice flow to form a stable neck. They assumed the total volume of the neck was given by the following:

\[ V_{total-neck} = V_{throat} r_{throat} l_{throat} \quad (2.9) \]

Then, the minimum time for snap-off to occur becomes:

\[ t_{snap-off} = \min \left[ \frac{V_{total-neck,i} - V_{actual-neck,i}}{q_{crev,i}} \right] \quad (2.10) \]

Therefore, the time of event is then given as \( t_{event} = \min(t_{piston-like}, t_{snap-off}) \), and they updated the volume of the neck using the following expression:

\[ V_{actual-neck,new} = V_{actual-neck,old} + t_{event} q_{crev} \quad (2.11) \]

They concluded that the capillary number, aspect ratio and contact angle have a significant influence on the competition between piston like advance and snap-off. Snap-off was the dominant mechanism for capillary numbers in range \( 10^{-8} \rightarrow 10^{-7} \). They suggested that a high oil recovery can be obtained by invoking the effect of flow rate, non-zero contact angle and existence of large-range correlation in the pore structure.

Singh and Mohanty (2003) developed a dynamic model to simulate drainage two-phase flow. They used a cubic network with cubic bodies (pores) and square cross-section
In their model they assumed strongly water wet-system (i.e $\theta = 0$). From the description of their work, it is clear that there were two different models used to study the fluid flow, one at high and intermediate capillary number and the other at low capillary number.

In the case of high and intermediate capillary number, their model consists of two separate sub-models, one used to study fluid flow in the bulk of the pore elements and the other dealt with wetting layer flow. They ignored snap-off because—as they claimed—it is not very important in the drainage process. The bulk flow model is very similar to the one introduced by Van der Marck et al. (1997). However in their model, the interface is free to move in either direction along the throat connecting two pores containing different fluids. They reported that the explicit and more accurate way for incorporating wetting layer flow is to solve two linear flow problems, one for the bulk flow and the other for wetting layer flow. The two pressure solutions are related through capillary pressure that will be used to define the new radius of curvature of the interfaces through an iterative method. Because this process requires expensive and time-consuming computation, they developed a heuristic scheme to implement wetting layer flow in the model. Their procedure involves computation of a potential at each interface location. The potential was defined as the capillary pressure drop across the interface. Then, an amount of wetting phase was removed from the interface in proportion to the interface potential at that location. In this procedure the total amount of the wetting fluid that is removed can be specified a priori either as a fraction of the bulk flow volume or as constant. In their model, the volume flowing through wetting layers was set to be 1% of the volume flowing through the bulk.

In the case of low capillary number ($C_a < 10^{-6}$), they used what they called pseudo-percolation model. In this model, all interfaces were assigned a potential that is equal to the pressure drop across the interface. At each stage, the throat with the highest potential was assigned a non-zero conductance that was estimated from local pore and throat sizes. This led to pressure build up in the invading fluid and opened the throat for the fluid to drain into.

The model was used to study primary drainage with a constant inlet flow rate. Saturation and relative permeability were computed as a function of capillary number, viscosity
ratio and pore-throat size distribution.

2.6 Summary and project objective

A great effort has been spent to develop fully dynamic models for modelling drainage and imbibition processes. However, a number of key physical effects have yet to be captured accurately. In particular, previous work did not account for the swelling of wetting layers in both drainage and imbibition that allows snap-off, as observed in micromodel experiments (Lenormand and Zarcone (1984)). Snap-off is the key process by which the non-wetting phase becomes trapped, and determines, for instance, the amount of oil that is left unrecovered after waterflooding. Wetting layers were either ignored or treated in an approximate manner with some fixed conductance assumed throughout the displacement.

Related to this lack of physical realism is a controversy in the literature over the generic nature of multiphase flow in porous media. The conventional picture, based largely on quasi-static approaches to modeling, assumes that for typical reservoir displacements, each phase occupies its own connected sub-network through the porous medium. The hydraulic conductance of these sub-networks determine the multiphase flow properties, specifically the relative permeability (Dullien (1992), Blunt et al. (2002)). Disconnected regions do not flow unless the capillary number is very high. In contrast, Payatakes et al. (Vizika et al. (1994), Valavanides et al. (1998), Valavanides and Payatakes (2001)) suggest based on micromodel experiments and network modeling that the typical scenario for multiphase flow is significant transport via disconnected ganglia even at reservoir flow rates. However, their work can be criticized for not accommodating wetting layer flow and consequently substantially restricting the connectivity of the wetting phase.

In this work we introduce a conceptually simple dynamic model that explicitly simulates the dynamics of wetting layer swelling and snap-off. We are then able to address whether or not multiphase flow involves significant transport of disconnected non-wetting phase, even at typical reservoir flow rates, or whether this phenomenon is restricted to high capillary numbers.
Chapter 3

Model description

3.1 Introduction

We will describe a dynamic pore scale model of drainage and imbibition, including reasonable assumptions and accurate formulae, that will reproduce the events that are observed in the micromodel experiments such as the swelling of wetting layers and meniscus oscillations.

3.2 Principles of the model

In this model, we assume laminar flow with a linear relation between pressure drop and flow between a pore and throat; that there is no accumulation of oil within a single element. This means the model can not capture some exotic effects that might be seen at high Reynolds number such as ganglion deformation. The model is based on three main principles:

1. The amount of each phase in each pore or throat is known at each time step. The volume of each phase (with the contact angle) controls the configuration of fluids. This in turn determines the curvature of the oil-water interface and the pressure difference between these two fluids in each pore or throat.

2. By using equivalent networks of electrical resistors, the hydraulic resistances of the
fluids between pore and throat centres are calculated and used in a volume balance equation to obtain the fluid pressures at pore and throat centres. Using an equivalent resistor network simplifies the problem and makes it no more complex than solving the material balance equations for single-phase flow.

3. The pressure difference between the pore and throat centres and the previously computed hydraulic resistances of each phase are used to move phases between pores and throats and hence to update the fluid volumes. The simulation then returns to step 1.

3.3 Network and pore geometry

The porous medium is represented by a square lattice of pores and throats. In cross-section each pore or throat is a scalene triangle. The inscribed radius of a pore or throat varies sinusoidally, as shown in Figure 3.1 (Man and Jing (1999)). Each pore is divided into several branches (equal to the number of connected throats), which are considered as extensions of the throats they are connected to. All the pore branches meet at the centre of the pore that is treated as a volumeless joining point. This feature is introduced for two reasons. First, we believe it is more realistic than assuming a straight channel (i.e. uniform inscribed radius along the length). Second, it allows us to assign a unique and continuously varying capillary pressure as the interface (meniscus) moves in a pore or throat. The inscribed radius ($R$) at any point between the pore and the connecting throat centres is given by:

$$R = \left( \frac{R_p + R_t}{2} \right) + \left( \frac{R_p - R_t}{2} \cos \left( \frac{2\pi x}{l_p + l_t} \right) \right)$$

(3.1)

where $R_p, R_t$ are the pore and throat centre radii respectively, $l_p, l_t$ are the pore and throat lengths respectively. $x=0$ at the pore centre and $x = (l_p + l_t)/2$ at the throat centre.

3.3.1 Selection of pore and throat sizes

In many porous media, the grains are packed in a disordered manner, rather than in regular lattices. Theoretical treatments of the properties of these media often assume that the pore
space between the grains is completely spatial disordered. Bryant et al. (1996) published a study that analyses a real granular medium which showed that the pore space is spatially correlated. The origin of this pore space correlation is the strong spatial correlation of grain locations. Their analysis—as they reported—relied on physically representative
Table 3.1: Parameters used to determine pore and throat geometries.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t,_{min}$</td>
<td>0.2 $\mu$m</td>
</tr>
<tr>
<td>$R_t,_{max}$</td>
<td>100 $\mu$m</td>
</tr>
<tr>
<td>$l_{min}$</td>
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<td>$a_{min}$</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>1.6</td>
</tr>
</tbody>
</table>

network models of the pore space constructed from knowledge of the grain locations. They reported that simulated drainage experiments on these networks agreed with mercury porosimetry experiments in simple sandstones, whereas simulations in uncorrelated but otherwise identical networks do not. Therefore they concluded that deriving pore size distribution from mercury porosimetry without considering spatial correlation can give misleading results.

In our model, we ignore spatial correlation in the distribution of throat size and assume that the throat sizes can be obtained according to the truncated Weibull distribution. We use this distribution because it is easy to implement. The inscribed radius of the centre of any throat –that is assigned at random according to a truncated Weibull distribution– is given by the following equation:

$$R_t = (R_{t,_{max}} - R_{t,_{min}}) \left[ -\delta \ln(z(1 - e^{-\frac{1}{\varrho}}) + e^{-\frac{1}{\varrho}}) \right]^\frac{1}{\varrho} + R_{t,_{min}} \tag{3.2}$$

where $R_t$ is a radius of a throat centre and $z$ is a random number between zero and one. The parameters used in the distribution are shown in Table 3.1. The pore radius at its centre must be greater or equal to the maximum radius of the connecting throats. Therefore, the pore radius is given by the following expression:

$$R_p = \max\{R_{ti} | i=1, \ldots, n\} \times a \tag{3.3}$$

where $n$ is the number of the connecting throats, and the aspect ratio ($a$) is the ratio between the pore radius and the maximum radius of the connecting throats. It is obtained
by using Equation 3.2, replacing $R_{t,max}$ and $R_{t,min}$ by the maximum and minimum aspect ratios provided in Table 3.1. Although we use a topologically square lattice, we do allow the pore and throat lengths to vary – this corresponds physically to a distorted lattice, although we do not check if the network is physically realizable in two-dimensional space.

We used the same parameters for selecting the pores and throat lengths, replacing $R_{t,max}$ and $R_{t,min}$ by $l_{max}$ and $l_{min}$ provided in Table 3.1 in Equation 3.2. Once $l_p$, $l_t$, $R_p$ and $R_t$ are known, Equation 3.1 is used to determine the inscribed radius as a function of distance between the pore and throat centres.

### 3.3.2 Determination of the pore cross section and corner half angles

Each pore and throat has a scalene triangular cross-section with corner angles selected at random. The triangular cross-section of an element is determined through two parameters: the inscribed radius (described above), and the shape factor $F = A/P^2$, where $A$ is the cross-sectional area and $P$ is the perimeter (Mason and Morrow (1991)). The shape factor is used to determine the corner half angles for a triangle. In our model, we assume the shape factor for each pore and throat is chosen according to the truncated Weibull distribution (Equation 3.2) for a range of shape factor between zero and $\sqrt{3}/36$ (equilateral).

\[
F = \frac{A}{P^2} = \frac{1}{4\sum_{i=1}^{3}\cot(\alpha_i)} = \frac{1}{4} \tan \alpha_1 \tan \alpha_2 \cot(\alpha_1 + \alpha_2) \tag{3.4}
\]

where $\alpha_1$ and $\alpha_2$ are the two corner half-angles subtended at the longest sides of the triangle. It is clear from the above equation that for a single value of the shape factor, there are ranges of corner half-angles and triangular shapes. Therefore in this model, we followed the procedure of Patzek (2001) to select the non-unique solution for corner half-angles, which is as follows:

1. Select the upper and lower limits of the second largest corner half-angle. These two limits are given according to the following equations:

\[
\alpha_{2,min} = \arctan\left[\frac{2}{\sqrt{3}} \cos\left(\frac{\arccos(-12 \sqrt{3}F)}{3} + \frac{4\pi}{3}\right)\right] \tag{3.5}
\]
\[ \alpha_{2,\text{max}} = \arctan\left( \frac{2}{\sqrt{3}} \cos\left( \frac{\arccos(-12\sqrt{3}F)}{3} \right) \right) \] (3.6)

2. Pick randomly a value of \( \alpha_2 \) between the two limits. Here, we assume \( \alpha_2 \) is given by the following expression:

\[ \alpha_2 = \alpha_{2,\text{min}} + (\alpha_{2,\text{max}} - \alpha_{2,\text{min}}) \times z, \] where \( z \) is a random number between zero and one.

