INVESTMENTS AND NETWORK COMPETITION

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Investments and network competition

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This paper analyzes the impact of two-way access charges on the incentives to invest in networks with different levels of quality. When quality has an impact on all calls initiated by customers (destined both on-net and off-net), we obtain a result of “tacit collusion” even in a symmetric model with two-part pricing. Firms tend to under-invest in quality, and this is exacerbated if they can negotiate reciprocal termination charges above cost. When the quality of off-net calls depends on the interaction between the quality of the two networks, no network has an incentive to jump ahead of its rival by investing more.

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1. Introduction

This article addresses the issue of network competition between telecommunications operators when they have to invest in their own facilities. This situation should be described as one of two-way access, in the sense that each operator needs access to the rival’s network in order to terminate calls originated by its own customers but destined to subscribers belonging to the other network. Recent literature shows that regulatory concerns under two-way network competition are reduced, compared with one-way competition. Since network A wants its customers to be able to complete their calls to customers belonging to network B, and vice versa, A and B have a “double coincidence of wants” that makes the interconnection terms less problematic. The foreclosure problem typical of one-way access disappears. This does not necessarily imply that regulation is unnecessary in such an environment, since there may be other concerns. Access prices could be used as an instrument of tacit collusion because of a ‘raise-each-other’s-cost’ effect (Armstrong, 1998; Laffont, Rey, and Tirole, 1998 - hereafter ALRT - see Armstrong, 2002a, and Vogelsang, 2003, for excellent surveys). However, this problem is drastically reduced when operators compete over non-linear prices (e.g., two-part tariffs) to attract consumers.

A relevant result of the literature is the so-called “profit neutrality” result. In standard symmetric models with full consumer participation, it is generally found that profit is a constant that does not depend on the interconnection fee when firms compete in two-part tariffs. When the termination charge is increased, firms will increase call prices but, at the same time, they will reduce the fixed component to keep market shares. This is because higher termination revenues make firms more willing to compete for customers. These effects cancel out and firms are indifferent towards the level of (reciprocal) termination charges.

The model that we propose builds on the framework of ALRT but challenges the result of “profit neutrality”. We depart from ALRT by allowing firms to invest so as to improve the quality of their network. A better quality network is more expensive but may give an important edge to operators when competing with their rivals. As long as the level of investments affects the number of calls made by a subscriber of network A, and some of these calls are destined to subscribers of network B (off-
net calls), it is clear that the termination charge paid by network A for the termination of off-net calls will have an impact on the investment made by A. We believe that this dynamic aspect of access charges is an interesting and important question that has not yet been addressed by the literature. This paper offers two main contributions. Firstly, we study the role of access charges with asymmetric firms, where the asymmetry derives from quality choices that affect the amount of calls that customers are willing to make at a given price. In particular, we study two versions of a basic model: one where a network’s quality influences all the calls made both on-net and off-net, and another where quality has a “bottleneck” feature, i.e., the off-net quality depends on the minimum level of the qualities chosen by the two competing networks. Our second and more important contribution concerns the impact access charges have on the incentives to invest in networks of differing quality (quality is then the source of asymmetries in later stages of competition). Although there are some recent papers that have also studied the role of access charges with asymmetric firms in different settings, there is much less literature related to our second contribution.

We extend the framework of ALRT by introducing an investment stage, prior to price competition. We show that the incentives to invest are influenced by the way termination charges are set. When the quality of a network has the same impact on all calls initiated by its own users (terminated both on-net and off-net), we obtain a result of “tacit collusion” even in a symmetric model with two-part pricing. Firms tend to under-invest in quality, which would be exacerbated if they could negotiate reciprocal termination charges above cost. Investment decisions are strategic substitutes and above-cost access charges induce firms to invest less. Intuitively, when termination charges are above cost, an increase of own quality relative to the rival creates an access deficit making an operator reluctant to invest. This has a positive impact on operators’ profits since they avoid a costly battle over investments. We also show that when the quality of off-net calls depends on the interaction between the quality of the two networks, there is an additional problem, namely that no network has an incentive to jump ahead of its rival by investing more.

The paper is organized as follows. Section 2 describes the basic model when quality affects both on- and off-net calls. The last stage of the game (price competition) and the investment stage are solved in Sections 3 and 4. In Section 5 we discuss the results and formulate policy recommendations.
in favor of a regime of “bill-and-keep”, i.e., a zero interconnection rate. Section 6 solves a game where quality has a bottleneck feature. Section 7 concludes.

2. The model

Demand structure

We consider two operators that may decide to enter a telecommunications market. The two networks are differentiated à la Hotelling. A unit mass of consumers is uniformly located on the segment [0,1] while the network operators are located at the two extremities. We denote by 1 (respectively 2) the firm located at the origin (respectively at the end) of the line.

Network operators use non-linear tariffs. Since the customers’ demand function is known, network \( i = 1, 2 \) cannot do better than offering a two-part tariff \( T_i(q) = F_i + p_i q \), where the fixed fee \( F_i \) can be interpreted as a subscriber line charge and \( p_i \) as the price for each one of \( q \) calls.

When a consumer located at \( x \) buys from firm \( i \) located at \( x_i \), (s)he enjoys a utility given by:

\[
y + v_0 - |x - x_i| (2\sigma) + v_i(p_i) - F_i
\]

(1)

where \( y \) is the income, \( v_0 \) is a fixed surplus component from subscribing (“option value”, assumed to be large enough so that all customers always choose to be connected to a network) and \( v_i(p) \) is the indirect utility derived from making calls at a price \( p \). The parameter \( \sigma \) represents an index of substitutability between the networks, and its function is to regulate the intensity of price competition.

In order to discuss the important notion that there may be a benefit from having large investments in infrastructure, despite the associated costs, we introduce a parameter \( k_i \geq k > 0 \) that is increasing in operator \( i \)'s investment and affects quantities and utilities. \( k \) is some minimum quality level that
operators have to supply. Both quantity and indirect utility are increasing in \( k_i \); in particular, we assume that they can be expressed in a multiplicative form: \( q_i(p) = k_i q(p) \), \( v_i(p) = k_i v(p) \).

This notion captures the idea that investment is a quality parameter that consumers enjoy, *ceteris paribus*, since they would have easier access, faster delivery, less congestion problems, and so on. For instance, a utility function that satisfies our assumption is \( u_i(q) = k_i (q / k_i)^{1-\eta} / (1 - 1/\eta) \). This represents the utility from making \( q \) calls from a network of quality \( k_i \), and it generates a constant elasticity (\( \eta \)) demand for calls.\(^3\)

In the first part of this article, we assume that the quality supplied is the same for on-net and for off-net services. This describes a situation where the rival’s quality does not affect consumer behavior, once the decision to subscribe has occurred. For instance, the number of files sent or downloaded depends more on the bandwidth supported by the subscriber’s final link. This also applies to video-on-demand. Another relevant interpretation is where \( k \) represents a certain number of functionalities, such as number of terminals, ease of access etc., that would induce the subscriber to call more, independent of what happens at the terminating end. However, if one wants to emphasize the simultaneity of a conversation, then this assumption is not satisfactory. In Section 6 we will introduce an alternative model where off-net quality depends on the interaction with the rival’s quality.

**Market shares**

The consumer who is indifferent between the two networks determines the market share of the two firms. Firm \( i \)'s share is \( \alpha_i \) where:

\[
\alpha_i = \alpha(p_1, p_2, F_1, F_2) = 1/2 + \sigma(w_i - w_j)
\]

(2)

where \( w_i = v_i(p_i) - F_i = k_i v(p_i) - F_i \) is the net surplus for customers connected to network \( i \).
Cost structure

Both networks have full coverage. Coverage is defined as the fraction of customers who can but are not necessarily served by the network. Serving a customer involves a fixed cost \( f \) of connection and billing. All calls have to be originated and terminated. The marginal cost is \( c \) per call at the originating end and \( t \) at the terminating end. The total marginal cost for a call is thus \( c + t \). Networks pay each other an exogenous negotiated or regulated two-way access charge, denoted by \( a \), for terminating each others’ calls. Finally, each network incurs a convex cost \( I(k) \) to provide a service of quality \( k \), \( I'(\cdot) > 0 \) and \( I''(\cdot) > 0 \).

The timing of the game is as follows. Interconnection terms are set at stage 0. In stage I operators invest in quality. Finally, in stage II operators compete in two-part prices.

We are mostly interested in cost-based regulation, which is the current regulatory benchmark in many countries. Our status quo is cost-based regulation \( (a = t) \) and we analyze how a small departure from this benchmark affects firms’ profits and consumer surplus. The understanding is that, should our results show that both firms’ profits are positively affected by setting reciprocal charges above cost, firms would then negotiate them above cost if they are left unregulated. Conversely, should our result show that welfare is positively affected by setting reciprocal charges above cost, then they would be set above cost when regulated by a benevolent social planner. We will return to optimal regulation in Section 5.

3. Solution of stage II

Price competition
As shown above, market shares $\alpha_i$ are directly determined by net surplus $w_i$ and it is analytically convenient to imagine firms compete in $p_i$ and $w_i$ rather than in $p_i$ and $F_i$. In the last stage investments are fixed, hence network $i$ has to solve:

$$\max \Pi_i = \max_{w_i, p_i} \pi_i - I(k_i)$$
$$\pi_i = \alpha_i \left[(p_i - c - t - \alpha_j(a - t))k_i q(p_i) + v_i(p_i) - w_i - f\right] + \alpha_i \alpha_j(a - t)k_i q(p_i) =$$
$$= \alpha_i \left[(p_i - c - t)k_i q(p_i) + v_i(p_i) - w_i - f\right] + \alpha_i \alpha_j(a - t)\left[k_i q(p_i) - k_j q(p_j)\right]$$

We have at equilibrium:

$$p_i^* = c + t + (a - t)\alpha_j^*,$$

(3)

i.e., the usage fee is equal to the ‘perceived’ marginal cost. From eq. (3) one can easily note that, when access charges are set above cost, then both firms would charge a mark up above marginal cost. In addition, the firm with a market share above 50% would charge less than its rival. This is due to the fact that, being the larger firm, it would terminate more calls on-net than its rival, and therefore the perceived marginal cost for the larger firm would be smaller. The fixed fee $F_i$ at equilibrium is:

$$F_i^* = f + \alpha_j^*/\sigma - (p_i^* - c - t)k_i q(p_i^*) - (\alpha_j^* - \alpha_j^*)(a - t)\left(k_i q(p_i^*) - k_j q(p_j^*)\right).$$

(4)

After substitution into eq. (1), market share of firm $i$ at equilibrium is:

$$\alpha_i^* = 1/2 + \sigma \left(k_i W(p_i^*) - k_j W(p_j^*)\right)/3.$$ 

(5)
where $W(p) = v(p) + (p - c - t)q(p)$ is the total welfare generated by a network of unit quality.

