Pore-Scale Modelling of Non-Newtonian Flow

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This work is dedicated with pride to my parents...

...and with love to Audrey
Abstract

The majority of complex fluids used in oilfield applications are non-Newtonian polymeric solutions exhibiting shear-thinning (pseudoplastic) behaviour in solution. The bulk macroscopic properties of these solutions, mainly their viscosity/shear-rate dependency, are well understood and characterized using established models. Ideally, one wishes to extend this knowledge to the flow of non-Newtonian fluids in porous media and predict their macroscopic behaviours without any further experimental work.

This work focuses on the study of single- and multi-phase flow properties of inelastic shear-thinning fluids in porous media. Existing theoretical models as well as experimental findings are first reviewed before describing the approach chosen to resolve some of the outstanding issues highlighted in the literature review. The model uses networks that represent the disordered topology of real rocks. Realistic representations of porous media that capture the geometry and topology of sands and sandstones are used as input to our flow model. By modifying an existing state-of-the-art pore-scale network modelling computer program to account for non-Newtonian rheology, the results of four single-phase experiments from different sources in the literature are successfully predicted. The network model is also used to understand the physics of non-Newtonian flow in porous media and the impact of pore structure and rheology on the macroscopic flow behaviour. The method is also extended to analyze multi-phase flow transport and displacement properties. Guidelines are offered to the reader as to expected single- and multi-phase behaviour from which macroscopic flow parameters can be inferred. Finally, extension of this approach to investigate more rheologically complex fluids such as viscoelastic fluids is discussed.
Acknowledgements

As I put the final touches to my thesis, I realise this work marks the end of both my PhD but also my time at Imperial and there are a number of people I wish to thank for making it all possible.

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The last seven years spent wandering in the corridors of this ever-changing department will undoubtedly remain with me as I embark on a scary journey leading to the “real” world. More than anything, my memories will be associated with the various people I have met here. There are too many to acknowledge and rather than forgetting to mention some of them, I will gratefully thank all of them here for their contributions, large of small, undergraduate and Msc students, fellow PhD colleagues, teachers and all Earth Science and Engineering departmental staff in general.

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Nomenclature

Greek Letters

$\alpha$ Scaling Parameter (Dimensionless)
$\beta$ Cannella et al. Scaling Parameter [4](Dimensionless)
$\Delta P$ Pressure Drop (Pa)
$\delta$ Exponent in Weibull Distribution (dimensionless)
$\dot{\gamma}$ Shear Rate ($s^{-1}$)
$\dot{\gamma}_{\text{app}}$ Apparent/In-Situ Shear Rate ($s^{-1}$)
$\dot{\gamma}_c$ Critical Shear Rate ($s^{-1}$)
$\eta$ Exponent in Weibull Distribution (dimensionless)
$\lambda$ Cross Model Time Constant (s)
$\mu$ Shear Viscosity (Pa.s)
$\mu_0$ Zero Shear Viscosity (Pa.s)
$\mu_\infty$ Infinite Shear Viscosity (Pa.s)
$\mu_{\text{eff}}$ Effective Viscosity (Pa.s)
$\nabla P$ Pressure Gradient (Pa.m$^{-1}$)
$\phi$ Porosity (Dimensionless)
$\psi$ Porous Medium Tortuosity (Dimensionless)
$\rho$ Fluid Density (kg.m$^3$)
$\sigma$ Minimization Parameter (dimensionless)
$\tau$ Shear Stress (Pa)
$\xi$ Weibull Distribution Property
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<td>Shape Factor</td>
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<td>$P$</td>
<td>Pore/Throat Perimeter</td>
<td>m</td>
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<td>$A$</td>
<td>Cross-Sectional Area</td>
<td>$m^2$</td>
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<td>$a$</td>
<td>Carreau-Yasuda dimensionless parameter</td>
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<td>Consistency Index</td>
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<td>Fluid Hydraulic Conductance</td>
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<td>Dimensionless</td>
</tr>
<tr>
<td>$N_{\text{NN}}$</td>
<td>Non-Newtonian Capillary Number</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>$P$</td>
<td>Fluid Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric Flow Rate</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$q$</td>
<td>Darcy Velocity: Flow Rate Per Unit Area</td>
<td>m.s$^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of circular capillary</td>
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</tr>
<tr>
<td>$R_{\text{equ}}$</td>
<td>Equivalent Radius</td>
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</tr>
<tr>
<td>$v$</td>
<td>Fluid Velocity</td>
<td>m.s$^{-1}$</td>
</tr>
<tr>
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<td>Normalized Random Number</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$Z$</td>
<td>Average Connection Number</td>
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</tr>
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Chapter 1

Introduction

The oil industry is at the heart of energy issues faced by modern societies. The world’s demand for energy is very large and is set to continue to grow over the next decades. In 2003, oil accounted for about 40% of that demand for energy [18] and is likely to remain the main source of energy in the near future. Extracting more oil out of existing reservoirs is therefore paramount if the industry is to meet the future growth of energy consumption.

As production from mature oil reservoirs starts declining, the need for enhanced oil recovery processes (EOR) becomes increasingly important. In many of these reservoirs, water flooding has been implemented and is presently approaching its economic limit. Yet, a large proportion of the oil originally in place may remain trapped in the reservoir. One of the most important aspects for revitalization of mature reservoirs is the application of EOR processes. Activity in enhanced recovery techniques is already significant, and is expected to increase dramatically in the next decade. Although the classification of a scheme as “improved” or “enhanced” oil recovery is somewhat arbitrary, it is commonly accepted to qualify as EOR any period of tertiary oil production (i.e. following secondary water flooding) [1]. The methods used vary greatly in the physical and chemical processes they involve (Figure 1.2), as well as in their efficiencies and costs (Figure 1.1).

The work presented here focuses on understanding the flow of polymers in porous media. During water flooding, high molecular weight polymers are dissolved in the aqueous phase to increase its viscosity, resulting in a more stable displacement of oil by the injected water and enhanced sweep efficiency [6].

Besides EOR applications, there are many aspects of hydrocarbon production processes where understanding the flow of non-Newtonian fluids in porous media is also of great interest. These include well drilling and construction, reservoir stimulation and production [19]. Polymers in solution can be used in oil production wells
to block excessive water production from a watered-out high permeability layer. In water injection wells, using polymers in treatment fluids helps to initialize and stabilize fractures hence increasing injectivity. The presence of surfactant in the water phase also lowers the surface tension between the aqueous and oil phases, lowering the residual oil saturation and increasing oil recovery [6,20–22]. More generally, understanding the flow of rheologically complex fluids through porous media is important in many other engineering applications. Knowledge of the behaviour of polymer solutions in rocks can be applied in ground water hydrology, soil mechanics, industrial pollutant infiltration and chemical flooding processes.

These polymeric fluids exhibit complex behaviour such as time and shear rate dependency of their macroscopic properties and reactions with oil, water or the porous medium itself. The main rheological bulk properties of non-Newtonian fluids will be presented in Chapter 2. In oilfield applications, the majority of complex fluids used are non-Newtonian polymeric solutions exhibiting shear-thinning (pseudoplastic) behaviour in solution. Yet, most of the current understanding of flow in porous media has been developed for purely Newtonian fluids, where the viscosity is independent of shear rate. Modelling and predicting the transport properties of such solutions is therefore of great interest.

In Chapter 3 I will review the existing knowledge on non-Newtonian flow in porous media. Most of the past and current research remains limited to the study of single-phase displacements and mainly deals with power-law shear-thinning flu-
ids, such as Xanthan gum. Essentially, the majority of studies have been devoted to relating easily measurable properties (such as the solution bulk properties) to macroscopic transport quantities in an effort to understand and model complex fluids behaviour in porous media. This approach requires an appropriate and convenient description of the medium in which flow is taking place. Until recently, the most common approach undertaken was to represent the medium as a bundle of capillary circular tubes. However the method fails to predict consistently the macroscopic in-situ behaviour of shear-thinning fluids from the knowledge of their bulk properties. Part of the explanation lies in that, although numerous ways were developed to account for the complexity of the medium, the so-called “capillary bundle” approach fails to capture correctly the fundamental topological complexities that control flow at the pore-scale.

An appealing approach to predict the rheological behaviour of complex fluids is to use network models that represent the topological features of porous media. These models have been extensively used to analyze and understand multi-phase transport phenomena [9–11, 23–30]. The porous medium is represented as a collec-
tion of interconnected elements (pores and throats), where each element is assigned some idealized geometry with effective properties that capture the characteristics of real rocks. A method is presented in Chapter 4 to extend the predictive capabilities of network models to the study of shear-thinning fluids in porous media.

Adequately describing the complexity of the flow paths is key towards making correct predictions of fluid displacements. In Chapter 5 I will show how the approach chosen, unlike many others, does not require any fitting parameter and is capable of reproducing several published experimental data from different sources.

More importantly, the work undertaken here offers the opportunity to model and understand physically transport phenomena that have not been thoroughly studied before. In Chapter 6 the model is used to investigate the validity of the standard semi-analytical approach that introduces a length scaling to relate bulk rheograms to properties measured in-situ. In Chapter 7, the approach is further extended to multi-phase displacement and used to predict the impact of non-Newtonian rheology on relative permeability. Finally, the main results and findings of this work are summarised in Chapter 8, along with some discussion about possible extensions of the method and future work on non-Newtonian flow in porous media.

The following articles have used material presented in this thesis:


Chapter 2

Non-Newtonian Rheology

2.1 Rheological Background

2.1.1 Introduction

Rheology is usually defined as the branch of physics that studies the deformation and flow of matter [31]. In simpler terms, a rheological measurement indicates how “fluid-like” or “solid-like” a material is. For liquids, this is usually achieved by imposing a shearing flow on the liquid and measuring the resulting stress. Under laminar conditions in a circular capillary, the viscosity can be defined as the fluid resistance to shear (Eq. 2.1).

\[ \tau \equiv \mu \dot{\gamma} = -\mu \left( \frac{dv}{dr} \right) \]  

(2.1)

The fluids for which this shear resistance, or viscosity \( \mu \), is not a constant for all rates of shear \( \dot{\gamma} \) are known as non-Newtonian fluids. In practice, almost all polymer solutions exhibit non-Newtonian behaviour and their viscosity is a function of shear rate, \( \mu = \mu(\dot{\gamma}) \). The exact form of the shear stress/shear rate relationship depends on the nature of the polymeric solution. Figure 2.1 illustrates the most common rheological behaviours exhibited by non-Newtonian solutions.

Fluids showing an increase in their effective viscosity (slope of the shear stress/shear rate curve) with increasing shear rate are said to be shear-thickening or dilatant. This type of behaviour is often observed in slurries and mud used during drilling operations. Other fluids can sustain an applied (non zero) stress without flowing and are generally known as Bingham-type of fluids. The stress at which the fluid first flows is called the yield stress. Once the stress applied exceeds its yield value, the fluid can behave as pseudo-Newtonian, plastic or dilatant. However, the majority of dilute polymeric solutions, and especially those used in EOR applications, are shear-thinning solutions.
This work will focus on the flow of shear-thinning solutions in porous media. Extensions of the method to other rheological behaviour is discussed in Chapter 8.

### 2.1.2 Xanthan Solution Properties

The two main polymers used in the oil industry for hydrocarbon recovery are synthetic polyacrylamide (in its partially hydrolyzed form, HPAM) and Xanthan biopolymer [6]. Although in practice more applications are performed with HPAM, the fluid model in this work was based on Xanthan properties since most experimental results available in the literature are also based on Xanthan solutions [4, 5, 12, 13, 20, 21, 32–42].

![Molecular mechanisms of shear-thinning behaviour.](image1)

Xanthan gum (UK food additive E415) is a natural microbial dessication-resistant polymer prepared commercially by fermentation from the bacteria Xanthomonas campestris. The main macroscopic behaviour exhibited by Xanthan in bulk solution is a dependency of its viscosity on shear rate. As already mentioned, Xanthan solutions are shear-thinning, which means their viscosity decreases with increasing...
rate of shear. Physically, this phenomenon is linked to the alignment and concentration of polymer molecules in solution (Figure 2.2). As the rate of shear is increased, these molecules align with the direction of flow, hence reducing the drag between fluid layers and resulting in a decrease of effective viscosity. At low shear rate, polymers molecules are disordered and obstruct the flow resulting in high effective viscosity values. At high rates of shear, the majority of these molecules are aligned with the direction of flow, and the effective viscosity tends to approach that of the solvent.

The elastic properties of these solutions are often related to the degree of rigidity of the polymer chains. For instance, flexible polymers such as HPAM exhibit visco-elastic behaviour when flowing in porous media. This is thought to be due to the series of contractions and expansions in the porous medium through which the polymer molecules are flowing, resulting in an additional pressure drop. The additional term is due to the capacity of HPAM molecules to deform and elongate while flowing. However, because of its structure (Figure 2.3), the molecule of Xanthan can be to a good approximation treated as a “rigid rod”. This implies that visco-elastic effects associated with the flow of Xanthan in porous media are much less significant than for HPAM, which makes it easier to study. In this work, visco-elastic effects were neglected and Xanthan solutions were assumed to exhibit only a shear-dependent viscosity.

Figure 2.3: Xanthan gum molecular structure [2].
2.2 Shear-Thinning Behaviour

2.2.1 Formulation

Many shear-thinning fluids will exhibit Newtonian behaviour at extreme shear rates, both low and high. For such fluids, the shape of the effective viscosity curve against shear rate (on log-log axes) is shown in Figure 2.4. In the power-law region, the variation of effective viscosity can be described by the Ostwald-de Waele relationship (Eq. 2.2) for $n < 1$ [43, 44].

\[ \mu_{\text{eff}} = C \dot{\gamma}^{(n-1)} \]  

This model has been widely used in many engineering and scientific applications. Its main advantage lies in its simplicity since it only contains two parameters. It is relatively easy to incorporate into analytical solutions to many problems in simple and defined flow fields, and can be made to fit almost any data over a limited range of shear rate.

Figure 2.4: Rheological behaviour of shear-thinning fluids.

However, the power-law model cannot describe the behaviour outside the shear-thinning region, that is, for small or large values of shear rate, where it predicts infinite values for viscosity. One way to resolve the issue is to define the shear stress/shear rate relationship through a truncated power-law (Eq. 2.3), which ensures physical boundary conditions are satisfied.

\[ \mu_{\text{eff}} = \text{MAX}\{\mu_\infty, \text{MIN}\{C \dot{\gamma}^{(n-1)}, \mu_0\}\} \]
Several other constitutive equations have been developed in the past that capture the full bulk rheological behaviour of pseudoplastic solutions. Only the two most important ones, the Cross and the Carreau-Yasuda model, are presented here. The Cross model [45] is a four-parameter equation that describe more accurately the variation of effective viscosity over the entire range of shear rate (Eq. 2.4).

\[ \mu_{\text{eff}} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + (\lambda \dot{\gamma})^m} \]  

(2.4)

The Cross model exponent, \( m \), is related to the power-law one via \( m = 1 - n \). The time constant, \( \lambda \), describes the transition region in the viscosity curve. Specifically, \( \dot{\gamma} = \frac{1}{\lambda} \) is the shear rate at which the fluid changes from constant viscosity to shear-thinning behaviour, also called critical shear rate, \( \dot{\gamma}_c \). The critical shear rate can be related to the truncated power-law parameters in the following way.

\[ \dot{\gamma}_c = \left( \frac{\mu_0}{C} \right)^{\frac{1}{n-1}} \]  

(2.5)

In the limits, Eq. 2.4 extrapolates to the zero and infinite shear viscosity values, \( \mu_0 \) and \( \mu_\infty \), which are both assumed to be finite. For intermediate shear rates:

\[ \frac{1}{1 + (\lambda \dot{\gamma})^m} \approx \lambda^{-m} \dot{\gamma}^{-m} = \lambda^{(n-1)} \dot{\gamma}^{(n-1)} \]  

(2.6a)

\[ \mu_{\text{eff}} - \mu_\infty \approx (\mu_0 - \mu_\infty) \lambda^{(n-1)} \dot{\gamma}^{(n-1)} \]  

(2.6b)

Hence, for \( \mu_{\text{eff}} \gg \mu_\infty \), the model reduces to Eq. 2.2 with \( C = (\mu_0 - \mu_\infty) \lambda^{(n-1)} \).

The Carreau-Yasuda model [46,47] in its general form has an extra dimensionless parameter, \( a \), describing the transition region between the zero shear-rate viscosity region and the shear-thinning region (Eq. 2.7).

\[ \mu_{\text{eff}} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + (\lambda \dot{\gamma})^a}^{\frac{1-n}{a}} \]  

(2.7)

The time constant, \( \lambda \), and the shear-thinning exponent, \( n \), are equivalent to those from the previous formulations. For many shear-thinning fluids, \( a \) is approximately equal to 2 and hence the Carreau-Yasuda model is often reduced to its expression when \( a = 2 \), also called simply the Carreau model.

Figure 2.5 illustrates the difference in the predicted effective viscosity/shear rate curves from the truncated power-law (Eq. 2.3), Carreau (Eq. A.1) and Cross (Eq. 2.4) models using the same four parameters, \( \mu_0, \mu_\infty, C \) and \( n \). As can be seen, all three models give the same predictions for effective viscosity at intermedi-
ate shear rates, i.e. in the shear-thinning region. The main difference between the models resides in their predictions at low and high shear rate. While the truncated power-law and Carreau models give similar results at low shear rate, they differ at high shear rate values where the Cross model is now in close agreement with the Carreau results.

2.2.2 Experimental Behaviour

Escudier et al. [3] performed a set of experiments on Xanthan gum that provides a basis for modelling the bulk rheology of the non-Newtonian phase. These experiments were conducted over a wide range of temperature, pH and solvent salinity. Figure 2.6 shows the viscosity/shear rate results they obtained from seven different solutions at a concentration of 2500ppm. Xanthan is generally very stable in solution and its viscosity properties are relatively insensitive to variations of pH and/or salinity. Figure 2.6 also presents the predicted results using the Carreau-Yasuda (Eq. 2.7) and Cross models (Eq. 2.4). The parameters used to match the bulk rheological data are listed in Table 2.1. As seen from Figure 2.6, both models can predict very accurately the bulk rheological behaviour of Xanthan solutions observed experimentally.

However, it is not always feasible to obtain bulk rheological measurements corresponding to the Xanthan solution one may wish to consider. Table 2.2 presents Carreau parameters corresponding to bulk rheology experiments from five literature
sources [3–5, 12, 48]. Where the authors did not quote these parameters, a best fit to the Carreau model was used to infer them if possible. These results were used to establish correlations between $n$, $C$, $\mu_0$ and polymer concentration $C_P$, as shown in Fig. 2.7, Fig. 2.8 and Fig. 2.9. Note that the infinite shear viscosity, $\mu_\infty$, is often very difficult to measure in practice and was not taken into account here. The actual value of $\mu_\infty$ is usually very close to that of the solvent and can easily be inferred if it has not been measured directly.

![Viscometric behaviour of Xanthan gum solutions (2500ppm) together with Carreau-Yasuda (- -), Cross (–) model fits and experimental data (●) [3].](image)

<table>
<thead>
<tr>
<th>Bulk Parameters</th>
<th>Carreau-Yasuda Model</th>
<th>Cross Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>11.6</td>
<td>11.6</td>
</tr>
<tr>
<td>$\mu_\infty$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$2.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\dot{\gamma}_c = \frac{1}{\lambda}$</td>
<td>0.01</td>
<td>0.017</td>
</tr>
<tr>
<td>$n$</td>
<td>0.38</td>
<td>0.33</td>
</tr>
<tr>
<td>$a$</td>
<td>1.7</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2.1: Bulk rheological parameters of Carreau-Yasuda and Cross model match to Escudier et al. experimental data (2500ppm) [3].

Although there is some scatter in the data, the following correlations were established. These correlations can be used to deduce the Carreau parameters ($a = 2$) of a Xanthan solution where no experimental results are available. For convenience, Table A.1 in the Appendix provides the necessary Carreau model parameters for
various polymer concentrations, based on Equations 2.8a, 2.8b and 2.8c.

\[ n = \frac{1}{1 + (1.35 \times 10^{-3}C_P^{0.967})} \]  

(2.8a)

\[ C = 0.095 \left( \frac{1 - n}{n} \right)^{2.37} \]  

(2.8b)

\[ \mu_0 = 1 \times 10^{-3}e^{3.79 \times 10^{-3}C_P} \]  

(2.8c)

<table>
<thead>
<tr>
<th>Source</th>
<th>Concentration (ppm)</th>
<th>( \mu_0 )</th>
<th>( C )</th>
<th>( n )</th>
</tr>
</thead>
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<td>Escudier et al. [3]</td>
<td>2500</td>
<td>13.2</td>
<td>0.766</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>2500</td>
<td>10.7</td>
<td>0.758</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>2500</td>
<td>8.89</td>
<td>0.770</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>2500</td>
<td>15.3</td>
<td>0.845</td>
<td>0.30</td>
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<tr>
<td></td>
<td>2500</td>
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<td>0.854</td>
<td>0.30</td>
</tr>
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<td>2500</td>
<td>15.8</td>
<td>0.911</td>
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</tr>
<tr>
<td></td>
<td>2500</td>
<td>13.0</td>
<td>0.728</td>
<td>0.31</td>
</tr>
<tr>
<td>Cannella et al. [4]</td>
<td>300</td>
<td>0.0086</td>
<td>0.017</td>
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</tr>
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<td>0.043</td>
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</tr>
<tr>
<td></td>
<td>1200</td>
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<td>0.195</td>
<td>0.48</td>
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<td></td>
<td>1600</td>
<td>1.0</td>
<td>0.620</td>
<td>0.35</td>
</tr>
<tr>
<td>Hejri et al. [12]</td>
<td>1000</td>
<td>N/A</td>
<td>0.181</td>
<td>0.418</td>
</tr>
<tr>
<td></td>
<td>1500</td>
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<td></td>
<td>2000</td>
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<td>0.0115</td>
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</tr>
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<td></td>
<td>700</td>
<td>0.027</td>
<td>0.026</td>
<td>0.60</td>
</tr>
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<td></td>
<td>1000</td>
<td>0.069</td>
<td>0.05</td>
<td>0.59</td>
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<tr>
<td>Helmreich et al. [48]</td>
<td>100</td>
<td>0.0016</td>
<td>0.009</td>
<td>0.63</td>
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<td></td>
<td>500</td>
<td>0.0075</td>
<td>0.039</td>
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<tr>
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<td>0.033</td>
<td>0.095</td>
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<tr>
<td></td>
<td>2000</td>
<td>0.505</td>
<td>0.337</td>
<td>0.38</td>
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</table>

Table 2.2: Carreau model parameters for Xanthan solutions from literature sources.
Figure 2.7: Power-law exponent variation with polymer concentration from various sources (see Table A.1), and trendline (Eq. 2.8a).

Figure 2.8: Consistency index variation with power-law exponent from various sources (see Table A.1), and trendline (Eq. 2.8b).
Figure 2.9: Zero shear viscosity variation with power-law exponent from various sources (see Table A.1), and trendline (Eq. 2.8c).
Chapter 3

Literature Review

3.1 Single-Phase Flow in Porous Media

Modelling and measuring the bulk properties of polymeric solutions presented in Chapter 2 is a standard and reliable experimental procedure. Quite naturally then, efforts have been made to extend the laws of motion for purely Newtonian fluids in porous media (Darcy’s law) to rheologically complex ones using easily measurable bulk properties such as the effective viscosity/shear rate behaviour.