3. The corresponding value of the largest corner half-angle \( \alpha_1 \) can be found from:

\[ \alpha_1 = -\frac{1}{2} \alpha_2 + \frac{1}{2} \arcsin\left( \frac{\tan \alpha_2 + 4F}{\tan \alpha_2 - 4F} \sin \alpha_2 \right) \] (3.7)

4. The smallest corner half angle is then obtained from: \( \alpha_3 = \frac{\pi}{2} - (\alpha_1 + \alpha_2) \).

### 3.3.3 Restricting snap-off to throats

The snap-off phenomenon can be considered as a function of the aspect ratio (ratio between the pore and throat radii). In the throats, when the oil/water interface passes through the throat centre and moves into the other side of the throat, there is a possibility for the water in the wetting layers to snap off at the centre of the throat producing what is called by snap-off. This can happen if the radius at the interface is more than twice the radius at the throat centre –see the analysis provided later.

In theory it is also possible for snap-off to occur in pores. Consider a pore that is connected to two throats. One branch has a wide pore/throat boundary radius and the other has a small pore/throat boundary radius. It is possible that a meniscus passing the wide boundary will cause snap-off at the narrow boundary. This is an unphysical effect. Therefore, we limit snap-off to the throats. This can be done by ensuring that the radius at the pore/throat boundary is greater than half the radius at the pore centre, Equation 3.8.

\[ R_{\text{boundary}} > \frac{1}{2} R_P \] (3.8)
Substituting Equation 3.1 in Equation 3.8

\[
\left( \frac{R_p + R_t}{2} \right) + \left( \frac{R_p - R_t}{2} \right) \cos \left( \frac{2\pi x}{l_p + l_t} \right) > \frac{1}{2} R_p
\]  

(3.9)

Re-arranging

\[
\frac{1}{2\pi} \arccos \left( \frac{R_t}{R_t - R_p} \right) > \frac{x}{l_p + l_t}
\]

(3.10)

where \( x \) is equal to \( l_p / 2 \) the boundary location. If ‘\( > \)’ sign is used in Equation 3.10 instead of ‘\( = \)’, the final pore length can be given by the following expression:

\[
l_{p\text{-final}} = 2x = 2(l_p + l_t)\left( \frac{1}{2\pi} \arccos \left( \frac{R_t}{R_t - R_p} \right) \right)
\]

(3.11)

where \((l_p + l_t)\) represents the spacing between the pore and the throat. If Equation 3.10 is not satisfied, the pore length is reduced using Equation 3.10 with \((l_p + l_t)\) held constant.

3.4 Fluid flow through the network

Initially, the system is assumed to be completely filled with a defending fluid with viscosity \( \mu_1 \). The invading fluid with viscosity \( \mu_2 \) is injected into the system from the inlet side with a constant injection rate. For example, in simulating primary drainage, the non-wetting fluid (oil) is the invading fluid and the wetting fluid (water) is the defending fluid. The fluids are assumed to be immiscible and there is no dispersion of the phases, so that there is a well-defined interface between the two phases. In addition, the fluids are assumed to be incompressible. We assume we have a water-wet system with an oil/water contact angle \( \theta = 0 \).

3.4.1 Determination of fluid configurations

We start by assuming the volume of each phase in each pore or throat is known. Then, with known contact angles, the fluid configuration and the local capillary pressure can be computed.

The total cross-sectional area \( (A_t) \) for an element (pore or throat) filled with a single
phase is
\[ A_t = R^2 \sum_{i=1}^{n} \cot(\alpha_i) = CR^2 \] (3.12)

where \( n \) represents the number of the corners, \( \alpha \) is the corner half angle and \( C \) is a constant defined by Equation 3.12. Therefore, the volume for a fluid that occupies the whole cross-section can be given by the following equation:
\[ V = \int_{x_1}^{x_2} CR^2(x)dx = C \int_{x_1}^{x_2} R^2(x)dx \] (3.13)

where \( R(x) \) is given by Equation 3.1. Here, \( x_1 \) and \( x_2 \) are the limits of the integration that are determined by the location of the fluid interfaces. For example in Figure 3.2a, the water fills the pore from the centre up to the first oil-water interface, so here, \( x_1 = 0.0 \) and \( x_2 \) is the location of the interface.

In the case of water in wetting layers and the oil in the centre, the wetting layer volume is equivalent to the volume of a cylinder that has cross-sectional area equal to the wetting layer cross-section and length equal to the difference of the location of the fluid interfaces.
\[ A_{ci} = r^2[\cos \theta(\cot \alpha_i \cos \theta - \sin \theta) + \theta + \alpha_i - \frac{\pi}{2}] \] (3.14)

where \( A_{ci} \) is the cross-sectional area of the \( i^{th} \) wetting layer, \( r \) is the radius of the curvature of the wetting layer and \( \theta \) is the contact angle. The total cross-sectional area for all the wetting layers is
\[ A_c = \sum_{i=1}^{n} A_{ci} \] (3.15)

where \( n \) represents the number of the corners.

The oil volume will be the difference between the total volume given by Equation 3.13 and the wetting layer volume. For example in Figure 3.2, the wetting layer volume between the interface \( (x = x_2) \) and the pore/throat boundary is obtained by computing the wetting layer cross-sectional area at the location of the interface \( (x = x_2) \) using Equation 3.15 and multiplying it by the difference between the location of the interface and the pore/throat boundary. Equation 3.13 is used to find the volume between the interface
(x = x_2) and the pore/throat boundary. The oil volume then will be the difference between these two volumes. For simplicity, in the volume calculations we assume that fluid interfaces, except in the wetting layers, are flat. However, interfacial curvature is accounted for in the computation of capillary pressure.

Figure 3.2: Illustration of a fluid configuration: oil occupies the pore/throat boundary. (a) Side view: the inscribed radius of the pore element ($R(x)$) varies sinusoidally with the length. (b) Cross-sectional view at the pore/throat boundary: the pore element has a triangular cross-section. Oil is in the centre and water is in the corners (wetting layers). $\theta$ is the contact angle and $r$ is the radius of curvature of the wetting layers. We assume that $r$ is same in all the three corners and constant in each pore or throat, but varies between pores and throats and over time.
3.4.2 Computing the fluid resistance

Calculating the fluid resistance is potentially a complicated problem, but using an equivalent electrical resistors network helps to simplify the computations. Before giving a detailed description of how the equivalent hydraulic resistance is obtained, it is worthwhile to state the expressions used in finding the resistance of each phase. A phase may occupy: the whole element cross-section, the corners with the other (non-wetting) phase in the centre, or the centre of the element with the other (wetting) phase in the corners. The general form of the fluid hydraulic resistance ($W$) for the three regions is:

$$W = \mu_f \int_{x_1}^{x_2} \frac{dx}{G(x)_{\text{region}}}$$  (3.16)

where the subscript \textit{region} stands for the three regions: whole cross-section, centre or wetting layers; $x_1$ and $x_2$ are the location of the interfaces; $G(x)$ is the fluid conductance per unit length and $\mu_f$ is the fluid viscosity. The conductance per unit length of a fluid occupying the whole cross-section is given by the following approximation based on Poiseuille’s law for flow in a circular cylinder (Hui and Blunt (2000)):

$$G = \frac{\pi}{128} \left( \frac{A_t}{\pi} + R \right)^4$$  (3.17)

where $A_t$ is the cross-sectional area given by Equation 3.12 and $R$ is the inscribed radius given by Equation 3.1. Since $R$ in Equation 3.17 is function of $x$, the integration in Equation 3.16 is evaluated numerically.
In the case where fluid occupies the corners with $\theta + \alpha < \frac{\pi}{2}$, the conductance per unit length is given by the following approximation (Zhou et al. (1997)):

$$G = \sum_{i=1}^{n} \left[ \frac{A_{ci}^2(1 - \sin \alpha_i)^2}{12(\sin \alpha_i)^2(1 - \phi_3)^2(\phi_2 + f \phi_1)^2} \right]$$  \hspace{1cm} (3.18)$$

where $\phi_1 = \frac{\pi}{2} - \alpha_i$, $\phi_2 = \cot \alpha_i \cos \theta - \sin \theta$, $\phi_3 = (\frac{\pi}{2} - \alpha_i) \tan \alpha_i$ and $A_{ci}$ is the corner area given by Equation 3.14. $f$ is used to indicate the boundary condition at the fluid/fluid interface, $f = 1$ represents a no-flow boundary, while $f = 0$ is a free boundary. In Equation 3.18, we assume $f = 1$. The curvature of the wetting layer is assumed to be constant in a single element (i.e $G$ is not function of $x$), although it varies between pores and throats. Thus, we can write the hydraulic wetting layer flow resistance ($W_l$) as:

$$W_l = \frac{\mu_f (x_2 - x_1)}{G}$$  \hspace{1cm} (3.19)$$

ignoring the curvature of the pores and throats in the $x$ direction.

When fluid occupies the centre of an element with wetting phase in the corners, the conductance per unit length is given by Equation 3.17 replacing $A_i$ by $A_{cen}$, where $A_{cen} = A_i - \sum_{i=1}^{n} A_{ci}$, $A_{ci}$ is the corner area, $n$ represents the number of corners.

Using these formulae enables the model to handle any number of fluid interfaces between pore and throat centres. The use of the electrical resistors diagram and equivalent hydraulic resistance simplifies and clarifies this approach. For example, Figure 3.2a shows a fluid configuration where oil occupies the centre at the pore/throat boundary. Its equivalent electrical resistors diagram is given in Figure 3.3. Thus, the equivalent hydraulic resistance of the fluids between the interfaces ($W_{eq\;interface}$) is given by the following equation:

$$\frac{1}{W_{eq\;interface}} = \frac{1}{W_{p-l}^{w}} + \frac{1}{W_{T-l}^{w}} + \frac{1}{W_{p-c}^{o}} + \frac{1}{W_{T-c}^{o}}$$  \hspace{1cm} (3.20)$$

where $W_{p-l}^{w}$ is the pore wetting layer resistance, $W_{T-l}^{w}$ is the throat wetting layer resistance, $W_{p-c}^{o}$ is the pore oil resistance and $W_{T-c}^{o}$ is throat oil resistance. Then, the equivalent hydraulic resistance for the fluids between pore and the throat centres ($W_{eq}$) can be obtained.
through the following expression:

\[ W_{eq} = W_{p-c}^w + W_{e\text{interface}} + W_{T-c}^w \]  \hspace{1cm} (3.21)

where \( W_{p-c}^w \) is the pore water resistance and \( W_{T-c}^w \) is the throat water resistance. This is only one example of how the equivalent hydraulic resistance is computed.

Appendix A lists the expressions used for the different fluid configurations considered in the model. We only allow up to two menisci to be present in each throat and one meniscus in each pore branch.

### 3.5 Solving for the fluid pressure

Since the equivalent hydraulic resistance is used in the volume conservation equations, the model can be considered as solving a single phase flow problem in which the phase conductance between the pore and throat centres is found from the equivalent hydraulic resistance calculated in the previous step. Therefore, the total flow rate between the pore centre and throat centre will be:

\[ Q_{\text{total}} = \frac{P_p - P_t + P_c}{W_{eq}} \]  \hspace{1cm} (3.22)

where \( P_p \) and \( P_t \) are the pore centre pressure and throat centre pressure respectively, \( P_c \) is the sum of the capillary pressures of the menisci between the pore and throat centres. For instance, in Figure 3.2, \( P_c = P_{c1} + P_{c2} \), where, \( P_{c1} \) is the capillary pressure at the first interface \((x = x_2)\) and \( P_{c2} \) is the capillary pressure at the second interface \((x = x_3)\).

The absolute value of the capillary pressure at any meniscus is given by the following expression:

\[ P_c = \frac{2\sigma \cos(\theta + \gamma)}{R(x)} \]  \hspace{1cm} (3.23)

where \( \gamma \) is the inclination angle; \( \tan \gamma = \frac{dR(x)}{dx} \). The sign of the capillary pressure at any meniscus depends on the location of the non-wetting fluid (oil). If it is on the right of the meniscus, the sign is positive, otherwise it will be negative.
For a system of $m$ throats and $n$ pores, we have $(m+n)$ unknowns. These unknowns are determined by applying volume conservation for both the pores and throats. The conservation equation for pore $j$ with $n$ connecting throats, labelled $i$ is:

$$\sum_{i=1}^{n} Q_{\text{total}}^{ij} = \sum_{i=1}^{n} \frac{P^i_j - P^j_i + P_{\text{eq}}}{W_{\text{eq}}^{ij}} = 0 \quad (3.24)$$

For a throat $i$, where $R$ and $L$ label the left and right pores:

$$\frac{P^R_i - P^L_i + P_{\text{eq}}^R}{W_{\text{eq}}^{Ri}} + \frac{P^L_i - P^R_i + P_{\text{eq}}^L}{W_{\text{eq}}^{Li}} = 0 \quad (3.25)$$

We use Equation 3.25 to find the throat pressures that are then put in Equation 3.24 to obtain a series of linear equations for pore pressures only. Once the pore pressures have been found, Equation 3.25 can be used to compute the throat pressures. Equation 3.24 is solved using a standard iterative matrix solver.