Finally, the equilibrium profit (gross of investment) of network operator $i$ is given by:

$$\pi_i = \sigma_i^2 / \sigma - \alpha_i^2 (a - t) (k, q(p_i) - k, q(p'_i)).$$

(6)

We can now state the following proposition.5

**Proposition 1.** One of the following conditions is sufficient for the firm with the higher investment to always have more than 1/2 of the market, independently from the way access charges are set: a) access charges are sufficiently close to termination costs; b) products are sufficiently differentiated; c) demand is sufficiently rigid.

**Proof.** See the Appendix.

An immediate implication is that, if firm $i$ is the higher quality (larger) firm, then the second term in eq. (6) is negative if access is regulated above cost, while the opposite holds true for the smaller firm.

**Comparative statics: effects of access charges and quality on equilibrium shares and prices**

Cost-based regulation is a very common regulatory benchmark. Our aim is to consider what happens to the equilibrium in the last stage when access charges are slightly increased above termination costs.

The following proposition summarizes the results:

**Proposition 2.** A small variation of the access charge above its marginal cost has:
• a positive impact on prices: \[ \frac{\partial p_i^*}{\partial a} \Bigg|_{a=t} = \alpha_j^*; \]

(7)

• no local effect on market shares: \[ \frac{\partial \alpha_i^*}{\partial a} \Bigg|_{a=t} = 0; \]

(8)

• if products are sufficiently differentiated, the bigger firm’s market share reaches a maximum at \( a = t \).

When access is regulated at cost, prices are not affected by quality, while market shares are:

• \[ \frac{\partial p_i^*}{\partial k_i} \Bigg|_{a=t} = \frac{\partial p_j^*}{\partial k_j} \Bigg|_{a=t} = 0; \]

(9)

• \[ \frac{\partial \alpha_i^*}{\partial k_i} \Bigg|_{a=t} = \sigma \nu(c + t) / 3 = \frac{\partial \alpha_i^*}{\partial k_j} \Bigg|_{a=t}. \]

(10)

**Proof.** See the Appendix.

A small increase in the termination charge implies that the perceived marginal cost increases, which explains eq. (7), and also why the effect is magnified for the smaller firm (since it terminates relatively more calls off-net than its rival). As far as the neutral result on market share is concerned, there are two effects that offset each other. An access mark up pushes prices up, implying a lower indirect utility \( k_i \nu(p_i) \) for the customer resulting from calls. At the margin this effect is equal to \(-k_i q(p_i^*) \partial p_i / \partial a\). At the same time, from eq. (4), the main effect on fixed fees from allowing an access mark up is to reduce them (see the third term in eq. (4); note also that the fourth term in eq. (4) is the same for both firms): this gives a benefit to the user that is again equal to \( k_i q(p_i^*) \partial p_i / \partial a \) at the margin. Therefore, locally at \( a = t \) the net surplus of customers does not change and market shares are
unaffected, giving the result summarized by eq. (8). When access is set at cost, the perceived marginal cost is equal to the true marginal cost, independent from the firm’s share of the market, which explains eq. (9). However, quality has an impact on customers’ indirect utility, and a higher quality allows a firm to obtain a bigger share of the market (eq. (10)) when access is regulated at cost.

**Effect of the access charge on gross profits**

We can state the following result when quality is treated exogenously:

*Proposition 3.* In a neighborhood of \( a = t \), a small increase in the access charge increases the profit of the lower quality (smaller) network and decreases the profit of the higher quality (larger) network. Thus, the “profit neutrality” result does not hold.

*Proof.* See the Appendix.

The reason for this result is that, when the access charge is slightly increased above cost, the effect on market shares is negligible. However, the higher quality (larger) firm becomes relatively cheaper than the rival, and therefore relatively more calls per unit of quality will be placed on the rival network than received from it, reinforcing the net outflow of calls that amounts to \( k_i q(p_i) - k_j q(p_j) \). This creates a net loss on access that negatively affects the larger firm. On the contrary, the smaller firm terminates more calls, earning a small margin on them.

This result complements the “profit neutrality” result that has been found in the seminal papers of ALRT. Carter and Wright (2003) also find that asymmetries matter for profit neutrality. The type of asymmetries that they consider (brand loyalty) is given exogenously, while in this paper asymmetries may arise from the endogenous decision of firms to invest in quality. Moreover, our results differ from Carter and Wright (2003) since our quality variable affects customers’ quantities. This implies that when the access charge is increased above cost, there is a first-order effect that does not arise in
their paper where brand loyalty only has an impact on the decision to subscribe and not on calling behavior. This is why in our model the effects of the access charge are pushing in opposite directions for the larger and for the smaller firm, while the analysis of Carter and Wright (2003) relies only on second-order effects.

In the context of our model, we have thus shown that for given investment levels, a small increase in the access charge would give benefits to the smaller firms. However, the type of asymmetries considered here arises from the decision of firms to invest in quality. Since access charges do not change very frequently (they have to be filed and approved with regulators), it is realistic to think that access charges have an impact on endogenous investment levels. Does the smaller firm have more incentives to invest when the access charge is set above cost? This is what we study in the next section.

4. Investments

Access is regulated at cost

Suppose that $a = t$. In stage I network $i$ has to solve $\max_k \Pi_i = \pi_i^* - I(k_i)$, where $\pi_i^* = \alpha_i^*/\sigma$ and $\alpha_i^*$ results from simplifying eq. (5), i.e., $\alpha_i^*\big|_{a=t} = 1/2 + \sigma(c + t)(k_i - k_j) / 3$. We fix the investment level of firm $j$ and consider the best reply of firm $i$:

$$\frac{\partial \Pi_i}{\partial k_i} = 2\alpha_i^* v(c + t) / 3 - I'(k_i) = v(c + t) / 3 + 2\sigma v(c + t)^2 (k_i - k_j) / 9 - I'(k_i) = 0$$

(11)

with SOC $\frac{\partial^2 \Pi_i}{\partial k_i^2} = 2\sigma v(c + t)^2 / 9 - I''(k_i) < 0$ that we assume to be satisfied at equilibrium. Notice how convexity of the cost function is needed to ensure the existence of an equilibrium, otherwise there would be an escalation of investments and existence would fail. Implicitly
differentiating eq. (11) we obtain the slope of the best reply \( \frac{\partial k_i}{\partial k_j} = \frac{1}{(1 - 9I^*/(2\sigma v^2))} < 0 \), i.e., a network would *decrease* its quality when its rival invests more. To understand this result of strategic substitutability, notice that gross profit is proportional to the square of market share, and market share is linear in the quality difference \( \Delta = k_i - k_j \). Hence, the marginal benefit is constant for a given \( \Delta \), while the marginal cost to ensure the same \( \Delta \) increases with the rival’s \( k_j \) since the investment function is convex. Therefore a firm will supply a lower investment the higher the rival’s investment.

If firms choose their investments simultaneously, then eq. (11) simplifies to \( v(c + t)/3 - I'(k_j) = 0 \) at the symmetric equilibrium. It is easy to see that, although equilibrium profits are affected by the substitutability parameter, investments are not. Investment levels would increase with indirect utility \( v \) (hence they would decrease in \( c \) and \( t \)), having a negative impact on net profits since market shares would be fixed at 1/2 in a symmetric equilibrium. Also note the under-investment feature: when firms are located at the opposite ends of the line the socially optimal investment would be \( I'^{-1}(v/2) \), which is higher than the privately chosen investment \( I'^{-1}(v/3) \).

**Access is not regulated at cost**

In stage I, the FOC with respect to quality of firm \( i \) is the following:

\[
\frac{\partial \Pi_i}{\partial k_i} = 2 \frac{\alpha_i^*}{\sigma} \frac{\partial \alpha_i^*}{\partial k_i} - (a - t) \frac{\partial \Omega}{\partial k_i} - I'(k_i) = 0 \quad \text{where} \quad \Omega = \alpha_i^* \left( k_i q(p_i^*) - k_j q(p_j^*) \right).
\]

(12)

As before, we are interested in seeing if, starting at \( a = t \), a firm would increase or decrease quality for a given rival’s quality when the access charge is slightly increased above cost:

**Proposition 4.** Imagine that the access charge is slightly increased above cost. Then the optimal investment level of firm \( i \) changes in the following way:
\[
\frac{d k_i}{d a} \bigg|_{a_{\text{opt}}} = \alpha_i \left[ \frac{1}{2} + \sigma(v(c + t)(k_j - k_i))g(c + t) \right] \frac{\sigma(c + t)}{\frac{1}{2} \sigma(v(c + t)^2 - l^*(k_j))}.
\]

(13)

**Proof.** See the Appendix.

An immediate implication of Proposition 4 is:

*Corollary 1.* If the access charge is set slightly above cost, a firm that is larger that its rival would always decrease its investment level. This would also be true if a firm is smaller than its rival and products are sufficiently differentiated.

*Proof.* See the Appendix.