3.1.1 In-Situ Rheology

Many researchers have attempted to relate the bulk properties of complex fluids to their behaviour in a porous medium [4–6,12,13,20,21,32–35,40–42,44,49–55]. Experimental results suggest that the apparent viscosity measured in a porous medium as a function of Darcy velocity, \( q, \mu_{\text{app}}(q) \), has a similar shape to the bulk \( \mu_{\text{eff}}(\dot{\gamma}) \).

In practice, pressure drop and Darcy velocity are often the measured quantities, and the apparent viscosity is determined through Darcy’s law (Eq. 3.2), where the absolute permeability, \( K \), is measured independently from an experiment using a Newtonian fluid of known viscosity. In general, the flow rate measured, \( Q \), will not be a linear function of the pressure gradient imposed, \( \nabla P \), for non-Newtonian fluids.

\[
Q = -\frac{KA}{\mu_{\text{app}}} (\nabla P - \rho g) \tag{3.1}
\]

For instance Figure 3.1 shows experimentally measured apparent viscosity (defined through Darcy’s law Eq. 3.2) versus Darcy velocity curves corresponding to the flow of 300 ppm and 600 ppm Xanthan solutions through a 264 mD Berea sandstone core (after Cannella et al. [4]). Darcy velocity is defined as the total flow rate through the medium per unit area.

\[
q = \frac{Q}{A} \tag{3.2}
\]
The relative simplicity of the behaviour observed in porous media led many authors to investigate how to infer \textit{in-situ} rheograms from the experimentally measured one. This is usually achieved through the definition of an apparent shear rate in the porous medium by combining a mathematical description of the medium with a formulation of the bulk rheology. Using dimensional analysis there is a length that relates velocity ([L]/[T]) to shear rate (1/[T]). Physically this length is related to the pore size. One estimate of this length is the square root of the absolute permeability times the porosity, $K\phi$ \cite{4,5,12,13,20,32,33,35,56,57}. This allows the determination of \textit{in-situ} rheograms from the bulk measured $\mu_{\text{eff}}(\dot{\gamma})$.

\begin{equation}
\mu_{\text{app}}(q) = \mu_{\text{eff}} \left( \dot{\gamma}_{\text{app}} = \frac{q}{\sqrt{K\phi}} \right)
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.1.png}
\caption{Apparent viscosity versus Darcy velocity of 300ppm and 600ppm Xanthan solutions in a 264mD Berea sandstone core (after Cannella \textit{et al.} \cite{4}).}
\end{figure}

This approach is only valid if the absolute permeability is constant. However, as discussed later, adsorption and pore-blocking phenomena can in certain circumstances reduce the observed permeability, resulting in higher apparent viscosity. Often this is neglected and Eq. 3.3 is used to define an “apparent” or “\textit{in-situ}” shear rate experienced by the fluid in a porous medium, from the velocity/apparent viscosity measurements, and then compared to the bulk shear rate/viscosity relationship (see for instance Figure 3.2).
However, as noted by many authors [4,5,12,13,20,32,33,35,56,57] these rheograms appear to be shifted from the bulk curve by a constant factor, $\alpha$.

$$\mu_{app}(q) = \mu_{eff} \left( \dot{\gamma}_{app} = \alpha \frac{q}{\sqrt{K_{abs} \phi}} \right)$$  \hspace{1cm} (3.4)

Figure 3.2: Comparison between effective viscosity/shear rate curves measured in-situ and in bulk solution (after Fletcher et al. [5]): the in-situ rheograms are shifted by a constant factor $\alpha$.

Reported values for $\alpha$ vary depending on the approach chosen, but experimental results suggest it generally lies in the range 1 to 15. From this observation, it appears that the knowledge of $\alpha$ along with a measurement of the medium’s permeability and porosity is sufficient to infer in-situ rheograms from the bulk ones via Eq. 3.4. To predict $\alpha$ however, one must have a mathematical description of the porous medium. By far the most common approach used to determine the value of this scaling parameter is the so-called “capillary bundle model”.

### 3.1.2 Capillary Bundle Model

From the discussion above, it is clear that the bulk rheological properties influences the flow behaviour in porous media. The molecular structure of Xanthan and its shear-thinning properties affect the overall macroscopic flow. Equally important however is the structure of the medium in which the polymer solutions are flowing. Heterogeneous media with complex topology are unlikely to give rise to the same
flow behaviour observed in well-defined systems such as rheometers.

In order to describe single-phase non-Newtonian flow in porous media, a macroscopic flow rate/pressure drop relationship must be developed. This requires having an appropriate representation of the porous medium that captures its macroscopic and microscopic properties. A common approach consists of representing the medium by a bundle of parallel capillary tubes with the same overall resistance to flow as the porous medium replaced, also known as the “capillary bundle model”. The flow in the capillaries is assumed to be fully developed and an empirical tortuosity factor is introduced. For instance, the well-known Blake-Kozeny-Carman equation introduces a tortuosity of $25/12$ in the case of a Newtonian fluid [58].

This approach has been extended to purely viscous shear-thinning fluids and expressions relating pressure drop and Darcy velocity have been developed (see for instance Savins [59]). Although the complexity of the models based on this approach can vary depending on the tortuosity and constitutive rheological model chosen, the basic principles and rationale behind them remain similar. They allow the definition of an average radius that is dependent on macroscopic properties of the medium such as porosity, absolute permeability and some measure of tortuosity, $\psi$. For example, Savins proposed the following [59]:

$$R_{equ} = \sqrt{\frac{8K\psi}{\phi}} \tag{3.5}$$

As a result, analytical expressions of flow properties can be derived from Stokes’ law [6, 22, 44, 59, 60]. Usually flow is solved in a single circular capillary of average radius $R_{equ}$. For instance, consider the flow of an incompressible fluid under steady-state conditions in a circular tube.

The motion is described using spherical coordinates.

$$v_\theta = v_r = 0 \quad \text{and} \quad v_z(0) = v_z(L)$$
The rate of momentum in and out, across the cylinder is:

\[
(2\pi r L \tau_r z) \big|_r - (2\pi r L \tau_r z) \big|_{r+\delta r}
\]  

Likewise, the rate of momentum in and out, along the cylinder is:

\[
(2\pi r \delta r v_z) (\rho v_z) \big|_{z=0} - (2\pi r \delta r v_z) (\rho v_z) \big|_{z=L}
\]

Gravity acting on the fluid:

\[
(2\pi r \delta r L \rho g)
\]

Pressure:

\[
(2\pi r \delta r P) \big|_{z=0} - (2\pi r \delta r P) \big|_{z=L}
\]

The overall momentum balance is equal to 0, hence neglecting gravity and taking the limits when \(\delta r \to 0\), one obtains for (3.6) + (3.7) + (3.8) + (3.9):

\[
\frac{\partial}{\partial r} (r \tau_r z) = r \frac{\partial P}{\partial z}
\]

Hence,

\[
\tau_r z = \frac{\Delta P}{2L} r
\]

Then, using a mathematical description of the fluid rheology (such as power-law, Carreau or Cross models), several authors [4, 12, 13, 20, 21, 33, 35, 44, 56, 57, 61] have derived expressions to define an apparent shear rate experienced by the fluid in the porous medium from the Darcy velocity. Sorbie [6] provided a comprehensive review of the different approaches.

For an infinite power-law fluid (Eq. 2.2), Eq. 2.1 and Eq. 3.11 can be used to relate shear rate to pressure drop along the capillary.

\[
\tau_r z = C\dot{\gamma}^n = \frac{\Delta P}{2L} r
\]
Therefore,

\[ \dot{\gamma}^n \equiv \left( -\frac{dv_z}{dr} \right)^n = \frac{1}{2} \frac{\Delta P r}{C L} \]  

(3.13)

The fluid velocity in the z direction is found after integrating Eq. 3.13 with zero velocity at the wall.

\[ v_z(r) = \left( \frac{\Delta P}{2CL} \right)^{\frac{1}{n}} \frac{n}{n+1} \left[ R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}} \right] \]  

(3.14)

The average velocity \(< v_z >\) is:

\[ < v_z(r) > = \frac{\int_0^{2\pi} \int_0^R v_z(r) r dr d\theta}{\int_0^{2\pi} \int_0^R r dr d\theta} = \frac{2}{R^2} \int_0^R v_z(r) r dr \]  

(3.15)

Hence,

\[ < v_z(r) > = \left( \frac{\Delta P}{2CL} \right)^{\frac{1}{n}} \frac{n}{3n+1} R^{\frac{n+1}{n}} \]  

(3.16)

The volumetric flow rate \(Q\) is:

\[ Q = \pi R^2 < v_z(r) > = \frac{\pi n}{3n+1} \left( \frac{\Delta P}{2CL} \right)^{\frac{1}{n}} R^{\frac{3n+1}{n}} \]  

(3.17)

The effective viscosity, \(\mu_{\text{eff}}\), can then be defined through a pseudo-Poiseuille law (Eq. 3.18).

\[ Q = \frac{\pi R^4 \Delta P}{8 \mu_{\text{eff}} L} \]  

(3.18)

Consequently,

\[ \mu_{\text{eff}} = C \left( R \Delta P \right)^{\frac{n-1}{n}} \]  

(3.19)

As mentioned earlier, the capillary radius \(R\) can replaced by an equivalent one, for instance Eq. 3.5, based on the overall macroscopic properties of the medium. The subtlety lies here in trying to establish a macroscopic definition of fundamentally microscopic quantity such as the shear rate. As a consequence, various expressions were proposed for \(\dot{\gamma}_{\text{app}}\) as a function of the measured Darcy velocity.

The Darcy velocity can be calculated from the average one [57]:

\[ q = < v_z(r) > \phi = \left( \frac{\Delta P}{2CL} \right)^{\frac{1}{n}} \frac{\phi n}{3n+1} R^{\frac{n+1}{n}} \]  

(3.20)

Savins [59] defined the apparent porous shear rate by evaluating Eq. 3.13 at the
wall \((r = R)\).
\[
\dot{\gamma}_{\text{app}} = \left( \frac{\Delta P R}{2CL} \right)^{\frac{1}{n}} = \frac{< v_z(r) >}{R} \left( \frac{3n + 1}{n} \right)
\] (3.21)

Therefore, assuming a tortuosity of 1 and combining Eq. 3.5 and Eq. 3.20, one can obtain an expression of the same form as Eq. 3.4 [6], with \(\alpha = \sqrt{2} \left( \frac{3n+1}{4n} \right)\).
\[
\dot{\gamma}_{\text{app}} = \left( \frac{3n + 1}{n} \right) \frac{q}{\sqrt{8K\phi}}
\] (3.22)

For most common Xanthan solutions the shear-thinning exponent \(n\) is in the range of 0.3 – 0.6, hence from Eq 3.22 \(\alpha\) varies from 1.65 – 2.24. As already mentioned, experimental evidence showed values of \(\alpha\) in the range of 1 – 15 depending of the fluid and porous media properties. Clearly the example just described cannot predict experimental results observed for Xanthan solutions in all cases. Generally speaking, the model tends to underestimate (small value of \(\alpha\)) the shear rate experienced by the fluid in complex porous media.

Chauveteau & Zaitoun [20] and Chauveteau [61] performed single-flow experiments in bead-packs and sandstone cores. The found experimentally values for \(\alpha\) in the range of 1.5 – 2.5 for bead-packs and 2 – 20 for various sands. Their study highlighted the dependence of the scaling factor, \(\alpha\), on the medium topology and structure. They found the more complex and tortuous the medium, the greater the scaling factor.

Cannella et al. [4] investigated the flow behaviour of Xanthan in porous media for different polymer concentrations, rock lithologies, residual oil saturation and rock permeability. They used experimental results and effective medium theory to relate Darcy velocity to in-situ shear rate. They derived an expression for the scaling factor that is both dependent on the porous medium structure and the fluid properties (via the shear-thinning index \(n\)).
\[
\dot{\gamma}_{\text{app}} = \beta \left( \frac{3n + 1}{n} \right)^{\frac{n}{n-1}} \frac{q}{\sqrt{K\phi}}
\] (3.23)

Hence,
\[
\alpha = \beta \left( \frac{3n + 1}{n} \right)^{\frac{n}{n-1}}
\] (3.24)

However, as with previous authors, they introduced an empirical correction factor, \(\beta\), in order to match their experimental results with the capillary bundle predictions.
Nonetheless, they matched all their experimental flow data using a single value for their empirical constant \((\beta = 6)\).

Based on experimental results, Duda and Klaus \[62\] showed that the bulk rheological behaviour of purely shear-thinning fluid (such as Carbobymethyl cellulose, “CMC”, or Xanthan gum) was not well predicted by the capillary bundle model. They used a porous media viscometer to determine the pressure drop versus flow rate relationship for different solutions of Xanthan and CMC. Data of friction factor times Reynold’s number versus Reynold’s number clearly showed discrepancies between the capillary model predictions and experimental results, particularly at high Reynolds numbers. The study showed that the measured pressure drop is a stronger function of Reynolds number than predicted by the capillary model. Greater pressure drops than those predicted by the capillary model appeared at low \(n\) values. This deviation between the model predictions and the experiments was seen to increase with decreasing power-law exponent.

One possibility is that non-uniform pores and throats (i.e. sudden contractions and expansions) generate excess pressure drops that are not captured by the theoretical model. Although solutions for non-uniform cross section capillaries exist for Newtonian flow, it remains difficult to extend them to even simple shear-thinning fluids. Even so, these changes should not lead to significant deviations from the theoretical solutions. Boger et al. \[63\] demonstrated that for a pure power-law fluid flowing through a sudden contraction, the excess pressure drop created is proportional to the fully developed pressure drop at low Reynolds number and therefore should not explain the observed differences. Furthermore, real polymer solutions do not behave like pure power-law fluids. The transition between the shear-thinning region and a constant viscosity plateau at low shear is particularly important under creeping flow conditions, and exist everywhere in the system (since a zero shear rate exists at the centre of the pore). Duda and Klaus \[62\] noted that this transition led to a different relationship between the excess pressure drop and flow rate than that between the fully developed pressure drop and flow rate in a uniform conduit. They concluded that a model based on fully developed flow in cylindrical tubes was therefore not applicable. To be adequate, the model should account for non-uniform bonds in the medium, as well as for the transition to Newtonian behaviour at low shear rates.

### 3.1.3 Effective Medium Theory

Theoretical approaches, such as effective medium theory (EMT) and homogenisation, can also be used to analyze flow of complex solutions in porous media. EMT was originally developed to calculate the effective electrical conductivity of a net-
work of linear resistors with randomly distributed conductivity [64]. Numerical simulations on a 3D cubic lattice network indicated that EMT is adequate to describe flow properties in a network with narrow pore size distribution [65]. This is not unexpected since the method was developed for regular electrical networks and is therefore not suitable when large variations in properties, whether pore size or connectivity distribution, are present. By contrast, critical path analysis, which assumes that most of the flow is carried by a connected cluster, appeared to give adequate results only in systems with a wide pore size distribution [65].

Effective medium theory was also used to study theoretically the flow of a power-law solution in porous media [4]. This approach is superior to the traditional capillary bundle model, which cannot describe the connectivity of flow channels and their variable cross section. An empirical tortuosity factor is usually introduced in the model in order to account for the complexity of flow paths. Accounting formally for these two features in the capillary bundle model has proven difficult due to the accompanying mathematical complexities. EMT offers the opportunity to study how variations in cross section and connectivity might affect the flow properties of power-law fluids. However EMT predictions are known to be poor near the percolation threshold at which continuous flow channels cease to exist.

From EMT, Cannella et al. [4] derived a relationship between apparent viscosity and Darcy velocity for a purely power-law fluid flowing in a regular network. The apparent viscosity showed a dependency on the fluid bulk parameters, the radii distribution, the average radius, the permeability and porosity. They noted that these parameters were characteristic of the system and could therefore be defined independently. However the relationship also showed a dependency on the effective radius for a Newtonian fluid as well as its ratio to the effective radius for a power-law fluid. In a real system, these parameters are not obtainable a priori. Therefore their values must be assigned heuristically hence reducing predictive capabilities to the method. Nonetheless, their approach showed the impact of some of the other parameters, such as the bulk rheological ones, on the apparent viscosity/Darcy velocity relationship.

Their method was also used to investigate the impact of variable tube cross-sections and non-uniform connectivity via two simple cases. First EMT was applied to a regular network of constant coordination number ($z = 2$) made up of a series of constrictions and expansions. The results showed that the scaling between Darcy velocity and shear rate was indeed sensitive to the radii ratio as well as the fraction of wider tube elements. The same conclusion applied when studying a network with constant tube radii but where a fraction of the elements were closed to flow.
The network thereby obtained had uniform tube radii, but the tubes were randomly connected and flow paths tortuous.

These two examples showed the limiting conditions represented by the classic capillary bundle model, i.e. uniform tube radii and constant connectivity, underestimation of the scaling factor between shear rate and velocity. This is the basis for the introduction of an empirical scaling factor, usually named $\alpha$. In the general case of a bimodal distribution of tube radii, the results also showed that most of the flow would be carried by a connected cluster made of larger tubes. Below a critical radius ratio, most of the smaller elements appeared to have a negligible impact on the overall flow.

### 3.1.4 Continuum Approach

Pearson and Tardy [60] reviewed the main continuum models used for describing transport in porous media with special emphasis on the effects of non-Newtonian rheology. After reviewing the different mathematical approaches used to describe non-Newtonian flow in porous media, they concluded that none of the present continuum models give accurate estimates of macroscopic transport properties: the scaling factor, $\alpha$, is a function of both the bulk rheology and the pore structure and currently there is no theory that can predict its value reliably. Although simple, the capillary bundle model approach cannot describe and account for the complex geometry and topology of real porous media.

However, they also showed that a macroscopic law for the flow of power-law fluid in the limit of small Reynolds number could be derived through homogenisation (up-scaling microscopic laws to a macroscopic description). They demonstrated that the single-capillary power-law between flow rate and pressure gradient also applied at the macroscopic scale [60]. Hence, expressions derived from a single-capillary could be used to describe locally the flow.

Recently, Fadili et al. [66] presented a 3-D filtration law for the flow of pure power-law fluids in heterogeneous media. They concluded that modelling accurately the macroscopic flow behaviour of very shear-thinning fluids required a precise statistical description of the medium structure and topology. Therefore, approaches such as the capillary bundle model may never achieve accurate predictions. They also pointed out that the number of flow paths was found numerically to decrease for small values of $n$. 

3.1.5 Network Models

Network models were originally developed to study multi-phase flow of purely Newtonian fluids. However the same approach has been used by several authors to investigate transport phenomena of power-law fluids.

Sorbie et al. [39] investigated the rheology of Xanthan using a network modelling approach. From their review of past macroscopic approaches, they concluded that capillary bundle models did not allow an interpretation of the flow of polymer solutions at the pore scale. They used a two-dimensional network of capillary tubes on a regular rectangular lattice with randomly distributed radii to represent the porous medium. The results from their network calculations showed that even for Newtonian fluids, the medium-sized pores carried the bulk of the flow. This effect is more pronounced for non-Newtonian fluids and highlights the importance of cooperative effects within a network and more generally within a complex medium. Using expressions based on capillary bundle considerations and their network model, they found that although the approach gave a reasonable estimate of the flow rate/pressure drop relationship in a single bond, an empirical correction factor was needed to predict their macroscopic simulation results. They calculated this scaling factor to be in the range of $1.2 - 1.7$ for their system.

Pearson and Tardy [60] considered the flow of a power-law fluid in a two-dimensional square capillary bundle network. A number of conclusions were drawn from their calculations. Firstly, the power-law exponent, $n$, remained unchanged in the porous medium for infinite power-law fluids, and the Darcy velocity was found to vary linearly with $\left(\frac{\Delta P}{L}\right)^{1/n}$. The number of flow paths appeared to decrease with decreasing values of $n$, which strongly suggested streaking patterns. The empirical factor $\alpha$ was dependent on both the fluid (exponent $n$) and the medium topology. Furthermore, $\alpha$ was seen to increase with increasing medium complexity or tortuosity, which significantly affected transport properties. Both these modelling studies, Sorbie et al. [39] and Pearson & Tardy [60] used topologically simple networks and so were not able to make predictions of properties in natural porous materials.

Balhoff and Thompson [67] applied network modelling to the problem of predicting transport properties of yield-stress type fluids in porous media. By using three-dimensional networks based on a packed bed of spheres and the bulk rheological behaviour of yield-stress fluids, they determined flow velocities as a function of pressure drop applied. They showed that the macroscopic Darcy velocity/pressure drop curve obtained from their network was “shifted” by a constant factor from the analytical solution based on Darcy’s law and a capillary bundle method (a type of $\alpha$ factor). They attributed this shift to distribution of sizes in the network, connec-
tivity and diverging-converging geometries, although they did not quantify it. This shift essentially corresponded to a lower threshold pressure gradient observed in the network than predicted by the capillary bundle model.

### 3.2 Phenomenological Effects in Porous Media

Several phenomenological effects may be of importance when considering the flow of Xanthan solutions. In many applications of polymer flow in porous media, the size of these long-chained molecules approaches that of the dimension of the pores. As a consequence, complex rock-fluid interactions, such as adsorption, mechanical entrapment and inaccessible pore volume, may appear. These phenomena may be of importance when pore sizes approach that of the polymer molecules (typically $10^{-9}$ to $10^{-8}$ m) or for very shear-thinning solutions (i.e. high polymer concentration). To circumvent these issues, one may consider the flow of polymer in high permeability porous media, where these interactions can be neglected. However, even then, complications may arise with some polymer solutions (HPAM for instance) due to elastic phenomena resulting in excess pressure drops. Also, at high velocity, inertial effects complicate the behaviour of all polymer solutions. Ideally, one would start by studying polymer solutions with short relaxation times flowing at low velocity in high permeability media.

#### 3.2.1 Depleted Layer Phenomena

The first phenomenological effect of importance tends to exclude polymer molecules from the surface of the pores hence creating an apparent slip effect. This is usually attributed to entropic exclusion near the rock surface resulting in part of the pore volume being unavailable to flow [7, 8, 41]. Next to the wall, the molecules cannot rotate freely and as a result are forced into flowing towards the center of the pore. Sorbie provided a theoretical basis to account for this phenomenon both in a single capillary [7] and in network models [8]. This apparent slip effect causes the fluid in the center of the pore to travel faster than if no depleted layer effects were present. Models have been developed to relate analytically the effective viscosity in a given layer to polymer concentration. Hence the velocity enhancement can be calculated. Results on a two-dimensional square network suggested this effect could be significant, even for relative small-sized depleted layers, especially when the mean pore size approaches that of the polymer molecules. In particular, the presence of this phenomenon appears to significantly shift the higher Newtonian plateau (at low shear rates) to lower viscosity values. The impact on the slope of the shear-thinning region appeared much less pronounced (only decreased slightly).
This effect has also been observed experimentally in both adsorbing and non-adsorbing media. Teeuw and Hesselink [34] investigated the flow behaviour of a biopolymer in Bentheim sandstone cores. From their experiments they inferred that, for large polymer molecules, the exclusion of the polymer from the pore walls was likely to lead to an apparent slip effect whereby the molecules travel at a higher flow rate through the medium than the solvent.

### 3.2.2 Adsorption

The second mechanism of importance when considering non-Newtonian flow in porous media is polymer adsorption. Adsorption is the process of transferring material from a fluid phase (in our case the aqueous phase) to a solid phase (rock). Due to interactions between the surface and the molecules, part of the polymer is bound to the pore wall. This results in fewer polymer molecules flowing in the bulk of the solution, as opposed to the depleted layer effect.