The pressures are used to compute the phase flow rates across the pore/throat boundaries. For example, from the equivalent resistors diagram of Figure 3.2a, the water flow rate across the pore/throat boundary is the wetting layer flow rate and it is given through the following equation:

$$Q_{\text{water}} = \frac{(P_p - P_t + P_c)}{W_{\text{eq}}(W_{p-c}^w + W_{T-c}^w)(W_{eq} - (W_{p-c}^w + W_{T-c}^w))} \quad (3.26)$$

The oil flow rate is:

$$Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}} \quad (3.27)$$

### 3.5.1 Solving for constant injection rate

While we set up the pressure equation for constant inlet and outlet pressures we want to simulate flow with a constant injection rate. Aker et al. (1998) have shown how to achieve a constant injection rate in a dynamic network model. However, their method involves solving for the pressure field twice at each time step. Here we use an approximate technique that only requires a single pressure solution. Over time the pressure drop across
the network changes. We assume that between time steps the change in pressure drop necessary to maintain a constant injection rate is small. Hence, we simply adjust the pressure drop at each time step to maintain a constant value of $Q$ as follows:

- For the $n^{th}$ time step, the pore and throat pressures are computed as described above, with a pressure drop $\Delta P^n$.

- The total injection rate ($Q^n$) is then obtained by summing all the flow rates between the inlet throats and their connected pores.

- For the next pressure solve, for the $(n+1)^{th}$ time step we use a pressure drop:

$$\Delta P^{n+1} = \Delta P^n \times (1 + \beta \left( \frac{Q_{\text{desired}} - Q^n}{Q_{\text{desired}}} \right)) \quad (3.28)$$

where $Q_{\text{desired}}$ is the desired, target injection rate and $\beta$ is a constant parameter, which we set to 0.5. This method maintains $Q^n$ to within 0.5% of $Q_{\text{desired}}$ for the cases we consider.

### 3.6 Selection of the time step

We choose a time step ($\Delta t$) according to the following formula:

$$\Delta t = \min \{5 \times 10^{-5} \text{s}, \min \{ \frac{V_i}{2Q_i} | i = 1, \ldots, n \} \} \quad (3.29)$$

where $n$ is the number of the elements in the pore network, $V_i$ is the $i^{th}$ element volume (i.e. the maximum amount of fluid that can be held in the $i^{th}$ element) and $Q_i$ is the total flow rate into the $i^{th}$ element. Equation (3.29) ensures that an element cannot be completely filled in a single time step. The time step value of $5 \times 10^{-5}$ s ensures that in most cases only a small fraction of a volume of a pore or throat is filled with invading fluid in a time step.
3.7 Updating fluid volumes

Once the fluid volumes are determined, the configuration of the phases in the elements is adjusted. There are two steps in this process. First, the pressure computation only determines the total flow of oil and water between pore and throat centres—we need to use the fluid configuration to determine the flow rates of each phase from pore to throat. Having done this the water volume in each element is updated. From the new fluid volume the configuration of each phase in each element is determined and a new total fluid resistance can be found, the pressure recomputed and the simulation continues. For example, consider the fluid configuration shown in Figure 3.2a. By assuming the total flow rate across the pore/throat boundary given by Equation 3.22 and the flow direction is towards the throat, the water flow rate in the throat \( Q_{\text{in water}} \) is given by Equation 3.26 and is small, since this is only flow in layers. Similarly we can compute the flow rate of water out of the throat at the pore/throat boundary to the right of the throat centre \( Q_{\text{out water}} \). The new water volume in the throat for time level \( n \) is given by the following expression:

\[
V_{\text{water}}^n = V_{\text{water}}^{n-1} + (Q_{\text{in water}}^n - Q_{\text{out water}}^n) \times \Delta t
\]  

(3.30)

Then Equation 3.13 is reused to find the new location of the interface, with \( V = V_{\text{water}}^n \) and \( x_2 = \frac{l_p + l_t}{2} \) (i.e the throat centre). It is clear that the interface location cannot be obtained from a direct substitution, and so an iterative method must be used to obtain an interface location consistent with the imposed change in volume.

3.8 Computing the total phase saturations

The network wetting phase saturation \( S_w \) can be given by the following equation:

\[
S_w = \frac{\sum_{i=1}^{n} V_{wi}}{\sum_{i=1}^{n} V_i}
\]

(3.31)

where \( n \) is the number of elements in the network, \( V_{wi} \) is the volume of water in the \( i^{th} \) element and \( V_i \) is the volume of the \( i^{th} \) element.
The volume of the wetting phase in each element can be obtained by updating the phase volume every time step using Equation 3.30 or using the current fluid configuration. By knowing the fluid configuration, the location of the interfaces can be determined and then the phase volume can be computed. For example, from the fluid configuration shown in Figure 3.2a, the water volume in the pore branch can be given by the following expression:

\[ V_{w\ ji} = V_{w\ ji-portion} + V_{w\ ji-layer} \]  

(3.32)

where \( V_{w\ ji} \) is the total water volume in the \( j^{th} \) branch of the \( i^{th} \) pore, \( V_{w\ ji-portion} \) is the volume of the portion of the pore branch that is totally filled with water. Equation 3.13 is used to find this volume between the pore centre (\( x_1 = 0 \)) and the current location of the water/oil interface in the pore branch (\( x = x_2 \)). \( V_{w\ ji-layer} \) is the wetting layer volume.

The total pore water volume (\( V_{wi} \)) then can be given as follows:

\[ V_{wi} = \sum_{j=1}^{m} V_{w\ ji} \]  

(3.33)

where \( m \) is the number of branches in the \( i^{th} \) pore.

### 3.9 Summary

Transporting fluids between the pores can be summarized in the following points:

1. Obtain the conductance matrix whose elements represent the overall conductances between the pore centres.

2. Use some computer package to solve the pore pressures (pressure solver).

3. Compute the volumetric rate and updating the fluid configurations.

4. Update the fluids saturations (if it is necessary).

5. Repeat the above steps (1-4), until some stop criterion has been reached.

Any displacement process (i.e primary drainage or imbibition) starts with assuming that the network is fully saturated with defending fluid. An interface is a located at the
centre of the inlet throats to indicate the start of the displacement process. Then, we will start moving the fluids between the pores by implementing the above four steps.
Chapter 4

Primary drainage

4.1 Introduction

Reservoir rock is initially fully saturated with water before primary oil migration. Oil migration is a primary drainage process where a non-wetting phase (oil) invades a medium that is full of wetting phase (water).

4.2 Micro-flow mechanisms of primary drainage

The fluid displacement mechanisms can be divided into two main types: piston-like and snap-off mechanisms. The purpose of this section is to explain how the model can be used to simulate these pore-level processes that have been seen in micromodel experiments (Lenormand and Zarcone (1984)).

4.2.1 Piston-like displacement

This is the process by which the invading fluid pressure is high enough to allow it to enter the bulk (centre) of the element (pore or throat) by pushing the displaced fluid in front of it. There are three types of piston-like invasion: invasion of a single pore branch or throat side, invasion of a pore centre and menisci fusion.
Chapter 4. Primary drainage 4.2. Micro-flow mechanisms of primary drainage

**Invasion of a single pore branch or throat side**

This type of invasion occurs if the pressure of the element that contains the invading fluid is higher than the pressure of the connected element that contains the displaced fluid plus the capillary pressure at the fluid interface. For example, Figure 4.1a shows a pore branch whose centre is filled with oil is connected to a throat that is fully water saturated. If the pore pressure is sufficiently high, the oil advances towards the throat centre (Figure 4.1b). The amount of oil that enters the throat is controlled by the pressure difference between the pore and the throat and the pore-throat geometries. The oil continues moving until it passes the throat centre and starts filling the other side of the throat (Figure 4.1c).

This mechanism is modelled by changing the fluid configurations according to the location of the oil/water interface. Figure 4.2 shows a simplified flow chart of how this is done. From Figure 4.1, it is clear that this mechanism involves two fluid configurations: where the meniscus is between the left pore and the throat centre, and where the meniscus is between the throat and the right pore centre. The equations for the fluid equivalent resistance, phase fluxes and field pressures for both configurations are provided in Appendix A. If the volume of oil that enters the system (displaced volume) is positive, the oil will enter the throat, otherwise, the fluid configuration remains unchanged. If oil enters the throat, the volume of the oil in the throat and the new location of the oil/water interface are determined according to the procedure explained in the previous chapter. If the new interface can be located within the current fluid configuration, there will be no change in the fluid configuration, otherwise, the fluid configuration will be changed to one in which the location of the new interface has moved beyond the centre of the throat, as shown in Figure 4.1.

**Invasion of a pore centre**

When an interface reaches the pore centre, it is moved into the neighbouring flow channels. In our model, the same method to do this is used for both oil invasion (drainage) and water invasion (imbibition).

Consider the diagram shown in Figure 4.3a. If the oil volume entering from the lower branch is more than the water that is displaced from the branch, the remaining oil will be
Chapter 4. Primary drainage  

4.2. Micro-flow mechanisms of primary drainage

Figure 4.1: Invasion of a single pore branch or throat side. a) Oil is filling the pore branch. b) Oil is entering the throat side. c) Oil is passing the throat centre.

The remaining oil is distributed so that there is the same capillary pressure at the interfaces in all the connected branches. Then in subsequent time steps, the oil will move according to the flow direction in each branch, Figure 4.3c. In summary:

1. Check if the remaining oil is more than the summation of the connected branches volume. If this is so, the whole pore is filled with oil, otherwise we move to the
next step.

2. Sort the branches according to their size. The branch with the smallest size is at the top of the list and the one with the largest size is at the bottom of the list.

3. The branch with the smallest size will be used as a reference point. In other words,
the procedure will start by assuming an initial guess of the location of the interface in the branch of the smallest size. The location of this interface then will be used to find the capillary pressure in the branch. Then, by maintaining the same capillary pressure in all other branches, the location of the interfaces in these branches will be found using an iterative method.

4. The total volume can be given by the following expression:

\[ V = \sum_{i=1}^{m} [V_i(x)]_{x_1=0}^{x_2=x_i} \]  \hspace{1cm} (4.1)

where \( V_i(x) \) is given by Equation 3.13, \( x_i \) is the location of the interface in the \( i^{th} \) branch obtained from step 2 and \( m \) is the number of branches.

5. Check if the volume obtained in step 4 is equal to the volume of the remaining oil.

If it is so, these are the right locations for the interfaces, otherwise the steps 3 – 4 are repeated again until the correct locations are reached.

Sometimes, the solution might be reached and one or two of the interfaces in the branches of the smallest size are beyond the pore/throat boundary. In this case, we assume these branches are completely filled with oil and their volumes are subtracted from the original remaining oil. Then the above procedure is repeated to distribute the remaining oil among the remaining branches.

**Menisci fusion**

Menisci fusion is an oil invasion mechanism in which a throat that connects two fully oil-filled pore branches holds two menisci back-to-back that come together. The fusion of two menisci in a throat is straight forward to model and occurs when the interface locations coincide, at which point all the water volume is accommodated in layers. However, more complex situations may arrive if both oil and water are flowing in the same direction through a throat, as shown in Figure 4.4. This is the typical situation at high capillary numbers. Oil displaces water through a throat in a piston-like fashion until the oil/water meniscus reaches the pore/throat boundary, Figure 4.4b. In reality further displacement...
Figure 4.3: Oil invasion of a pore centre. a) Oil approaching the pore centre. b) When the volume of oil flowing into the pore exceeds the volume of the lower branch, oil is distributed in all branches, regardless of flow direction. The capillary pressure in all the branches is same. c) In the subsequent time steps the oil interfaces are updated according to the local flow direction.

would lead to there being two menisci in the pore branch. However, this is not allowed in our model. Instead we freeze the meniscus at the pore/throat boundary. We assume that the flux from the element centre into pore two is from oil. However, we keep the interface location fixed until the water volume is too small to be accommodated in the pore centre and the two menisci fuse leaving water only in layers.
4.2. Micro-flow mechanisms of primary drainage

Figure 4.4: Oil invasion. a) Oil begins to invade a throat. b) Invasion may continue until the meniscus reaches the pore/throat boundary. We do not allow two menisci in a single pore branch. Hence the meniscus remains frozen in place until the water volume in pore two is insufficient to support occupancy of the element centre. Then the menisci fuse, and water only occupies layers.