This result can be applied to a situation where investments occur sequentially so that there is an “incumbent” and an “entrant”. Corollary 1 states that if the incumbent has a low-quality network and the entrant decides to come into the market with a higher-quality network, then the results of Proposition 3 would translate into similar incentives to invest, i.e., an access charge below cost would give higher profits and higher incentives to invest. However, this would not be true with an entrant that has to challenge an incumbent with an existing network of high quality: in this case an access charge below cost would induce higher investments but would diminish the overall profit for the entrant. This result can be rephrased by noting that a policy designed to help a small entrant in stage II (i.e., by setting \(a > t\)) would *not* translate into higher incentives for the entrant to enter on a bigger scale with higher investments.

Suppose that investment levels are chosen simultaneously. We know from the previous section that when access is regulated at cost, then the reaction function is downward sloping. What happens in a symmetric situation if firms are left to negotiate termination charges?
Proposition 5. In a symmetric equilibrium, firms always have an incentive to negotiate reciprocal access charges above cost. This is detrimental to social welfare.

Proof. See the Appendix.

The result is intuitive since market shares would be split equally in a symmetric equilibrium, and given two-part pricing, the profit neutrality result would apply to gross profits. What is new here is that there is no neutrality with respect to investments: an access charge above cost would induce both firms to invest less. Hence the key to understanding Proposition 5 lies in Corollary 1: above-cost termination charges have an investment-reduction effect. The intuition for this effect can be grasped by imagining first that \( a = t \) and firms are in a symmetric equilibrium. That is, taking \( k_j = k^* \) as given, the marginal cost for firm \( i \) of providing \( k_i = k^* \) is equal to the marginal benefit arising from that quality level. Imagine now \( a \) is set slightly above cost. Taking the same \( k_j \) as given, why is firm \( i \) now more reluctant to invest? If firm \( i \) goes slightly above \( k^* \), it would now create its own access deficit, since it would become ‘bigger’ than its rival. Therefore the marginal benefit at \( k^* \) is now decreased, and firm \( i \) would want to invest less than \( k^* \) which is the optimal choice at \( a = t \). By agreeing on a common mark up above cost, firms are able to credibly commit not to fight over costly investments: they save on investment costs without losing anything from gross profits. In other words, we can restore “tacit collusion” even in a symmetric equilibrium under two-part pricing.

5. Discussion

The results derived in the previous sections point to an important influence that termination charges have on the firms’ incentives to invest. As shown in Proposition 5, if access terms were left unregulated, firms would naturally tend to agree on above-cost reciprocal charges as a way to avoid a costly investment battle. Since our results also imply that free negotiations would run against the
In general, there are three allocations that should be taken into account for social welfare: a) calls, b) market shares, c) investments.

The efficient allocation of calls arises when the call price is equal to the marginal cost \( c + t \). Optimal market shares depend on transportation costs and asymmetries between networks. Clearly, in a symmetric equilibrium, each firm should get the closest half of the line. The latter allocation is actually reached by private firms in a symmetric equilibrium, and we can therefore neglect it. Optimal regulation should then take into account only a) and c). In the absence of any effect from investments, a) would call for \( a = t \) since, with two-part tariffs, firms price each call at the perceived marginal cost that would correspond to the true marginal cost. However, because of the investment effect, a departure of termination below cost would always be optimal. Firms invest “too” little even when \( a = t \), hence the first-order gain arising from higher investments when \( a \) is set below \( t \) always prevails over the second-order loss from an inefficiently high number of calls being made.

This result has a local nature, since it has been derived in a neighborhood around cost-based access prices. However, it is also valid under large deviations from cost-based access prices if one uses specific functional forms. We have obtained closed-form solutions for an example of linear demand \( v(p) = (1 - p)^2/2 \) and quadratic investment costs \( I(k) = k^2/2 \). The resulting expressions are rather cumbersome and are not reported here. However, from the left panel of figure 1, one can see that welfare is (globally) maximized for below-cost charges. The algebraic expression of the welfare-maximizing termination charge \( a \) does not provide additional insights, hence we prefer to describe the plots. In this linear example, the parameters that can influence the optimal charges are \( \sigma, c \) and \( t \). The substitutability parameter has a negligible impact on investments, and in the right panel of figure 1 we plot the optimal percentage discount below termination cost, \( (t - a)/t \), as a function of the two remaining parameters.

[Insert Figure 1 about here]
The percentage discount increases as the marginal cost of a call decreases. In fact, for low enough marginal costs, the optimally regulated charge would be zero (100% negative mark up), since prices cannot be negative. To understand this result, remember that the fundamental trade-off for regulated charges to depart from cost is higher investments versus the misallocation of calls. Also bear in mind that, starting with cost-based termination, the private investment chosen by a firm is increasing in the unit indirect utility \( v(\cdot) \). When \( v \) is high, the distortion to induce extra investments will be quite substantial. On the contrary, if \( v \) is low, there is not much reason to induce higher investments. In the linear case, there is a choke-off point when the price is high enough such that \( v = 0 \). If access is regulated at cost and the total marginal cost \( c + t \) is equal to the choke-off price (the intercept 1 in our linear example), then only in this limit case a social planner would want to shut down the market, and \( a = t \) would be optimal (zero discount). As \( c + t \) is lowered below the choke-off point (i.e., when the problem starts making economic sense), discounts arise. For high total marginal cost, a big distortion on call allocations is needed to generate high investments, hence the mark up would be negative but small. As \( c + t \) decreases even further, negative mark ups increase, and they may reach 100%. Note that with the development of high-speed networks, the cost of carrying traffic on a network is very low.

The optimal regulation of termination charges has to be assessed considering three important assumptions that are embedded in our approach and which are needed to generate our results:

1. The amount and quality of information at the regulator’s disposal;
2. Its ability to commit to access rules before investment takes place;
3. The multiplicative effect of quality in consumer surplus.

**Information: a “bill-and-keep” regime?**

Optimal charges can be calculated only if the regulator knows demand and cost parameters. Although some engineering network models are available (e.g., the cost-proxy model used by the FCC in the US, see Gasmi et al., 2002) and demand elasticities for most services are known (at least their order of
magnitude), it is contentious that, in practice, regulators could *ex ante* calibrate access charges. In our opinion, this type of regime would be quite intrusive, leading to the paradoxical result that more rather than less regulation would be needed as competition develops in the telecommunications industry. The policy implications that we want to gather from our analysis suggest at least three main messages:

- Many countries (including the US since the Telecommunications Act of 1996) have put into practice regulations where the calling party’s network should pay the called party’s network a termination fee based on the Long Run Incremental Cost (LRIC) of the traffic sensitive facilities of the receiving network used to terminate the call. Our model suggests that LRIC is not a good benchmark when the incentives to invest are taken into consideration;

- Free negotiations would lead to agreements which are detrimental to social welfare. The concern that firms might use negotiations over termination to reduce the intensity of competition seems to be well-placed;

- Below-cost charges are optimal but also difficult to calibrate. Given that one needs a regulatory system that has good theoretical properties but is also simple to implement, one practical suggestion is to impose a regime based on reciprocal “bill-and-keep” arrangements, i.e., a zero interconnection rate. We are in favor of “bill-and-keep” not because the optimal interconnection should be zero (although it may be with low marginal costs), but because it would be easy to put into practice and it would provide higher incentives to invest.

**Commitment: LRIC vindicated?**

The optimal access charges derived above assume that access charges are set before investments take place. Clearly, the regulator may not credibly commit *ex ante* to complicated access charge rules before investments are made due to legal, political and practical constraints on the regulator. However, without commitment, a regulator may want to change access charges *ex post*. In this case, the optimal termination charges (without commitment) would take investments as given and would only have to mediate among two of the allocative problems described at the beginning of this section,
i.e., calls and market shares. The efficient allocation of market shares, for given investments, is
\[ \alpha_i^w = 1/2 + \sigma \left( k_i W(p_i^*) - k_j W(p_j^*) \right) > \alpha_j^*, \]
where \( \alpha_j^* \) is given by eq. (5). Therefore a social planner may want \textit{ex post} to distort termination charges away from cost if such distortions allow for improvement in the distribution of market shares. However, we know from Proposition 2 that a small variation of the access charge around cost will have a zero impact on market shares. Thus we can state that, locally, \( \alpha = t \) is optimal for given investments, even when firms are asymmetric. A small change would not help in the allocation of market shares but would just make call consumption depart from its optimal level.

In fact this local result is stronger and in many cases it is a global result: the only reason to distort termination from cost is to induce the high quality firm to gain market share but this may not be possible. In fact, from Proposition 2 we also know that, when \( \sigma \) is low (which is needed for existence), \( \alpha_j^* \) reaches a maximum at \( \alpha = t \). Hence, any departure from cost would worsen the allocation of market shares. In this respect, the common regulatory benchmark LRIC is ‘vindicated’ if one believes that regulators have not taken investment considerations into account or if they lack the \textit{ex ante} ability to commit to access rules before investments take place. It should be noted that, in the presence of appropriate legal and regulatory institutions, a “bill-and-keep” system is less vulnerable to commitment problems. It limits detailed regulatory intervention, it simplifies the regulator’s task, and it can be written \textit{ex ante} in regulatory contracts as a simple and enforceable access rule stating that no compensation is required for terminating other calls originated on rival networks.