Experimental results from Cannella et al. [4] indicated an apparent viscosity higher at low velocity than the zero shear viscosity. This phenomenon could be due to polymer molecules entanglement and interactions with the porous medium itself at low velocity. The non-alignment of the long-chained molecules with the flow field may cause additional resistance to flow, especially in constrictions. However, Newtonian like behavior in porous media at low velocity was obtained experimentally for more dilute solutions as well as being previously observed in other studies. A
constant viscosity plateau at low velocity was also observed in the porous media when the experiments were conducted at higher temperatures (80°C). Hence it is likely that significant polymer adsorption occurred at lower temperatures.

Similarly, Greaves and Patel [35] conducted polymer-flooding experiments on high permeability Elginshire sandstone rocks. They observed a higher polymer effective viscosity in the porous medium than in the beaker at low flow rates. They suggested that the interaction of polymer molecules with the solid could explain their results. However, in polymer gel systems it is also possible for the aqueous phase to become essentially solid. In this case displacement occurs through deformation of the gel phase, as studied in micro-model experiments [68]. Such cases were not considered here, and the aqueous phase was assumed to behave as a liquid, albeit a non-Newtonian one.

3.3 Multi-Phase Flow in Porous Media

In the section above, the various approaches proposed to study the single-phase flow of polymeric solutions in porous media have been presented. Whether on analytical or numerical models, there has been extensive work on the subject since the 1960’s. By contrast, to date, very little research has been presented on multi-phase flow phenomena, involving at least one non-Newtonian phase. As mentioned in the introduction, this is despite the fact that there are a number of oilfield and engineering applications where the simultaneous flow of Newtonian and non-Newtonian phases may occur.

Typically, Xanthan solutions are used in oilfield applications to enhance the displacement efficiency of the oil phase by the aqueous phase. In the past, this was modeled with a direct extension of the approach used for two-phase Newtonian immiscible displacements. Normally, relative permeability functions from the Newtonian displacements are used directly while the non-Newtonian effects are usually incorporated through a change of mobility ratio. For instance, Wu and Pruess [69] presented a numerical method in order to investigate transport phenomena of non-Newtonian fluids through porous media. They concluded that since the flow behaviour of non-Newtonian fluids was highly dependent on the porous medium structure and the solution bulk properties, developing a universal numerical approach for all cases was not achievable. Nonetheless, they developed a fully implicit three-dimensional finite difference model capable of modelling the displacement of a Newtonian phase by power-law and Bingham type fluids.
However, there is some experimental evidence that the relative permeability curves of the non-Newtonian phase may be different from that of the Newtonian one. In the case of a shear-thinning fluid this means the viscosity behaviour cannot be accounted solely by changing the viscosity ratio, and a new set of relative permeability curves must be defined for the non-Newtonian phase. Schneider and Owens [70] conducted steady-state measurements of relative permeability for polymer-oil systems in sandstone cores. From their experimental results, they found that the relative permeability of the aqueous phase was substantially reduced when polymer was added to the injected water. These findings were observed for different polymers flowing in both water-wet and oil-wet media. More surprisingly, they also observed a reduction in the oil relative permeability during polymer-oil displacement in an oil-wet core. However this reduction appeared to be significantly variable from one experiment to another and was therefore difficult to quantify.

Yi [71] derived an extended Buckley-Leverett model for the displacement of a yield-stress fluid by a Newtonian phase. The method was applied to the problem of fracture cleanup where oil or water displaces a Herschel-Bulkley type of fluid (fracturing fluid). The author developed an idealized one-dimensional methodology to solve fracturing fluid cleanup flow problems. They found there was a minimum pressure gradient below which the non-Newtonian phase would not flow.

To date only one attempt has been made at modelling multi-phase flow involving a non-Newtonian phase using network models. Shah [65] used two-dimensional square and three-dimensional cubic lattices to study the flow of power-law and Bingham fluids. Firstly, he found that, for single-phase flow with small values of the power-law exponent \( n \), flow occurred only through a small subset of the percolating cluster. This subset became even smaller as \( n \) was decreased. This was found to be sensitive to the distribution of conductances in the network. The wider the distribution, and hence the more heterogeneous the network, the smaller the subset of pores and throats participating to flow. The author also provided numerical network simulations of immiscible displacement of oil by Xanthan gum. His results were compared visually with flow experiments performed in a Hele-Shaw cell. The network results were found to be in good qualitative agreement with experiments. However no direct numerical comparison with experimental results was undertaken.
Chapter 4

Non-Newtonian Network Model

4.1 Work Motivation

The literature review undertaken in Chapter 3 highlighted some of the outstanding issues yet to be resolved in order to understand and predict the flow of polymer solutions in porous media.

Several conclusions can be drawn from the various experimental work performed in the past. In bulk solution, the vast majority of shear-thinning polymers, such as Xanthan gum, exhibit two constant-viscosity plateaux at high and low shear rate. The observed *in-situ* behaviour suggests these two plateaux also exist in the porous medium. Therefore, the rheological behaviour cannot be modelled satisfactorily by use of the classic power-law. Furthermore, although of importance in specific experimental settings, it was shown that, to a good approximation, elastic effects, as well as adsorption and exclusion phenomena, can be neglected when considering the flow of Xanthan-like solutions.

Despite many attempts, theoretical modelling remains unsatisfactory. The classic capillary bundle approach suffers from several drawbacks: it fails to account for the complexity and heterogeneity of real porous media, and most of the results derived are only applicable to infinite power-law fluids. Yet, in simple systems such as bead-packs, the model was found to give surprisingly good results, given its simplicity. It is clear however, that the scaling of bulk rheograms cannot so far be applied successfully to a variety of fluid and porous media settings. Both theoretical and experimental work have highlighted that this scaling factor between the bulk and *in-situ* rheograms is a function of both the fluid and the medium properties, and currently there is no theory that can predict it reliably.
From the observations above, several questions remain unanswered. For instance, can $\alpha$ be predicted \textit{a priori} for any shear-thinning fluid in different types of porous media? Is the scaling of bulk rheograms always applicable? If not, is there any other approach that could be used to predict the flow behaviour of polymer solutions? As already mentioned, several authors showed the macroscopic description of polymer flow behaviour requires a detailed representation of the medium in which flow takes place. Hence, an appealing approach to predict the \textit{in-situ} rheological behaviour of complex fluids is to use network models that represent the topological features of porous media. By combining realistic descriptions of porous media with the right physics at the pore-scale, one hopes to be genuinely predictive and answer some of the questions laid out before.

In this work, we seek to extend the predictive capabilities of a state-of-the-art two-phase network model [11] to cope with non-Newtonian rheology. In particular, we will show how network modelling can be used to predict different experiments involving a range of shear-thinning fluids in various porous media. Once the predictive capability of the method has been demonstrated, the model will be used to gain physical insights into various unresolved issues, such as the scaling factor dependencies and observed streaking behaviours. Finally, results will be shown on the multi-phase flow of polymer solutions that not only confirm experimental results but provide the first physical explanations of the observed behaviour.

4.2 Network Modelling Background

Network models have been extensively used to analyze and understand multi-phase transport phenomena [9–11, 23, 25, 26, 28–30]. The porous medium is represented as a collection of interconnected elements (pores and throats), where each element is assigned some idealized geometry with effective properties that capture the characteristics of real rocks. Macroscopic properties, such as capillary pressure or relative permeability, can then be estimated across the network by applying the appropriate rules that govern the transport and arrangement of fluids at the pore-scale. Adequately describing the complexity of the flow paths is key towards making correct predictions of fluid displacements.

4.2.1 Network Generation

The first step to be undertaken in network modelling is to find a way to replace the exact geometrical description of the void space (the space that can be occupied by fluid) in the medium by a simpler one based on pores and throats. In this study, we use networks derived from three different porous media: a sand-pack, a Berea sandstone and a heterogeneous North Sea sandstone.
The basis for generating these networks are three-dimensional digital images of their pore spaces. For example, Fig. 4.1 shows a digital representation of a 27 mm$^3$ sample of Berea sandstone. Although, there are several ways to obtain these rock images [25], the networks used here were constructed from voxel images generated by simulating the geological processes by which the porous medium was formed [9,10]. This approach was first used by Bryant and co-workers [28,29] and extended by Øren et al. [9,10]. The sedimentation, compaction and diagenesis of grains of different sizes making up the medium is simulated explicitly as well as the presence of clays.

From this, a topologically equivalent network model is built (Fig. 4.2) with pore sizes, shapes, volumes and connectivity based on the three-dimensional representation [9,10]. The skeleton of the voxel image corresponds to the points on the boundary surfaces that have neighbouring voxels from three or more grains with pore locations being points with four or more neighbouring grains [9]. The area, perimeter, radius and volume of each pore and throat are calculated directly from the voxel representation. A network constructed using this method is shown in Fig. 4.2.
The actual shapes assigned to pores and throats are also significant in determining transport properties, particularly during multi-phase displacements. Real porous media are highly irregular and during multi-phase flow the wetting phase resides in the grooves and crevices of the pore space. Flow through these wetting layers has been observed experimentally and can have a significant impact on the displacement [25]. However, representing every complexity of the porous medium is difficult. Hence, pores and throats can be represented in the network by geometrical elements that can accommodate wetting layers. In this study, pores and throats are modeled as having a square, circular or triangular cross section depending on their assigned shape factors, $G$ [10].

$$G = \frac{A}{P^2}$$  \hspace{1cm} (4.1)

This shape factor, the ratio of cross-sectional area to perimeter squared, is calculated directly from the voxel image. The three networks used are mainly composed of irregular-shaped triangular elements (Fig. 4.3). Having network elements with angular shapes allow for the non-wetting phase to occupy the center of the element while the wetting phase remains in the corners [11].
Chapter 4 Non-Newtonian Network Model

Xavier Lopez

Figure 4.3: The network elements (triangular, circular or square cross-section) have the same shape factor as that measured on the irregular voxel representation, after Valvatne & Blunt [11].

The statistical properties of the networks used here can be found in the Appendix. The standard deviation of the pore and throat size distributions divided by the mean is smallest for the sand-pack and largest for the heterogeneous sandstone. The three networks span a plausible range of properties encountered in porous media from an unconsolidated system with a relatively uniform distribution of pore size to a highly heterogeneous reservoir sandstone.

One should point out that in this study the networks are simply read as input to our flow model. Details about the creation and development of these networks are described by other authors [9–11]. This approach to representing porous media has proved very successful in predicting single and multi-phase flow properties of porous media, including permeability [29], relative permeability and capillary pressure [9, 10, 28, 30], two- and three-phase relative permeability experimental data [11, 23] and the impact of wettability variation on reservoir performance [72]. Blunt et al. [25, 26] provide recent reviews on how to describe and represent multi-phase flow through topologically complex porous media.

4.3 Extending Network Models to Shear-Thinning Rheology

Although these network models were originally developed to study multi-phase flow of purely Newtonian fluids, the approach is extended here to non-Newtonian phenomena. As already discussed, we consider polymer solutions, such as Xanthan, whose bulk rheology is well-described using a truncated power-law (Eq. 2.3). The
advantage of this simpler formulation over more accurate constitutive models, such as Cross or Carreau, is that analytical solutions to relate effective viscosity to pressure drop based on single-phase non-Newtonian flow in a circular capillary can be developed. Furthermore, this model not only ensures the right boundary conditions are respected, but also provides satisfactory results when compared to experimental data.

For a purely Newtonian fluid of constant viscosity $\mu$ flowing in a circular capillary of radius $R$, the Hagen-Poiseuille law (Eq. 3.18) applies. For a truncated power-law fluid (Eq. 2.3), equivalent equations that relate the volumetric flow rate to the rheological parameters ($C$ and $n$) and capillary tube dimensions ($R$ and $L$) can be derived for each of the network elements.

4.3.1 Analytical Solution for a Truncated Power-Law Fluid in a Circular Capillary

Just as in Chapter 3, consider the flow of an incompressible fluid under steady-state conditions in a circular tube. By considering the overall momentum balance, one can write Equation 3.11.

$$\tau_{rz} \equiv -\mu_{eff} \frac{\partial v_z}{\partial r} = \frac{\Delta P}{2L} r$$

Because of the truncation, three separate regimes need to be considered, that lead to three different expressions for the volumetric flow rate. At the centre of the capillary, there is no shear rate and hence $\mu_{eff} = \mu_0$. For a truncated power-law fluid, the effective viscosity remains equal to $\mu_0$ until some radius $r_1$. Between $r_1$ and $r_2$, the fluid experiences a power-law regime. Between $r_2$ and $R$, the high shear rate Newtonian plateau is reached and $\mu_{eff} = \mu_\infty$.

In the most general case, a truncated power-law fluid may experience any value of viscosity within $[\mu_0; \mu_\infty]$ across the capillary.
Assume $r_1$ and $r_2$ are known and $r_1 < r_2 < R$ and $\mu_{eff}$ is defined through a truncated power-law (Eq. 2.3). Starting at the wall, $r_2 < r < R$, with $v_z = 0$ at $r = R$, Eq. 2.3 and Eq. 4.2 lead to
\[
\frac{\partial v_z}{\partial r} = -\frac{\Delta P}{2\mu_\infty L} r
\] (4.3)
and
\[
v_z(r) = \frac{\Delta P}{4\mu_\infty L} (R^2 - r^2)
\] (4.4)

In the power law region, for $r_1 < r < r_2$, since $v_z$ is continuous
\[
v_z(r) = \left(\frac{\Delta P}{2CL}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) \left[ r_2^{n+1} - r_{n+1}^{n+1} \right] + \frac{\Delta P}{4\mu_\infty L} (R^2 - r^2)
\] (4.5)

Also, at $r = r_2$, $\partial v_z/\partial r$ is continuous, therefore using Eq. 4.3 and Eq. 4.4
\[
r_2 = \left( \frac{2L}{\Delta P} \right) \frac{C^{\frac{1}{n}}}{\mu_0^{\frac{1}{n}}}
\] (4.6)

Similarly, for $0 < r < r_1$, since $v_z$ is continuous
\[
v_z(r) = \left(\frac{\Delta P}{2CL}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) \left[ r_2^{n+1} - r_{n+1}^{n+1} \right] + \frac{\Delta P}{4L} \left[ \frac{r_1^2 - r^2}{\mu_0} + \frac{R^2 - r_2^2}{\mu_\infty} \right]
\] (4.7)

Likewise, at $r = r_1$, $\partial v_z/\partial r$ is continuous, hence using Eq. 4.3 and Eq. 4.5
\[
r_1 = \left( \frac{2L}{\Delta P} \right) \frac{C^{\frac{1}{n}}}{\mu_0^{\frac{1}{n}}}
\] (4.8)

Clearly, depending on the existence of $r_1$ and $r_2$, three distinct regimes may be obtained. Note that in all cases, $v_z(r)$ is maximum at the centre of the tube. Hence the shear rate must be zero at $r = 0$, by definition. This means that for a non-Newtonian fluid described by a truncated power-law, $r_1$ always exists and is less than or equal to $R$.

1st REGIME:

For $R \leq r_1$, i.e. $R \leq \left( \frac{2L}{\Delta P} \right) \frac{C^{\frac{1}{n}}}{\mu_0^{\frac{1}{n}}}$. 

Eq. 4.7 applies with $r_2 = r_1 = R$, therefore
\[
v_z(r) = \frac{\Delta P}{4\mu_0 L} (R^2 - r^2)
\] (4.9)

and
\[
<v_z(r)> = \frac{R^2 \Delta P}{8\mu_0 L}
\] (4.10)
This leads to Poiseuille’s flow equation:

\[ Q = \pi R^2 \langle v_z(r) \rangle = \frac{\pi R^4 \Delta P}{8\mu_0 L} \]  

(4.11)

Therefore

\[ \mu_{\text{eff}} = \mu_0 \]  

(4.12)

2nd REGIME:

Just as before, for \( r_1 < R \leq r_2 \), i.e. \( \frac{2L}{\Delta P} \frac{C^{1+n}}{\mu_0} < R \leq \frac{2L}{\Delta P} \frac{C^{1+n}}{\mu_\infty} \).

Eq. 4.5 applies with \( r_2 = R \) and \( r_1 \) defined by Eq. 4.8, therefore

\[ \mu_{\text{eff}} = \frac{R^4 \Delta P}{\frac{r_1^4 \Delta P}{\mu_0} + 8L \left( \frac{\Delta P}{2CL} \right)^{1/n} \left( \frac{n}{n+1} \right)} \times \Gamma_1 \]  

(4.13)

where \( \Gamma_1 \) is

\[ \Gamma_1 = \left[ r_1^{\frac{n+1}{n}} (R^2 - r_1^2) + r_1^2 \left( R^{\frac{n+1}{n}} - r_1^{\frac{n+1}{n}} \right) - \left( \frac{2n}{3n+1} \right) \left( R^{\frac{3n+1}{n}} - r_1^{\frac{3n+1}{n}} \right) \right] \]  

(4.14)

3rd REGIME:

Likewise, for \( r_1 < r_2 < R \), i.e. \( \frac{2L}{\Delta P} \frac{C^{1+n}}{\mu_0} < \frac{2L}{\Delta P} \frac{C^{1+n}}{\mu_\infty} < R \).

Eq. 4.4 applies with \( r_2 \) and \( r_1 \) defined by Eq. 4.6 and Eq. 4.8 respectively, therefore

\[ \mu_{\text{eff}} = \frac{R^4 \Delta P}{\frac{r_1^4 \Delta P}{\mu_0} + \frac{\Delta P(R^4 - r_1^2)}{\mu_\infty} + 8L \left( \frac{\Delta P}{2CL} \right)^{1/n} \left( \frac{n}{n+1} \right)} \times \Gamma_2 \]  

(4.15)

where \( \Gamma_2 \) is

\[ \Gamma_2 = \left[ r_2^{\frac{n+1}{n}} (r_2^2 - r_1^2) + r_1^2 \left( r_2^{\frac{n+1}{n}} - r_1^{\frac{n+1}{n}} \right) - \left( \frac{2n}{3n+1} \right) \left( r_2^{\frac{3n+1}{n}} - r_1^{\frac{3n+1}{n}} \right) \right] \]  

(4.16)

The derivation presented above shows that an exact analytical solution relating effective viscosity to pressure drop can be derived for the flow of a truncated power-law fluid in a circular capillary. This solution is dependent on the size of the capillary and some of the bulk fluid parameters (\( C \) and \( n \)). However, our network models are mainly composed of irregular triangular-shaped pores and throats, while Eqs. 4.12, 4.13, and 4.15 were derived for a circular cylinder. To account for non-circular pore shapes, we replace \( R \) in Eqs. 4.12, 4.13, and 4.15 with an appropriately defined equivalent radius, \( R_{\text{equ}} \). An empirical approach is used to define \( R_{\text{equ}} \) based on the conductance of the pore or throat, \( G \). This equivalent radius is determined using...
Poiseuille’s law (Eq. 3.18).

\[
Q = \frac{\pi R^4 \Delta P}{8 \mu_{eff} L} = \frac{G}{\mu_{eff}} \nabla P \tag{4.17}
\]

Hence,

\[
R_{equ} = \left( \frac{8G}{\pi} \right)^{1/4} \tag{4.18}
\]

For circular elements, Eq. 4.18 reduces to Eq. 4.17. For non-circular pores and throats (mainly triangular shaped in our networks) the expressions for \( G \) are more involved: – these are given in the next section.

### 4.3.2 Pore-Scale Modelling

Consider two adjacent pores \( i \) and \( j \) connected by a single throat. The flow rate \( Q_{ij} \) between these two pores is given by Eq. 4.19:

\[
Q_{ij} = \frac{G_{ij}}{\mu_{eff,ij}} \frac{L_i}{L_{ij}} (P_i - P_j) \tag{4.19}
\]

The conductance between two pore bodies is taken to be the harmonic mean of each individual conductance - the two pores, \( i, j \), and the connecting throat, \( t \) (see Fig. 4.4).

\[
\frac{L_{ij}}{G_{ij}} = \frac{L_i}{G_i} + \frac{L_t}{G_t} + \frac{L_j}{G_j} \tag{4.20}
\]

As mentioned before, for a circular tube the conductance \( G \) is given analytically by Poiseuille’s law (Eq. 4.17). More generally,

\[
G = A^2 G \xi \tag{4.21}
\]

where \( \xi \) is a constant that varies depending on the shape of the element (Fig. 4.3). Analytical expressions are available [9] to determine \( \xi \) for circles (\( \xi = 1/2 \)), equilateral triangles (\( \xi = 3/5 \)) and squares (\( \xi = 0.5623 \)). From numerical simulations Øren
et al. [9] found that the conductance for irregular triangles was closely approximated by the above equation, using the same constant $\xi$ as for an equilateral triangles, $3/\sqrt{5}$.

Once the conductances of each element is computed, Eq. 4.18 is used to find the equivalent radius and this is used in Eqs. 4.12, 4.13, and 4.15 to find the effective viscosity. In a network of pores and throats one does not know each pressure drop $\Delta P$ a priori. Hence, computing the flow and effective viscosities requires an iterative approach [39]. An initial guess is made for the effective viscosity in each network element. The choice of this initial value is rather arbitrary but does influence the rate of convergence, although not the final results. As a general rule, when one is interested in solving for only one flow rate across the network, the initial viscosity guess can be taken as the limiting boundary condition, $\mu_0$ (i.e., the viscosity at very low shear rates). However, when one tries to explore results for a range of increasing flow rates, the convergence process can be significantly speeded up by retaining the last solved solution for viscosity. Once each pore and throat has been assigned an effective viscosity and conductance, the relationship between pressure drop and flow rate across each element $i$ can be found.

$$Q_i = G_i \frac{\Delta P_i}{\mu_{eff,i} L_i}$$ (4.22)

When conservation of volume in each pore with appropriate inlet and outlet boundary conditions (constant pressure) is invoked, the pressure field is solved across the entire network using standard techniques (see, for instance, Blunt et al. [25]). As a result the pressure drop in each network element is now known, assuming the initial guess for viscosity. Then, using Eq. 4.22, the effective viscosity of each pore and throat is updated and the pressure recomputed. The method is repeated until satisfactory convergence is achieved. In our case, convergence must be achieved simultaneously in all the network elements.

The tolerance used for the simulation studies was set such that the pressure would be recomputed if the flow rate in any pore or throat changed by more than 1% between iterations. The total flow rate across the network $Q_T$ is then computed and an apparent viscosity is defined as follows,

$$\mu_{app} = \mu_N Q_N$$ (4.23)

where $Q_N$ is the total flow rate for a simulation with the same pressure drop with a fixed Newtonian viscosity $\mu_N$. The Darcy velocity is obtained from $q = Q_i / A$, where $A$ is the cross-sectional area of the network.
4.4 Summary

A method was described to extend the predictive capabilities of a state-of-the-art two-phase network model to cope with shear-thinning rheology. The model uses topologically disordered networks of irregularly shaped pores connected by throats that represent different porous media of interest. Analytical solutions were derived for the flow of truncated power-law fluids in circular capillaries. This is used as a basis for flow calculations in more irregular-shaped elements. Using an iterative method, the pressure field can be solved across the network and consequently the total flow rate can be predicted for a given pressure drop.