4.2.2 Snap-off

Snap-off is a mechanism that is controlled by wetting layer flow. Water accumulates in layers until oil no longer contacts the solid and water spontaneously fills the centre of the throat separating the oil into two droplets. The accumulation of water in the wetting layers is function of the contact angle and the pore/throat aspect ratio.

In our model, there are two types of snap-off. Snap-off that occurs to the oil that invades a fully water-saturated throat will be called snap-off in drainage. The second snap-off occurs in a throat that is already filled with oil. Here, the water starts accumulating in the wetting layers as a result of a drop in the capillary pressure. Many authors have described this process (see for instance, Blunt et al. (2002) and Mogensen and Stenby (1998)). This type of snap-off will be called snap-off in imbibition, since it is common in imbibition. However, there is nothing preventing it from happening in drainage as mentioned by Toledo et al. (1994a,b).
Snap-off in drainage

When the non-wetting phase invades a fully water-saturated throat, the wetting phase will remain in the corners and crevices of the throat space to form wetting layers. As the non-wetting fluid passes the narrowest section of the throat, the radius of the curvature of the wetting layers will be at its smallest value. Then, with increasing oil volume in the throat, the meniscus is pushed into wider regions, the capillary pressure decreases and the wetting phase in the layers starts to accumulate (see Figure 4.5). This process may continue until the wetting fluid cannot be held in the wetting layers any more. At this point, it will snap-off at the narrowest region of the throat separating the oil into two droplets providing two oil-water interfaces one to the right of the centre of the throat and the other to the left, as shown in Figure 4.5c. The determination of the location of the interfaces is based on knowing the volume of the water at the throat centre and assuming a constant capillary pressure, where the capillary pressure is given by Equation 3.23.

If the volume of water flowing into the throat is more than the water flowing out in Figure 4.5c, the volume of water in the centre of the throat will increase. This means that oil will enter the throat but it cannot penetrate it. If the water flow into the throat is less than that flowing out, the volume of water in the centre will shrink until the two oil menisci meet, reconnecting the oil and the oil then continues flowing to the next pore.

Snap-off in imbibition

Micromodel observations in water-wet systems with zero initial water saturation for low capillary number show that flow through wetting layers advances ahead of the bulk flow through fully water-saturated elements (Lenormand and Zarcone (1984)). Here, when water in wetting layers invades an oil-filled throat, the layers swell (Figure 4.6a). This accumulation will continue until the water can no longer be accommodated in layers. Then snap-off happens and the throat at its narrowest point will be rapidly filled with water (Figure 4.6b).

Our strategy for modelling this type of snap-off is illustrated in Figure 4.7. Snap-off occurs when the volume of water at the narrowest part of the throat is too large to be accommodated in layers (i.e when the water no longer is in contact with the solid surface).
Figure 4.5: Snap-off in drainage. a) Oil is approaching the narrowest region of the throat. b) Water in wetting layers accumulates when the capillary pressure drops as oil advances into wider regions of the throat space. c) Water snaps-off at the centre of the throat separating the oil into two droplets.
At this point water spontaneously and instantly fills the centre of the throat and the oil is separated into two. Notice that when this happens the local capillary pressure increases. The snap-off capillary pressure \( P_{c,\text{snap-off}} \) is obtained by the flowing expression:

\[
P_{c,\text{snap-off}} = \frac{\sigma}{R} \left( \cos \theta - \frac{2 \sin \theta}{\cot \alpha_1 + \cot \alpha_2} \right)
\]

where \( R \) is the throat radius at the centre.

---

Figure 4.6: Modeling snap-off in imbibition. a) Water accumulates in wetting layers. b) Water snaps-off at the centre of the throat.
Figure 4.7: Modelling snap-off in imbibition. Capillary pressure is shown as a function of water volume for an example throat. When the water volume in the throat is sufficiently large, the water can only be accommodated by having the center of the throat completely filled with water - this represents snap-off.
4.3 Primary drainage results

The flow is characterised by two dimensionless numbers: the capillary number, $C_a$, Equation 2.2, and the viscosity ratio, $M$, which is defined as the ratio between the defending fluid viscosity $\mu_1$ and the invading fluid viscosity $\mu_2$.

\[
M = \frac{\mu_1}{\mu_2}
\]  \hspace{1cm} (4.3)

We divide our results under three categories: the influence of capillary number on the dynamic fluid movement, the relation between the capillary number (flow rate) and snap-off and the influence of viscosity ratio.

4.3.1 Influence of capillary number on the dynamic fluid movements

In this subsection, we change the capillary number by varying the injection flow rate. The general understanding of oil invasion is that at high capillary numbers the oil fills all the pore elements (in all directions) and moves towards the outlet face in a piston-like fashion (Lenormand and Zarcone (1984)). However, at low capillary number, the oil flows through a pathway of larger pores and throats with the lowest capillary entry pressure. This leads to a ramified, invasion percolation-like displacement that can be simulated readily with quasi-static models (Toubou et al. (1987), Blunt et al. (1992)). In this section we will test whether or not our model reduces to the quasi-static limit as the flow rate is decreased.

Figure 4.8 shows the fluid distribution for simulations at different capillary number (oil is shown in grey and water in black). In all the simulations, we used a 2D network of 9×9 pores, a unit viscosity ratio (fluid viscosity of 1 Pa.s) and 0.05 N/m interfacial tension. The selection of this network size is based on optimization of the computation time that is required to complete the runs. The run for the lowest capillary number ($C_a = 3.1 \times 10^{-5}$) took around 55 hours on a standard PC. The runs were stopped at first oil breakthrough. The Darcy oil velocity is obtained by dividing the oil injection rate by the inlet cross-sectional area. The oil injection rate is the sum of the oil flow rates between the inlet
throats and the connecting pore branches. The inlet cross-sectional area (0.52 mm$^2$) is the product of the length of the inlet (2600 µm) and the mean thickness of the lattice which is taken to be the average diameter of the pores (200 µm). Each simulation was performed at a constant injection rate. In the pictures shown in Figure 4.8, each pore is connected to four throats and a small black line is used to distinguish between the pore branch and the throat. In addition, when snap-off occurs there are throats having two interfaces (water occupies the throat centre). Figure 4.8a shows the results of a quasi-static model that ignores rate-dependent effects for the same network (Nickel and Wilkinson (1983)) to compare it with the dynamic one at the lowest capillary number $C_a = 3.1 \times 10^{-5}$ studied (Figure 4.8b).

As the capillary number is increased, the oil flows through more of the inlet pores and sweeps more of the network, although there is an increasing frequency of snap-off. At the lowest capillary numbers, the displacement is dendritic and the sequence of pores and throats are filled is largely controlled by their entry capillary pressure (determined by the minimum inscribed radius of the throat). The displacement patterns for $C_a = 3.1 \times 10^{-5}$ and $C_a \to 0$ are similar, although not identical, since the perturbative effect of viscous forces does affect the exact pathway of filled pores and throats, especially away from the inlet. As the flow rate increases, viscous forces become more significant and small pores near the inlet may be filled in preference to larger pores or throats near the outlet, because of the significant pressure drop across the network. Furthermore, dynamic events, such as snap-off, become more common, and the oil is not necessarily connected to the inlet, although it is still flowing. At the highest capillary number studied, 0.33, oil moves largely indiscriminately through pores and throats of any size as a train of generally disconnected ganglia.

**Fractional flow curves**

The fractional flow of phase $i$ in a multiphase system can be given through the following expression:

$$f_i = \frac{Q_i}{Q_{total}}$$  \hspace{1cm} (4.4)
Figure 4.8: Fluid distributions for simulations of primary drainage at different capillary number ($C_a$). In this and subsequent figures, the water in the centres of pores and throats is shown in black and oil is shown in grey. The fine black lines separate pores and throats. The distributions at oil breakthrough are shown. a) A quasi-static model, representing the limit of $C_a \to 0$, b) A run for $C_a = 3.1 \times 10^{-5}$, c) $C_a = 3.8 \times 10^{-4}$, d) $C_a = 0.33$.

where $Q_i$ is the flow rate of phase $i$ and $Q_{total}$ is the total flow rate (of both phases).

In our model, we used a small 2D network ($17 \times 17$ pores) to study the influence of the
capillary number on the fractional flow curves. The computer time needed to model the
dynamics of wetting layer flow precluded the use of a larger network. The average saturation
was computed in a slice of 4-pore length distance at the middle of the network. The phase and total flow rates were obtained at the centre of the slice by using the equations provided in the section on solving the fluid pressure.

Figure 4.9 shows the water fractional flow as a function of water saturation for different capillary numbers. If viscous forces were completely dominant, with oil and water flowing together, then we expect \( f_w = S_w \). (Dullien (1992), Koplik and Lasseter (1985), Blunt et al. (1992)). Even at the higher capillary numbers, the effect of wetting layer flow and the wide pore size distribution prevent the fractional flow becoming linear. However, as \( C_a \) increases, \( f_w \) does tend towards a straight line.

At low capillary number, when capillary forces dominate, the oil and water occupy different pathways –see Figure 4.8– with little movement at most oil/water interfaces. As capillary number increases, more menisci become mobile. This explains why the fractional flow decreases with decreasing capillary number and has an S shape, characteristic of low rate experimental measurements at the lowest values (Dullien (1992)). While the fractional flow curves for quasi-static model, \( C_a \to 0 \), and \( C_a = 6.2 \times 10^{-5} \) have a similar shape, the quasi-static curve appears to be shifted to lower water saturation which is a result of the difference of the water saturation in wetting layers. In the quasi-static model the capillary pressure is the maximum local entry pressure reached during a displacement. This means that wetting layers tend to carry relatively little water. In contrast, the dynamic model allows locally lower capillary pressures with large amounts of water retained in layers. However, these layers carry relatively little flow. For instance, Figure 4.7 shows that up to half the total volume of a typical throat may be filled with water in layers. In other words, for a specific value of fractional flow in Figure 4.9, the water saturation in wetting layers for the quasi-static model is lower by around 0.2 than that for the dynamic model (\( C_a = 6.2 \times 10^{-5} \)). While our model may tend to over-estimate the effects of wetting layer flow, it does indicate that the amount of water contained in layers is very sensitive to dynamic effects and may not be accurately predicted by static models.
Figure 4.9: Water fractional flow as a function of water saturation for different capillary numbers.

**Relative permeability**

Relative permeability can be defined as follows (Dullien (1992), Knudsen et al. (2001)):

\[
\frac{Q_{nw}(S_w)}{A} = \frac{k_{rnw}(S_w)k}{\mu_{nw}} \times \frac{\Delta P(S_w)}{l} \quad (4.5)
\]

\[
\frac{Q_{w}(S_w)}{A} = \frac{k_{rw}(S_w)k}{\mu_{w}} \times \frac{\Delta P(S_w)}{l} \quad (4.6)
\]

where \(k_{rnw}\) and \(k_{rw}\) are the non-wetting and wetting relative permeability respectively, \(\Delta P\) is the network pressure drop, \(l\) is the length of the network in the flow direction and \(k\) is the absolute permeability of the network for single-phase flow. With some mathematical
manipulation, the relative permeability equations become:

\[
k_{rnw}(S_w) = \frac{Q_{nw}(S_w)}{Q_s} \times \frac{\mu_{nw}}{\mu_s} \times \frac{\Delta P_s}{\Delta P(S_w)} \quad (4.7)
\]

\[
k_{rw}(S_w) = \frac{Q_{w}(S_w)}{Q_s} \times \frac{\mu_w}{\mu_s} \times \frac{\Delta P_s}{\Delta P(S_w)} \quad (4.8)
\]

where the subscript \(s\) stands for single phase flow. Because of the presence of the capillary force, the sum of the relative permeabilities is not unity since interfaces block the flow (i.e. \(k_{rnw} + k_{rw} \neq 1\)).

Figure 4.10a shows two-phase relative permeability curves at unity viscosity ratio for two capillary numbers. The relative permeability curves were obtained by using the same procedure that is followed in obtaining the fractional flow curves. There is a difference in the total mobility \((k_{rnw} + k_{rw})\) between the two curves. This is because at low capillary number, there are more capillary-blocked pores which reduces the mobility of the oil. The water relative permeability appears to be insensitive to capillary number. Figure 4.10b shows similar plots of experimental results in primary drainage for an artificial sintered medium (Lefebvre du Prey (1973)) for unity viscosity ratio. The two figures show good agreement, particularly for the oil relative permeability.