\textbf{Quality: additive or multiplicative?}

Our result regarding optimal below-cost charges relies on the multiplicative specification of quality in the utility function. To see why, let us take an alternative view and assume that quality enters in an additive way: \( y + v_0 - \sqrt{y} \sqrt{\frac{2\sigma}{v(p)}} + k - F_i \). Since a similar specification is used by Carter and Wright (2003), we can use some of their results and go one step further by endogenizing the
quality choice also in this context. For convenience, we report here the stage-II profit function for

firm 1:

\[
\pi_i^* = \frac{\alpha_i^2}{\sigma - \alpha_i^2} (a - t)(q(p_i^*) - q(p_j^*))
\]

\[
\alpha_i^* = \frac{1}{2} + \frac{\sigma}{3} \left[ k_i - k_j + v(p_i^*) - v(p_j^*) + (a - t)(\alpha_i^*, q(p_i^*) - \alpha_i^*, q(p_j^*)) \right], \quad \text{with} \quad \left. \frac{\partial \alpha_i^*}{\partial a} \right|_{a^*} = 0 .
\]

The important difference to note is that now, compared to eq. (5), when termination is regulated around cost there is no traffic imbalance even in the presence of asymmetric networks. Thus, if a firm increases its own quality compared to the rival, it will affect market shares but no direct effect will arise from access deficits (or gains). On the contrary, in our multiplicative version there is a direct effect arising from call imbalance: a firm that does better than the rival in terms of quality faces an access deficit if \( a \) is set slightly above cost.

In stage I firm \( i \) maximizes \( \Pi_i = \pi_i^* - I(k_i) \). Also in this case the reaction function, when \( a = t \), is downward sloping for the same reasons explained in Section 4.1. We can also show that, when regulation is at cost, \( \left. \frac{\partial^2 \Pi_i}{\partial k_i \partial a} \right|_{a^*} = 0 . \) Instead in our multiplicative specification this term is negative because of the first-order effect arising from the access deficit created by increasing quality relative to the rival. With some additional effort it can be shown that investment reaches a \( \text{maximum} \) when \( a = t \). This is because when \( a = t \), stage-II (gross) profits are maximized for the high-quality firm (see Proposition 1 in Carter and Wright, 2003), and therefore there is the strongest incentive to compete, via higher investments, for such stage-II profits. However, if \( a \) is either below or above cost, stage-II (gross) profits are lower and firms are more reluctant to invest. Since the neutrality result applies to gross profits in a symmetric equilibrium, this finding immediately implies that firms' profits are \( \text{minimized} \) when \( a = t \). Once (additive) asymmetries are endogenized, we can thus conclude:
In a symmetric equilibrium a social planner would set termination at cost (investments are still “too” low since they are privately chosen, but any charge other than cost would make firms invest even less; moreover no further call distortions are introduced with $a = t$);

Unregulated firms would always agree not to set termination rates at cost. The model, however, is elusive as to whether firms would go above or below cost, since the profit function is U-shaped with a minimum at $a = t$.

In summary, both the additive and the multiplicative specification find that unregulated agreements over termination charges would be detrimental to social welfare once investments are endogenized, but the multiplicative specification is crucial in generating below-cost optimal charges. Determining which model resembles reality more closely is an empirical matter. We believe that a multiplicative model, i.e., a specification where quality has an impact on call quantities, is realistic. In recent years, call minutes have increased in an exponential way. Part of this increase can be attributed to lower prices and to more people having subscribed to the services (this is particularly true for mobile telephony). However, many networks are now mature in terms of users, yet minutes are growing by more than price decreases alone would justify. The additional effect comes from higher-quality networks: users, say with a high-speed or ISDN/DSL connection, may surf the net and call at the same time, hence higher quality feeds into higher call minutes at a given price. This effect may also arise from other sources: easier access, wider coverage, etc. are all features that are improved by network investments and which allow people to make more calls per unit of time. These traits are captured by a multiplicative specification but would be absent in an additive model.

Given the central role played by the multiplicative specification, it is important to see if our fundamental result on “collusive” charges is robust to alternative (multiplicative) specifications which may also reflect realistic environments. This is taken into consideration in the next section.

6. Bottlenecks and interacting qualities
In our main section we have used a rather special model specification where own quality controls both on- and off-net calls. In this section we consider an alternative model where off-net calls depend on the interaction between the quality of the two networks. In particular, we imagine that the quality for services supplied off-net depends on the minimum quality provided by the two networks, \( \min\{k_i, k_j\} \). This assumption describes a typical situation in the telecommunications sector where the simultaneity of a conversation plays a central role. While a network is in control of the on-net behavior of its customers as in the previous model, now off-net calls depend on the quality of the bottleneck facility that results from the non-cooperative choice of the two networks.

**Market shares**

Without loss of generality suppose that network 1 provides the higher-quality network, \( \min\{k_1, k_2\} = k_2 \). The consumer who is indifferent between the two networks determines the market share of the two firms. In particular firm \( i \)'s share is still given by eq. (2) where now:

\[
\begin{align*}
  w_i &= v_i(p_i) - F_i = \alpha_i k_1 v(p_i) + \alpha_i k_2 v(p_i) - F_i \\
  w_2 &= v_2(p_2) - F_2 = \alpha_2 k_2 v(p_2) + \alpha_i k_2 v(p_2) - F_2 = k_2 v(p_2) - F_i
\end{align*}
\]

Note that the net surplus for consumers connected to network 2 does not change with respect to the previous case, since on-net and off-net calls have the same quality; this is not true for net surplus of customers connected to network 1 which also depends on the lower quality of interconnection that decreases the volume of off-net calls.

**Stage II competition**

Imagine firms compete in \( p_i \) and \( F_i \). In the last stage network 1 solves:
\[
\max_{\pi_1, k_1} \Pi_1 = \max_{\pi_1, k_1} \pi_1 - I(k_1) \\
\pi_1 = \alpha_1 \left[ (p_1 - c - t)q(p_1)(\alpha_1 k_1 + \alpha_2 k_2) + F_1 - f \right] + \alpha_2 (a - t)k_2 (q(p_2) - q(p_1)) 
\]

For network 2, since the quality for on-net and off-net calls remains the same, profit is:

\[
\max_{\pi_2, k_2} \Pi_2 = \max_{\pi_2, k_2} \pi_2 - I(k_2) \\
\pi_2 = \alpha_2 \left[ (p_2 - c - t)q(p_2) + F_2 - f \right] + \alpha_2 (a - t)k_2 (q(p_1) - q(p_2)) 
\]

One should note that both expressions for profits are different from those obtained in Section 3.1. Firm 1’s expression differs for the reasons described above (which influence both indirect utility and quantities). Firm 2’s expression differs only for the quantity of incoming calls received from the rival network, which are now reduced compared to the previous model.

In the Appendix we characterize the equilibrium.\(^9\) Here we report only the expressions for call prices:

\[
p_1^* = c + t + \alpha_2^* (a - t)k_2 / \bar{k}, \quad \text{with} \quad \bar{k} = \alpha_1^* k_1 + \alpha_2^* k_2, \\
(14) \\
p_2^* = c + t + \alpha_1^* (a - t) \\
(15)
\]

where \(\alpha_1^*\) is the market share of network 1 at the equilibrium. Call prices are set to maximize joint surplus between a network and its customers (and fixed fees are then set to split the surplus created, depending on the intensity of price competition). Since own quality affects joint surplus for firm 2 in a multiplicative way, this explains why it still charges the perceived marginal cost, as eq. (3) in Section 3. However, joint surplus for firm 1 is weighted down for off-net calls by the lower rival’s quality,
which results in the additional “quality-adjusted factor” $k_z/\bar{k} < 1$ in eq. (14).\textsuperscript{10} In particular, when $a = t$, we have $p_1^* = p_2^* = c + t$ and:

$$\alpha_i^* \bigg|_{a=t} = \frac{1/2 - \sigma(c + t)(k_i - k_z)/3}{1 - \sigma(c + t)(k_i - k_z)}.$$  

(16)

In order for an interior equilibrium to exist, it is required that products are sufficiently differentiated (or that quality differences are not too big): $\sigma(c + t)(k_i - k_z) < 3/4$. It is straightforward to show that, whenever $k_1 > k_2$, it is always $\alpha_i^* \bigg|_{a=t} > 1/2$. We are able to show that this result is more general and we obtain a result parallel to Proposition 1:

**Proposition 6.** One of the following conditions is sufficient for the firm with the higher investment to always have more than 1/2 of the market: a) access charges are sufficiently close to termination costs; b) products are sufficiently differentiated; c) demand is sufficiently rigid.

**Proof.** See the Appendix

Despite the fact that the larger network is being penalized by the quality of the bottleneck, it still gets more than 50% of the market, as in Section 3. The comparative statics are also similar to those in Section 3 (Propositions 2 and 3) with two notable differences:

**Proposition 7.** A small variation of the access charge above its marginal cost has:

- a positive impact on prices: $\frac{\partial p_i^*}{\partial a} \bigg|_{a=t} = \alpha_i^* > 0$;
- a positive impact on the market share of the larger firm: $\frac{\partial \alpha_i^*}{\partial a} \bigg|_{a=t} > 0$. 
When access is regulated at cost, prices are not affected by investments, while market shares are:

\[
\frac{\partial p^*_j}{\partial k_i} \bigg|_{a=t} = 0; \quad \frac{\partial \alpha^*_j}{\partial k_i} \bigg|_{a=t} = \frac{\frac{v}{2}(c + t)}{[1 - \sigma v(c + t)(k_1 - k_2)]^2} = -\frac{\partial \alpha^*_j}{\partial k_j} \bigg|_{a=t}.
\]

In a neighborhood of \( a = t \), a small increase in the access charge increases the gross profit of both networks: \( \frac{\partial \pi_i}{\partial a} \bigg|_{a=t} > 0. \)

**Proof.** See the Appendix

The key novelty is that an increase in the access charge above cost has an impact on shares. As before, the impact of a higher termination charge is to increase both prices, which leads to lower indirect utility, and to reduce fixed fees since part of the termination mark up is passed on to customers. Contrary to the previous case, the overall effect is negative for customers. However, the reduction in fixed fees is almost the same for both networks, while the increase in price is diluted for firm 1 - compared to the previous case - by the “quality-adjusted factor” in eq. (14). This gives an edge to firm 1 that is able to acquire more customers. This is also important in understanding why both firms would be better off with termination charges above cost, for given investment levels. For firm 1, the market expansion effect dominates the increased access deficit. For firm 2, despite the loss of market share, the traffic imbalance of off-net calls still generates a net positive effect.