However, there are a number of assumptions in the method. Again, the rheology of the non-Newtonian phase is assumed to be well-described by a truncated power-law model. Effects of adsorption, surface exclusion, and elasticity were not considered. Furthermore, the expressions for the hydraulic conductivity of the non-Newtonian phase in irregular-shaped elements used an empirical expression for effective radius and were based on solutions for a circular tube.
Chapter 5

Predicting Single-Phase Non-Newtonian Flow

In the previous chapter, a method was developed to extend the predictive capabilities of pore-network models to non-Newtonian flow. The network model is used to predict the relationship between apparent viscosity and flow rate in porous media. This model is now validated by predicting several experimental results. First, the model predictions are tested against two experiments in sand-packs and then against two sandstone data-sets.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$C_p$</th>
<th>$C$</th>
<th>$n$</th>
<th>$\mu_0$</th>
<th>$\mu_\infty$</th>
<th>$\phi$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hejri et al. [12] (Sand-pack)</td>
<td>1000ppm</td>
<td>0.181</td>
<td>0.418</td>
<td>0.5</td>
<td>0.0015</td>
<td>0.35</td>
<td>0.893</td>
</tr>
<tr>
<td>Vogel &amp; Pusch [13] (Sand-pack)</td>
<td>—</td>
<td>0.04</td>
<td>0.57</td>
<td>0.1</td>
<td>0.0015</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>Cannella et al. [4] (Berea Sandstone)</td>
<td>1200ppm</td>
<td>0.17</td>
<td>0.53</td>
<td>0.1</td>
<td>0.0015</td>
<td>0.2</td>
<td>0.264</td>
</tr>
<tr>
<td>Fletcher et al. [5] (Berea Sandstone)</td>
<td>500ppm</td>
<td>0.0115</td>
<td>0.73</td>
<td>0.0115</td>
<td>0.0015</td>
<td>0.2</td>
<td>0.263</td>
</tr>
<tr>
<td>(Brent Sandstone)</td>
<td>1000ppm</td>
<td>0.05</td>
<td>0.59</td>
<td>0.069</td>
<td>0.0015</td>
<td>0.33</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 5.1: Truncated power-law parameters used to fit the bulk experimental data.

5.1 Sand-pack Predictions

5.1.1 Hejri et al. Experiments

Hejri et al. [12] studied the flow of Flocon$^{TM}$ 4800 solution (Xanthan biopolymer) in various unconsolidated sand-packs covering a wide range of permeability values.
They conducted single-phase displacement experiments for three polymer concentrations. The bulk shear-rate dependence of the viscosity of their 1000 ppm solution is shown in Fig. 5.1. The corresponding rheological parameters used as input in the model were determined by fitting a truncated power-law (Eq. 2.3) to the digitalised experimental data [12]. These are summarized in Table 5.1 for each experiment. The match between the truncated power-law model predictions and the experimental results is obtained by minimization of $\sigma$ over the range of interest,

$$
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( 1 - \left( \frac{\mu_i}{\mu_{\text{net},i}} \right) \right)^2}
$$

(5.1)

where the subscript net stands for network results and the label $i$ runs over all $N$ data points. Note, however, that their experiments only covered the shear-thinning region, which means that the assignment of high- and low-viscosity cutoffs is somewhat arbitrary. In this case, we make use of the correlations previously derived in Chapter 2 (Eqs. 2.8a, 2.8b & 2.8c) to infer the missing parameters.

![Figure 5.1: Rheology of 1000 ppm Flocon$^{TM}$ 4800 MX solution, experimental data (o) and truncated power law-fit (−) (after Hejri et al. [12]).](image)

To reproduce experimental conditions as closely as possible, we used a network based on a sand-pack whose main characteristics are presented in Appendix C. Simulations were run covering a range of flow rates corresponding to the experimental conditions. The predicted apparent viscosity results are shown in Fig. 5.2. The
The shape of the curve is correct, but it is shifted to higher flow rates. The reason for this is obvious: the network sand-pack has a much higher permeability (101D) than the experimental system (0.893D). This means that in the network the pores are, on average, larger with lower shear rates for a given velocity, resulting in higher apparent viscosities. We can easily account for this effect by realizing that simply re-scaling the network size will result in a porous medium of identical topological structure, but different permeability. A new network was generated with all lengths scaled by a factor $\sqrt{K_{\text{exp}}/K_{\text{net}}}$, where the superscripts $\text{exp}$ and $\text{net}$ stand for experimental and network, respectively, and the simulations re-run. By construction the re-scaled network now has the same permeability as the experimental sand-pack, but otherwise has the same structure as before. The results now almost perfectly overlie the experimental data (Fig. 5.2). Note that this is not an *ad hoc* procedure since this scaling is based on the experimentally measured permeability.

While the length re-scaling works well, it is somewhat cumbersome since it involves the generation of a new network for each case considered. Another approach is to re-scale the $\mu_{\text{app}}(q)$ curve using Eq. 3.3. If Eq. 3.3 were valid, then to obtain the correct apparent viscosity for a medium with a different permeability and porosity.
than the network, we need to re-scale the network flow rates as follows:

\[ q^{re\text{-scaled}} = q^{net} \sqrt{\frac{K^{exp}}{K^{net}}} \frac{\phi^{exp}}{\phi^{net}} \]  

(5.2)

Applying this post-priori re-scaling gives identical results to re-adjusting all the lengths in the network (Fig. 5.2). This is despite the fact that Eq. 3.3 is not strictly valid in this case since the shapes of the bulk and porous medium viscosity curves are different. The introduction of a porosity ratio in Eq. 5.2 has negligible effect since the experimental and network porosities are similar. Since the scaling in Eq. 5.2 is easy to apply and gives the same results as re-scaling the network, we will use Eq. 5.2 for all our subsequent predictions. Note again that this scaling is based on experimentally measured parameters and involves no adjustable constants.

Figure 5.3: Comparison between the experimental data (○) of Hejri et al. [12] and an empirical relation (—) using the bulk rheology and an empirical scaling factor \( \alpha \) (Eq. 3.3).

Figure 5.3 compares the rheology measured in the porous medium with a prediction based on Eq. 3.3 with a best fit to the data (using Eq. 5.1) obtained for \( \alpha = 1.05 \). Notice two problems associated with this empirical approach. First, as discussed before, \( \alpha \) cannot be predicted a priori. Second, Eq. 3.3 cannot match the low-velocity behaviour accurately: this is because the results are affected by the low-rate cutoff in the viscosity that affects some of the smaller pores in the sand-pack, even though it is not evident in the bulk properties. However, as will be shown later,
provided that \( \alpha \) could be quantified \textit{a priori}, the predictions of \textit{in-situ} rheograms from Eq. 3.3 can be improved by use of a different bulk rheological model, such as the Carreau model.

### 5.1.2 Vogel & Pusch Experiments

Vogel & Pusch [13] conducted flow experiments on sand-packs using three different types of polymer: a polysaccharide solution, a hydroxyethylcellulose (biological origin) solution, and a polyacrylamide solution (HPAM). As before, a network based on a sand-pack (Appendix C) was used to predict the flow of their polysaccharide solution (Xanthan-like properties). As for Hejri \textit{et al.}, the bulk rheological parameters were determined from a best fit (Eq. 5.1) to the digitalised data from the authors (see Fig. 5.4 and Table 5.1).

![Rheology of a polysaccharide solution](image)

Figure 5.4: Rheology of a polysaccharide solution, experimental data (○) and truncated power-law fit (–) (after Vogel & Pusch [13]).

After re-scaling using Eq. 5.2, good agreement was found between the experimental results and the model simulations (Fig. 5.5). The experimental data were also compared with the inferred \textit{in-situ} rheogram obtained from Eq. 3.3 with \( \alpha = 1.34 \) (Fig. 5.6). This time the agreement is good with the appropriate value of \( \alpha \), although it is now different from its previous value.
Figure 5.5: Comparison between network simulations (- - & —) and the experimental data (◦) of Vogel & Pusch [13]. The re-scaled network results are identical and match the experimental data, if all the lengths are multiplied by a constant factor to match the experimental permeability or if the flow rate is re-scaled using Eq. 5.2.

Figure 5.6: Comparison between the experimental data (◦) of Vogel & Pusch [13] and an empirical relation (——) using the bulk rheology and an empirical scaling factor $\alpha$ (Eq. 3.3).
5.2 Sandstone Predictions

5.2.1 Cannella et al. Experiments

Cannella et al. [4] investigated the flow behaviour of Xanthan in porous media for different polymer concentrations, rock lithologies, residual oil saturations, and rock permeability. They used experimental results and effective medium theory to relate Darcy velocity to \emph{in-situ} shear rate (Eq. 3.23). They provided us with a very good case for comparison since they used the same type of fluid in the same type of rock (Berea sandstone) as our simulations. To analyze their findings, a fluid rheology was used in the network simulations (see Table 5.1), corresponding to their 1200ppm Xanthan solution (Fig. 5.7).

![Rheology of 1200 ppm Xanthan solution, experimental data (○) and truncated power-law fit (−) (after Cannella et al. [4]).](image)

Figure 5.7: Rheology of 1200ppm Xanthan solution, experimental data (○) and truncated power-law fit (−) (after Cannella et al. [4]).

The sand-pack network was first used to address the need for using a more elaborate porous medium representation. Simulation results (Fig. 5.8) showed that even after re-scaling the model was not able to reproduce the experimental results. For a given Darcy velocity, the apparent viscosity was overestimated by the network. This is what one would expect intuitively since for a given pressure gradient, the more homogeneous and simple the porous medium, the greater the flow. The sand-pack fails to capture the complexity of a highly tortuous consolidated sandstone. When a Berea network was used (see Appendix B), the results were in good agreement with
the experiment (Fig. 5.8). This emphasizes the importance of capturing both the pore-scale physics and the correct pore structure to make accurate predictions.

Figure 5.8: Comparison between network simulations (--- & ---) and the experimental data (○) of Cannella et al. [4]. Note that to match experimental data in a sandstone, a sandstone network (---) needs to be used – the sand-pack results (---) fail to predict the data accurately.

However, the network simulations did not match the experimental data at very low flow rates where the apparent viscosity does not reach a maximum value. The experiments suggested a higher viscosity at low flow rates in the porous medium than in the rheometer. This is possibly due to pore blocking by adsorption, which is not accounted for in the model. However, the Berea network model predictions were found to be in good agreement for the shear-thinning region. For very high Darcy velocities, the model predictions deviated moderately from the experimental points, although the appropriate limiting conditions were reached.

The in-situ rheogram inferred from the bulk using Eq. 3.23 best matches the experimental data for $\alpha = 4.8$ (Fig. 5.9). Note the scaling factor is now much greater than for the sand-pack experiments. As explained in the next chapter, this is due to the more complex topology and greater level of heterogeneity of the Berea sandstone.
Figure 5.9: Comparison between the experimental data (○) of Cannella et al. [4] and an empirical relation (—) using the bulk rheology and an empirical scaling factor $\alpha$ (Eq. 3.3).

### 5.2.2 Fletcher et al. Experiments

Fletcher et al. [5] also studied the flow of Flocon$^\text{TM}$ 4800MXC solutions in various reservoir sandstones. They conducted several coreflooding experiments to investigate the effects of core preparation, polymer concentration, temperature and flow rate on the overall macroscopic results. The bulk rheogram of their 500 ppm and 1000 ppm Xanthan solutions measured experimentally is shown in Fig. 5.10. Again, the truncated power-law parameters used in the flow model were determined from a best fit to the experimental data (Table 5.1).

Fig. 5.11 shows the experimental results they obtained for the flow of their 500 ppm Xanthan solution in a Berea core (0.264 $D$). After re-scaling, the predictions made with the Berea network model capture correctly the trend of data but do appear to overestimate the apparent viscosity. This is thought to be a direct consequence of adsorption phenomena occurring in the porous medium. From their various experiments, the authors concluded that Berea sandstone exhibited significant polymer retention in porous media compared with higher permeability sandstones such as Clashach and Brent. They attributed this difference in pore-blocking phenomena to polymer adsorption due to the presence of clay in Berea.
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Figure 5.10: Rheology of 500 ppm & 1000 ppm Xanthan solutions, experimental data (○) and truncated power-law fit (−) (after Fletcher et al. [5]).

Figure 5.11: Comparison between network simulations (−), the experimental data (○) of Fletcher et al. [5] and an empirical relation (− −) using the bulk rheology and an empirical scaling factor $\alpha$ (Eq. 3.3).
To assess the extent to which polymer retention may have affected the in-situ results, the Berea network was also used to predict the experimental flow results of the 1000ppm Xanthan solution in a Brent sand core (1.32D). As can be seen from Fig. 5.12 once the results have been re-scaled using Eq. 5.2, the network predictions are in very good agreement with the measured data. In general pore-blocking phenomena are strongly affected by the average pore size, i.e. the higher the permeability, the less polymer retention, and by the presence of clays. Since the method does not model explicitly these phenomenological effects, the network simulations overpredict the flow through the medium. However, in the absence of such effects, the predictions from the network model are excellent.

Figure 5.12: Comparison between network simulations (—) and the experimental data (o) of Fletcher et al. [5].

Note that, although not measured in these two experiments, the authors observed a low velocity Newtonian viscosity plateau for various concentrations of Xanthan solutions in porous media. For low-retention media such as Brent and Clashach sands, both Newtonian viscosity plateaux were observed experimentally. This is strong experimental confirmation that, at least for the systems considered here and in the absence of adsorption or pore blocking (see Chapter 3), the observed in-situ behaviour could in principle be scaled from the bulk rheogram.
5.3 Discussion

5.3.1 Carreau Model Predictions

As mentioned in the previous chapter, the advantage of using a truncated power-law to model the bulk rheology of Xanthan is that it allows the derivation of exact analytical solutions for the flow of shear-thinning fluids in circular capillaries. These expressions are required in the network model to relate the effective viscosity of each pore and throat to pressure gradient. However, this method was found to lead to convergence problems for low values of the shear-thinning exponent, $n$.

This is illustrated here by looking at the flow of a high concentration solution of Xanthan in a sand-pack. The generic bulk rheology used in this example is taken from Table A.1 (Appendix A) representing a 2000 ppm Xanthan solution. Just as before, the sand-pack network is used to predict the apparent viscosity versus Darcy velocity curve for this system.

![Network Results (truncated power-law)](image)

Figure 5.13: Sand-pack network simulations results of a generic 2000 ppm Xanthan solution using a truncated power law ($n = 0.32$).

As can be seen from Fig. 5.13, the model appears to predict unphysical solutions. This is further illustrated by looking at the flow behaviour of individual pores and throats. Fig. 5.14 shows the effective viscosity values of two throats randomly chosen in the network, one with triangular and one with circular cross section, correspond-
Effective viscosity values of two network throats as a function of flow rate using a truncated power-law model ($n = 0.32$). Note that the effective viscosity vary non-monotonically with flow rate, indicating a poorly converged solution.

To a good approximation, the effective viscosity of a shear-thinning fluid flowing with flow rate $Q$ in a circular tube can be calculated via Eq. 5.3.

$$
\mu_{\text{eff}} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + (\frac{Q}{Q_{\text{crit}}})^2}^{\frac{1}{n}}
$$

(5.3)

Just as for the bulk rheological model, we define a critical flow rate, $Q_{\text{crit}}$, based on the critical shear rate defined by Eq. 2.5.

$$
Q_{\text{crit}} = \frac{\pi \gamma_{\text{crit}} R_{\text{equ}}^3}{3}
$$

(5.4)

The equivalent radius $R_{\text{equ}}$ for non-circular elements is defined as before, by Eq. 4.18.
Eq. 5.3 & Eq. 5.4 are used in the network to update viscosity values in each element, once the velocity field has been calculated. These expressions are found empirically to fit the previously derived analytical expressions for \( \mu_{eff} \) as a function of \( Q \) for higher values of \( n \) (Eq. 4.12, Eq. 4.13 and Eq. 4.15), as shown in Fig. 5.16. Furthermore, Eq. 5.3 being a smoother function, a significant increase in convergence speed and lower tolerance were achieved. At low \( n \) values, this approach eliminated completely convergence-related issues. As can be seen from Fig. 5.15, the macroscopic results for the flow of very shear-thinning fluids can now be predicted reliably.

Finally, to verify the validity of the method, one of the experiments previously presented is predicted using this approach. The measured bulk rheology corresponding to Cannella et al. experiments is now modeled using a Carreau law (Eq. 2.7). The parameters corresponding to a best fit to the data are used as input to the fluid model. Fig. 5.17 shows the comparison between their experimental results for the flow of their 1200 ppm Xanthan solution, and the Berea network simulations results. The predictions made using the Carreau law approach are almost identical to those made using the truncated power-law model and only deviate slightly at high velocities (Fig. 5.17), which is consistent with the bulk behaviour of both models (see Fig. 2.5).

Figure 5.15: Sand-pack network simulations results of a generic 2000 ppm Xanthan solution using a truncated power-law (\( \diamond \)) and a Carreau law (\( - \)) (\( n = 0.32 \)).
Figure 5.16: Effective viscosity values of two network throats as a function of flow rate using a Carreau (— & - - -) and truncated power-law model (△ & ◦) ($n = 0.54$).

Figure 5.17: Comparison between network simulations using a Carreau law (—) and truncated power-law (△) compared to the experimental data (◦) of Cannella et al. [4].
5.3.2 Scaling Factors From Experiments

As discussed in Chapter 3, the traditional approach in the literature is to introduce a length scaling to relate the viscosity/shear rate relationship to one between viscosity and Darcy velocity in a porous medium. At first sight it appears that we have done something similar, in introducing a permeability, or rather a length scaling, to our network model results. However, there is a significant difference: the method is capable of matching four experimental data sets on different systems without any arbitrary adjustable parameters – the permeability scaling comes from easily obtained independent experimental data. In contrast, all previous, more empirical, approaches have needed to introduce an extra factor that varies with different porous media and fluid systems. To emphasize this, Table 5.2 shows $\alpha$ factors (Eq. 3.3) and $\beta$ constants (Eq. 3.23) necessary to match the four data sets presented here.

Notice that even for the same porous medium, the values necessary to obtain a match vary. This illustrates the dependence of the scaling factor on the porous medium structure, but also on the fluid rheology as well as other phenomenological effects. With use of the network model that captures correctly the topological complexity of the porous media and its nonlinear relationship with flow, all the data is predicted a priori. However, in order to be in position to make macroscopic predictions of the flow of shear-thinning solutions, it is necessary to have a detailed characterization of the pore space.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Medium</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hejri et al. [12]</td>
<td>Sand-pack</td>
<td>1.05</td>
<td>1.3</td>
</tr>
<tr>
<td>Cannella et al. [4]</td>
<td>Berea Sandstone</td>
<td>4.8</td>
<td>6.0</td>
</tr>
<tr>
<td>Fletcher et al. [5]</td>
<td>Berea Sandstone</td>
<td>5.3</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>Brent Sandstone</td>
<td>2.8</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 5.2: Calculated $\alpha$ (Eq. 3.3) and $\beta$ (Eq. 3.23) values for the four experiments studied.
5.4 Summary

A pore-network model was used to predict the relationship between apparent viscosity and flow rate in porous media. The model uses topologically disordered networks of irregularly shaped pores connected by throats that represent different porous media of interest. Analytical expressions were used to find the effective viscosity in each pore and throat as a function of pressure drop, assuming a truncated power-law shear-thinning fluid. The Carreau model can also be used as rheological model but the subsequent equations used to relate effective viscosity to pressure drop are then derived empirically. An iterative procedure was used to find the flow throughout the network and from this an apparent viscosity could be defined. A length re-scaling was applied to predict results for porous media with different permeabilities than the network. A sand-pack and a Berea network were used to predict four different data sets in the literature. The model input data was the permeability and porosity of the system and its bulk rheological behaviour. The apparent viscosity of the fluid in the porous medium was then predicted as a function of flow rate. In all cases the agreement between simulation and experiment was good without resorting to any arbitrary adjustable parameters. In contrast, the traditional empirical approaches would not have been able to predict the data without resort to an ad hoc scaling parameter that varied with the fluid system and porous medium.
Chapter 6

Investigation of Single-Phase Polymer Flow

In Chapter 5, network modelling was used to predict the flow of shear-thinning fluids (such as Xanthan) in porous media. The approach successfully reproduced the results of different experiments in the literature, but did require a detailed topological description of the porous medium of interest, which is not always attainable in practice. Nonetheless, this work offers the opportunity to predict reliably the behaviour of non-Newtonian fluids in porous media and to investigate the flow behaviour of shear-thinning fluids for a variety of situations where experimental data may not be readily available. Of particular interest is to try and use the network model to evaluate and further develop empirical approaches such as the capillary bundle model, while also providing guidelines on how to predict flow behaviour of polymer solutions in different systems.

6.1 Bulk and In-Situ Rheology

In order to investigate the effect of varying bulk rheology on the macroscopic flow results while avoiding any convergence related issues, the constitutive Carreau equation will be used here as the rheological model (Eq. 6.1).

\[
\mu_{\text{eff}} = \mu_{\infty} + \frac{\mu_0 - \mu_{\infty}}{[1 + (\frac{\dot{\gamma}}{\gamma_{\text{crit}}})^2]^\frac{1-n}{2}}
\]  

(6.1)

where,

\[
\gamma_{\text{crit}} = \left(\frac{\mu_0}{C}\right)^{\frac{1}{n+1}}
\]  

(6.2)

As explained previously, this model is a good fit to most bulk rheological data and empirical solutions can be found to relate the effective viscosity of each pore and throat to pressure gradient. For the rest of this study a generic bulk rheology, presented in Table 6.1, is used as input to our flow model.
The standard semi-analytical approach introduces a length scaling to relate bulk rheograms to properties measured \textit{in-situ}. Again, these rheograms appear to be shifted from the bulk curve by a constant factor, $\alpha$ (Eq. 3.4), or $\beta$ as defined in Eq. 3.23. From the capillary bundle model formulation, $\alpha$ should be linked to a measure of tortuosity in the medium. Therefore one expects $\alpha$ to increase with media heterogeneity and complexity. However the variation of $\alpha$ with fluid properties appears less intuitive. Based on effective medium theory and their experimental results, Cannella \textit{et al.} [4] proposed the apparent porous medium shear rate in the shear-thinning region to be:

$$
\dot{\gamma}_{\text{app}} = \beta \left( \frac{3n + 1}{n} \right)^{\frac{n}{n-1}} \frac{q}{\sqrt{K\phi}} \quad (6.3)
$$

where $\beta$ is an experimentally determined constant that is independent of fluid properties. Note that a close relationship exists between Eq. 3.4 and Eq. 6.3.

$$
\alpha = \beta f(n) = \beta \left( \frac{3n + 1}{n} \right)^{\frac{n}{n-1}} \quad (6.4)
$$

However, when plotted (Fig. 6.1), the variations of Eq. 6.4 appear to be weakly dependent on $n$ for $0 < n < 1$. Nonetheless, the value of $\alpha$ or $\beta$ for a particular medium and fluid cannot be predicted theoretically a priori. However, one could attempt in principle to quantify its value experimentally by running a large number of experiments with different media and polymer concentrations. The approach presented here intends to do just that but without actually having to do the experimental work. We assume the results from the network model are close to the experimentally measured ones, and quantify $\alpha$ over a wide range of rheologies. We make use of three different networks to ascertain whether the observed behaviour is medium-related.

The variations of $\alpha$ with $n$ in a given medium are investigated with all other parameters remaining constant. However, varying the power-law exponent on its own may appear rather artificial since physically the values of all four parameters used in Carreau’s model are linked to the amount of polymer present in solution. For the sake of completeness, a short summary of experimentally measured bulk parameters from the literature is provided in Appendix A.
From the collected data, correlations were established that allow the reader to infer the probable bulk behaviour of a Xanthan solution for a given concentration. Although there is some scatter in the data, the correlations derived give a reasonable quantification of the bulk parameters where no experimental data are available. For simplicity we only present here results obtained by varying $n$ on its own. However, the behaviour is principally controlled by $n$ alone and the other parameters have little impact on the results, except for low values of $n$ – these cases are discussed in detail later.