### 4.3.2 Connected and disconnected flow

Snap-off causes oil to become disconnected in primary drainage. The resultant oil ganglia can flow through the network. The contribution of ganglion transport to the overall flow of oil is represented by the difference between the total oil and connected oil fractional flow curves in Figure 4.11. The connected oil fractional flow only considers the flow of oil that is connected to the inlet. It is clear that disconnected flow is only appreciable for the largest flow rates, and it is insignificant for most typical reservoir displacements. This contradicts the finding of Payatakes and co-workers who studied two-phase flow behavior in imbibition and found significant ganglion transport for low capillary numbers (Valavanides et al. (1998), Valavanides and Payatakes (2001)). Our model, however, has a wider pore size distribution and accounts explicitly for wetting layer flow and simulates...
4.3.3 Influence of viscosity ratio on fluid movement

In this subsection, we study the effect of viscosity ratio on displacement patterns and the degree of snap-off. For illustrative purposes we ran simulations with a capillary number of $8.3 \times 10^{-2}$ on the same $9 \times 9$ network as before. We also ran a series of simulations on
Figure 4.11: The oil fractional flow and oil fractional flow from oil connected to the inlet as a function of oil saturation for several capillary numbers. a) $C_a = 0.24$. b) $C_a = 2.1 \times 10^{-3}$. c) $C_a = 6.2 \times 10^{-5}$.
a statistically similar 30 × 30 network with a capillary number of $1.1 \times 10^{-2}$.

At high flow rates and viscosity ratios less than 1 (oil more viscous than water), it would be expected that the displacement of oil by water would be stable, with a relatively flat front progressing through the system. Figure 4.12a illustrates a displacement for $M=0.1$ that confirms this. However, there is a significant amount of snap-off and a significant proportion of the oil moves as disconnected ganglia. The reason for large amounts of snap-off can be explained by studying the fluid resistance of a pore element that contains oil in the centre and water in wetting layers. As explained in the previous chapter, the phase resistance is a combination of the geometry (i.e whether the phase occupies the centre, layer or whole pore) and viscosity. Therefore, with an oil viscosity ten times higher than the water viscosity, in the large pores, the oil resistance will be of the same order of magnitude as the wetting layers. This means that the oil flow is relatively slow compared to the accumulation of water in wetting layers. Thus water has sufficient time to accumulate and snap-off at the centre, leading to the generation of the oil ganglia seen in Figure 4.12a.

Figure 4.12: Influence of viscosity ratio on fluid movement. a) 2D image of fluid distribution for $M = 0.1$. b) 2D image of fluid distribution for $M = 10$.

For a viscosity ratio greater than one ($M=10$), the oil fingers through the water (Figure 4.12b), since the displacement is now unstable. In addition, the wetting layer flow is
less significant compared to oil flow in the pore centres, since the water is relatively more viscous, and as a consequence there is less snap-off.

Figure 4.13: Influence of viscosity ratio on fluid movement. a) The total oil fractional flow and connected oil fractional flow curves for $M = 0.1$. b) The total oil fractional flow and connected oil fractional flow curves for $M = 10$.

Figure 4.13 shows the total oil fractional flow and the oil fractional flow considering...
only the movement of oil that is connected to the inlet for a 2D network of size of $30 \times 30$, for (a) $M = 0.1$; and (b) $M = 10$. For $M = 0.1$, snap-off is common and an appreciable amount of oil transport is from disconnected oil (Figure 4.13a). In the case of $M = 10$, the oil fingers into the water through the larger pores and throats, which means a high oil fractional flow is reached at low oil saturation. In addition, there is not much difference between the total oil fractional flow curve and the continuous one which indicates less occurrence of snap-off.

![Figure 4.14: Influence of viscosity ratio on relative permeability curve. a) Relative permeability curve for $M = 0.1$. b) Relative permeability curve for $M = 10$.](image)

Figure 4.14 shows relative permeability as a function of oil saturation for (a) $M = 0.1$; and (b) $M = 10$. Here we see a dramatic influence of mobility ratio on the water relative
permeability. The figure shows that the higher the viscosity of one fluid is (oil), the lower is the relative permeability of the other fluid (water). These results agree with the results published by Lefebvre du Prey (1973) (Figure 2.6).
Chapter 5

Imbibition

5.1 Introduction

The most common technique used to displace oil in a reservoir is to inject water. Water flooding a water-wet medium is imbibition, where the wetting phase displaces the non-wetting phase. The microscopic mechanisms for imbibition are more complicated than for drainage (Øren et al. (1998)). Imbibition involves three types of water invasion (Lenormand and Zarcone (1984)).

- The first type is piston-like or connected advance mechanism, in which the fluid advances in a connected front occupying the centre of the pore space. It is most likely to occur in the throats as illustrated by Figure 5.1a.

- The second type of water invasion is called pore body filling, in which the threshold capillary pressure is controlled by the largest radius of curvature required to invade the pore body. This radius depends on the size of the pore body and the number of connecting throats. For example, if the co-ordination number is Z, there will be (Z-2) pore body filling mechanisms as shown in Figure 5.1(b-d). Here, as the number of connecting throats that filled with oil increases, the pressure required for filling the pore body decreases (i.e $P_c(2) > P_c(3) > P_c(4) > ...$).

- The third type of water invasion is called snap-off, as the capillary pressure decreases, the radius of the curvature of the fluid interface increases and the wetting
layers in the crevices start to swell. The swelling of the wetting layers continues until a point where further filling of the crevices causes the interfacial curvature to decrease which leads to fluid instability and the centre of the pore space is spontaneously filled with wetting fluid. The snap-off mechanism is shown in Figure 5.2.

Figure 5.2: Snap-off in imbibition. After Lenormand and Zarcone (1984)
Chapter 5. Imbibition 5.2. Micro-flow mechanisms of imbibition

5.2 Micro-flow mechanisms of imbibition

In imbibition, we use the same mechanisms that were used in primary drainage, except for snap-off in drainage because it is restricted to the drainage process. We will show that snap-off in imbibition controls the fluid displacement and the trapping of oil. This section will focus on the methods used for determining the initial water saturation, displacing trapped oil and rearranging fluid configurations to avoid having more than two menisci between the pore and throat centres.

5.2.1 Determination of initial water saturation

The imbibition process starts with a network that contains oil in the bulk of the pore elements and water in wetting layers. The water in the wetting layers is obtained by applying the same capillary pressure to all the elements. In other worlds imbibition starts with a fixed capillary pressure initially applied in the network. In summary:

1. The initial water saturation is obtained by using the maximum capillary pressure ($P_{cmax}$) in the network which is given through the following expression:

$$P_{cmax} = \frac{2\sigma}{R_{min}} \quad (5.1)$$

where $\sigma$ is interfacial tension and $R_{min}$ is the radius at the centre of the smallest throat in the network. The initial water volume ($V_{initial}$) for the smallest element is:

$$V_{initial} = A_c l \quad (5.2)$$

where $A_c$ is the wetting layer cross sectional area given by Equation 3.15 with wetting layer radius of curvature equal to half the radius of the throat centre.

2. For the other elements, the radius of the curvature of the wetting layers ($r$) is obtained from knowing the capillary pressure through the following equation:

$$r = \frac{\sigma}{P_{cmax}} \quad (5.3)$$
The initial water volume is obtained by using Equation 5.2 with radius of curvature $r$ obtained from Equation 5.3. In our simulations the initial water saturation in the network is 12%.

3. Once imbibition has started, water will start flowing in wetting layers which may lead to an increase in water volume in these layers. Updating the water volume in these layers for each element is done by evaluating the difference in volume between the water flowing in and that flowing out of these layers. If this volume cannot be accommodated in layers, then snap-off occurs. However, the new volume will be used to evaluate the radius of the curvature of the wetting layers which will later be used to obtain the hydraulic resistance.

### 5.2.2 Displacing trapped oil

There are two mechanisms that lead to disconnected or trapped oil: bypassing and snap-off. Bypassing occurs when water displaces oil from the centres of the pore space and surrounds a region of oil-filled elements. Snap-off is the more significant contributor to oil trapping. Where the wetting layers swell ahead of the displacement front composed of fully water-saturated elements, snap-off may occur in the narrower throats, trapping oil in larger elements.

In a dynamic model trapped oil ganglia may be displaced due viscous forces. We may need to assign oil originally in a throat to a pore branch. The method we use is illustrated in Figure 5.3. The principal difficulty is how to model movement between a throat and a neighbouring pore when there is a disconnected oil ganglion between them (Figure 5.3b). In summary:

1. If oil is in one or more pore branches and there is oil flow out of the pore (Figure 5.3a), this is considered as a continuous displacement and the technique described in section 4.2.1 is used to displace this oil. If there is no continuous oil flow, such as when all the other pore branches are filled with water or water fills the pore branch close to the pore centre (Figure 5.3b), the oil is assumed to be trapped and a discontinuous displacement is needed to displace the oil.
2. The discontinuous displacement is performed if there is at least one pore branch completely filled with water, otherwise, the oil remains trapped in the pore branch until one of the other pore branches becomes available for displacement.

3. The discontinuous displacement starts by placing an interface at the pore centre in a fully water saturated branch or (if there are more than one water branch), the branch that provides the highest-pressure drop. In the next time step, the oil will start flowing through this branch and the injecting flux will be exactly same as the displacing flux. The injection process will continue until the centre of the whole branch is filled with oil.

4. The final step is to follow the oil segment injected in the previous step by a segment of water. The water segment is in the same size as the pore branch to ensure that the next oil injection does not start until the pore branch becomes fully water-saturated.

5. The last three steps are repeated until all the trapped oil is displaced.

![Diagram](image-url)

Figure 5.3: Displacing trapped oil. a) Oil is connected across the pore. Continuous oil displacement is applied. b) All the outlet branches of the pore contain oil at the pore/throat boundary, thus, the oil in the lower branch is considered trapped.
5.2.3 Rearranging fluid configurations

In our model, we only allow up to two interfaces to exist between the pore and throat centres. There are some cases where the fluid configurations need to be rearranged to avoid having more than two interfaces, which is done by allowing the oil droplets to jump within the single pore element (i.e. pore branch or throat side). In this subsection, we will explain the technique that is used to rearrange the fluid configuration for two cases: displacing a disconnected oil droplet between the pore and throat centres and rearranging the fluid configuration after snap-off when the water-oil interface reaches the pore/throat boundary.

Displacing a disconnected oil droplet between the pore and throat centres

Figure 5.4a shows oil invading a water-saturated pore branch. First continuity of the oil flow is checked (i.e. whether oil is injected from the other pore branches). If there is a continuous oil flow, piston-like displacement –subsection 4.2.1– is assumed and the oil-water interface is moved forwards toward the pore/throat boundary. For discontinuous oil flow, the fluid configuration will change to a new one where the oil is assumed to occupy the pore branch portion up to the pore/throat boundary and water occupies the pore centre, as shown in Figure 5.4b. Then, the displacement process will be carried on in a piston-like fashion until the whole oil droplet is pushed inside the throat (Figure 5.4c). If the volume of the oil droplet is less than the volume of the left side of the throat, the oil is assumed to occupy the throat portion that from the throat centre and water occupies the pore/throat boundary (Figure 5.4d). Figure 5.5 presents a flowchart of the technique used.

Rearranging the fluid configuration after snap-off

As presented in section 4.2.2, after snap-off water spontaneously fills the centre of the throat and the oil is separated into two (Figure 5.6a). Therefore, when the water-oil interface reaches the left pore/throat boundary (Figure 5.6b), a new fluid configuration is required in which the water is placed at the left pore/throat boundary and the two oil droplets are combined together and assumed to be displaced in a piston-like fashion (Fig-
Figure 5.4: Displacing a disconnected oil droplet between the pore and throat centres. a) Oil at the pore centre. b) Oil at the pore/throat boundary. c) Oil inside the throat. d) Oil at the throat centre.

Figure 5.6c). The volume of oil is conserved in this rearrangement.

5.3 Imbibition results

Our results of imbibition are divided into three categories as in section 4.3: the effect of capillary number (flow rate) on fluid movement, the relation between the capillary number and snap-off and the influence of viscosity ratio.