**Investment decisions**

Imagine \( a = t \). It matters if a firm is smaller or larger than its rival. Firm 1’s net profit is:

\[
\Pi_1 = \alpha_i^2 (1/2 + M_2)/\sigma - I(k_i)
\]
\[ \alpha^*_i = \begin{cases} 
\frac{1}{2} - \sigma v(c + t)(k_1 - k_2) / 3 & \text{if } k_1 > k_2 \\
1 - \sigma v(c + t)(k_1 - k_2) & \text{if } k_1 \leq k_2
\end{cases} \]

Consider first the best reply from above \((k_1 > k_2)\). The expression for profit can be written as

\[ \Pi_i = \alpha^*_i \left[ \frac{1}{2} - \sigma v(c + t)(k_1 - k_2) / 3 \right] \sigma - I(k_i) \]

and its derivative with respect to \(k_1\) is:

\[ \frac{\partial \Pi_i}{\partial k_1} = -\frac{\alpha^*_i v(c + t)}{3} \left[ \alpha^*_i - \frac{2}{3} - \frac{\sigma v(c + t)(k_1 - k_2)}{1 - \sigma v(c + t)(k_1 - k_2)} \right] - I'(k_i). \]

\[ (17) \]

Firm’s profit is decreasing in its own quality (a sufficient condition is that products are sufficiently differentiated, even without taking into account investment costs). Hence the best reply from above is to match the rival’s quality. The best reply from below is obtained from maximizing

\[ \Pi_i = \alpha^*_i \left[ \frac{1}{2} - \sigma v(c + t)(k_2 - k_1) / 3 \right] \sigma - I(k_i) : \]

\[ \frac{\partial \Pi_i}{\partial k_1} = \frac{5\alpha^*_i v(c + t)}{6} \frac{1 - \frac{4}{3} \sigma v(c + t)(k_2 - k_1)}{1 - \sigma v(c + t)(k_2 - k_1)} - I'(k_i) = 0. \]

\[ (18) \]

For an interior equilibrium to exist it should be that \(\sigma v(c + t)(k_2 - k_1) < 3 / 4\); thus, the first term in the previous FOC is always positive. Therefore there are two cases. Either the solution to eq. (18) (denoted as \(\hat{k}_i(k_2)\)) lies to the left of the rival’s quality, or – if it does not – the best reply from below is to match the rival’s quality. Putting all this together, results in the following best response (an example of a typical reaction function is presented in figure 2):

\[ k^*_i = \min \{ k_2, \hat{k}_i(k_2) \}. \]
If we imagine that investments occur sequentially and we imagine firm 1 is the entrant, this result is saying that an entrant will never want to jump ahead of the incumbent. Either it will match the rival, or it will invest strictly less.

If investments occur simultaneously, there is a *continuum* of symmetric equilibria, where the highest investment occurs when eq. (18) is satisfied, 
\[ 5v(c + t)/12 - I'(k_1) = 0, \]

\[ k_1^* \in [k, I^{-1}(5v(c + t)/12)], \]

where \( k \) is some minimum quality level that operators have to supply. This result is quite worrying. Not only are firms investing less than the efficient level (however, they may invest more that in Section 4, *ceteris paribus*), but they also have a serious coordination problem: if a rival invests very little, also the other firm will do the same. The preferred equilibria for the firms are those corresponding to low quality levels.

The very last question that we address is whether the access charge could still be used as a device of “tacit” collusion. We focus only on the case of simultaneous investments since it is cumbersome to characterize the firm’s best reply for any level of rival’s investments. In a symmetric equilibrium, we can prove that under two-part pricing “tacit collusion” occurs also in the case of “bottleneck” quality:

**Proposition 8.** In a symmetric equilibrium, firms decrease their investments when access charges are set slightly above cost:

\[
\left. \frac{dk_i}{da} \right|_{a=t}^{a>\tilde{a}} = -\frac{-q(c+t)/24}{\frac{1}{\pi} \sigma v(c+t)^2 - I''(k_i)} < 0.
\]

Firms have an incentive to negotiate access charges above cost, which would be detrimental to social welfare.

**Proof.** See the Appendix

The multiplicity of equilibria that we described with \( a = t \), would still be present with \( a > t \). However, by agreeing to do so, firms reduce the upper range of equilibrium investment levels, getting rid of the equilibria that are “good” from the point of view of social welfare but “bad” from the firms’ point of view.
We have studied two specifications that model quality choice and its impact on calls. These specifications capture two features that are relevant for telecommunications. In the first one (studied at length in Sections 2-4), quality is related to coverage, ease of access, and so on, all quality features that would induce a customer to call more at a given call price, both on-net and off-net. In the second specification (studied in this Section), quality is related to clarity and fidelity of sound, reduced eavesdropping, i.e., quality features that for off-net calls depend on the lowest quality provided by the two networks.

Quality of course may include quite a vast array of other attributes as well. In particular, a problem that is of great concern relates to congestion over interconnected networks, such as the Internet. In this context, investment occurs over transmission capacity in order to control congestion of infrastructures, usually measured as delays or loss of service.\textsuperscript{12}

An Internet provider endowed with more capacity would be able to offer a better quality of service (QoS) to its customers. However, since providers are interconnected in order to ensure global connectivity, the degree of congestion is not controlled by any particular provider alone. To get from point A to point B, most Internet traffic has to pass across multiple networks. Our model can be re-interpreted to provide some insights into the question of access pricing and investment incentives for the Internet.

It is well known that when individual users download documents, the amount of traffic flow they send is quite negligible compared to the traffic flow they receive. For simplicity we therefore assume that users only download documents, i.e., they only receive traffic. We also assume that documents can be downloaded from other users. Some of these other users may be customers of the same provider or may belong to the other provider. Now we interpret our quality parameter \( k_i \) as the QoS (the inverse of congestion/delay) offered by provider \( i \). This QoS is ensured for all on-net downloads. However, the QoS for off-net downloads will depend on the capacity of both networks. In particular,
if the delay suffered for off-net transmissions is a weighted average between on-net and off-net transmissions, we can assume that the QoS for off-net downloads that a customer of provider $i$ attempts from network $j$ is \[ \Lambda_i = \delta k_i + (1 - \delta)k_j \], where $\delta \in [0, 1]$ is an exogenous parameter that reflects the network architecture, i.e., the way networks interconnect and exchange traffic. $\delta = 1$ corresponds to the situation where all downloads from provider $j$ are immediately passed to provider $i$ and is the same as our basic model introduced in Section 2. We have worked out the solution for this extension, and we have found that the collusive role of above-cost access charges is robust so long as $\delta$ is high enough. When access is regulated at cost, firms still invest too little (the problem is exacerbated since by supplying higher capacity a firm does not internalize the fact that it also improves the QoS of downloads requested by rival users). If $\delta$ is sufficiently high, firms would want to collude on above-cost charges for the same reasons described in the previous sections. On the other hand, if $\delta$ is very low the result can be reversed. To see why, imagine in the limit $\delta = 0$ and $a = t$. If $a$ is increased by a small amount and a firm unilaterally decides to invest a bit more in capacity, it will not induce its customers to download more files off-net, since a $\Lambda_i$ does not change, on the contrary the rival users have their $\Lambda_j$ increased and will make more requests off-net. Firm $i$ will be compensated for the resulting traffic imbalance, and thus will have a higher incentive to invest. This result then points to the crucial role played by the $\delta$ parameter, i.e., the way traffic is passed from one network to another.

On the Internet, a system called “hot-potato routing” characterizes many interconnection arrangements. Backbone providers do not want to incur the cost of carrying traffic meant for another provider’s customers and so they try to dump the data off as soon as possible: they forward along packets very quickly - like a hot potato. Hence “hot-potato routing”, congestion and bill-and-keep are interrelated aspects for the Internet that may help explain why Internet slowdowns have not been eliminated to date.

We want to stress the fact that our model is best interpreted and applied for telecommunications networks and it is too simple a description for the Internet. For instance, we have not considered the capacity of peering points. In addition, users and content providers hosted by service providers do not
coincide: backbones may refuse to peer with backbones hosting a high proportion of content providers. In our specification, a higher QoS implies that users would want to download more files; however it would also take less time, but we have neglected this aspect. However, we have shown how a simple description of a network architecture (in itself an endogenous choice) and economic settlements (access charges) are closely linked and deserve to be further investigated.

7. Conclusions

This article has analyzed the incentives that network operators have to invest in facilities with different quality levels. We have extended the framework of ALRT by introducing an investment stage, prior to price competition. Quality is costly but is beneficial to customers that would be calling more. We have shown that the level of termination charges has important implications in terms of investment incentives. In particular, the standard result of profit neutrality no longer holds true. When the quality of a network influences both on-net and off-net calls, we have shown that:

- For given investment levels, a small firm would benefit from a charge above cost while the opposite would occur for a large firm;
- Once investment is endogenized, the larger firm would always react to higher termination charges by reducing investment; this would also be the case for the smaller firm as long as products are sufficiently differentiated;
- Since net profits in a symmetric equilibrium are simply \( \frac{1}{4\sigma} - I(k) \), this implies that firms would negotiate an access charge above cost.