### 6.2 Network Simulations

In this section, use is made of networks derived from three different porous media: a sand-pack, a Berea sandstone and heterogeneous North Sea sandstone. As mentioned previously, the networks are mainly composed of irregular-shaped triangular elements whose characteristics are derived from the three-dimensional representation of the medium. Once again, notice from their properties in Appendix B, C & D that the standard deviation of the pore and throat size distributions divided by the mean is smallest for the sand-pack network and largest for the heterogeneous sandstone. The three networks span a plausible range of properties encountered in porous media. Details about the flow model are described in Chapter 4 & 5.
Fig. 6.2 shows the flow behaviour predicted by the model for the sand-pack network with the rheology in Table 6.1 where $n = 0.5$, and rheogram predictions based on Eq. 3.3. The best fit to the data is obtained for $\alpha = 1.389$.

![Sand-pack network results: $n = 0.5$](image)

Figure 6.2: Sand-pack network (○) and predicted rheogram (—) results for $n = 0.5$.

Just as before, the match between the network results and Eq. 3.3 is obtained by minimization of $\sigma$ in Eq. 5.1 over the shear-thinning region. As seen in Fig. 6.2, agreement between results from Eq. 3.3 and the network are excellent with a fitted value of $\alpha$. Hence, we can now predict macroscopic results for a wide range of $n$ values. For each $\mu_{app}(q)$ curve, a value of $\alpha$, as defined in Eq. 3.3, can be determined, so that the bulk rheogram matches the in-situ results. It is assumed here that these two curves do represent the observed experimental behaviours, both in the bulk and in the porous medium.

Fig. 6.3 shows the values of $\alpha$ (found through minimization of Eq. 5.1) for the sand-pack network over a range of $n$ values. To a reasonable approximation $\alpha$ is constant for large values of $n$. Also shown on the graph is a curve using Eq. 6.4 with a fixed value of $\beta$ chosen to match the high $n$ results. Using $\beta = 1.35$ gives a good prediction of $\alpha$ for all values of $n$. 

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Figure 6.3: Variation of the scaling factor $\alpha$ for the sand-pack network. The unfilled triangles represent cases where the scaling of the bulk rheogram to predict the rheology in the porous medium does not provide accurate results. Also indicated is the $\alpha$ value for a match to Hejri et al. [12] ($\alpha = 1.05, n = 0.418$) and Vogel & Pusch [13] ($\alpha = 1.34, n = 0.57$) experiments. The solid line shows results using Eq. 6.4 with $\beta = 1.35$.

Figure 6.4: Sand-pack network predictions (- - -), Vogel & Pusch [13] experimental data (○) and predicted results for $\alpha = 1.34$ using Eq. 3.3 (- -).
Fig. 6.3 also shows the value of $\alpha$ corresponding to a match to Vogel & Pusch’s experiments [13]: $\alpha = 1.34$ for $n = 0.57$ (Fig. 6.4). The network simulations overestimated $\alpha$ by about 4%. As can be seen from Fig. 6.4, this discrepancy is small and well within the likely experimental error. For the other sand-pack experiment the prediction is not so good, although still acceptable.

The analysis is repeated for the Berea sandstone network (Fig. 6.5). As expected, a higher value of $\alpha$ than for the sand-pack is necessary in order for the bulk rheogram to match the in-situ results. Unlike the previous example, the variations of $\alpha$ with $n$ appear to be somewhat less than those predicted by Eq. 6.4 using $\beta = 2.96$.

![Figure 6.5](image.png)

Figure 6.5: Variation of the scaling factor $\alpha$ for the Berea network. The unfilled circles represent cases where the scaling of the bulk rheogram to predict the rheology in the porous medium does not provide accurate results. Also indicated is the $\alpha$ value for a match to Cannella et al. [4] ($\alpha = 4.8, n = 0.53$) and Fletcher et al. [5] ($\alpha = 2.8, n = 0.59$) experiments. The solid line shows results using Eq. 6.4 with $\beta = 2.96$.

While the network predicts an $\alpha$ value of 3.0 for $n = 0.48$, Cannella et al. [4] experiments are best matched for $\alpha = 4.8$ (Fig. 5.9) - the network model underpredicts $\alpha$ by 62%. However, the actual predictions of apparent viscosity versus Darcy velocity from the network model are still in reasonable agreement with the experimental results (Fig. 5.9), especially considering the range of error and uncertainty associated with each experiment and the possible effects of polymer adsorption in their experiments. This corresponds to the worst $\mu_{app}$ versus $q$ predictions from the model.
For a more complex and heterogeneous system, the variations in $\alpha$ are more significant (Fig. 6.6).

The values of $\alpha$ necessary to predict in-situ rheograms are higher for a more heterogeneous medium. Variations with $n$ are larger in amplitude than previously observed, and do not seem to be satisfactorily captured by Eq. 6.4. However, for relatively large values of $n$ (depending on the complexity and heterogeneity of the system), it is possible to define a value for $\alpha$ so that the bulk and in-situ rheograms match. The scaling factor, $\alpha$, varies with $n$ in a manner that appears to be broadly consistent with effective medium theory. The range of $\alpha$ values found from the network simulations is consistent with previous studies in the literature [4–6,12].
### 6.3 Predictions for low values of $n$

The white-filled symbols on Figs. 6.3, 6.5 & 6.6 indicate the values of $n$ for which significant difference between the porous medium and bulk rheograms are observed. For instance, Fig. 6.7 shows the predicted results from the heterogeneous sandstone network and using Eq. 3.3 for $n = 0.01$. The predicted bulk rheogram departs significantly from the network results. The apparent porous medium $n$ appears to be larger than the bulk “measured” one. The value of $n$ at which this difference is first observed increases with the heterogeneity of the pore space.

![Figure 6.7: Heterogeneous sandstone network (○-)and predicted rheogram (—) results using Eq. 3.3 for $n = 0.01$. In this example the apparent porous medium power-law exponent is slightly larger than inferred from the bulk rheogram.](image)

The error in the predicted rheogram using Eq. 3.3 is significant and only a couple of points along the curve are accurately matched. This behaviour can be explained by looking at the distribution of effective viscosity in the network. We start with the case where $n = 0.5$. Even for the more heterogeneous sandstone network, the predicted rheogram using Eq. 3.3 is in very good agreement with the network results. We look at an individual point along the $\mu_{app}(q)$ curve corresponding to an apparent viscosity of $3.6 \times 10^{-2}$ Pa.s with a Darcy velocity of $10^{-6}$ m.s$^{-1}$. If one looks at a histogram of effective viscosity for each throat in the network (Fig. 6.8), one expects...
to see that most of the network elements are in the shear-thinning regime.

![Effective viscosity histogram for the heterogeneous sandstone network](image_url)

Figure 6.8: Effective viscosity histogram for the heterogeneous sandstone network for a Darcy velocity of $10^{-6}$ m.s$^{-1}$ ($n = 0.5$). The average, apparent viscosity $\mu_{\text{app}}$ is indicated.

Most elements show a distribution of values centered around the overall effective viscosity of $3.6 \times 10^{-2}$ Pa.s. This is what one would expect intuitively despite the fact that there is no obvious formal relationship between the apparent viscosity of the network and individual elements’ effective viscosity. There are some elements where there is very little flow and as a consequence have a high viscosity, indicating the low shear rate Newtonian plateau. However, these elements have little impact on the overall viscosity.

A two-dimensional slice taken from the middle of the heterogeneous sandstone network is shown to illustrate this effect (Fig. 6.9). Pores and throats are coloured according to their effective viscosity values. The size of the bonds on the figure is proportional to the amount of fluid carried through the element. As can be seen from Fig. 6.9 most of the elements participating to flow have an effective viscosity in the range $3 \times 10^{-2}$ Pa.s to $6 \times 10^{-2}$ Pa.s. A large number of flow pathways are visible and viscosity contrasts between them are small.
We now look at the case where $n = 0.01$. To make a meaningful comparison a Darcy velocity that gives the same apparent viscosity as before, $3.6 \times 10^{-2}$ Pa.s, is chosen. The histogram of effective viscosity (Fig. 6.10) now appears rather different from the previous case, despite the fact it is the same medium with the same apparent viscosity. Note that the large peak at 0.5 Pa.s indicates that a majority of the pores and throats experience a low pressure drop and have a viscosity equal to the low shear rate plateau value $\mu_0$. For low $n$ values, the viscosity histogram is bi-modal. Very few network elements have a viscosity close to the apparent one in the shear-thinning regime.
Figure 6.10: Effective viscosity histogram for the heterogeneous sandstone network for a Darcy velocity of $2.17 \times 10^{-7}$ m.s$^{-1}$ ($n = 0.01$). The average, apparent viscosity $\mu_{\text{app}}$ is indicated.

This essentially reflects a “streaking” pattern in the medium, with a population of non-flowing pores and throats and a population of elements carrying most of the flow. These elements have a viscosity close to $\mu_\infty$ and therefore overall, not all the pores and throats are in the shear-thinning regime. This leads to an apparently higher power-law exponent and consequently the predicted rheogram from the bulk curve cannot match satisfactorily the porous medium results. This phenomenon is clearly illustrated by Fig. 6.11. As before, a two-dimensional slice of the network is shown here, where each element has been coloured according to effective viscosity values. Notice that unlike previously, many fewer throats have their viscosity within the range of interest. A few preferred flow pathways appear and their size show that they carry a substantial amount of the total flow through the network.

As already mentioned in Chapter 3, Pearson and Tardy [60] performed two-dimensional pore-network simulations of the flow of shear-thinning fluids and also showed this apparent streaking effect for low values of the power-law exponent $n$. When this effect is significant using a re-scaled bulk rheogram to predict the porous medium properties only gives an approximate estimate of the behaviour, even if an appropriate value of $\alpha$ can be estimated. In this regime the porous medium rheogram depends on $n$ and the plateau viscosities and cannot be readily predicted from the bulk rheology alone.
Figure 6.11: Two-dimensional slice of the heterogeneous sandstone network \((q = 2.17 \times 10^{-7} \text{ m/s and } n = 0.01)\). Pores and throats are coloured according to their effective viscosity values. The width of a pore or throat is proportional to the amount of fluid carried by the element.

6.4 Scaling Factor Dependency on Porous Media Structure

For relatively large values of \(n\), one can define a more general type of rheogram, valid for a given rheology, by looking at the variations of apparent viscosity versus \(q/\sqrt{K\phi}\) for different media type. For instance in Fig. 6.12, we compare the porous media behaviour of our four systems (bulk, sand-pack, Berea and heterogeneous sandstone) for a given value of \(n\) \((n = 0.5)\). As explained in the previous section, the value of \(\alpha\) necessary to match each curve with the bulk one can be assumed to be constant for relatively large values of \(n\) \((n > 0.6)\).
From the figure one can see that the scaling factor between the bulk and in-situ rheogram can be interpreted as some sort of “measure” of the level of complexity and heterogeneity of a porous medium. However, it is far from obvious which of the media topological features (pore and throat size distributions, connectivity, . . . ), if any, has the biggest impact on the value of $\alpha$. As of yet, there is no theory that can predict $\alpha$ a priori and although the analysis undertaken in this chapter aims at providing some general guidelines on how to estimate its value, it still limited to a relatively small subset of porous media of known structure.

6.4.1 Cubic Network Generation

To ascertain whether $\alpha$ depends on some measurable medium property, and hence can be predicted, we use a series of networks based on cubic lattices. The networks generated are based on a regular cubic lattice and although they are not based on real three-dimensional systems, their topological features can easily be changed to mimic those of real rocks.
All the cubic networks used here are based on a $20 \times 20 \times 10$ lattice size. Initially, the average connection number, $Z$, is equal to 6 and the network comprises of 4,000 pores and 12,200 throats. Fig. 6.13 shows a representation of the basic lattice used to generate the cubic networks. All pores and throats are assigned a constant radius and have circular shapes.

As discussed in Chapter 4, during multi-phase displacements, angular pore shapes allow the wetting phase to reside in the corners, while the non-wetting phase fills the centers. Although only single-phase flow is being considered here, pores and throats are still assigned non-circular shapes. The shape of pores and throats in the networks are determined according to their shape factor, $G$. Triangular elements have shape factors ranging from 0.0 (slit-shaped) to 0.04811 (equilateral). Square and circular elements have fixed shape factors of 0.0625 and 0.07958 ($1/(4\pi)$), respectively. The proportion of triangle, square and circular elements is taken to be similar to the Berea network. Since these networks are not based on a real rock sample, the shape factors of triangular elements are distributed according to a truncated Weibull distribution (Eq. 6.5).

$$\xi = (\xi_{max} - \xi_{min}) \left( -\delta \ln \left[ x \left( 1 - e^{-1/\delta} \right) + e^{-1/\delta} \right] \right)^{1/q} + \xi_{min}$$

(6.5)

where $x$ is a random number between 0 and 1 and $\xi$ the property of interest (here the shape factor).
Throat sizes and lengths are assigned so as to reproduce the statistical properties of the Berea network (see Appendix B). For instance Fig. 6.14 illustrates how the cumulative frequency of the Berea network throat radii is reproduced hence creating a similar frequency distribution for the cubic network but for a different number of elements.

Figure 6.14: Cumulative frequency (a) and frequency (b) distributions of throat radius of Berea network and a statistically equivalent cubic network.

The cubic network thereby obtained has now equivalent statistical properties to the Berea one but is based on a regular lattice with connection number of 6. By removing randomly throats across the lattice, one can artificially reduce the average connection number of the network. A series of networks with equivalent statistical properties to the Berea one are generated for various average connection numbers. For instance, Fig. 6.15 shows a representation of a cubic network with equivalent size distributions as Berea and average connection number of 4.
6.4.2 Cubic Network Results

Just as before, single-phase flow of a Carreau-like fluid is now simulated through the generated networks and $\alpha$ calculated in each case. The bulk rheology used as input to the flow model is given in Table 6.1 with $n = 0.5$.

Results for $\alpha$ versus average connection number are shown in Fig. 6.16. Also indicated on the figure is the value of $\alpha$ for the Berea network under the same flow conditions. As can be seen, the cubic networks failed to predict the correct value for $\alpha$ in the Berea network and systematically predicted a higher value. These results are surprising since, intuitively, the cubic networks generated might be expected to be more homogeneous than the Berea network. One would expect the results from the cubic networks to underestimate the scaling factor, i.e. predict a value of $\alpha$ in the range of 1-3. However, in the cases presented here, for the same pressure gradient and fluid bulk rheology, smaller flow rates were predicted through the cubic networks. This is despite the fact that the pore and throat size distributions of all the networks are reproduced from the Berea network. It is clear from the results that, although the pore and throat size distributions play an important role, this information alone is not sufficient to predict $\alpha$ and does not adequately represent
the topology and complexity of a real rock such as Berea.

![Alpha vs Average Connection Number](image)

**Figure 6.16:** Variations of $\alpha$ with average connection number (---) for the cubic networks. Also indicated is the value of $\alpha$ for the original Berea network.

The scaling factor is approximately constant for high values of $Z$ but starts increasing dramatically as the networks become less and less well connected. As throats are removed across the network, the flow paths become more tortuous hence reducing the overall flow.

Some statistical information about the spatial distribution of the geometrical parameters that characterizes the Berea network topology is still missing in the cubic networks. Okabe [73] measured the autocorrelation function of Berea sandstone on a three-dimensional image of the rock obtained by micro-CT scanning. Fig. 6.17 shows there exists some spatial correlation in the Berea sample beyond $100 \times 10^{-6}$ m. From the information given in Appendix B, this represents at least 2 pore sizes in the Berea network (largest pores and throats in the network: $\approx 50 \times 10^{-6}$ m). Clearly this would not be captured by the way in which the cubic networks are generated. Because the assignment of pore size and connectivity is entirely done at random, there is no spatial correlation beyond a single pore size. In a network based on a real rock sample, it is less likely to have large contrasts in size over a few pore lengths, i.e. the largest pores and unlikely to be directly connected to the smallest ones. In the cubic networks, some of the natural flow pathways might not be reproduced accurately, hence increasing tortuosity.
Nonetheless, these results could have been anticipated by looking more closely at the statistical properties of the three networks in Appendix B, C & D. All three networks have an average connection number between 4 & 5, similar ranges of pore and throat sizes and yet very different flow properties. For instance, it was already shown in Fig. 5.8 that unlike the Berea network, the sand-pack failed to predict the macroscopic results of a sandstone flooding experiment. However, the three networks do have different ratios of standard deviation of pore and throat size distributions to the mean. Therefore, the analysis is repeated but will now look at the effect of varying this ratio on the value of $\alpha$. 

Figure 6.17: Autocorrelation function of a 3D micro-CT image of Berea sandstone (after Okabe [73]).
Just as before, the Berea network is taken as reference and we investigate whether its value of $\alpha$ can be predicted using cubic networks. A new series of networks are generated this time with constant coordination number of 4.2 (that of the Berea network) but varying size distributions. The throat sizes are distributed using truncated Weibull functions (Eq. 6.5) with $R_{\min} = 4 \times 10^{-6} \text{m}$ and $R_{\max} = 2 \times 10^{-4} \text{m}$. Table 6.2 summarizes the main properties of the networks generated using Eq. 6.5 covering a range of standard deviation to mean ratios.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\eta$</th>
<th>$\delta$</th>
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<th>Std Deviation</th>
<th>Std Deviation / Mean</th>
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<td>$9.87 \times 10^{-6}$</td>
<td>0.717</td>
</tr>
</tbody>
</table>

Table 6.2: Throat size distribution parameters in cubic networks with $Z = 4.2$.

Using the same rheology as before, flow simulations are run and $\alpha$ determined. Fig. 6.18 shows the results obtained for all the networks in Table 6.2 and include the values of $\alpha$ for our three original networks (Berea, sand-pack and heterogeneous sandstone).

Again, all values of $\alpha$ determined from the cubic networks are greater than those from the networks based on real rocks. From the cubic networks’ results, $\alpha$ appears to increase with increasing ratio of throat size standard deviation to the mean. This trend is broadly consistent with results from the previous analysis. For small ratios, the networks are mainly composed of pores and throats with low contrast in size. Being well connected ($Z = 4.2$), the networks thereby obtained are quite homogeneous and consequently $\alpha$ is small. As the ratio of standard deviation of pore and throat size distributions to the mean increases, so does the heterogeneity of the medium hence giving rise to greater values of $\alpha$. Note that the same trend is also observed for the three networks based on real rock systems but with lower values of $\alpha$. 
From the previous analysis it is clear that accurate predictions of single-phase flow of shear-thinning solutions in porous media require a detailed topological description of the medium of interest. Having similar pore and throat size distributions or average connection number is not sufficient to reproduce exactly and capture all media complexities and heterogeneities. In our examples, the three networks based on real porous media had different spatial distribution of their topological properties. Capturing spatial correlations such as the spatial distribution of connectivity through the medium is key to be genuinely predictive. Similar conclusions were reached by Sok et al. [74] and Arns et al. [75] when looking at Newtonian multi-phase flow. They concluded that correlations in rock micro-structure had a major effect on computing multi-phase flow properties such as residual saturation or relative permeability. The same conclusion can be extended to the single-phase flow of non-Newtonian solutions. However, as already mentioned, with a limited number of representative systems, one should still be able to estimate an acceptable value of the scaling factor $\alpha$ from the results discussed above.
6.5 Single-Phase Flow of Viscoelastic Fluids

In the final section of this chapter, we look at the flow of a viscoelastic solution in sandstones. Although, the results are by no means a thorough analysis of macroscopic flow properties of such solutions, they nonetheless provide a basis for further work. Moreover, the work undertaken also illustrates the relevance of network modelling as a powerful tool for understanding complex phenomena such as the flow of non-Newtonian solutions in porous media.

All the experimental work presented here was performed in the Schlumberger Cambridge Research laboratory facilities and the data kindly provided by John Crawshaw.

6.5.1 Viscoelastic Solution Bulk Rheology

The fluid studied is a surfactant solution. As explained in Chapter 1, surfactants are often injected in hydrocarbon reservoirs prior to shear-thinning solutions such as Xanthan to lower the surface tension between the aqueous and oil phases, lowering the residual oil saturation and increasing oil recovery. The mixture is composed of a potassium salt of linoleic acid (70%) and oleic acid (30%). The solution is made up of 3% weight of surfactant and 8% weight of potassium chloride (KCl) in deionised water.

Most viscoelastic solutions (VES) exhibit shear-thinning behaviour of both their viscosity and elasticity in bulk solution. Fig. 6.19 shows the viscosity versus shear rate bulk measurements obtained for our VES solution. The experimental data are modelled by use of a Carreau law (Eq.6.1) with the parameters shown in Table 6.3. Note the very low value of the shear-thinning exponent, \( n = 0.065 \), and the high viscosity value at zero shear rate.

<table>
<thead>
<tr>
<th>( C )</th>
<th>( n )</th>
<th>( \mu_0 )</th>
<th>( \mu_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>45.7</td>
<td>0.065</td>
<td>20.65</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 6.3: Carreau bulk parameters used to fit the experimentally measured VES rheogram.
6.5.2 In-Situ Measurements

Flooding experiments were conducted with the VES solution on two different cores: a Berea ($K = 220$ mD) and a Bentheimer sandstone ($K = 840$ mD). All experiments were performed at $25^\circ$C and cores were initially brine-saturated then displaced with a large number of pore volumes of VES before the apparent flow curves were measured. Fig. 6.20 shows the experimental results for the flow of VES in Berea. Unlike in bulk solution, the in-situ results show some evidence of extensional behaviour. The rheological properties of the fluid changes with flow geometry. In the porous medium, the converging and diverging channels bring out both the extensional and shear properties of the solution.

Above a Darcy velocity of $2.5 \times 10^{-5}$ m.s$^{-1}$, the viscosity appears to stabilize to a pseudo-Newtonian value. However, as the flow is increased further, the viscosity starts decreasing again in a “power-law” manner. This behaviour cannot be predicted from the measurement of bulk properties.
Also shown on Fig. 6.20 are the Berea network predictions using the Carreau parameters given in Table 6.3 as input into the flow model. The network results were re-scaled using Eq. 5.2 to account for the permeability difference. As can be seen from the results, although the onset of the shear-thinning region is well predicted, the apparent power-law exponent is greater for the experimental data. The apparent power-law exponent predicted by the network is only slightly greater than the bulk one, but significantly different from the experimental curve. This can be explained by the relatively large difference (several orders of magnitude) between the two Newtonian viscosity values used as input into the model. Indeed, for low to medium pressure gradient imposed to the network, a large number of pores and throats are still in a power-law regime. They have not yet reached their physical viscosity limit because it is still several orders of magnitude lower. As explained in previous sections, the change in apparent power-law slope is controlled by the number of elements that have reached this lower viscosity limit.

However, the experimental curve shows an apparent plateau behaviour at medium to high Darcy velocity. This is attributed to elastic effects in the porous medium,
which, beyond a threshold velocity, generates an extra-pressure gradient, hence increasing the apparent viscosity. To account for this, we could assume the lower viscosity value to be equal to that of the apparent viscosity plateau observed experimentally (here 0.35 Pa.s). The resulting apparent rheogram predicted from the network will obviously not predict the high Darcy velocity results (i.e. beyond the plateau observed), but should be able to predict the data up to the stabilisation of apparent viscosity.