5.3.1 Influence of capillary number on fluid movement

As in subsection 4.3.1, the capillary number is changed by varying the injection flow rate. Figure 5.7 shows the fluid distribution for simulations at different capillary number (oil is shown in grey and water in black). In all the simulations, we used a 2D network of...
Figure 5.5: Flow chart for displacing a disconnected oil droplet shown in Figure 5.4. The fluid configurations mentioned are explained in Appendix A.
Chapter 5. Imbibition

5.3. Imbibition results

Figure 5.6: Rearranging fluid configurations after snap-off, when a water/oil interface reaches a pore/throat boundary. a) The throat after snap-off with water in the centre. b) Water-oil interface at the left pore/throat boundary. c) Rearranging the fluid configuration with water at the left pore/throat boundary and oil at the center of the throat.

21 × 21 pores, a unit viscosity ratio (fluid viscosity of 1 Pa.s) and 0.05 Nm⁻¹ interfacial tension. The Darcy oil velocity is obtained by dividing the oil injection rate by the inlet cross-sectional area. The oil injection rate is the sum of the oil flow rates between the inlet throats and the connecting pore branches. The inlet cross-sectional area (1.54 mm²) is the product of the length of the inlet (7700 µm) and the mean thickness of the lattice which is taken to be the average diameter of the pores (200 µm).

Figure 5.7a shows fluid flow pattern at a high capillary number ($C_a = 0.35$). Water is flowing in the bulk of the pores and displacing oil in a piston-like fashion. Water is also flowing in corners, however, this flow does not have sufficient time to cause accumulation of water in throat centres and cause snap-off. As the capillary number decreases, the influence of the wetting layer flow becomes visible as it is seen in Figure 5.7b.
In the case of low capillary numbers, the water flow in wetting layers has sufficient time to cause snap-off. Figure 5.7c shows the fluid flow pattern for a low capillary number \((C_a = 4.2 \times 10^{-5})\), where narrow throats have been filled by snap-off well ahead of the connected front.

**Fractional flow and relative permeability**

To study the influence of capillary number (flow rate) on the fractional flow curve for two-phase imbibition, we used a 2D network of size \(21 \times 21\). The fractional flow curves were obtained by using the procedure explained in section 4.3.1. Figure 5.8 shows water fractional flow as a function of water saturation for different capillary numbers. In Figure 5.8a, a fractional flow curve at a high capillary number \((C_a = 0.12)\) is presented. At high capillary number the viscous force is dominant and the fractional flow curve is expected to be a diagonal straight line \((f_w = S_w)\) since oil and water flow together. The slight deviation from a straight line in Figure 5.8a is due to the capillary forces and the presence of wetting layer flow. As the capillary number decreases, the fractional flow curve becomes S-shaped. The presence of trapped oil with a low mobility means that the water fractional flow is close 1 at high water saturation. At low water saturation, low conductance water layers means that \(f_w < S_w\).

At low capillary number, the capillary force dominates and the influence of the wetting layer flow is significant (Figure 5.7c). It is clear from Figure 5.8c that the slope of the fractional flow curve is sharp and the oil residual saturation is high. This is due to snap-off which is the main process by which oil becomes trapped.

In Figure 5.9 we present the relative permeability curves as a function of water saturation for the same capillary numbers that are used in obtaining the fractional flow curves (Figure 5.7). Figure 5.9a shows the relative permeability curves for the highest capillary number studied \((C_a = 0.12)\). From the curve, it is clear that the relative permeability curves are close to straight lines, but not exactly diagonal which is due to the wetting layer flow. The initial water saturation in Figure 5.9a is about 12\% (obtained by the procedure explained in section 5.2.1). However the water relative permeability is very small \((< 0.01)\) which because of the low conductance of the wetting layers. In other words,
Figure 5.7: Fluid distributions for simulations of imbibition at different capillary number ($C_a$). In this and subsequent figures, the water in the centres of pores and throats is shown in black and oil is shown in grey. The fine black lines separate pores and throats. a) $C_a = 0.35$. b) $C_a = 2.3 \times 10^{-3}$. c) $C_a = 4.2 \times 10^{-5}$.

The reason of having very low water relative permeability at low water saturation is that most of the water flow is in wetting layers. The figure also shows that the oil relative permeability at the initial water saturation is about 0.66 which might be seem low. How-
Figure 5.8: Water fractional flow as a function of water saturation for different imbibition capillary numbers. The solid line indicates the infinite capillary limit, $f_w = S_w$. a) $C_a = 0.12$. b) $C_a = 1.6 \times 10^{-3}$. c) $C_a = 3.3 \times 10^{-5}$.

ever, wetting layers reduce the oil cross-sectional area in each element that significantly reduces the conductance.

As the capillary number decreases, the capillary force increases which leads to more deviation in the relative permeability curves from the diagonal lines (Figure 5.9b). At the lowest capillary number ($C_a = 3.3 \times 10^{-5}$), the capillary force dominates and snap-off is significant which is indicated by the sharp reduction in the oil relative permeability curves. Water as the wetting phase is confined to the narrowest region of the pore space in a capillary-controlled displacement, leading to a low relative permeability. The oil relative permeability drops rapidly as the water saturation increases, because of snap-off (Figure 5.9c).
Figure 5.9: The effect of capillary number on imbibition relative permeability for unit viscosity ratio. a) $C_a = 0.12$. b) $C_a = 1.6 \times 10^{-3}$. c) $C_a = 3.3 \times 10^{-5}$. 
5.3.2 Connected and disconnected flow

Snap-off is the major reason for having disconnected oil in imbibition. The resultant oil ganglia can flow through the network. The contribution of ganglion transport to the overall flow of oil is represented by the difference between the total oil and connected oil fractional flow curves in Figure 5.10. The connected fractional flow is the fraction of the total flow carried by oil that is connected to the outlet. It is clear that disconnected flow is appreciable for all the flow rates studied, even for the lowest capillary number that is typical of reservoir displacements. At low oil saturation essentially all the oil flow is from ganglion transport, regardless of capillary number. This result agrees with the finding of Payatakes and co-workers who found significant ganglion transport for low capillary numbers (Valavanides et al. (1998), Valavanides and Payatakes (2001)) and contradicts the conventional quasi-static conceptualization of multiphase flow (Dullien (1992)).

5.3.3 Influence of viscosity ratio on fluid movement

This subsection studies the influence of viscosity ratio on the displacement patterns and the degree of snap-off. We used a network of $21 \times 21$ pores and ran simulations with capillary number of $C_a = 1.6 \times 10^{-3}$. We used the same intermediate capillary number that we used to obtain the fractional flow curves at unit viscosity ratio to compare the results with the ones produced with unit viscosity ratio (see the previous subsection).

At low viscosity ratio (water is more viscous than oil), the displacement is expected to be stable with a relatively flat front progressing through the system. The wetting layer flow is very small due to the high resistance of water since it is the more viscous fluid. This means the water bulk flow is much higher than the wetting layer flow which leads to less snap-off. Figure 5.11a shows a displacement for $M = 0.1$ that confirms this. For viscosity ratios greater than 1 (oil is more viscous than water), water flows more easily in wetting layers which leads to a high degree of snap-off as it is shown in Figure 5.11b for $M = 10$. For the comparison between Figure 5.11 and Figure 5.7b, it becomes clear that snap-off is a strong function of viscosity ratio, where at high viscosity ratio the wetting layer flow is significant and as a consequence snap-off is common.
Figure 5.10: The oil fractional flow and oil fractional flow from oil connected to the outlet as a function of oil saturation for several capillary numbers. a) $C_a = 0.12$, b) $C_a = 1.6 \times 10^{-3}$, c) $C_a = 3.3 \times 10^{-5}$. 
Figure 5.11: Influence of viscosity ratio on fluid movement for imbibition process with $C_a = 1.6 \times 10^{-3}$. a) 2D image of fluid distribution for $M = 0.1$. b) 2D image of fluid distribution for $M = 10$.

In Figure 5.12 we present the total oil fractional flow and the connected oil fractional flow (i.e considering the flow of oil that is only connected to the outlet) as a function of oil saturation for (a) $M = 0.1$ and (b) $M = 10$. For the low viscosity ratio ($M = 0.1$), the total oil fractional flow varies rapidly at intermediate saturation consistent with the flat frontal displacement observed in Figure 5.7a. There is not much difference between the total oil fractional flow and the connected one since there is little trapping of oil. However, at high viscosity ratio ($M=10$), the total oil fractional flow varies more linearly with saturation. There is a large gap between the total oil fractional flow and the connected one even at high oil saturation (Figure 5.12b) which indicates the high degree of snap-off and ganglion transport.

Relative permeability curves are shown in Figure 5.13 for (a) $M = 0.1$ and (b) $M = 10$. The water relative permeability for $M = 0.1$ is high compared with that for $M = 10$. For $M = 0.1$ the water relative permeability is very high, indicating that water occupies the centres of all the elements carrying significant flow. For an unstable displacement the water relative permeability is much lower, since water flows in layers in the narrower regions of the pore space. These observations are consistent with the experimental results of Lefebvre du Prey (1973) (Figure 2.6).
Figure 5.12: Influence of viscosity ratio on fluid movement for imbibition process for $C_a = 1.6 \times 10^{-3}$. a) The total oil fractional flow and connected oil fractional flow curves for $M=0.1$. b) The total oil fractional flow and connected oil fractional flow curves for $M=10$. 

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Figure 5.13: Influence of viscosity ratio on relative permeability for $C_a = 1.6 \times 10^{-3}$. a) Relative permeabilities curve for $M=0.1$. b) Relative permeabilities curve for $M=10$. 
Chapter 6

Conclusions and recommendations

6.1 Conclusions

A dynamic pore-scale model of drainage and imbibition has been developed to understand the fundamentals of multiphase flow in porous media. In particular we have challenged the conventional view of multiphase flow that is based on quasi-static displacement where capillary effects dominate at the pore scale. By modelling wetting layer flow and the dynamics of snap-off we have shown that viscous effects are significant in many cases.

- From the numerical primary drainage results we conclude:

1. At low capillary number \( C_a \leq 10^{-5} \), oil tends to flow through the larger pores that have the smallest capillary entry pressures and the displacement pattern is similar to that predicted using a quasi-static model. With increasing capillary number, the oil can enter pores and throats of all sizes and the displacement is less ramified.

2. More oil ganglia are formed by snap-off as the capillary number increases. However, the contribution of ganglion transport to the overall flow of oil is insignificant except at very large capillary numbers, \( C_a > 0.1 \). The fractional flow is a function of flow rate for capillary numbers greater than \( 10^{-5} \). The dynamic model, even at the lowest capillary number studied (\( 6.2 \times 10^{-5} \)), predicted a much greater saturation of water in layers than an equivalent quasi-
static model ($C_a \to 0$). This tended to shift the computed water fractional flow curves to the right.

3. With a viscosity ratio less than one (oil more viscous than water) and high flow rates, a flat frontal displacement is observed but with a large number of oil ganglia. These ganglia are formed by snap-off, which is favoured due to the comparatively low flow resistance in wetting layers.

4. For a viscosity ratio greater than one the oil fingers through the water, there is less snap-off and the oil is well connected.

- From the numerical imbibition results we conclude:

1. At the highest capillary number studied ($C_a = 0.12$), the influence of wetting layer flow is insignificant and water displaces oil in a piston-like fashion. The water fractional flow curve for unit viscosity ratio is close to a diagonal line.

2. As the capillary number decreases, the influence of wetting layer flow becomes more significant and more oil ganglia are formed by snap-off.

3. The contribution of ganglion transport to the overall flow of oil is significant even at low capillary numbers (i.e. typical reservoir flowrates).

4. The amount of snap-off is a strong function of viscosity ratio. At viscosity ratios greater than 1 (water is less viscous than oil), wetting layer flow is significant and snap-off is common which leads to more oil ganglia. For viscosity ratios less than 1, a piston-like displacement was observed with a relatively flat front. Snap-off is less common.

### 6.1.1 Model realizations and validation

The results that are presented in this thesis were generated by using a small two-dimensional networks ($17 \times 17$ for drainage and $21 \times 21$ for imbibition) and we did not check the realization of the model. In other words, we did not study the influence of changing throat size distribution which might be result in reducing the scatters that are seen in the fractional flow and relative permeability curves. In addition, because we use a small
two-dimensional networks, the residual oil saturation that is shown in the results might not be representative. Vizika et al. (1994) showed that network size has a great influence on the residual saturation (see Figure 2.11). Another drawback of the model is that the use of two-dimensional networks where real porous media are three-dimensional (see next section for more details). In three-dimensional networks, fluids are more connected since the coordination number (conductivity) in these networks is larger that that in two-dimensional networks. However, oil ganglion transports will still be significant especially at low oil saturation as reported by Payatakes and co-workers (see chapter two).