In other words, we can restore a “tacit collusion” story à la ALRT even in a symmetric equilibrium with two-part pricing. Tacit collusion does not arise as a result of a “raise each other’s cost” effect, but is instead due to an effect that we may describe as “diminish each other’s incentives to invest”. Although gross profits are fixed at the Hotelling level, an access charge will be used to induce firms to invest less, i.e., to save on costs. This is detrimental to social welfare.
We have also shown that when the quality of off-net calls depends on the interaction between the quality of the two networks (i.e., quality is a “bottleneck”), neither network has an incentive to jump ahead of its rival. “Tacit collusion” with two-part pricing in symmetric models also holds true in this case.14

To conclude, this paper sheds some light on the normative issue of how to regulate termination charges. In particular, our results suggest that private negotiations over reciprocal access charges would not be efficient, and that firms would strictly prefer to set access charges above termination costs. On the contrary, in order to induce firms to invest in an efficient manner, access charges should be set below costs. Hence our results show that the current understanding of interconnection charges, typically based on LRIC, may not be right in a dynamic perspective. Of course, setting the “right” level of access charges below cost would be a very delicate job in practice, since regulators may not be able to observe the cost structure of a network and it is difficult to give operators incentives to report cost truthfully. The Federal Communications Commission has recently sought comments on implementing a regime based on “bill-and-keep”. Such systems have been advocated as a way of efficiently sharing the value created by a call when customers derive utility from receiving calls (DeGraba, 2002). Our results reinforce their good dynamic properties.
Appendix

Proof of Proposition 1. Start from eq. (5) and consider $W(p)$. $W$ is maximized when $p = c + t$. Without loss of generality, let firm 1 be the larger firm, $k_1 > k_2$. Imagine first $a > t$. If we conjecture that firm 1 has more than 50% of the market share, then from eq. (3) both firms charge a mark up on costs and firm 1 is relatively cheaper. Hence $W(p_1^*) > W(p_2^*)$ and the bracket in eq. (5) is unambiguously positive. Similarly, if $a < t$, then both prices are charged below costs, and firm 1 is relatively more expensive, i.e. it is closer to marginal costs. Hence unit welfare for firm 1 is still greater than for firm 2 and the bracket is still positive. On the contrary, imagine $a > t$ and let us conjecture that firm 1 is the smaller firm (a similar argument would apply if $a < t$). Then firm 1 is the more expensive firm and both firms charge a mark up above costs. However, since $p_1 - p_2 = (a - t)(1 - 2\alpha_i)$, if $a$ is close to $t$, then the price differential is small, therefore $W(p_1^*) \approx W(p_2^*)$, implying that the bracket in the expression for market share will be positive, violating our initial conjecture. Similarly, if products are sufficiently differentiated, then market shares will be close to 1/2, making the price differential close to zero, and the same violation would occur. Finally, unit welfare would not change with price differences if demand is perfectly rigid. Hence, if demand is sufficiently rigid, then it cannot be the case that the bigger firm has less than 50% of the market. $Q.E.D.$

Proof of Proposition 2. Denote with $s_i$ the gross utility minus the cost of calls provided by network $i$: $s_i = k_iW(p_i^*)$. From eq. (5), in a neighborhood of $a = t$:

$$\frac{\partial \alpha_i^*}{\partial a} \bigg|_{a=t} = \frac{\sigma}{3} \left( \frac{\partial s_i}{\partial \alpha} - \frac{\partial s_j}{\partial \alpha} \right) \bigg|_{a=t} .$$

(A1)
We have \( \frac{\partial s_j}{\partial a} = k_i \frac{\partial^2 W}{\partial p_i \partial a} \), where \( \frac{\partial p_i^*}{\partial a} = \alpha_j^* - \frac{\partial \alpha_j^*}{\partial a} (a - t) \), and \( \frac{\partial W}{\partial p_i} = (p_i^* - c - t)q'(p_i^*) \).

Evaluating the last expression in \( a = t \), since \( p_i^* = c + t \), we have \( \frac{\partial W}{\partial p_i} \bigg|_{a=t} = 0 \). So, we obtain \( \frac{\partial \alpha_j^*}{\partial a} \bigg|_{a=t} = 0 \). Market share reaches an optimum in \( a = t \). We do not know yet if this optimum is a maximum or a minimum. Let us consider the second derivatives:

\[
\frac{\partial^2 s_i}{\partial a^2} = k_i \frac{\partial^2 W}{\partial p_i^2} = k_i \left( \frac{\partial^2 W}{\partial a \partial p_i^*} + \frac{\partial^2 W}{\partial p_i^* \partial a^2} \right) = k_i \left[ (q'(p_i^*) + (p_i^* - c - t)q''(p_i^*)) \left( \frac{\partial p_i^*}{\partial a} \right)^2 + \frac{\partial W}{\partial p_i^*} \frac{\partial^2 p_i^*}{\partial a^2} \right]
\]

\[
\left. \frac{\partial^2 s_i}{\partial a^2} \right|_{a=t} = k_i q'(p_i^*) \left( \frac{\partial p_i^*}{\partial a} \bigg|_{a=t} \right)^2 = \alpha_j^* k_i q'(c + t).
\]

Imagine now firm 1 is the larger firm. If \( \left. \frac{\partial^2 s_1}{\partial a^2} \right|_{a=t} < \left. \frac{\partial^2 s_2}{\partial a^2} \right|_{a=t} \), then market share reaches a maximum and thus a small departure of the access charge from the marginal cost of termination yields a decrease in the market share of firm 1. This inequality can be rearranged as \( (1 - \alpha^*_1) k_1 > (\alpha^*_2) k_2 \), which is satisfied when \( \alpha^*_1 < 1/(1 + \sqrt{k_2/k_1}) \). Notice that as \( \sigma \to 0 \) the LHS tends to 1/2, while the RHS is always above 1/2 as \( k_1 > k_2 \), hence the above inequality will always be satisfied for sufficiently low substitutability.

We now turn to the impact of stage-I investment levels on equilibrium in stage II:

\[
\frac{\partial \alpha_j^*}{\partial k_i} = \frac{\sigma}{3} \left( \frac{\epsilon s_j}{\epsilon k_i} - \frac{\epsilon s_i}{\epsilon k_i} \right)
\]

(A2)

where
\[
\frac{\partial s_i}{\partial k_i} = W(p_i^\ast) + k_i(p_i^\ast - c - t)q'(p_i^\ast) \frac{\partial p_i^\ast}{\partial k_i}
\]
\[
\frac{\partial s_j}{\partial k_i} = k_j(p_j^\ast - c - t)q'(p_j^\ast) \frac{\partial p_j^\ast}{\partial k_i}
\]

\[\text{(A3)}\]

while the effects of the own and rival quality on the equilibrium prices are:
\[
\frac{\partial p_i^\ast}{\partial k_i} = -\frac{\partial \alpha_i^\ast}{\partial k_i} (a - t), \quad \frac{\partial p_j^\ast}{\partial k_j} = -\frac{\partial \alpha_j^\ast}{\partial k_j} (a - t). \quad Q.E.D.
\]

**Proof of Proposition 3.** From eq. (6) we compute:
\[
\frac{\partial \pi_i}{\partial a} = 2\frac{\alpha_i^\ast}{\sigma} \frac{\partial \alpha_i^\ast}{\partial a} - \alpha_i^2 (k, q(p_i^\ast) - k, q(p_i^\ast)) -
\]
\[
- (a - t) \left[ 2\alpha_i^\ast \frac{\partial \alpha_i^\ast}{\partial a} (k, q(p_i^\ast) - k, q(p_i^\ast)) - (\alpha_i^\ast)^2 \left( k, q'(p_i^\ast) \frac{\partial p_i^\ast}{\partial a} - k, q'(p_i^\ast) \frac{\partial p_j^\ast}{\partial a} \right) \right]
\]

Using eq. (7) and (8) when evaluating the previous expression in \(a = t\), it results:
\[
\frac{\partial \pi_i}{\partial a} \bigg|_{a=t} = -\alpha_i^2 (k, q(p_i^\ast) - k, q(p_i^\ast)) = -\alpha_i^2 (k_i - k_j)q(c + t). \quad Q.E.D.
\]

**Proof of Proposition 4.** We start by implicitly differentiating the FOC given by eq. (12):
\[
2 \left[ \left( \frac{\partial \alpha_i^\ast}{\partial k_i} \right)^2 dk_i + \alpha_i^\ast \frac{\partial^2 \alpha_i^\ast}{\partial k_i^2} dk_i + \frac{\partial \alpha_i^\ast}{\partial k_i} \frac{\partial \alpha_i^\ast}{\partial a} da + \alpha_i^\ast \frac{\partial^2 \alpha_i^\ast}{\partial k_i \partial a} da \right] - \frac{\partial \Omega}{\partial k_i} da -
\]
\[
- (a - t) \left( \frac{\partial^2 \Omega}{\partial k_i^2} dk_i + \frac{\partial^2 \Omega}{\partial k_i \partial a} da \right) - I^*(k_i) dk_i = 0.
\]
\[\text{(A4)}\]
Since we want to calculate all the previous expressions at \( a = t \), we can use the results in Proposition 2 to simplify calculations:

\[
\frac{\partial \Omega}{\partial k_i} \bigg|_{a=t} = 2\alpha_i^*\sigma(c+t)(k_i - k_j)q(c+t)/3 + \alpha_i^{*2}q(c+t) = \alpha_i^*\left[1/2 + \sigma(c+t)(k_i - k_j)\right]q(c+t).
\]

We still have to compute two terms in (A4) (the second derivatives). From (A2) we compute

\[
\frac{\partial^2 \alpha_i^*}{\partial a \partial k_j} \bigg|_{a=t} = \frac{\sigma}{3} \left( \frac{\partial^2 s_j}{\partial a \partial k_j} - \frac{\partial^2 s_i}{\partial a \partial k_i} \right) \bigg|_{a=t}.
\]

From (A3) we also get:

\[
\frac{\partial^2 s_i}{\partial a \partial k_j} = \frac{\partial W(p_i^*)}{\partial a} + k_i q'(p_i^*) \frac{\partial p_i^*}{\partial k_i} \frac{\partial p_i^*}{\partial a} + k_j (p_i^* - c - a) \left( q''(p_i^*) \frac{\partial p_i^*}{\partial k_i} \frac{\partial p_i^*}{\partial a} + q'(p_i^*) \frac{\partial^2 p_i^*}{\partial k_i \partial a} \right) .
\]