Therefore, the network simulations are re-run with a different flow input. We assume that an artificial truncation exists below the viscosity value of 0.35 Pa.s. The fitting parameters used in the Carreau law are now different to ensure as many points as possible above the truncation are captured by the model, especially after the onset of the shear-thinning region (Table 6.4).

<table>
<thead>
<tr>
<th>$C$</th>
<th>$n$</th>
<th>$\mu_0$</th>
<th>$\mu_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.005</td>
<td>20.65</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 6.4: Truncated Carreau bulk parameters used in Berea experimental comparison.

Figure 6.21: Comparison between network simulations (—) using rheology from Table 6.4 and experimental data ($\circ$) of VES in Berea core (courtesy of Schlumberger Cambridge Research). The re-scaled network results are re-scaled using Eq. 5.2.
Fig. 6.21 shows the predicted rheogram from the network simulations using the bulk rheological parameters in Table 6.4. The results predicted from the network are now in relatively good agreement with the experimental data up to the onset of the second shear-thinning region. The slope of the apparent viscosity curve predicted from the network is now almost identical to that observed experimentally. This change in predicted apparent shear-thinning exponent is due to preferred flow pathways through the network, and the existence of a lower Newtonian viscosity limit.

The analysis is repeated in a similar manner for the Bentheimer core flooding experiment. The network used for the predictions is still based on a Berea sandstone, but is now re-scaled to an absolute permeability of 840mD. Initially the original bulk rheogram, and its corresponding Carreau parameters (see Table 6.3), was used as input to our flow model. As can be seen from Fig. 6.22, the in-situ results obtained are similar to those for the Berea experiment.

Figure 6.22: Comparison between network simulations (—) using rheology from Table 6.3 and experimental data (○) of VES in Bentheimer core (courtesy of Schlumberger Cambridge Research). The re-scaled network results are re-scaled using Eq. 5.2.
Again, the onset of the shear-thinning region is well predicted but the apparent shear-thinning slope of the two curves are different. Just as before, an artificial low viscosity truncation is imposed to our flow model and the network simulations re-run with the following Carreau parameters.

\[
\begin{array}{cccc}
C & n & \mu_0 & \mu_\infty \\
48 & 0.05 & 20.65 & 0.25 \\
\end{array}
\]

Table 6.5: Carreau bulk parameters used in Bentheimer experimental comparison.

This time the agreement between the experimental data and the network predictions are excellent up to the apparent plateau observed at intermediate Darcy velocities.

Figure 6.23: Comparison between network simulations (—) using rheology from Table 6.5 and experimental data (◦) of VES in Bentheimer core (courtesy of Schlumberger Cambridge Research). The re-scaled network results are re-scaled using Eq. 5.2.
The quality of the predictions made by the network model for the first part of the experimental curve does illustrate the physics and the nature of the elastic effects in porous media. In essence, this effect can be seen as a limiting lower viscosity value for the polymeric solution when flowing in porous media. For low to intermediate Darcy velocity, the solution behaves like a pure shear-thinning solution with an artificially high lower viscosity value. A plateau is therefore observed in the apparent viscosity curve. However, once the pressure gradient imposed (and hence the Darcy velocity) exceeds a threshold value, a secondary shear-thinning regime appears.

As hinted previously, this artificial lower Newtonian viscosity may be the result of extra-pressure drops in the porous medium generated by the diverging-converging geometry of the flow channels. The results suggest that viscous and elastic pressure drops may be of similar order of magnitude at low to intermediate velocity. At high velocity however, the viscous terms dominate and the flow becomes purely shear-thinning again.

However, other phenomenological phenomena might also participate in the observed in-situ behaviour. Viscoelastic fluids sheared in their linear region can show time-dependency because their molecular microstructure may take time to respond to the flow. At short times the molecular structure of the solution cannot respond quickly, and an elastic response is observed, while for longer times, the system can adjust itself continuously, showing viscous effects. As already mentioned this is linked to the degree of flexibility and deformation incurred by the system. When observed over all time scales, the system is viscoelastic [44].

By contrast, the microstructure of non-linear viscoelastic fluids (i.e. also showing shear-thinning behaviour) can take time to respond to flow, but is also changed by the flow itself. This is referred as thixotropy [76] and differs from the time dependency exhibited by linear viscoelastic systems in that the molecular structure is broken down by deformation as well as responding to it. As already discussed in Chapter 2, shear-thinning behaviour usually results from the alignment of rod-like particles in the direction of flow. These structural changes will always take some time to occur. However, thixotropy in porous media will be significant when the time-scale over which it is seen becomes significantly longer than the average time it takes for the fluid to flow through the medium. Whenever this is the case, the viscosity of the fluid will typically decrease as a function of time upon shearing, as illustrated in Fig. 6.24.
Clearly, changes of viscosity in the system will influence macroscopic flow results, and may participate in the \textit{in-situ} behaviour observed in Fig. 6.20. Furthermore, the apparent viscosity plateau observed at intermediate velocities may also be the results of oscillations in the flow field in a way analogous to some of the non-converged results presented in Chapter 5 (see Fig. 5.15 for instance).

Proper predictions of the properties of viscoelastic solutions flowing in porous media, that capture the full $\mu_{app}$ vs $q$ behaviour, will have to be the subject of future work.

\section*{6.6 Summary}

Previous work was extended in order to investigate the use of a length scaling to predict the \textit{in-situ} rheogram from the bulk measured one. The method is restricted to those fluids whose bulk rheology is well described by the Carreau model. Initially, the polymer solutions are assumed to be purely viscous, inelastic and adsorption phenomena are not considered.

An empirical expression based on the Carreau model was used to relate the effective viscosity to pressure drop in each bond. For a generic rheology, we predicted \textit{in-situ} rheograms for a range of values of the shear-thinning exponent $n$ in each medium. Results were compared with predicted ones using a length scaling. In
all three networks, for relatively large values of $n$ (greater than 0.4), the approach appeared to be satisfactory once a value of the scaling factor $\alpha$ was determined. Effective medium theory gave good predictions of the variation of $\alpha$ with $n$, particularly for the least heterogeneous system studied - the sand-pack. The more complex and heterogeneous the medium, the greater $\alpha$. We confirmed that where experimental data were available the network model predictions were close to measured values.

However at low $n$ values, the in-situ rheogram appears to have a larger apparent shear-thinning exponent and re-scaling the bulk rheology no longer gives reliable predictions, even if a best-fit value of $\alpha$ is determined. This is attributed to a bi-modal distribution of viscosities in the system, resulting in a “streaking behaviour” where a few pores and throats with very low viscosity carry most of the flow, while most elements experience little pressure drop and have a high viscosity at the low shear rate plateau. The more complex and heterogeneous the medium, the more pronounced this phenomenon.

The dependence of the scaling factor $\alpha$ on some of the medium topological properties was investigated by use of networks based on a regular cubic lattice. It was shown that in-situ results cannot be predicted from the knowledge of pore and throat size distributions or average connection number. Accurate flow predictions require a detailed topological description of the system of interest. Nonetheless, this work offers the reader some guidelines as to the expected behaviour of simple shear-thinning solutions in porous media. With the results presented here, for a given polymer concentration, one can estimate the bulk and porous medium behaviour of pseudo-plastic solutions in sand-packs, and homogeneous and heterogeneous sandstones. With some knowledge of the pore structure, it should be possible to use the results in this chapter to estimate the in-situ rheogram with reasonable accuracy.

Finally, we show how network modelling can be used to analyze the flow of more rheologically complex fluids such as viscoelastic surfactants. We used a simple empirical approach where viscoelastic effects are assumed to give an artificially high lower Newtonian viscosity plateau. The use of a length scaling to infer in-situ results is in principle applicable to more complex rheologies, provided the physical phenomena involved at the pore-scale are accounted for.
Chapter 7

Multi-Phase Flow Analysis

The analysis undertaken so far has been limited to describing, understanding and predicting single-phase macroscopic flow properties of shear-thinning solutions in porous media. However, there are many aspects of hydrocarbon production processes where understanding multi-phase flow involving at least one non-Newtonian phase is of great interest.

In particular, polymeric solutions such as Xanthan gum have been used for water-control purposes or to enhance oil recovery during secondary water flooding of reservoirs. For reservoir engineering purposes, one must have a prior knowledge of the macroscopic behaviour, i.e. flow and transport properties, of such solutions in order to optimize any field applications. Assessing the efficiency of displacing oil by Xanthan polymer will normally require running field-scale reservoir simulations. To do so, the appropriate bulk and in-situ macroscopic behaviours of the polymeric solution must be given as input to the simulation model. By applying the right physics at the pore-scale, one can describe the macroscopic flow properties of polymer flooding and subsequently use these results to design field applications.

In this chapter, we use the multi-phase network model described by Valvatne & Blunt [11]. The only change to the model is the incorporation of non-Newtonian properties to the aqueous phase. For multi-phase flow, the model can simulate any sequence of oil or water flooding with any distribution of oil/water contact angles. The Berea network is used to predict multi-phase displacements in sandstone reservoir rocks. The properties of the network can be found in Appendix B.
7.1 Model Description

7.1.1 Multi-phase Newtonian Flow

This section briefly describes the two-phase network model that forms the basis of this work. The model is presented in greater details by other authors [11, 15, 25].

The flow is assumed to be capillary dominated (quasi-static). Hence, the pore-scale configurations of fluid in the model are entirely controlled by capillary forces — the pressure gradient is assumed to have no effect on the arrangement of oil and water in the pore space, and in particular the residual oil saturation remains constant.

Initially, all the elements (pores and throats) in the network model are filled with water. Therefore, displacement can only occur through piston-like displacement whereby the centre of an element can only be filled if it has an adjacent element containing oil. All the throats along the inlet face are assumed to be connected to a reservoir of oil. The capillary pressure $P_c$ is increased by increasing the pressure in the oil phase $P_o$ (keeping the water phase pressure $P_w$ constant). The elements are then filled in order of increasing capillary entry pressure. This process continues until some predefined saturation is reached, or all elements have been filled by oil. Once an element has been filled by oil, water remains in the corners. This ensures the wetting phase is connected throughout primary oil flooding, since escape to the outlet is always possible through wetting layers.

Following primary drainage, the oil pressure is held constant at the outlet and water is injected from the inlet ($P_c$ decreases). The model accounts for trapping and wettability alteration where the network elements or part of them are in direct contact with the oil. However, the corners and elements still only containing water remain strongly water-wet (see Fig. 7.6). In addition to wettability alteration, contact angles are affected by the direction of flow due to surface roughness. With wettability alteration and water in corners the mechanisms by which water can displace oil become more complex. They are described in [11].

In primary drainage the oil/water contact angle is assumed to be zero. For secondary flooding we assume that the oil/water advancing contact angles are distributed at random uniformly from 30° to 90°. It has already been shown that using these contact angles accurately predicts drainage and imbibition relative permeabilities for Newtonian flow in Berea [11]. Fig. 7.1 shows the experimental relative permeabilities for Berea sandstone and the predictions from the network model.
7.1.2 Multi-phase Non-Newtonian Flow

To be representative, the bulk rheology of the non-Newtonian phase is taken from Cannella et al. [4] experiments in Berea sandstone and is given below. These parameters correspond to a best fit of the Carreau model (Eq.6.1) to their 1200ppm Xanthan experimental data.

\[
C = 0.195, \quad n = 0.48, \quad \mu_0 = 0.102, \quad \mu_\infty = 0.0015
\]

Table 7.1: Carreau bulk parameters corresponding to Cannella et al. [4] experimental data.
The results presented in Chapter 5 illustrated the suitability of the model to predict reliably the single-phase flow of Xanthan-like solutions in porous media (see for instance Fig. 5.12). The accuracy of the predictions for multi-phase Newtonian (Fig. 7.1) and single-phase non-Newtonian flow give some confidence in our results for multi-phase non-Newtonian flow. However in this case, we will not have any experimental data to compare against.

We now simulate two-phase flow displacements (both primary drainage and secondary imbibition) where the wetting phase (water + polymer, called just water for simplicity) is assumed to be non-Newtonian while the non-wetting phase (oil) remains Newtonian. Throughout the displacement cycles, a constant pressure gradient is applied across the network (boundary condition). Single-phase Newtonian flow of oil (viscosity of $1.4 \times 10^{-3}$ Pa.s) is then computed across the network, for that given pressure gradient. Using Darcy’s law (Eq. 3.2), the absolute permeability of the network can then be determined (3.1 D for our Berea network).

Single-phase non-Newtonian flow of water (i.e. $S_w = 100\%$) is then simulated, where the network is assumed to be completely filled with a shear-thinning aqueous solution of Xanthan polymer. We start by applying a relatively small pressure gradient across the network (74 Pa/m). The viscosity of each element is initially taken as the upper Newtonian one measured in the bulk solution. After convergence a solution for the velocity field is found and the total flow rate can be obtained. The flow rate thereby obtained will be used as reference for the relative permeability calculations. Corresponding to this velocity field is a new set of effective viscosities in each of the network elements. These are used as subsequent initial guesses when simulating primary drainage. Although the overall pressure gradient does not change when $S_w$ decreases, the pressure field may. Again iterations are necessary to compute the velocity field corresponding to the same overall pressure gradient but with $S_w \neq 100\%$. Water relative permeability is defined in the following way:

$$k_{rw} = \frac{Q_{\text{water (}S_w \neq 100\%\text{)}}}{Q_{\text{water (}S_w = 100\%\text{)}}}$$

(7.1)

where the flow rates are measured for the same pressure gradient.

Primary drainage is then simulated from 100\% water saturation to irreducible water saturation (where the water flow rate is zero) and secondary water flooding (imbibition) to residual oil saturation for a range of pressure gradients. The pressure gradient across the network is increased in increments. The viscosities computed from the previous single-phase results (at a lower pressure gradient) are used as initial guesses for calculations at a higher pressure gradient.
7.2 Primary Drainage

Fig. 7.2 presents primary drainage relative permeability results for pressure gradients ranging from 74 Pa/m to $1.5 \times 10^8$ Pa/m. For relatively small pressure gradients across the network, the relative permeability curve of the water phase is almost identical to the Newtonian case. However, its value decreases when the pressure gradient is increased.

For gradients greater than $1.5 \times 10^5$ Pa/m, the relative permeability curve increases again from its minimum value until almost equaling the Newtonian case at very high pressure gradients (Fig. 7.3).
Note that for the range of pressure gradients studied here (that give Darcy velocities covering the entire shear-thinning region) the non-Newtonian wetting phase relative permeability is always less than the Newtonian wetting phase curve. To quantify this reduction in relative permeability, we look at the ratio between the relative permeability for the non-Newtonian wetting phase and its Newtonian case value as a function of pressure gradient (Fig. 7.4).

Plotted on the same graph are the apparent viscosity values corresponding to the non-Newtonian single-phase displacements over the same range of pressure gradients. The reduction in relative permeability is most pronounced at low saturation and for a pressure gradient that coincides with the inflexion point of the single-phase apparent viscosity curve. To understand how the single-phase behaviour conditions the multi-phase results, we look at a point along one of the saturation curves of Fig. 7.4, say for an intermediate saturation, $S_w = 0.496$. 

![Figure 7.3: Primary drainage relative permeability results from network simulations (pressure gradient from $1.5 \times 10^5$ Pa/m to $3.7 \times 10^8$ Pa/m, $n = 0.48$).](image-url)
For a relatively small pressure gradient, for instance $7.5 \times 10^3$ Pa/m, the non-Newtonian relative permeability approaches that of the Newtonian case. Fig. 7.5 shows the viscosity histogram of completely water-filled elements in the network for four different pressure gradients, both for $S_w = 0.496$ (grey) and the single-phase case, $S_w = 1.0$ (white). As expected, for a small pressure gradient (Fig. 7.5a) most of the water-filled elements have a high viscosity value, close to the low-rate Newtonian plateau for the bulk fluid (0.102 Pa.s). The distributions of viscosity are similar for both saturations, except, of course, that fewer elements are filled with water when $S_w = 0.496$. The flow rate is similar to that predicted using the single-phase non-Newtonian viscosity and the Newtonian relative permeability. Hence the relative permeability ratio is close to 1.

As the overall pressure gradient across the system is increased, the situation changes (Figs. 7.5b to d). The distribution of water viscosities when $S_w = 0.496$ deviates significantly from that when $S_w = 1.0$. For intermediate pressure gradients (between $1 \times 10^4$ Pa/m and $1 \times 10^5$ Pa/m), the single-phase water viscosity histogram ($S_w = 1.0$) shifts towards the lower Newtonian viscosity value because of the shear-
Figure 7.5: Viscosity histograms for water-filled elements, $S_w = 0.496$ (grey) and single-phase flow, $S_w = 1.0$ (white); a) Pressure gradient of $7.5 \times 10^5$ Pa/m; b) Pressure gradient of $1.5 \times 10^5$ Pa/m; c) Pressure gradient of $3.7 \times 10^6$ Pa/m; d) Pressure gradient of $3.7 \times 10^8$ Pa/m.

thinning behaviour of the fluid resulting in a large increase in overall flow rate. However, for $S_w = 0.496$, the water-filled elements experience less shear and have a higher viscosity due to the presence of the oil. This causes a sharp reduction in relative permeability. This reduction is maximum when the two viscosity histograms are the furthest apart, here for a pressure gradient of $1.5 \times 10^5$ Pa/m (Fig. 7.5b). When the pressure gradient exceeds $1.5 \times 10^5$ Pa/m, the shear rates experienced by the water-filled elements are sufficient for them to carry significant amount of flow despite the presence of oil. As can be seen from Figs. 7.5c and d, the difference in the histograms decreases, until they eventually coincide at very high pressure gradients where viscosities are close to the high flow rate plateau.
Water as the wetting phase occupies the smaller pores and throats in the network. Furthermore, where oil resides in the center of the pore space, water remains in wetting layers in the corners [11] (Fig. 7.6).

Figure 7.6: Possible fluid configurations. (a) Initially the element is water-filled and strongly water-wet. (b) Following primary oil flooding the part of the element in contact with oil will alter its wettability. (c) During water flooding the element might again become completely water-filled. (d) If wettability alteration was large enough, oil might become sandwiched as a layer between water in the corner and the centre, after Valvatne [15].

The shear rate is proportional to the flow rate of the water divided by a characteristic length. This length will be the thickness of the wetting layer, or the radius of a pore that is fully water-saturated. The flow rate is proportional to the pressure gradient multiplied by the conductance, $G$. For wetting layers this conductance scales as the square of the characteristic length and as radius to the power four for fully-filled elements [9,11]. Thus the shear rate decreases with decreasing characteristic length. In multi-phase flow, water is forced to flow through narrower regions of the pore space - on average - than in single-phase flow. This results in lower shear rates giving higher viscosities. This is the physical origin of the decrease in relative permeability. This effect is most significant where most of the water flow is in the smallest throats and in thin layers - at low water saturation, as is evident in Fig. 7.4.

One may also consider defining non-Newtonian relative permeabilities as a ratio of pressure gradients for single and two-phase flow at the same water flow rate. However, this definition was found to result in much more dramatic changes in relative permeability than using a definition based on a fixed pressure gradient. In particular, at low water saturation, the relative permeability ratio became much greater than 1. The definition chosen here is based on a flow rate ratio at the same pressure gradient (boundary condition) since it gives relative permeabilities that are
fairly close to the Newtonian values (as opposed to a pressure gradient ratio at a fixed flow rate).

### 7.3 Secondary Imbibition

The same trend in the behaviour of the relative permeability is seen for secondary imbibition (Figs. 7.7 & 7.8). Although not shown here, the analysis of effective viscosity histograms during the imbibition displacement was repeated and lead to similar conclusions as previously described for drainage. For pressure gradients up to $1.5 \times 10^5$ Pa, the water relative permeability decreases and then increases again when greater pressure gradients are applied across the system.

![Secondary imbibition relative permeability results from network simulations (pressure gradient from 74 Pa/m to $1.5 \times 10^5$ Pa/m, $n = 0.48$).](image)

**Figure 7.7:** Secondary imbibition relative permeability results from network simulations (pressure gradient from 74 Pa/m to $1.5 \times 10^5$ Pa/m, $n = 0.48$).

### 7.4 Discussion

One of the most fundamental assumptions of the network model is that the flow is capillary dominated (or quasi-static). This means that during multi-phase displacements the viscous pressure drop across the network must be negligible in comparison to typical capillary pressures [25]. To assess whether this condition is satisfied or not, it is common to define a capillary number that measures the ratio of viscous
Figure 7.8: Secondary imbibition relative permeability results from network simulations (pressure gradient from $1.5 \times 10^5$ Pa/m to $3.7 \times 10^8$ Pa/m, $n = 0.48$).

to capillary forces in the system. However, this number is usually defined for a Newtonian phase of constant viscosity.

For the purpose of this work, a non-Newtonian capillary number is defined as follows:

$$N_{cap}^{NN} = \frac{\mu_{app} q}{\sigma} = \frac{K \Delta P}{\sigma L}$$  \hspace{1cm} (7.2)

where the apparent viscosity and flow rate are taken from the single-phase flow simulation.

The assumption of capillary dominance is usually taken to be valid for capillary numbers less than $10^{-4}$ [27]. In the results presented above, for a typical interfacial tension of $55 \times 10^{-3}$ N/m, the condition holds for pressure gradients less than $3 \times 10^5$ Pa/m. This means that in principle, one should be able to observe the reduction of relative permeability followed by the beginning of the increase in relative permeability ratio. However, the predictions at larger pressure gradients are unlikely to be valid, since dynamic effects will also affect the fluid configurations at the pore scale.
Figure 7.9: Primary drainage relative permeability results from network simulations (pressure gradient from $7.2 \times 10^2$ Pa/m to $1.8 \times 10^4$ Pa/m, $n = 0.01$).

Figure 7.10: Primary drainage relative permeability results from network simulations (pressure gradient from $1.8 \times 10^4$ Pa/m to $7.2 \times 10^5$ Pa/m, $n = 0.01$).
For moderately shear-thinning fluids, such as those studied so far, the viscosity/shear rate and apparent viscosity/flow rate curves have similar shapes (see, for instance Figs. 5.10 & 5.11). However, as already explained in Chapter 6, for strongly shear-thinning fluids with shear-thinning exponents, $n$, close to zero, the bulk and porous medium behaviour may be very different because most of the flow is taken by a few low viscosity elements [60]. We ran a multi-phase flow simulation for $n = 0.01$ to investigate whether or not the behaviour was the same as seen for less shear-thinning fluids. Figs. 7.9 & 7.10 show the relative permeability drainage curves for a range of pressure gradients. Initially, for small pressure gradients, the water relative permeability decreases in a similar manner as before (Fig. 7.9), although the magnitude of this reduction is now significantly larger. However, whereas before the water relative permeability would increase again for all saturation values, for pressure gradients greater than $1.8 \times 10^4$ Pa/m, part of the water relative permeability curve is still decreasing. For water saturations less than 0.6, the water permeability values continue to decrease until very large pressure gradients are achieved.

This difference in behaviour is because at low saturation many of the elements containing water in the corner have high viscosities close to the low shear rate plateau with most of the flow taken by just a few low-viscosity elements. Fig. 7.11 shows the relative permeability ratios. The relative permeability ratio only increases for very high pressure gradients when water in the corners flows sufficiently fast to enter the shear-thinning regime. However, over the same range of pressure gradients but for $S_w$ greater than 0.6, the water relative permeability values increase again towards their Newtonian limit at moderate pressure gradients. This is because most elements are now completely water-filled and all have similar flow rates.