In this thesis, we have not shown any validation of the model except we have compared the trend of relative permeability curves with the result published by Lefebvre du Prey (1973), that is because of the limited time for this study. However, the validity of the model can be checked by performing micromodel experiments and looking at individual fluid movements. Where the Occurrence of snap-off, pore filling, meniscus oscillations and fluid movements within a single element can be compared.

The capillary pressure in our model is dynamic capillary pressure, since we assume that the local (i.e. pore) capillary pressure is a function of fluid saturation (see section 1.3.3). However, we did not check if our results lie in a dynamic regime, thus, we did not calculate the dynamic capillary coefficient ($\tau$) in our result. Hassanizadeh et al. (2002) showed that ($\tau$) can be estimated according to experimental or theoretical studies and its value range from $10^4$ to $10^7$ (ms)$^{-1}$. They reported that pore-scale network models can be used to estimate the dynamic capillary coefficient. Their procedure is as follows:

1. Preform two or more drainage simulations with different network pressure drop. They used very low pressure drop in some simulations to represent a static or equilibrium flow.

2. Obtain capillary pressure–saturation ($P_c – S$) relationship for all the simulation with recording the time for each data point in the ($P_c – S$) relation.

3. Subtract the $P_c – S$ curves for low and high network pressure drop from each other to eliminate the static capillary pressure show in equation 1.7. then, the capillary
pressure equation becomes:

\[ p_{\text{high}} - p_{\text{low}} = -\tau \left( \frac{\partial S_{\text{high}}}{\partial t} - \frac{\partial S_{\text{low}}}{\partial t} \right) \] (6.1)

4. The dynamic capillary coefficient (\(\tau\)) is given as the slope between the data points \((p_{\text{high}}^{\text{high}} - p_{\text{low}}^{\text{low}})\) and \((\frac{\partial S_{\text{high}}^{\text{high}}}{\partial t} - \frac{\partial S_{\text{low}}^{\text{low}}}{\partial t})\).

6.2 Applications and further possible simplifications of the model

Our model simulated two-phase flow in a strongly water-wet system. However, this model could be modified to simulate three-phase flow in a mixed wettability medium.

Simulating three-phase flow with our model would be very difficult but not impossible. It would be necessary to determine all possible fluid configurations in a pore or throat as a function of phase volume. A simple example of a three-phase fluid configuration is shown in Figure 6.1 where gas occupies the pore centre, water is at the pore/throat boundary and oil is at the throat centre. This fluid configuration is equivalent to the two-phase fluid configuration (G) that is shown in appendix A, replacing the oil resistance at the pore centre with the gas resistance, otherwise, all other equations are same.

In the case of mixed-wet systems, there are many factors affecting the wetting alteration such as fluid composition, temperature, duration of aging and fluid saturations (Buckley et al. (1995)). A range of contact angle between \(0^\circ\) and \(180^\circ\) could be distributed among the pores and throats for water-flooding. In addition, in water-flooding oil layers may form sandwiched between water in corners and water in the centre. These layers play a remarkable role in reducing the residual oil saturation as long as oil has a pathway to the outlet (Kovscek et al. (1993)). In our model, a new set of phase resistance equations will be required to account for wettability change, since the equations we have used in this work are restricted to water-wet systems. In addition, an extra term will be added in the calculation of the equivalent hydraulic resistance to account for the presence of oil layers.
6.2. Simplifications of the model

Figure 6.1: Fluid configuration for three-phase flow system. Gas occupies the pore centre, water is at the pore/throat boundary and oil occupies the throat centre.

6.2.1 Further simplifications of the model

However, the major limitation of this model is the computation time needed that has limited our studies to relatively small 2D systems. We will discuss how to simplify the model to enable larger 3D networks to be considered.

In the work we have proposed, we assumed that a pore is divided into several branches (i.e. equal to the number of connected throats) and the pore centre is treated as volume-less joining point. The pore branches are considered as extension of the throats they are connected to (i.e. their shape is sinusoidal). In addition we allowed four interfaces to exist between the pore centres at the same time –two in the throat and one in each pore branch. The location of these interfaces is obtained via an iterative method which is time consuming. The simplification of the model might be made by reducing the number of the interfaces between the pore centres to two and by ignoring the pore branches. This simplification has no effect on the methodology of our model. All that would be required would be to reduce the distance between the pore centres from \( \left( \frac{l_1 + l_2}{2} + l_t \right) \) to \( l_t \) and with this reduction in the distance two interfaces have been removed. With this simplification
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6.2. Simplifications of the model

the computation time will be reduced to half of the original time needed. With the new change, the radius equation and fluid configurations need to be changed. The next subsection highlights the main changes in the model that result from this new simplification.

**Change in the pore geometry and fluid configurations**

In the network and pore geometry section 3.3, throats have a triangular cross-section and their radius varies sinusoidally. The radius \( R(x) \) is given by the following equation

\[
R = \left( \frac{R_n + R_t}{2} \right) + \left( \frac{R_n - R_t}{2} \right) \cos \left( \frac{2\pi x}{l_t} \right)
\]  

(6.2)

where the subscript \( n \) stands for the node that connects several throats, \( x \) varies between zero (at the node) and \( \frac{l_t}{2} \) (at the throat centre). It is clear from Equation 6.2 that the throat is connecting two nodes with different radii which means the two sides of the throat have different sizes.

There is no change needed in the subsection on the selection of pore and throat sizes except instead of using pore radii, node radii should be used. In the section on fluid flow through the network –section 3.4– the main change is the fluid configurations. Since there are no pore branches in the new model, the fluid configurations presented in Appendix A need to be changed. The change in the fluid configurations comes from ignoring the pore branch. For example, fluid configuration A will still be same but with equivalent resistance equal only to the throat side resistance. In configuration C the oil enters the throat side from the left so the fluid configuration will be as shown in Figure 6.2a and its equivalent resistor diagram is shown in Figure 6.2b. The change we made here is assuming that the node is located at the pore/throat boundary of the diagram shown in Appendix A –fluid configuration C. The same change should be applied to the other configurations.

**Solving for pressure and computing flow rates**

The same principle for finding the pressure should be used with the new model, the difference here is that the volume conservation should be applied across the nodes. We do need to compute the node and throat centre pressures exactly as explained in section 3.5.
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Figure 6.2: Schematic diagram of a fluid configuration for the new model. a) Fluid configuration: Oil enters the throat from the left side. b) The equivalent electrical resistors diagram for Figure 6.2a used to compute hydraulic resistance.

The throat pressures are needed to account for the water accumulation in the wetting layers. The phase flow rates are obtained as the phase flow rate that enters/leaves the throat from the node. The following example clarifies this issue. The diagram presented in Fig-
Figure 6.3: Schematic diagram is used in computing field pressure and phase fluxes. a) Fluid configuration: Water at the centre of the throat. b) The equivalent electrical resistors diagram used to compute hydraulic resistance.

Thus, the equivalent hydraulic resistance of the fluids in the left side of the throat is

$$W_{eq1} = (W_{T-11}^w / W_{T-c1}^w) + W_{T-c1}^w$$ (6.3)

and for the right side is

$$W_{eq2} = W_{T-c2}^w + (W_{T-12}^w / W_{T-c2}^w)$$ (6.4)

then the total resistance between the nodes is the summation of the two equivalent resistances. The total flow rate between the nodes is given through the flowing expression

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6.2. Simplifications of the model

\[ Q_{\text{total}} = \frac{P_{n1} - P_{n2} + P_c}{W_{eq1} + W_{eq2}} \quad (6.5) \]

where \( P_{n1}, P_{n2} \) are the pressure at the first node and second node respectively, \( P_c \) is the total capillary pressure between the two nodes obtained as explained in section 3.5. After the node pressures computed the throat pressures need to be calculated in the same way as presented in section 3.5. Then the entering and leaving water flow rates can obtained through the following equations

\[ Q_{\text{water}}^{\text{in}} = \frac{P_{n1} - P_t + P_{c1}}{W_{T-I1}^w} \left( 1 - \frac{W_{T-I1}^w}{W_{eq1}} \right) \quad (6.6) \]

\[ Q_{\text{water}}^{\text{out}} = \frac{P_t - P_{n2} + P_{c2}}{W_{T-I2}^w} \left( 1 - \frac{W_{T-I2}^w}{W_{eq2}} \right) \quad (6.7) \]

The water volume in the throat then can be updated according to the procedure explained in section 3.7.

Micro-flow mechanisms

In the new model we have the same mechanisms: piston-like and snap-off. For piston-like advance there is no change in the section on invasion of a throat side. In the section on pore centre invasion, the change is that when an interface reaches a node, the fluid will be distributed in all neighbouring throats by the same procedure presented in section 4.2.1 or simply by assuming the fluid is distributed equally among the throats connecting that node.

The snap-off mechanisms can be modelled exactly as explained in section 4.2.2, the difference only is that snap-off (when water starts flowing in the bulk of the throat) happens when an interface in one of the throat side reaches the connected node.
Appendix A

Fluid Configurations

In all the following figures:

○ is the pore centre, □ is the throat centre,

\( P/T \) is the pore/throat boundary,

\( W_{P-c}^w \) is the pore water resistance,

\( W_{T-c}^w \) is the throat water resistance,

\( W_{P-c}^o \) is the pore oil resistance,

\( W_{T-c}^o \) is the throat oil resistance,

\( W_{P-i}^w \) is the pore wetting layer resistance,

\( W_{T-i}^w \) is the throat wetting layer resistance,

and \( a//b = 1/(\frac{1}{a} + \frac{1}{b}) \).

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1. Fluid configuration (A):

\[ W_{eq} = W_{P-c}^w + W_{T-c}^w \]  \hspace{1cm} (A.1)

\[ W_{eq}^w = W_{eq} \]  \hspace{1cm} (A.2)

\[ W_{eq}^w = 0.0 \]  \hspace{1cm} (A.3)

\[ Q_{total} = \frac{P_{p} - P_{T}}{W_{eq}} \]  \hspace{1cm} (A.4)

\[ Q_{water} = Q_{total} \]  \hspace{1cm} (A.5)

\[ Q_{oil} = 0.0 \]  \hspace{1cm} (A.6)

Figure A.1: a) Fluid configuration A: all water. b) The equivalent electrical resistors diagram.
2. Fluid configuration (B):

Figure A.2: a) Fluid configuration B: One oil/water interface (in the pore). b) The equivalent electrical resistors diagram.

\[
W_{eq} = \left( \frac{W_{p-l}^w}{W_{p-c}^o} \right) + W_{p-c}^w + W_{T-c}^w \quad (A.7)
\]

\[
W_{eq} = W_{p-l}^w + W_{p-c}^w + W_{T-c}^w \quad (A.8)
\]

\[
W_{eq}^o = W_{p-c}^o \quad (A.9)
\]

\[
Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \quad (A.10)
\]

\[
Q_{water} = Q_{total} \quad (A.11)
\]

\[
Q_{oil} = 0.0 \quad (A.12)
\]
3. Fluid configuration (C):

Figure A.3: a) Fluid configuration C: One oil/water interface (at the pore/throat boundary). b) The equivalent electrical resistors diagram.

\[ W_{eq} = (W_{w,l}^o/W_{p-c}^o) + W_{T-c}^o \]  \hspace{1cm} (A.13)
\[ W_{eq}^o = W_{p-l}^o + W_{T-c}^o \]  \hspace{1cm} (A.14)
\[ W_{eq}^w = W_{p-c}^w \]  \hspace{1cm} (A.15)
\[ Q_{total} = \frac{P_P - P_T + P_C}{W_{eq}} \]  \hspace{1cm} (A.16)
\[ Q_{water} = \left(\frac{P_P - P_T + P_C}{W_{p-l}^w}\right)(1 - \frac{W_{T-c}^w}{W_{eq}}) \]  \hspace{1cm} (A.17)
\[ Q_{oil} = Q_{total} - Q_{water} \]  \hspace{1cm} (A.18)
4. Fluid configuration (D):

Figure A.4: a) Fluid configuration D: One oil/water interface (in the throat). b) The equivalent electrical resistors diagram.