Then, \( \frac{\partial^2 s_i}{\partial a \partial k_j} \bigg|_{a=t} = 0 \). Similarly \( \frac{\partial^2 s_j}{\partial a \partial k_i} \bigg|_{a=t} = 0 \), implying that \( \frac{\partial^2 \alpha_i^*}{\partial a \partial k_i} \bigg|_{a=t} = 0 \). It is also immediate to show that \( \frac{\partial^2 \alpha_i^*}{\partial k_i^2} \bigg|_{a=t} = 0 \). As a result at \( a = t \), eq. (A4) simplifies to:

\[
( c + t )^2 dk_i - \alpha_i^* \left[1/2 + \sigma(c+t)(k_i - k_j)\right]q(c+t)da - I^*(k_i)dk_i = 0
\]

that results in eq. (13). \( Q.E.D. \)

**Proof of Corollary 1.** The denominator of eq. (13) is negative by SOC. Imagine first \( k_1 > k_2 \) (i.e., from Proposition 1 firm 1 is the larger network). From eq. (13) it is then immediate that \( k_1 \) would decrease with \( a > t \). Now imagine \( k_1 < k_2 \) (firm 1 is the smaller network). Using eq. (11) we can rewrite eq. (13) as: \( \text{sign}\left(\frac{dk_i}{da} \bigg|_{a=t}\right) = -\text{sign}(I'(k_i) - 2\sigma(c+t)/9) \). Substituting the FOC w.r.t. \( k_i \) into the expression for
market shares gives \( \alpha_i^* = 3I'(k_i)/(2v(c + t)) \). This places a restriction in order to have an interior solution: \( 0 < I'(k_i) < v(c + t)2/3 \). When products are sufficiently differentiated (\( \sigma \to 0 \)) then from eq. (11): \( I'(k_i) \to v(c + t)/3 \), implying that the overall sign is negative. \( Q.E.D. \)

*Proof of Proposition 5.* In a symmetric equilibrium net profits are \( \Pi_i = 1/(4\sigma) - I(k_i^*) \). From eq. (13) we know that in a symmetric equilibrium a small increase in the access charge will induce firms to invest less, which has a positive impact on profits. From eq. (11) this implies that \( k_i^* < I'^{-1}(v/3) \), while the socially optimal investment for each firm is \( k_i^* = I'^{-1}(v/2) > k_i^* \). Hence, by increasing the charge above costs, firms would depart even further away from the socially optimal levels. \( Q.E.D. \)

*Proof of Proposition 6.* We can write eq. (2) as:

\[
\alpha_i = \alpha(p_1, p_2, F', F) = \frac{M_1 + \sigma(F_2 - F_i)}{M_1 + M_2}
\]

(A5)

where \( M_1 = 1/2 + \sigma k_1(v(p_1) - v(p_2)) \), \( M_2 = 1/2 + \sigma(k_2v(p_2) - k_1v(p_1)) \). After maximization, we obtain at the equilibrium the following conditions for network 1:

\[
p_i^* = c + t + \alpha_i^* (a - t)k_i/\bar{k}, \ \text{with} \ \bar{k} = \alpha_i^* k_i + \alpha_i^* k_2, \]  

(A6)

\[
F_i^* = f + \frac{\alpha_i^*}{\sigma}(M_1 + M_2) - (p_i^* - c - t)\left[\bar{k} + \alpha_i^*(k_i - k_1)\right]q(p_i^*) - (\alpha_i^* - \alpha_i^*)a - t)k_2\left(q(p_i^*) - q(p_i^*)\right), \]  

(A7)

while for network 2 it results:
\[ p^*_2 = c + t + \alpha^*_2 (a - t), \]

(A8)

\[ F^*_2 = \frac{f + \alpha^*_2 (M_1 + M_2)}{\sigma - (p^*_2 - c - t)k_2 q(p^*_2) - (\alpha^*_2 - \alpha^*_1) (a - t) k_2 (q(p^*_2) - q(p^*_1))}. \]

(A9)

Substituting (A6), (A7), (A8) and (A9) into (A5), market share in equilibrium is given by:

\[ \alpha^*_1 = \frac{1}{3} + \frac{1}{3} \frac{M_1 + \sigma \Theta}{M_1 + M_2}. \]

(A10)

where \( M_1 \) and \( M_2 \) are now evaluated in \( p^*_1 \), and \( \Theta = (p^*_1 - c - t)q(p^*_1)k^2 + \alpha^*_2 (k_1 - k_2) - (p^*_2 - c - t)k_2 q(p^*_2). \)

Evaluating the equilibrium market share in \( a = t \), since \( \Theta\big|_{a=t} = 0 \), \( M_1\big|_{a=t} = 1/2 \) and \( M_2\big|_{a=t} = 1/2 + \sigma c (k_2 - k_1) \) and rearranging, we obtain eq. (16).

To prove Proposition 6, from (A10) we have that \( \alpha^*_1 > 1/2 \) if and only if \( M_1 + 2\sigma \Theta > M_2 \), which can be rearranged to obtain:

\[ 2k_2 W(p^*_1) + (k_1 - k_2) [v(p^*_1) + (p^*_1 - c - t)q(p^*_1)4\alpha^*_1] > 2k_2 W(p^*_2), \]

(A11)

We also know that the difference in usage fees is given by the following condition:

\[ p^*_1 - p^*_2 = (a - t)(\alpha^*_2 k_2 - \alpha^*_1 k_1) / k. \]

(A12)
Let us first conjecture that firm 1 has a bigger market share than the rival (remember that \( k_1 > k_2 \)), then, if \( a > t \), the RHS of (A12) is negative, both prices are above marginal cost and inequality (A11) is satisfied. If \( a < t \) we can work along the same line; however, we also need an additional sufficient condition to ensure that the square bracket in (A11) is positive:

\[
v(p_1^*) + (p_1^* - c - t)q(p_1^*)4\alpha_1^* = v(p_1^*) + (a - t)q(p_1^*)4\alpha_1^*k_2/\bar{k} > v(p_1^*) + (a - t)q(p_1^*) > 0\]

which is ensured when \( v(p_1^*) \) is sufficiently high. If we now conjecture that firm 1 is the smaller firm, we would always get a contradiction. Imagine \( a > t \). If \( a \) is close to \( t \), the price difference is small but (A11) would then be violated. If products are differentiated, then market shares would be close to \( \frac{1}{2} \), but then the price difference would be negative and (A11) would be violated. If demand is inelastic, (A11) is immediately violated. Similar contradictions would arise if \( a < t \).

To conclude the characterization of stage II, equilibrium gross profits are given by:

\[
\pi_1 = \alpha_1^* (M_1 + M_2) / \sigma + \alpha_1^* (a - t)k_2 (q(p_2^*) - q(p_1^*)) + \alpha_1^* (p_1^* - c - t)q(p_1^*)(k_2 - k_1)
\]

(A13)

\[
\pi_2 = \alpha_2^* (M_1 + M_2) / \sigma + \alpha_2^* (a - t)k_2 (q(p_1^*) - q(p_2^*)).
\]

(A14) \( Q.E.D. \)

Proof of Proposition 7. From (A6) and (A8) we have the following comparative statics on prices:

\[
\frac{\partial p_1^*}{\partial a} = \alpha_1^* \frac{k_2}{\bar{k}} - \frac{\partial \alpha_1^*}{\partial a} (a - t) \frac{k_2}{\bar{k}}, \quad \frac{\partial p_2^*}{\partial a} = \alpha_1^* + \frac{\partial \alpha_1^*}{\partial a} (a - t), \quad \frac{\partial p_1^*}{\partial k_1} \bigg|_{a_{eq}} = \frac{\partial p_1^*}{\partial k_1} \bigg|_{a_{eq}} = 0.
\]

Let us now analyze the effect of \( a \) on the equilibrium market share given by (A10):

\[
\frac{\partial \alpha_i^*}{\partial a} = \left[ \frac{\partial M_i}{\partial a} + \sigma \frac{\partial \Theta}{\partial a} (M_1 + M_2) - \frac{\partial (M_1 + M_2)}{\partial a} (M_1 + \sigma \Theta) \right] \left[ 3(M_1 + M_2)^2 \right].
\]

(A15)
Evaluating all the above terms in $a = t$, it results

$$\frac{\partial M_1}{\partial a} \bigg|_{a=t} = \sigma k_2 q(c + t)\left(\alpha'_1 - \alpha'_2 k_2 / \bar{k}\right),$$

$$\frac{\partial (M_1 + M_2)}{\partial a} \bigg|_{a=t} = \sigma q(c + t)\alpha'_2 (k_1 - k_2) k_2 / \bar{k}, \quad \frac{\partial \Theta}{\partial a} \bigg|_{a=t} = \alpha'_2 k_2 q(c + t)\left[1 + \alpha'_2 (k_1 - k_2) / \bar{k}\right] - \alpha'_1 k_2 q(c + t),$$

where $\alpha'_1$ is the equilibrium market share evaluated in $a = t$, given by eq. (16). Given that

$$\Theta \bigg|_{a=t} = 0, M_1 \bigg|_{a=t} = 1/2, M_2 \bigg|_{a=t} = 1/2 + \sigma v(c + t)(k_2 - k_1),$$

substituting in (A15) and rearranging it results at an interior equilibrium:

$$\frac{\partial \alpha'_1}{\partial a} \bigg|_{a=t} = \frac{\sigma \alpha'_2 k_2 q(c + t) k_1 - k_2}{3 [1 + \sigma v(c + t)(k_2 - k_1)]} = \frac{\sigma \alpha'_2 k_2 q(c + t) k_1 - k_2}{3 [1 - \sigma v(c + t)(k_1 - k_2)]} > 0.$$

It is immediate to obtain:

$$\frac{\partial \alpha'_1}{\partial k_1} \bigg|_{a=t} = -\frac{\partial \alpha'_1}{\partial k_2} \bigg|_{a=t} = \xi v(c + t)/\left[1 - \sigma v(c + t)(k_1 - k_2)\right]^2.$$

We can finally analyze the effects of access charges on profits (A13) and (A14):

$$\frac{\partial \pi_1}{\partial a} = 2\alpha'_1 \frac{\partial \alpha'_1}{\partial a} \left(\frac{M_1 + M_2}{\sigma}\right) + \frac{\alpha'_2}{\sigma} \left(\frac{\partial M_1}{\partial a} + \frac{\partial M_2}{\partial a}\right) + (a - t) \frac{\partial \left(\alpha'_2 k_2 (q(p_2) - q(p_1))\right)}{\partial a} +$$

$$+ \alpha'_2 k_2 (q(p_2) - q(p_1)) + \alpha'_2 q(p_1)(k_2 - k_1) \frac{\partial p_1}{\partial a} + (a - t) \frac{\partial \left(\alpha'_1 (1 - \alpha'_2) \frac{k_2}{k} q(p_1)(k_2 - k_1)\right)}{\partial a},$$

$$\frac{\partial \pi_1}{\partial a} \bigg|_{a=t} = \frac{2}{3} \alpha'_1 \alpha'_2 \frac{k_2}{k} q(c + t)(k_1 - k_2) > 0 \text{ for } k_1 > k_2.$$
**Proof of Proposition 8.** We want to analyze what happens to investments when $a$ is increased slightly above $t$. If we can prove the impact is negative, then Proposition 8 is proven since in a symmetric equilibrium gross profits are fixed and firms would benefit from lower investment costs. Moreover, firms would depart even further away from the socially optimal level.