The secondary imbibition relative permeability curves for the case $n = 0.01$ show similar behaviour to the drainage case (Figs. 7.12 to 7.14). The reduction in relative permeability is greatest at low water saturation and is only recovered when large pressure gradients are applied across the network.
Figure 7.11: Primary drainage relative permeability ratios versus pressure gradient from network simulations (all saturations, \( n = 0.01 \)).

Figure 7.12: Secondary Imbibition relative permeability ratios versus pressure gradient from network simulations (all saturations, \( n = 0.01 \)).
Figure 7.13: Secondary imbibition relative permeability results from network simulations (pressure gradient from $7.2 \times 10^2$ Pa/m to $1.8 \times 10^4$ Pa/m, $n = 0.01$).

Figure 7.14: Secondary imbibition relative permeability results from network simulations (pressure gradient from $1.8 \times 10^4$ Pa/m to $7.2 \times 10^5$ Pa/m, $n = 0.01$).
The results presented here show that the displacement efficiency of oil by polymer flooding would be affected by the shear-thinning nature of the aqueous phase. However, this effect does not just comprise of an overall decrease in apparent viscosity. Most current reservoir simulators assume no changes in relative permeability and incorporate the shear-thinning effect through a change of mobility of the water phase. In order to simulate accurately multi-phase flow displacements, the reservoir model must be able to capture both the change in single-phase viscosity and the likely reduction in relative permeability. However, when relative permeability is defined based on a single-phase viscosity taken for the same water pressure gradient (and not the same flow rate) the modification is small, except at low water saturation.

7.5 Summary

We used pore-scale network modeling to study the multi-phase flow properties of shear-thinning solutions in porous media. The fluid model was based on the behaviour of Xanthan-like solutions (pseudo-plastic), where elastic and adsorption effects are neglected and the network used was based on a three-dimensional representation of a Berea sandstone. The approach presented in previous chapters was extended to study multi-phase flow properties in a water-wet medium, where the aqueous phase was assumed to behave like a Carreau-type fluid while the oil phase remained Newtonian. Relative permeability was defined as the ratio of multi-phase to single-phase water flow rates with the same pressure gradient. As the pressure gradient applied across the network increased, the relative permeability of the non-Newtonian phase decreased gradually before increasing again towards its limiting Newtonian value. This was explained by looking at the difference between effective viscosity distributions in the network with and without the presence of oil. For very shear-thinning solutions, the reduction in relative permeability was more pronounced, particularly for low water saturations. The behaviour in primary drainage and secondary imbibition was similar.
Chapter 8

Conclusions and Future Work

8.1 Conclusions

Understanding the flow of rheologically complex fluids through porous media is important in many engineering applications, including many aspects of hydrocarbon production processes.

In this work, a state-of-the-art pore network model was extended to study and predict the flow behaviour of non-Newtonian fluids in porous media. The model uses topologically disordered networks of irregularly shaped pores connected by throats that represent different porous media of interest. Three different networks spanning a plausible range of properties encountered in real systems, from an unconsolidated system to a highly heterogeneous reservoir sandstone, were used to analyze various aspects of non-Newtonian flow in porous media.

A method was developed to predict the single-phase flow of a pseudo-plastic solution (shear-thinning), such as Xanthan gum, in various porous media. Expressions were presented in order to find the effective viscosity in each pore and throat as a function of pressure drop. Using an iterative procedure, the macroscopic flow throughout the network could be predicted for a given pressure gradient.

The approach was successfully validated by predicting several different data sets in the literature. The method was capable of reproducing flow experiments for porous media with different permeabilities than the network after a length re-scaling was applied. However, the model input data, the permeability and porosity of the system as well as the fluid bulk rheology, are all measured independently and therefore are not arbitrary adjustable parameters.
In contrast, it was shown how the traditional empirical approach that infers \textit{in-situ} rheograms from the bulk measured ones, cannot predict experimental data without resorting to an \textit{ad hoc} scaling parameter, even with some statistical information about the medium of interest. Accurate flow predictions require a detailed topological description of the pore space. We confirmed that the value of the scaling parameter, $\alpha$, increases with the level of complexity and heterogeneity of the medium.

However, results from the network model showed this scaling being no longer applicable for very shear-thinning solutions. At low $n$ values, the \textit{in-situ} rheogram shear-thinning exponent is larger than in the bulk and re-scaling no longer gives reliable flow predictions, even if a best-fit value of $\alpha$ can be determined. This was attributed to “streaking behaviour” in the network where most of the flow was carried through a small subset of pores and throats. This phenomenon was more pronounced for the more complex and heterogeneous networks. Nonetheless, guidelines were offered to the reader to estimate the value of the scaling factor $\alpha$. From the results presented in this work, one should be able to estimate the bulk and porous medium behaviour of pseudo-plastic solutions in sand-packs, homogeneous and heterogeneous sandstones.

The use of a length scaling to infer \textit{in-situ} results from the measurement of bulk properties is in principle applicable to more complex rheologies, provided the physical phenomena involved at the pore-scale are accounted for. For instance, we showed how network modelling could be used to analyze the flow of viscoelastic surfactants. A simple empirical approach that introduces an artificially high lower Newtonian viscosity plateau was used. Although the model was capable of reproducing the results up to the first apparent viscosity stabilization, further analysis would be required to infer \textit{a priori} the value of this artificial lower viscosity plateau from bulk measurements alone.

The model was extended further to study macroscopic flow properties of oil displacement by a shear-thinning fluid. Simulation results from the network model showed the impact of shear-thinning rheology on the water relative permeability. The relative permeability of the non-Newtonian phase was seen to decrease gradually as the pressure gradient applied across the network increased, before increasing again towards its limiting Newtonian value. This behaviour was explained by the offset in effective viscosity distributions with and without the presence of oil. The reduction in relative permeability was more pronounced for very shear-thinning solutions and low water saturation. Capturing these variations cannot be achieved simply by accounting for mobility change, and is important for field-scale reservoir simulations.
and the design of enhanced oil recovery schemes.

### 8.2 Recommendations For Future Work

There are several main areas of interest where the work presented here could be used to improve further our understanding of non-Newtonian flow in porous media:

- Despite making excellent predictions on the single-phase flow of shear-thinning fluids in porous media, the method could be extended further to account for some of the physical effects ignored so far, such as adsorption [41, 77] and depleted layer phenomena [7, 8]. As described in Chapter 3, entropic exclusion of the polymer molecules near the walls can be particularly significant when the polymer molecules size approach that of the pore space (Fig. 3.2.1). This effect has been observed experimentally in both adsorbing and non-adsorbing media. Sorbie provided a theoretical basis to account for this phenomenon both in a single capillary [7] and in network models [8]. In general, the effect of surface exclusion leads to a non-linear problem which must be solved numerically in each of the network elements. The methodology presented in [7] and [8] could be implemented in our network model as follows:
  - For a given pressure gradient, viscosity distribution, initial polymer concentration and bulk viscosity/shear rate relationship, the thickness of the depleted layer is fixed.
  - The concentration profile across each element is then inferred using existing theoretical models. Hence the flow in each capillary can be calculated.
  - The polymer concentration is subsequently updated until convergence is achieved.
  - The final velocity field is used to solve for the macroscopic flow in the network.

- The flow of viscoelastic fluids in porous media can be analyzed further. Elastic behaviour is usually linked to the degree of flexibility of the polymer molecules resulting in added pressure drop. Based on experiments, Rothstein & McKinley [16] showed that a substantial extra pressure drop appeared, well above the value observed for a Newtonian fluid with equal viscosity at the same flow rates, for polymer solutions flowing through abrupt axisymmetric contraction-expansions (Fig. 8.2). They studied the variations of this enhanced pressure drop over a wide range of Deborah numbers, $D_e$, which, for an expansion-contraction problem, is defined by Eq. 8.1:

$$D_e = \frac{\omega \langle v \rangle}{R}$$  (8.1)
where \( \omega \) is the elastic relaxation time of the fluid, \( < v > \) the average velocity in a contraction of radius \( R \) (see Fig. 8.1).

Figure 8.1: Converging–diverging flow geometry.

Figure 8.2: Dimensionless pressure drop measurements across several axisymmetric contraction–expansions as a function of Deborah number. The hollow symbols represent stable flow conditions while the filled symbols represent unstable flow conditions. After Rothstein & McKinley [16].

Expressions must be developed to relate this excess pressure drop to some of the fluid bulk rheological parameters and the geometry in which flow is taking place. In our approach, the geometry of the network is known and therefore \( D_c \) could be computed to find the corresponding added pressure drop.

The network model could also be used to capture the transient effects linked to the degree of elasticity of the fluid. In particular, these effects will be dependent on the initial state of the viscoelastic solution (short or long relaxation
times) and its residence time in the porous medium. Quite clearly, the macro-
soscopic flow behaviour will be different whether or not the fluid has had time
to completely relax in the medium.

- The approach developed in this work could be used to model the flow of heavy
  oils in hydrocarbon reservoirs. Experimental studies on the rheological beha-
 viour of crude oils have reported the existence of non-Newtonian behaviour
  for certain types of crude oil (see for instance [78] and [55]). In particular waxy
  crude oils exhibit shear-thinning behaviour in bulk solution. This is attributed
  to the presence of long-chained paraffin hydrocarbons. Paraffin particles are
  high molecular weight straight chain hydrocarbons that solidify at low tem-
  peratures. Essentially, these particles behave in a similar manner as polymer
  molecules: they align with the direction of flow at high rate of shear, hence
  reducing the solution viscosity.

![Figure 8.3: Rheology of a waxy crude oil measured experimentally by Al-Roomi et al. [17] at different temperatures.](image)

Fig. 8.3 shows the bulk rheology of a waxy crude oil measured experimentally by Al-Roomi et al. [17]. The crude oil is clearly shear-thinning and hence the
approach used in Chapters 4 and 5 could be extended to study the flow of a
non-Newtonian non-wetting phase in porous media. During multi-phase flow,
both wetting and non-wetting phases could be treated as non-Newtonian to
assess possible recovery efficiencies from heavy oil reservoirs.
In addition to their shear-thinning properties, heavy oils often behave as Bingham plastics. In general, the flow of yield stress fluids in porous media is of interest in many petroleum and groundwater engineering applications. Balhoff and Thompson [67] provided a theoretical basis for the inclusion of yield stress effects in network models.

- The bulk rheology of yield stress fluids is described using Bingham’s model:

\[
\tau_{xy} = \tau_0 + \mu_0 \left( \frac{\partial v_x}{\partial y} \right)
\]  

(8.2)

- The flow in a circular capillary is assumed to be given by

\[
q = \frac{g}{\mu_0} \Delta P \left[ 1 - \frac{4}{3} \left( \frac{\tau_0}{\tau_w} \right) + \frac{1}{3} \left( \frac{\tau_0}{\tau_w} \right)^4 \right]
\]

(8.3)

where \(\tau_w\) is given by Eq. 3.12 with \(r = R_{eq}\).

- As for shear-thinning fluids, an iterative method is needed to solve for the velocity field in each of the network elements. The macroscopic flow through the network is solved numerically once satisfactory convergence has been achieved.

The single-phase flow results could be validated through comparisons with some experimental data available in the literature. The multi-phase behaviour, however, will be more challenging due to the complexity in defining fluid stresses in corners and layers.

- Experimental confirmation of some of the results presented, either in micromodel studies or core experiments, would be of great value. Moreover, network modelling is the ideal tool to determine what experiments are needed to fully characterize the flow behaviour of complex fluids. For instance, this is particularly true for viscoelastic flow experiments. Network modelling could be used to minimize or isolate transient effects by identifying the critical system lengths and fluid properties to be considered when designing the experiments.

- The multi-phase flow analysis, involving one or several non-Newtonian phases, would require validation through experimental comparison. The results predicted by the network model could be implemented in conjunction with a field-scale simulator to assess the displacement efficiency of EOR schemes.
References


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[71] Yi, X., “Model for displacement of herschel-bulkley non-newtonian fluid by newtonian fluid in porous media and its applications in fracturing fluid cleanup”,

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Appendix A

Bulk Rheology

The following table provides the necessary Carreau model parameters for various polymer concentrations, based on Equations 2.8a, 2.8b and 2.8c. The bulk rheology of a Xanthan solution of given concentration can thereby be inferred in the absence of experimental data. Figure A.1 shows the bulk viscosity/shear rate relationships for different polymer concentrations. The bulk curves are calculated using parameters from Table A.1 in the Carreau model (Eq. A.1 & Eq. 2.5).

$$\mu_{\text{eff}} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{[1 + (\dot{\gamma})^2]^{\frac{1-n}{2}}}$$  \hspace{1cm} (A.1)

<table>
<thead>
<tr>
<th>Concentration (ppm)</th>
<th>(\mu_0)</th>
<th>(C)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2.114×10^{-3}</td>
<td>0.0028</td>
<td>0.82</td>
</tr>
<tr>
<td>400</td>
<td>4.469×10^{-3}</td>
<td>0.0139</td>
<td>0.69</td>
</tr>
<tr>
<td>600</td>
<td>9.448×10^{-3}</td>
<td>0.0351</td>
<td>0.60</td>
</tr>
<tr>
<td>800</td>
<td>1.997×10^{-2}</td>
<td>0.0678</td>
<td>0.54</td>
</tr>
<tr>
<td>1000</td>
<td>4.222×10^{-2}</td>
<td>0.1129</td>
<td>0.48</td>
</tr>
<tr>
<td>1200</td>
<td>8.926×10^{-2}</td>
<td>0.1715</td>
<td>0.44</td>
</tr>
<tr>
<td>1400</td>
<td>1.887×10^{-1}</td>
<td>0.2441</td>
<td>0.40</td>
</tr>
<tr>
<td>1600</td>
<td>3.989×10^{-1}</td>
<td>0.3314</td>
<td>0.37</td>
</tr>
<tr>
<td>1800</td>
<td>8.434×10^{-1}</td>
<td>0.4340</td>
<td>0.35</td>
</tr>
<tr>
<td>2000</td>
<td>1.783×10^{0}</td>
<td>0.5524</td>
<td>0.32</td>
</tr>
<tr>
<td>2200</td>
<td>3.769×10^{0}</td>
<td>0.6871</td>
<td>0.30</td>
</tr>
<tr>
<td>2400</td>
<td>7.968×10^{0}</td>
<td>0.8386</td>
<td>0.29</td>
</tr>
<tr>
<td>2600</td>
<td>1.684×10^{1}</td>
<td>1.0073</td>
<td>0.27</td>
</tr>
<tr>
<td>2800</td>
<td>3.561×10^{1}</td>
<td>1.1936</td>
<td>0.26</td>
</tr>
<tr>
<td>3000</td>
<td>7.528×10^{1}</td>
<td>1.3979</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table A.1: Generic Carreau model parameters derived from Equations (2.8a), (2.8b) and (2.8c) for different Xanthan concentrations.
Figure A.1: Generic Xanthan bulk rheology for different polymer concentrations, as predicted by the Carreau model using parameters from Table A.1.
Appendix B

Berea Network Properties

Figure B.1: Berea sandstone network.
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical Size</td>
<td>3.0mm x 3.0mm x 3.0mm</td>
</tr>
<tr>
<td>Number of pores</td>
<td>12349</td>
</tr>
<tr>
<td>Number of throats</td>
<td>26146</td>
</tr>
<tr>
<td>Average connection number</td>
<td>4.19</td>
</tr>
<tr>
<td>Number of connections to inlet</td>
<td>254</td>
</tr>
<tr>
<td>Number of connections to outlet</td>
<td>267</td>
</tr>
<tr>
<td>Number of triangular shaped elements</td>
<td>35518</td>
</tr>
<tr>
<td>Number of square shaped elements</td>
<td>2506</td>
</tr>
<tr>
<td>Number of circular shaped elements</td>
<td>473</td>
</tr>
<tr>
<td>Net porosity</td>
<td>0.190</td>
</tr>
<tr>
<td>Clay bound porosity</td>
<td>0.054</td>
</tr>
<tr>
<td>Absolute permeability (mD)</td>
<td>3148.19</td>
</tr>
<tr>
<td>Mean pore radius (10^{-6}m)</td>
<td>19.2</td>
</tr>
<tr>
<td>Pore distribution standard deviation (10^{-6}m)</td>
<td>8.47</td>
</tr>
<tr>
<td>Pore standard deviation / mean</td>
<td>0.442</td>
</tr>
<tr>
<td>Mean throat radius (10^{-6}m)</td>
<td>11.0</td>
</tr>
<tr>
<td>Throat distribution standard deviation (10^{-6}m)</td>
<td>7.03</td>
</tr>
<tr>
<td>Throat standard deviation / mean</td>
<td>0.641</td>
</tr>
</tbody>
</table>

Figure B.2: Berea network size distribution.
Appendix C

Sand-pack Network Properties

Figure C.1: Sand-pack network.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<td>Physical Size:</td>
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</tr>
<tr>
<td>Number of pores:</td>
<td>3567</td>
</tr>
<tr>
<td>Number of throats:</td>
<td>9923</td>
</tr>
<tr>
<td>Average connection number:</td>
<td>5.46</td>
</tr>
<tr>
<td>Number of connections to inlet:</td>
<td>195</td>
</tr>
<tr>
<td>Number of connections to outlet:</td>
<td>158</td>
</tr>
<tr>
<td>Number of triangular shaped elements:</td>
<td>12776</td>
</tr>
<tr>
<td>Number of square shaped elements:</td>
<td>210</td>
</tr>
<tr>
<td>Number of circular shaped elements:</td>
<td>506</td>
</tr>
<tr>
<td>Net porosity:</td>
<td>0.346</td>
</tr>
<tr>
<td>Clay bound porosity:</td>
<td>0.0</td>
</tr>
<tr>
<td>Absolute permeability ($mD$):</td>
<td>101870</td>
</tr>
<tr>
<td>Mean pore radius ($10^{-6}m$):</td>
<td>39.1</td>
</tr>
<tr>
<td>Pore distribution standard deviation ($10^{-6}m$):</td>
<td>15.8</td>
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<td>Pore standard deviation / mean:</td>
<td>0.404</td>
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<tr>
<td>Mean throat radius ($10^{-6}m$):</td>
<td>23.7</td>
</tr>
<tr>
<td>Throat distribution standard deviation ($10^{-6}m$):</td>
<td>11.2</td>
</tr>
<tr>
<td>Throat standard deviation / mean:</td>
<td>0.472</td>
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</table>

Figure C.2: Sand-pack network size distribution.
Appendix D

Heterogeneous Sandstone Network Properties

Figure D.1: Heterogeneous sandstone network.
<table>
<thead>
<tr>
<th>Physical Size:</th>
<th>10.0mm x 10.0mm x 10.0mm</th>
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</thead>
<tbody>
<tr>
<td>Number of pores:</td>
<td>9411</td>
</tr>
<tr>
<td>Number of throats:</td>
<td>20494</td>
</tr>
<tr>
<td>Average connection number:</td>
<td>4.27</td>
</tr>
<tr>
<td>Number of connections to inlet:</td>
<td>346</td>
</tr>
<tr>
<td>Number of connections to outlet:</td>
<td>376</td>
</tr>
<tr>
<td>Number of triangular shaped elements:</td>
<td>19692</td>
</tr>
<tr>
<td>Number of square shaped elements:</td>
<td>7371</td>
</tr>
<tr>
<td>Number of circular shaped elements:</td>
<td>2844</td>
</tr>
<tr>
<td>Net porosity:</td>
<td>0.196</td>
</tr>
<tr>
<td>Clay bound porosity:</td>
<td>0.04</td>
</tr>
<tr>
<td>Absolute permeability ($mD$):</td>
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</tr>
<tr>
<td>Mean pore radius ($10^{-6}m$):</td>
<td>68.6</td>
</tr>
<tr>
<td>Pore distribution standard deviation ($10^{-6}m$):</td>
<td>38.7</td>
</tr>
<tr>
<td>Pore standard deviation / mean:</td>
<td>0.565</td>
</tr>
<tr>
<td>Mean throat radius ($10^{-6}m$):</td>
<td>21.1</td>
</tr>
<tr>
<td>Throat distribution standard deviation ($10^{-6}m$):</td>
<td>22.3</td>
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<tr>
<td>Throat standard deviation / mean:</td>
<td>1.054</td>
</tr>
</tbody>
</table>

Figure D.2: Heterogeneous sandstone network size distribution.
Appendix E

Network Data File Structure

All network data properties are written in four different data files. The format of these files is that used by Statoil.

Throat Data

The data for the throats are read from the link files. The structure of the link files is as follows:

- *link1.dat

1. Total number of throats
2. Throat index
3. Pore 1 index
4. Pore 2 index
5. Throat radius
6. Throat shape factor
7. Throat total length (pore centre to pore centre)

<table>
<thead>
<tr>
<th>Example of *link1.dat file</th>
</tr>
</thead>
<tbody>
<tr>
<td>26146</td>
</tr>
<tr>
<td>1 -1 8 0.349563E-04 0.297308E-01 0.160000E-03</td>
</tr>
<tr>
<td>2 -1 53 0.171065E-04 0.442550E-01 0.211076E-04</td>
</tr>
<tr>
<td>3 -1 60 0.198366E-04 0.354972E-01 0.300000E-04</td>
</tr>
<tr>
<td>4 -1 68 0.938142E-05 0.323517E-01 0.100000E-04</td>
</tr>
</tbody>
</table>

- *link2.dat

1. Throat index
2. Pore 1 index
3. Pore 2 index
4. Length pore 1
5. Length pore 2
6. Throat length
7. Throat volume
8. Throat clay volume

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
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<td>2384</td>
<td>0.10E-04</td>
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<td>0.20E-04</td>
<td>0.32E-13</td>
<td>0.71E-14</td>
</tr>
</tbody>
</table>

**Example of °link2.dat file**

**Pore Data**

The data for the throats are read from the node files. The structure of the node files is as follows:

°node1.dat

1. Total number of pores, length, width and height of the network
2. Pore index
3. Pore X position
4. Pore Y position
5. Pore Z position
6. Pore connection number
7. Connecting links index (0: outlet; -1: inlet)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
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<td>0.30E-02</td>
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<td>0.70E-04</td>
<td>3</td>
<td>796</td>
<td>674</td>
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<tr>
<td>2</td>
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<td>3</td>
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<td>31</td>
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<tr>
<td>3</td>
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<td>0.10E-04</td>
<td>0.00E+00</td>
<td>1</td>
<td>392</td>
<td>0</td>
</tr>
</tbody>
</table>

°node2.dat

1. Pore index
2. Pore volume
3. Pore radius
4. Pore shape factor
5. Pore clay volume
### Example of *node2.dat* file

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tr>
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<td>0.74E-05</td>
<td>0.33E-01</td>
<td>0.21E-15</td>
</tr>
</tbody>
</table>
Appendix F

Two-Phase Non-Newtonian Network Simulation Code Manual

The program is based on the two-phase Newtonian code by Per Valvatne. The program uses a keyword based input file. Some keywords are optional and there is no necessary order to them. Whenever keywords are omitted, warnings will be given and default values will be used.

Comments in the data file are indicated by “%”, resulting in the rest of the line being discarded. All data should be on a single line following the keyword (possibly separated by comment lines). The input data file can be supplied as an argument to the executable:

Non_Newt_PERMsim_win32.exe
⇒ Please input data file: input_file.dat

NOTE: To avoid problems of paths definition, it is recommended that all files are placed in one folder (the executable, the network input data and the input file).

TITLE

This will be the title of given to the output files. In particular, the code will generate a file named “title.prt” copying all screen outputs and summarizing all the main results from the simulation. If omitted the title is taken to be the name of the input file.