\[ W_{eq} = \left( \frac{W_{P-l}^w + W_{T-l}^w}{W_{P-c}^o + W_{T-c}^o} \right) + W_{T-c}^w \]  \hspace{1cm} (A.19)

\[ W_{eq}^w = W_{P-l}^w + W_{T-l}^w + W_{T-c}^w \]  \hspace{1cm} (A.20)

\[ W_{eq}^o = W_{P-c}^o + W_{T-c}^o \]  \hspace{1cm} (A.21)

\[ Q_{total} = \frac{P_P - P_T + P_C}{W_{eq}} \]  \hspace{1cm} (A.22)

\[ Q_{water} = \frac{Q_{total} \left( \frac{(W_{P-l}^w + W_{T-l}^w)/(W_{P-c}^o + W_{T-c}^o)}{W_{P-l}^w + W_{T-l}^w} \right)}{W_{P-l}^w + W_{T-l}^w} \]  \hspace{1cm} (A.23)

\[ Q_{oil} = Q_{total} - Q_{water} \]  \hspace{1cm} (A.24)
5. Fluid configuration (E):

![Diagram of fluid configuration E]

\[ W_{\text{eq}} = \frac{(W_{p-l}^{w} + W_{T-l}^{w})}{(W_{p-c}^{o} + W_{T-c}^{o})} \quad (A.25) \]

\[ W_{eq}^{w} = W_{p-l}^{w} + W_{T-l}^{w} \quad (A.26) \]

\[ W_{eq}^{o} = W_{p-c}^{o} + W_{T-c}^{o} \quad (A.27) \]

\[ Q_{\text{total}} = \frac{P_{p} - P_{T} + P_{C}}{W_{eq}} \quad (A.28) \]

\[ Q_{\text{water}} = \frac{P_{p} - P_{T} + P_{C}}{W_{eq}^{w}} \quad (A.29) \]

\[ Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}} \quad (A.30) \]

Figure A.5: a) Fluid configuration E: Water in the corners, oil in the centre. b) The equivalent electrical resistors diagram.
6. Fluid configuration (F):

Figure A.6: a) Fluid configuration F: One oil/water interface (in the throat). b) The equivalent electrical resistors diagram.

\[ W_{eq} = W_{P-c}^w + W_{T-c}^w + (W_{T-l}^w/W_{T-c}^w) \]  \hspace{1cm} (A.31)

\[ W_{eq}^w = W_{P-c}^w + W_{T-c}^w + W_{T-l}^w \]  \hspace{1cm} (A.32)

\[ W_{eq}^o = W_{T-c}^o \]  \hspace{1cm} (A.33)

\[ Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \]  \hspace{1cm} (A.34)

\[ Q_{water} = Q_{total} \]  \hspace{1cm} (A.35)

\[ Q_{oil} = 0.0 \]  \hspace{1cm} (A.36)
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7. Fluid configuration (G):

Figure A.7: a) Fluid configuration G: Oil fills the throat. b) The equivalent electrical resistors diagram.

\[ W_{eq} = W_{P-c}^w + (W_{T-l}^w / W_{T-c}^o) \]  \hspace{1cm} (A.37)

\[ W_{eq}^w = W_{P-c}^w + W_{T-l}^w \]  \hspace{1cm} (A.38)

\[ W_{eq}^o = W_{T-c}^o \]  \hspace{1cm} (A.39)

\[ Q_{total} = \frac{P_P - P_T + P_C}{W_{eq}} \]  \hspace{1cm} (A.40)

\[ Q_{water} = Q_{total} \left( \frac{W_{eq} - W_{P-c}^w}{W_{T-l}^w} \right) \]  \hspace{1cm} (A.41)

\[ Q_{oil} = Q_{total} - Q_{water} \]  \hspace{1cm} (A.42)
Chapter A. Fluid Configurations

8. Fluid configuration (H):

Figure A.8: a) Fluid configuration H: One water/oil interface (in the pore). b) The equivalent electrical resistors diagram.

\[
W_{eq} = W_{p-c}^w + ((W_{p-l}^w + W_{T-l}^w)/(W_{p-c}^w + W_{T-c}^w)) \quad (A.43)
\]

\[
W_{eq} = W_{p-c}^w + W_{p-l}^w + W_{T-l}^w \quad (A.44)
\]

\[
W_{eq} = W_{p-c}^o + W_{T-c}^o \quad (A.45)
\]

\[
Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \quad (A.46)
\]

\[
Q_{water} = Q_{total}(\frac{W_{eq} - W_{p-c}^w}{W_{p-l}^w + W_{T-l}^w}) \quad (A.47)
\]

\[
Q_{oil} = Q_{total} - Q_{water} \quad (A.48)
\]
9. Fluid configuration (I):

\[ W_{eq} = W_{p-c}^w + (W_{T-l}^w/|W_{T-c}^w|) + W_{T-c}^w \]  

(A.49)

\[ W_{eq} = W_{p-c}^w + W_{T-l}^w + W_{T-c}^w \]  

(A.50)

\[ W_{eq}^o = W_{T-c}^o \]  

(A.51)

\[ Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \]  

(A.52)

\[ Q_{water} = Q_{total} \left( \frac{(W_{T-l}^w/|W_{T-c}^w|)}{W_{T-l}^w} \right) \]  

(A.53)

\[ Q_{oil} = Q_{total} - Q_{water} \]  

(A.54)

Figure A.9: a) Fluid configuration I: Oil only in the throat. b) The equivalent electrical resistors diagram.
10. Fluid configuration (J):

\[ W_{eq} = (W_{P-l}^w/W_{P-c}^w) + W_{P-c}^w + W_{T-c}^w + (W_{T-l}^w/W_{T-c}^w) \]  \hfill (A.55)

\[ W_{eq}^w = W_{P-c}^w + W_{T-c}^w + W_{T-l}^w \]  \hfill (A.56)

\[ W_{eq}^o = W_{P-c}^o + W_{T-c}^o \]  \hfill (A.57)

\[ Q_{total} = \frac{P_P - P_T + P_C}{W_{eq}} \]  \hfill (A.58)

\[ Q_{water} = Q_{total} \]  \hfill (A.59)

\[ Q_{oil} = 0.0 \]  \hfill (A.60)

Figure A.10: a) Fluid configuration J: Water at pore/throat boundary. b) The equivalent electrical resistors diagram.
11. Fluid configuration (K):

Figure A.11: a) Fluid configuration K: Oil at pore/throat boundary. b) The equivalent electrical resistors diagram.

\[ W_{eq} = W_{p-c}^w + \left( (W_{p-l}^w + W_{T-l}^w) / (W_{p-c}^w + W_{T-c}^w) \right) + W_{T-c}^w \]  
(A.61)

\[ W_{eq}^w = W_{p-c}^w + W_{p-l}^w + W_{T-l}^w + W_{T-c}^w \]  
(A.62)

\[ W_{eq}^o = W_{p-c}^o + W_{T-c}^o \]  
(A.63)

\[ Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \]  
(A.64)

\[ Q_{water} = \frac{(P_p - P_T + P_C)}{W_{eq}(W_{p-c}^w + W_{T-c}^w)} \left( W_{eq} - (W_{p-c}^w + W_{T-c}^w) \right) \]  
(A.65)

\[ Q_{oil} = Q_{total} - Q_{water} \]  
(A.66)
12. Fluid configuration (L):

Figure A.12: a) Fluid configuration L: Oil occupying the whole pore and the throat centre.
b) The equivalent electrical resistors diagram.

\[ W_{eq} = \left( \frac{W_{w_{P-l}}}{W_{p_{-c}}} \right) + W_{w_{T-c}} + \left( \frac{W_{w_{T-l}}}{W_{T_{-c}}} \right) \]  
(A.67)

\[ W_{eq} = W_{P-l} + W_{T-c} + W_{T-l} \]  
(A.68)

\[ W_{eq} = W_{p_{-c}} + W_{T_{-c}} \]  
(A.69)

\[ Q_{total} = \frac{P_p - P_T + P_C}{W_{eq}} \]  
(A.70)

\[ Q_{water} = Q_{total} \left( \frac{W_{P-l}}{W_{p_{-c}}} \right) \]  
(A.71)

\[ Q_{oil} = Q_{total} - Q_{water} \]  
(A.72)
13. Fluid configuration (M):

Figure A.13: a) Fluid configuration M: Oil occupying the whole throat and the pore centre. b) The equivalent electrical resistors diagram.

\[ W_{eq} = \left( \frac{W_{w_{p-l}}}{W_{w_{p-c}}} \right) + W_{w_{p-c}} + \left( \frac{W_{w_{T-l}}}{W_{w_{T-c}}} \right) \] (A.73)

\[ W_{w_{eq}} = W_{w_{p-l}} + W_{w_{p-c}} + W_{w_{T-l}} \] (A.74)

\[ W_{o_{eq}} = W_{o_{p-c}} + W_{o_{T-c}} \] (A.75)

\[ Q_{total} = \frac{P_P - P_T + P_C}{W_{eq}} \] (A.76)

\[ Q_{water} = Q_{total} \left( \frac{W_{w_{T-l}}}{W_{w_{T-c}}} \right) \] (A.77)

\[ Q_{oil} = Q_{total} - Q_{water} \] (A.78)
14. Fluid configuration (N):

**Figure A.14:** a) Fluid configuration N: Oil in the pore. b) The equivalent electrical resistors diagram.

\[
W_{eq} = W_{p-c}^{w} + (W_{p-l}^{w}/W_{p-c}^{w}) + W_{T-c}^{w} \tag{A.79}
\]

\[
W_{eq} = W_{p-c}^{w} + W_{p-l}^{w} + W_{T-c}^{w} \tag{A.80}
\]

\[
W_{eq}^{o} = W_{P-c}^{o} \tag{A.81}
\]

\[
Q_{total} = \frac{P_{p} - P_{T} + P_{C}}{W_{eq}} \tag{A.82}
\]

\[
Q_{water} = Q_{total} \left(\frac{W_{p-c}^{w}}{W_{p-l}^{w}}\right) \tag{A.83}
\]

\[
Q_{oil} = Q_{total} - Q_{water} \tag{A.84}
\]
15. Fluid configuration (O):

Figure A.15: a) Fluid configuration O: Oil at the pore/throat boundary and pore centre. b) The equivalent electrical resistors diagram.

\[
W_{eq} = (W_{w}^{w,p-l}/ W_{o}^{p-c}) + W_{w}^{w,p-c} + (W_{w}^{w,T-l}/ W_{o}^{T-c}) + W_{w}^{w,T-c} \tag{A.85}
\]

\[
W_{eq} = W_{w}^{w,p-l} + W_{w}^{w,p-c} + W_{w}^{w,T-l} + W_{w}^{w,T-c} \tag{A.86}
\]

\[
W_{eq} = W_{o}^{p-c} + W_{o}^{T-c} \tag{A.87}
\]

\[
Q_{total} = \frac{P_{p} - P_{T} + P_{C}}{W_{eq}} \tag{A.88}
\]

\[
Q_{water} = Q_{total} \left( \frac{W_{w}^{w,T-l}/ W_{o}^{w,T-c}}{W_{w}^{w,T-l}} \right) \tag{A.89}
\]

\[
Q_{oil} = Q_{total} - Q_{water} \tag{A.90}
\]
16. Fluid configuration (P):

Figure A.16: a) Fluid configuration P: Oil at the pore/throat boundary and throat centre. b) The equivalent electrical resistors diagram.

\[ W_{eq} = W_{P-c}^{w} + (W_{P-l}/W_{P-c}) + W_{T-c}^{w} + (W_{T-l}/W_{T-c}) \]  \hspace{1cm} (A.91)

\[ W_{eq} = W_{P-c}^{w} + W_{P-l}^{w} + W_{T-c}^{w} + W_{T-l}^{w} \]  \hspace{1cm} (A.92)

\[ W_{eq} = W_{P-c}^{o} + W_{T-c}^{o} \]  \hspace{1cm} (A.93)

\[ Q_{total} = \frac{P_{p} - P_{T} + P_{C}}{W_{eq}} \]  \hspace{1cm} (A.94)

\[ Q_{water} = Q_{total} \left( \frac{W_{P-l}^{w}}{W_{P-c}^{w}} \right) \]  \hspace{1cm} (A.95)

\[ Q_{oil} = Q_{total} - Q_{water} \]  \hspace{1cm} (A.96)
Bibliography


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