Recall from Section 6 that in a symmetric equilibrium with $a = t$, there is a continuum of equilibria. When $a$ is increased by an infinitesimal amount, such continuum of equilibria would still exists, where the lowest investment level is determined by the exogenously set $k$ while the highest level is determined by the FOC of the firm with the lower quality (best reply from below, see eq. (18)). Hence our analysis impinges upon determining the impact that a small increase in $a$ has on the interior solution of the best reply of the operator with the lower quality level, that we assume w.l.o.g. to be firm 2: $k_2 < k_1$. Then the gross profit of firm 2 is given by (A14) and the effect we want to study is

$$
\frac{dk_2}{da} \bigg|_{a=t, k_2=k_1} = -\frac{\partial^2 \Pi_2}{\partial k_2 \partial a} \bigg|_{a=t, k_2=k_1} \frac{\partial^2 \Pi_2}{\partial k_2^2} \bigg|_{a=t, k_2=k_1}.
$$

Differentiating (A14) with respect to $k_2$, we have:

$$
\frac{\partial \Pi_2}{\partial k_2} = \frac{1}{\sigma} \left[ \frac{\partial (M_1 + M_2)}{\partial k_2} \right] \alpha_2^* - 2\alpha_2^* \frac{\partial \alpha_2^*}{\partial k_2} (M_1 + M_2) + (a-t) \frac{\partial \left( \alpha_2^* k_2 (q(p^*) - q(p_1^*)) \right)}{\partial k_2} - I'(k_2).
$$

After simple but tedious calculations \(^{15}\) we obtain

$$
\frac{\partial^2 \alpha_2^*}{\partial k_2^2} \bigg|_{a=t, k_2=k_1} = \frac{\sigma^2 \nu^2}{3} > 0,
$$

and

$$
\frac{\partial^2 \Pi_2}{\partial k_2^2} \bigg|_{a=t, k_2=k_1} = \frac{\sigma \nu (c+t)^2}{18} - I''(k_2),
$$

which is assumed to be negative for SOC to hold. We also compute

$$
-\frac{1}{\sigma} \frac{\partial^2 \alpha_2^*}{\partial k_2 \partial a} \bigg|_{a=t, k_2=k_1} - \frac{q(c+t)}{8},
\frac{\partial^2 \alpha_2^*}{\partial k_2 \partial a} \bigg|_{a=t, k_2=k_1} = -\frac{\sigma q(c+t)}{12},
$$

implying

$$
\frac{\partial^2 \Pi_2}{\partial k_2 \partial a} \bigg|_{a=t, k_2=k_1} = -\frac{q(c+t)}{24}.
$$

As a result,

$$
\frac{dk_2}{da} \bigg|_{a=t, k_2=k_1} = -\frac{-q(c+t)/24}{\frac{1}{18} \sigma \nu (c+t)^2 - I'(k_2)} < 0.
$$

\textit{Q.E.D.}
References


Armstrong, M. “Network Effects: Competition in Prices or Utilities?,” Mimeo, Nuffield College, Oxford University, 2002b.


Footnotes

1 Dessein (2003) introduces customers’ heterogeneity in volume demand (light and heavy users). He shows that, under some conditions, profit neutrality still holds. Only when subscription demand is elastic, then firms would have strict preferences for a particular termination charge. When participation is not an issue (i.e., everybody subscribes), then symmetry matters for profit neutrality. Carter and Wright (2003) allow for some exogenous brand loyalty. Providing for this particular type of asymmetry implies that the larger network prefers the access charge to be set at the marginal cost of termination. See also Peitz (forthcoming). All these papers are static while our work allows for asymmetries to arise from endogenous choices.

2 Gans (2001) analyzes a model where the access pricing regime may give different incentives to invest; however there is no two-way interconnection feature in his model. DeGraba (2003) also discusses how termination charges can affect the investment decisions of carriers, but his main interest is on efficient interconnection arrangements when customers derive some utility from receiving calls.

3 This particular specification is used by Dessein (2003), but in his paper the author uses the exogenous parameter $k$ in order to differentiate customers according to their calling patterns. Here the endogenous parameter $k$ stands for a proxy of the endogenous investment made by the networks in setting (or upgrading) the quality of their infrastructures.

4 This substitution is only done for technical convenience and does not affect the results. Competition in prices rather than in net surpluses may give different results only in the presence of externalities. (Armstrong, 2002b).

5 The proof of existence and uniqueness would follow the lines of a similar proof of Laffont, Rey, and Tirole (1998), as extended to asymmetric settings by de Bijl and Peitz (2002).

6 To see why, recall from section 4.1 that, when $a = t$, $\sigma$ does not influence the privately chosen investment at all; while in general this is not true for $a$ different from $t$, its effect is small. This is quite useful since we can always choose a $\sigma$ low enough to ensure existence of an equilibrium.

7 This assumption is used by Economides (1999) to describe a long distance phone call that requires the use of long distance lines as well as local lines at both ends. In his example, the fidelity of sound of a call is the minimum of the qualities of the three services used. An interesting potential extension of our model could be to compare a situation of facility based competition (FBC) - where operators have their own facilities - with local loop unbundling (LLU) - where one firm (the incumbent, say firm 1) has full coverage, while the other firm (the entrant) leases the local loop from the incumbent. FBC corresponds to the model analyzed here (each firm can choose its own $k_i$ that is applicable to on-net calls, while off-net there would be an interaction between the two investments). On the other hand, under LLU, it is only the incumbent that invests in access facilities, hence there would be only $k_1$ in all the expressions.

8 Contrary to the analysis in Section 3, it now matters if firms compete in two-part prices or in net surplus and call prices. This is because there are externalities for customers of firm 1 (market shares enter their net surplus). We consider competition in two-part prices since this is economically more plausible (see Armstrong, 2002b). If
instead one considers competition in net surplus and call prices, the expressions for call prices derived below
would not change for a given market share, since it does not matter whether the competitor’s net utility or fixed
fee is taken as given. However, if one then substitutes the per minute prices by best responses, then Armstrong’s
argument in this reduced strategy space applies to the reduced profit functions where either net utility of
subscription fee is the only strategic variable. Equilibrium profits would be different (in particular they would be
higher since competition is tougher in the price setting game when network effects are positive).

9 As before, we refer to Laffont, Rey, and Tirole (1998) for the proof of existence and uniqueness of the
equilibrium.

10 The perceived marginal cost for on-net calls is \( c + t \), while it is \( c + a \) for off-net calls, and they would be the
optimal marginal prices if the firm could discriminate between the two types of calls. Without discrimination,
the FOC with respect to call price for firm 1 has to ‘weight’ these two prices by their market shares and quality
parameters, leading to:

\[
\alpha_1 k_1 (p_1 - c - t) + \alpha_2 k_2 (p_1 - c - a) = (\alpha_1 k_1 + \alpha_2 k_2) (p_1 - c - t) - \alpha_2 k_2 (a - t) = 0,
\]

which gives eq. (14).

11 In setting these values, we also made sure that the equilibrium exists and is unique. In particular, we checked
that it never pays to try to monopolize the market.

12 Quality of service can also be improved in ways other than infrastructure investment, e.g., implementing
caching technologies that aggregate traffic and reduce congestion.

13 This result is also confirmed for any value of \( \rho \) if one takes a symmetric CES expression for off-net calls,
\( (k^o_1 / 2 + k^o_2 / 2)^{1/\rho} \), where \( -\infty < \rho \leq 1 \) is a substitutability parameter describing the ‘quality’ of the
interconnection. Perfect connectivity \( \rho = 1 \) corresponds to the ‘weighted-average’ specification when \( \delta = 1/2 \); for
lower values of \( \rho \) the result still holds but its magnitude is diluted. All results are available from the authors.

14 We checked in a companion paper (Cambini and Valletti, 2003) the robustness of our main results when firms
offer two-part discriminatory prices (on-net and off-net call prices can differ). Previous literature has found that
– without taking into account asymmetries or investment choices – firms that offer two-part discriminatory
pricing would select access charges below cost (Gans and King, 2001). The investment effect that we have
discussed would still prevail as long as the investment function is not too convex.

15 Available from the corresponding author’s website.
FIGURE 1

OPTIMAL REGULATION OF RECIPROCAL TERMINATION CHARGES

Note: Left panel: $\sigma = 0.01$, $c = 0.1$, $t = 0.2$, $f = 0$, $v_0 = 15$; right panel: $\sigma = 0.01$
FIGURE 2

FIRM 1’S REACTION FUNCTION

Note: \( l(k) = k^2/2, \sigma = 0.01, \nu(\cdot) = 5, k_1 = 1 \)