TITLE
% Simulation title. Name of .prt file
Berea_simulation
NETWORK

This is a required keyword, specifying the files containing all network data. These data are located in four files, with a common filename that you enter after the keyword: “filename_node1.dat”, “filename_node2.dat”, “filename_link1.dat”, “filename_link2.dat”. Note that if these data are not located in the same folder you should precede the filename by its path location.

<table>
<thead>
<tr>
<th>NETWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berea</td>
</tr>
</tbody>
</table>

PRS_DIFF

A constant pressure difference is imposed across the network (boundary condition). The default value is 1Pa.

<table>
<thead>
<tr>
<th>PRS_DIFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Inlet Prs (Pa)</td>
</tr>
<tr>
<td>0.1</td>
</tr>
</tbody>
</table>

DRAINAGE

This is a required keyword that takes eight parameters (no default value).

1. Final water saturation target after drainage (fraction). A saturation of 0.0 is generally not feasible but will ensure that the residual saturation is reached.
2. The drainage phase can also be terminated when a maximum capillary pressure (Pa) is reached. If none of the conditions are reached, drainage will terminate when all pores and throats have been invaded by oil. Water will however still be present in the corners of the triangular pores and throats, ensuring non-zero water saturation.
3. Maximum step size in water saturation (fraction). The state of the model will be evaluated at first possible stable configuration after incremental target has been reached.
5. Should relative permeability be calculated? (true: “T” or false: “F”). If this is not of primary interest it can be very time saving to set this option to false (“F”) as more than 90% of the CPU time is spent solving for pressure, required for relative permeability calculations.
6. Calculate resistivity index? (true: “T” or false: “F”). CPU time for this calculation is the same as for relative permeability calculations.

7. Inject oil at the left hand side face (dimensionless x location of 0)? (true: “T” or false: “F”)

8. Inject oil at the right hand side face (dimensionless x location of 1)? (true: “T” or false: “F”). The displaced fluid (water in the case of drainage) is allowed to escape to both faces, hence referring to these faces as inlet or outlet becomes somewhat inaccurate. It is possible to set both item 7 and 8 to true. Only for relative permeability calculations, where a pressure differential is imposed across the network, is there any notion of direction in the fluid movement.

<table>
<thead>
<tr>
<th>DRAINAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>%FinalSat</td>
</tr>
<tr>
<td>0.00</td>
</tr>
</tbody>
</table>

**IMBIBITION**

This is also a required keyword that takes eight parameters (no default value).

1. Final water saturation target after imbibition (fraction). A saturation of 1.0 is again not attainable. If the network is water-wet (spontaneous imbibition), significant portions of the oil in the network may become trapped, resulting in high residual oil saturation. On the other hand, if the network becomes oil-wet during imbibition, the final water saturation target can become very close to 1.0. If the target water saturation is set to 1.0, imbibition will terminate when all available pores and throats have been imbibed.

2. Maximum capillary pressure reached after imbibition.

3. Maximum step size in water saturation (fraction). The state of the model will be evaluated at first possible stable configuration after incremental target has been reached.


5. Should relative permeability be calculated? (true: “T” or false: “F”) If this is not of primary interest it can be very time saving to set this option to false (“F”) as more than 90% of the CPU time is spent solving for pressure, required for relative permeability calculations.

6. Calculate resistivity index? (true: “T” or false: “F”). CPU time for this calculation is the same as for relative permeability calculations.

7. Inject oil at the left hand side face (dimensionless x location of 0)? (true: “T” or false: “F”)
8. Inject oil at the right hand side face (dimensionless x location of 1)? (true: “T” or false: “F”). Again oil is allowed to escape to both faces, which reduces the possibility of trapping. Injecting water refers to water invading the network through the bodies of the pores and throats. Water will also flow in the corners of the elements. It is therefore possible to set both item 7 and 8 to false. Piston-like displacements (which will preferably happen over snap-off events) will then only occur where elements were not initially drained. If all elements were in fact drained, the first imbibition event occurring will be a snap-off event.

<table>
<thead>
<tr>
<th>IMBIBITION</th>
<th>%FinalSat</th>
<th>maxPc</th>
<th>maxΔSw</th>
<th>maxΔPc</th>
<th>calcKr</th>
<th>calcI</th>
<th>injIn</th>
<th>injOut</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>-1.0E21</td>
<td>0.1</td>
<td>1.0E21</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

**SAT_CONVERGENCE**

This keyword controls the accuracy at which the incremental water saturation target is reached. It is generally not feasible to calculate water saturation after each filling event, as it would be computationally very expensive. When starting a new saturation step, an approximate number of required filling events is estimated. This estimate is however quite uncertain as the water saturation does not vary linearly with the number of filling events. The actual number of filling events performed before re-calculating water saturation should therefore be a relatively small fraction of the initial estimate. Subsequently, an updated estimate of the required number of filling events is made, using information from the previous step. Still, only a fraction of the estimated filling events should be performed before re-calculating water saturation, as the estimate remains uncertain.

1. Minimum number of filling events between two calculations of water saturation.
2. Fraction applied to the initial estimate of required number of filling events.
3. Fraction applied to subsequent estimates of required number of filling events.
4. Maximum increase factor in required number of filling events estimates.
5. Only solve for relative permeability when a stable capillary configuration is reached? (true: “T” or false: “F”).
PORE_FILL_ALG

During spontaneous invasion, the capillary entry pressure for pore bodies will depend on the number $n$ of adjacent oil filled throats (pore body filling). Several models have been proposed in the literature to capture this feature. The code lets you chose between four different ones.


   \[ P_c = \frac{2\sigma \cos \theta_a}{r} - \sigma \sum_{i=1}^{n} A_i x_i \]

   where $A_i$ are arbitrary weighting numbers and $x_i$ are random numbers between zero and one.

2. The second model ("blunt1") was proposed by M.J. Blunt in “Effects of heterogeneity and wetting on relative permeability using pore level modeling.”, *SPE Journal*, **2**, 70 – 87, 1997.

   \[ P_c = \frac{2\sigma \cos \theta_a}{r + \sum_{i=1}^{n} A_i x_i} \]

3. One can also implement the model proposed by Øren et al. ("oren1") in “Extending predictive capabilities to network models”, *SPE Journal*, **3**, 324 – 336, 1998.

   \[ P_c = \frac{2\sigma \cos \theta_a}{r + \sum_{i=1}^{n} A_i r_i x_i} \]

4. Extension of the previous model, correcting for non-circularity of pores, is also implemented ("oren2").

   \[ P_c = \frac{(1 + 2\sqrt{G\pi}) \sigma \cos \theta_a}{r + \sum_{i=1}^{n} A_i r_i x_i} \]
PORE_FILL_WGT
The weights $A_i$’s, corresponding to an $I_i$ event, used in the models for pore body filling will have an impact on the filling sequence during spontaneous invasion. These are specific to the chosen model specified in “PORE_FILL_ALG”.

1. For the default model (“blunt2”) $A_i$’s have a dimension of $m^{-1}$. Hence we choose to relate them to absolute permeability.

   $$A_2 - A_n = \frac{0.03}{\sqrt{K}}$$

   where $K$ is the absolute permeability of the sample (in $m^2$). When only one connecting throat contains oil ($I_1$ event) the process is similar to piston-like displacement and hence $A_1 = 0.0 \mu m^{-1}$.

2. The second model (“blunt1”) was proposed by M.J. Blunt in “Effects of heterogeneity and wetting on relative permeability using pore level modeling.”, *SPE Journal*, 2, 70 – 87, 1997.

   $$P_c = \frac{2 \sigma \cos \theta_a}{r + \sum_{i=1}^{n} A_i x_i}$$

3. For the “blunt1”) model, $A1$-$A6$ are $0.0, 50 \times 10^{-6}, 50 \times 10^{-6}, 100 \times 10^{-6}, 200 \times 10^{-6}$ and $500 \times 10^{-6}$.

4. Typical values for the “oren1”) and “oren2”) models for $A1$-$A6$ are $0.0, 0.5, 1.0, 2.0, 5.0$ and $10.0$. 

<table>
<thead>
<tr>
<th>Blunt2 Model</th>
<th>% A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150000</td>
<td>150000</td>
<td>150000</td>
<td>150000</td>
<td>150000</td>
<td></td>
</tr>
</tbody>
</table>
**REC_ANG_WEIB**

The receding contact angles, $\sigma_r$, are distributed according to a truncated Weibull distribution between a maximum and minimum value.

$$\sigma_r = (\sigma_{r,max} - \sigma_{r,min}) \left(-\delta \ln \left[x \left(1 - e^{-1/\delta}\right) + e^{-1/\delta}\right]\right)^{1/\eta} + \sigma_{r,min}$$

where $x$ is a random number between 0 and 1.

1. Minimum receding contact angle (degrees).
2. Maximum receding contact angle (degrees).
3. $\delta$ exponent.
4. $\eta$ exponent.

<table>
<thead>
<tr>
<th>REC_ANG_WEIB</th>
<th>% Min theta</th>
<th>Max theta</th>
<th>delta</th>
<th>etha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**ADV_ANG_WEIB**

This keyword distributes the advancing contact angles according to a Weibull distribution. It is analogous to that for receding angles.

1. Minimum advancing contact angle (degrees).
2. Maximum advancing contact angle (degrees).
3. $\delta$ exponent.
4. $\eta$ exponent.

<table>
<thead>
<tr>
<th>ADV_ANG_WEIB</th>
<th>% Min theta</th>
<th>Max theta</th>
<th>delta</th>
<th>etha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30.0</td>
<td>90.0</td>
<td>0.2</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**TRAPPING**

Trapping is an important aspect of imbibition, where clusters of oil might be prevented from flowing out of the model, resulting in residual oil saturation. However,
the options defined in this keyword are typically found to have little impact on the behaviour of the model.

1. Fluid can escape the model through the entry face? (true: “T” or false: “F”)
2. Fluid can escape the model through the exit face? (true: “T” or false: “F”)
3. Fluid can escape the model through both faces (entry and exit)? (true: “T” or false: “F”)
4. Allow drainage of dangling ends (pores with only one connecting throat) through wetting layers? (true: “T” or false: “F”)
5. Spontaneous imbibe connecting throats following imbibition of a pore? (true: “T” or false: “F”)
6. Conductance of water (m$^4$.Pa$^{-1}$.s$^{-1}$) in circular elements completely filled with oil.

| TRAPPING | % Fluid exit through
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>% entry exit both</td>
<td>allow drainage of dangling ends</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**RAND_SEED**

This is the seed to the random number generator. It should be a large positive integer.

<table>
<thead>
<tr>
<th>RAND_SEED</th>
<th>% Large positive integer</th>
</tr>
</thead>
<tbody>
<tr>
<td>6256257</td>
<td></td>
</tr>
</tbody>
</table>

**CALC_BOX**

When injecting fluid from either of the faces, the saturation of that phase will naturally be higher towards that face than the average network saturation. During relative permeability calculations, a constant pressure differential is imposed between the inlet and outlet boundary. If the whole network is used for these calculations, most of the pressure loss for the displaced fluid will occur towards the injection face, as most of the elements there will only contain the injecting fluid. In order to avoid these inlet effects, it is common to only use a fraction of the network (away from the
injecting face) for calculating saturation and relative permeability. If the keyword is omitted, the whole network will be used for the computations.

1. Dimensionless location for lower boundary.
2. Dimensionless location for higher boundary.

<table>
<thead>
<tr>
<th>CALC_BOX</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Dimensionless location for</td>
</tr>
<tr>
<td>% Lower boundary   Higher boundary</td>
</tr>
<tr>
<td>0.5                  0.95</td>
</tr>
</tbody>
</table>

---

**SOLVER_TUNE**

The pressure solver used for relative permeability calculations is an algebraic multi-grid solver.

1. The performance of the solver can be tuned by varying the solution tolerance. A lower tolerance will result in an increased number of required solver iterations. Note that this is not the tolerance of the non-Newtonian iterative scheme.
2. Memory allocation for the solver can be adjusted by the memory scaling factor. For large models it might be necessary to increase the factor above the default value (5).
3. Performance related information about the solver is written to the file “fort.11”. A value of 0 produces minimal information whereas a value of 3 will produce substantial information about tolerance, memory requirements...
4. Setting this flag to true (“T”) will output solver information to screen rather than to file.
5. In some cases it might be necessary to discard conductances below a certain threshold to ensure solver convergence. This option should be used with great care and only when problems are observed (typically when solving for the oil pressure in cycles greater than the secondary).

<table>
<thead>
<tr>
<th>SOLVER_TUNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Min Tolerance     Memory Scaling Solver Verb Verbo Conduc</td>
</tr>
<tr>
<td>1.0E-15          Factor output output Cut-Off</td>
</tr>
<tr>
<td>5                0    0    F     0.0</td>
</tr>
</tbody>
</table>
FLUID

This keyword assigns properties to the Newtonian phase. If the keyword is omitted, the default interfacial tension is 30 mN/m and all viscosities set to 1.0 cp. Note that both oil and water Newtonian properties are specified here, since by default, all phases will be considered as Newtonian.

1. Interfacial tension (mN/m).
2. Water Newtonian viscosity (cp).
3. Oil Newtonian viscosity (cp).
4. Water resistivity (Ohm.m).
5. Oil resistivity (Ohm.m).

<table>
<thead>
<tr>
<th>FLUID</th>
</tr>
</thead>
<tbody>
<tr>
<td>% interfacial</td>
</tr>
<tr>
<td>% tension</td>
</tr>
<tr>
<td>% (mN/m)</td>
</tr>
<tr>
<td>30.0</td>
</tr>
</tbody>
</table>

NON_NEWT

The bulk rheological parameters of the non-Newtonian phase are entered with this keyword. They correspond to both the power-law and the Carreau formulation. The two are related as follows.

\[
\mu_{eff} = \text{MAX}\{\mu_\infty; \text{MIN}\left(C\dot{\gamma}^{(n-1)}; \mu_0\right)\} \quad \text{(Truncated power-law formulation)}
\]

\[
\mu_{eff} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + \left(\frac{\dot{\gamma}}{\gamma_{crit}}\right)^{\frac{1}{n-1}}} \quad \text{(Carreau formulation)}
\]

where the critical shear rate \(\gamma_{crit}\) is given by

\[
\gamma_{crit} = \left(\frac{\mu_0}{C}\right)^\frac{1}{n-1}
\]

Note that by default the non-Newtonian phase is ALWAYS the wetting phase. If the keyword is omitted the wetting phase will be assigned its Newtonian values.

1. Shear-thinning exponent constant, C (Pa.s\(^n\)).
2. Shear-thinning exponent, n.
3. Infinite shear viscosity, \(\mu_\infty\) (cp).
4. Zero shear viscosity, \(\mu_0\) (cp).
Whenever this keyword is being used a set of viscosity output files will be created. These files are the converged viscosity values of all network elements at a given water saturation, for the pressure gradient imposed to the system. They consist simply of one column containing viscosity values (in Pa.s). They will be named according to the water saturation, e.g. “Mu_fileXXX.dat” (where “XXX” represent water saturation). These files, if present, are used as first guess for the effective viscosity distribution of the network at a given water saturation. If they do not exist, by default all network elements are assumed to have an effective viscosity corresponding to the zero shear one. The keyword also creates a file “Sw_list.dat” which records the last value of water saturation in the network.

<table>
<thead>
<tr>
<th>NON_NEWT</th>
<th>% Power Law Cst</th>
<th>Power Law Exp</th>
<th>Inf shear Viscosity (cp)</th>
<th>Zero shear Viscosity (cp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% C</td>
<td>% Default values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% 0.001</td>
<td>0.999</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.181</td>
<td>0.5</td>
<td>1.5</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

**OUTPUT**

Using this keyword, it is possible to create output files containing the properties of individual network elements in terms of radii, volumes, connection number, etc... (files named “title_pores.out” and “title_throats.out”), as well as reporting the distribution of water saturation in the network at every interval (“title_pores_Sw.out” and “title_throats_Sw.out”). Both set of files can become very large and consume a lot of memory space. The file format used is the same defined in the geostatistical package GSLIB (Deutsch, C. V., and A. G. Journel, “GSLIB Geostatistical Software Library and User’s Guide”, 2nd ed, Oxford University Press, Oxford, 1998). The data are stored in columns. This first row is some arbitrary file header, followed by the number of data columns and the title of each column.

1. Create files with properties of individual elements? (true: “T” or false: “F”)  
2. Create files with water saturation distribution of all pores and throats at every interval? (true: “T” or false: “F”)
MATLAB

This keyword is used to write the output files in a MATLAB format. If the keyword is omitted, or set to false, the output files will be formatted in the way previously described.

1. Write output files in a MATLAB format? (true: “T” or false: “F”)

MATLAB

% Format output files for input to MATLAB
F
Appendix G

Cubic Network Generation Code (netgen v2.0) Manual

This program is an extension of the “netgen” code originally written by Per Valvatne. The program uses a fixed format input file. The order in which the data appear in the input file must be respected.

Comments in the data file are indicated by “%” and should ONLY be placed after the data, resulting in the rest of the line being discarded. All data should be on a single line. The input data file can be supplied as an argument to the executable:

```
netgen_v2_win32.exe
⇒ Please input data file: netgen_input.dat
```

NOTE: To avoid problems of paths definition, it is recommended that all files are placed in one folder (the executable, the network input data and the input file).

This program will generate network data files with the same format (see Appendix E) used by Statoil for their reconstruction methods, and that can be supplied to the various network simulation codes. It is important to note that the networks generated here are not based on any real rock reconstruction, but on a regular cubic lattice (with possibility of reduced connectivity). The networks are constructed with periodic boundary conditions.

A base file name is specified first. The network data will be written to four files with names: “filename_node1.dat”, “filename_node2.dat”, “filename_link1.dat”, “filename_link2.dat”.

| cubic_BereaStats | % Base file Name |
The size of the three-dimensional cubic lattice \((X \times Y \times Z \text{ pores})\) is entered here.

\[
\begin{array}{ccc}
20 & 20 & 20 \\
\% \text{ Lattice size}
\end{array}
\]

In the case where the properties of the network to be created are not being copied from another existing network, they will be distributed according to a truncated Weibull distribution, between a maximum and a minimum value. We begin by specifying the distribution of the throats inscribed radii.

\[
r = (r_{\text{max}} - r_{\text{min}}) \left( -\delta \ln \left[ x \left( 1 - e^{-1/\delta} \right) + e^{-1/\delta} \right] \right)^{1/\eta} + r_{\text{min}}
\]

where \(x\) is a random number between 0 and 1, and radii are expressed in \(\mu\)m.

1. Minimum inscribed radius of a throat \((10^{-6}\text{m})\).
2. Maximum inscribed radius of a throat \((10^{-6}\text{m})\).
3. \(\delta\) exponent.
4. \(\eta\) exponent.

\[
\begin{array}{cccc}
3.0 & 73.0 & 0.1 & 1.0 \\
\% \text{ Min throat radius, max throat radius, delta exp, eta exp}
\end{array}
\]

The throat lengths are also distributed using a truncated Weibull distribution.

1. Minimum throat length \((10^{-6}\text{m})\).
2. Maximum throat length \((10^{-6}\text{m})\).
3. \(\delta\) exponent.
4. \(\eta\) exponent.

\[
\begin{array}{ccc}
1 & 260.0 & 0.5 \\
\% \text{ Min throat length, max throat length, delta exp, eta exp}
\end{array}
\]

The aspect ratios between pores and throats are also distributed using a truncated Weibull distribution. The inscribed radius of the pores connecting several throats is found by multiplying an aspect ratio to the average radius of the connecting throats.
A constraint is applied here so that the pore radius is always larger than the largest connecting throat radius.

\[ r_p = \text{MAX} \left( \alpha \sum_{i=1}^{n_c} r_i, \text{MAX} \left( r_i \right) \right) \]

The pore lengths are taken to be \( 2 \times r_p \).

1. Minimum aspect ratio.
2. Maximum aspect ratio.
3. \( \delta \) exponent.
4. \( \eta \) exponent.

| 1.3 | 3.3 | 0.2 | 3.0 | % Min aspect ratio, max aspect ratio, delta exp, eta exp |

The shape of pores and throats in the network are determined according to their shape factor, \( G \). The shape factor is a dimensionless variable relating the cross-sectional area, \( A \), of the element to its perimeter, \( P \).

\[ G = \frac{A}{P^2} \]

Triangular elements have shape factors ranging from 0.0 (slit-shaped) to 0.04811 (equilateral). Square and circular elements have fixed shape factors of 0.0625 and 0.07958 (1/\(4\pi\)), respectively. The shape factors of triangular elements are also distributed according to a truncated Weibull distribution.

1. Minimum shape factor.
2. Maximum shape factor.
3. \( \delta \) exponent.
4. \( \eta \) exponent.

| 0.001 | 0.04811 | 0.8 | 1.6 | % Triangles: Min G, max G, delta exp, eta exp |

Most of the elements in the networks will typically have a triangular cross-section. However, it is possible to have a certain proportion of elements with square or circular shape. Note that circular pores and throats can only contain one phase at a time.

1. Percentage of pores with a square cross-section.
2. Percentage of pores with a circular cross-section.

| 0.05 | 0.05 | % Pores: proportion of square and circular pores |

1. Percentage of throats with a square cross-section.
2. Percentage of throats with a circular cross-section.

| 0.15 | 0.05 | % Throats: proportion of square and circular throats |

When trying to reproduce experimental data, it is important to match the residual water saturation. It is possible here to add a uniform clay volume to all elements (as proportion of total elements volume). This volume is taken as containing water that is immobile, effectively indicating a residual water saturation of the network.

**NOTE:** This approach can only be used when the properties of the network to be created is not copied from another existing network. Therefore it must be set to 0.0 if the network sizes distribution is inferred from an external network.

| 0.0 | % Proportion of pore volume occupied by clay |

A regular cubic lattice will have a connection number of 6. It is possible here to reduce the overall connectivity of the network by randomly removing throats between pores. The desired average connection number of the network is specified here although the resulting number might be slightly different due to the averaging process.
It is possible to reproduce the statistical properties of another network (for example one based on a real rock) in the created cubic lattice. As mentioned before, the volume of clay must be set to zero to ensure the integrity of the network is respected. The statistical data to be used should be incorporated into an ASCII type of file (columns of data) and should correspond to the cumulative frequency of the property to be reproduced. The structure of the file should be as follow; the 1st column should contain the bin sizes (in $10^{-6} m$) from its minimum to its maximum value and be monotonically increasing; the 2nd column should contain the cumulative frequency and also be monotonically increasing.

**NOTE:** None of the two columns should include zero values. When the two following options are set to true, they override the previous options which distributed the throat radii and lengths according to a truncated Weibull distribution (warnings are emitted).

1. Copy the throat radii distribution from another network? (true: “T” or false: “F”)
2. Name of the file containing the cumulative frequency of the throat radii to be reproduced.

1. Copy the throat lengths distribution from another network? (true: “T” or false: “F”)
2. Name of the file containing the cumulative frequency of the throat lengths to be reproduced.
NOTE: When trying to reproduce the statistical properties of a disordered complex medium with a cubic lattice, one might encounter network integrity issues. Typically, this is expressed by a win32 error message. For some realisations of the network, it is not possible to reproduce these statistical properties on such a regular lattice without breaching the network integrity. The code will nonetheless attempt to do its task, in an infinite iterative manner, causing the heap memory to crash. This is NOT a serious error, nor is it due to a coding bug. One should simply re-run the code, for the same input file, until a satisfactory network realisation is obtained.