Power-added efficiency errors with RF power amplifiers

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This paper investigates the errors that can result when applying two different definitions of power-added efficiency to RF power amplifiers. The first definition is generic to all multiple-port networks and is simply the ratio of the total output power to total input power. The second definition is specific to RF power amplifiers and widely regarded as the industry standard. It has been found that, for power amplifier applications, the error can be highly significant.

1. Introduction

To a design engineer, a meaningful definition of power conversion efficiency is important. This is because in certain applications, such as mobile and satellite communications, DC power is at a premium. In modern communications and radar applications, the RF power amplifier may be the subsystem that consumes the most amount of DC power. For this reason, the efficiency of the RF power amplifier can, in certain applications, be critical to the design and performance of the overall system.

All amplifiers are essentially three-port networks: consisting of a main input DC port, an input signal port and an output signal port. The measured input DC power, $P_{\text{DC}}$, includes the power associated with all the bias lines of the amplifier. It is assumed here that no DC power enters or leaves the two signal ports. Also, the amplifier is assumed to be perfectly matched and have infinite reverse isolation. Therefore, the power measured at the input RF port, $P_{\text{IN}}$, corresponds to the power of the input signal at the fundamental frequency only. The power measured at the output RF port, $P_{\text{OUT}}$, corresponds to the power at the fundamental frequency and at all the spurious frequencies generated by the amplifier itself. Since all amplifiers have a finite power gain, the input signal power will contribute directly to the output signal power.

2. Conservation of energy

The proportion of the total input power (both at DC and RF) that is not converted into the output RF power is assumed to be dissipated as heat. This dissipated power, $P_{\text{DISS}}$, is thus defined as:

$$P_{\text{DISS}} = (P_{\text{DC}} + P_{\text{IN}}) - P_{\text{OUT}}$$

(1)

Since all of the power in the network is accounted for, the principle of conservation of energy is always obeyed.
3. Basic efficiency

At low signal frequencies, power amplifiers can have relatively high levels of power gain, \( G \), where \( G = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \). As a result, the direct contribution of the input signal power to the output signal power is insignificant and, therefore it is not included in the overall efficiency calculations of the power amplifier. The resulting basic efficiency, \( \eta \), is expressed as:

\[
\eta = \frac{P_{\text{OUT}}}{P_{\text{DC}}} \times 100\%
\]

If the gain of the power amplifier is relatively low, which can be found at RF, then the direct contribution of the input RF power to the output RF power becomes significant. As a result, the output RF power level can exceed the input DC power level when a significant proportion of the output RF power is supplied by the input RF power. This will cause the basic efficiency to have a numerical value greater than 100\%. Therefore, when input RF power is taken into account, basic efficiency loses its practical meaning and instead becomes just a convenient mathematical variable. It can be shown that basic efficiency exceeds 100\% whenever the power gain exceeds \( P_{\text{DC}}/P_{\text{IN}} \).

Since all networks must obey the principle of conservation of energy, equations (1) and (2) can be combined to give the upper theoretical limit on basic efficiency for a lossless system (i.e. with \( P_{\text{DISS}} \) set to zero):

\[
\eta_{\text{max}} = \frac{1}{\left(1 - \frac{1}{G}\right)} \times 100\%
\]

For example, a power amplifier with 3 dB gain will, theoretically, have a maximum value of basic efficiency of \( \eta_{\text{max}} = 200\% \).

4. Power-added efficiency

When input RF power is included in the efficiency calculations the modified expression is referred to as the power-added efficiency (PAE), \( \eta_{\text{ADD}} \). It will be shown that power-added efficiency can be defined in two different ways (Sweet 1990, Walker 1993), which under certain conditions can produce very different numerical results.

4.1. \( PAE - \) Definition 1

The first definition is simply the ratio of the total output power to total input power and can be applied to any multiple-port network, including RF power amplifiers (Sweet 1990):

\[
\eta_{\text{ADD}}^{\#1} = \frac{P_{\text{OUT}}}{P_{\text{DC}} + P_{\text{IN}}} \times 100\%
\]

Here, power-added efficiency can be interpreted as the efficiency of the network to convert the total input power (both at DC and RF) into output RF power. The upper theoretical limit on power-added efficiency can be obtained by combining equations (1) and (4) for a lossless system. Using this definition, power-added
efficiency can never exceed 100%. Equation (4) can be conveniently rearranged in terms of basic efficiency and RF power gain:

\[
\eta_{\text{ADD}}^{\#1} = \eta \times \frac{1}{\left(1 + \frac{\eta}{G}\right)} \times 100\%
\]  

(5)

Using equation (5), a three-dimensional surface plot of power-added efficiency is shown in Fig. 1, for typical values of RF power gain and all values of basic efficiency up to 200%. It can be seen that power-added efficiency can be zero at unity power gain, but only when the numerical value of basic efficiency is also zero.

4.2. PAE—Definition 2

This second definition is now widely regarded as the industry standard for RF power amplifiers (Walker 1993):

\[
\eta_{\text{ADD}}^{\#2} = \frac{P_{\text{OUT}} - P_{\text{IN}}}{P_{\text{DC}}} \times 100\%
\]  

(6)

Here, power-added efficiency can be interpreted as the efficiency of the network to convert the input DC power into the amount of the output RF power that is left over after the direct contribution from the input RF power has been removed. Again, the upper theoretical limit on power-added efficiency can be obtained by combining equations (1) and (6) for a lossless system. Once more, using this definition, power-added efficiency can never exceed 100%. Equation (6) can now be rearranged in terms of basic efficiency and RF power gain:

\[
\eta_{\text{ADD}}^{\#2} = \eta \times \left(1 - \frac{1}{G}\right) \times 100\%
\]  

(7)

The corresponding surface plot of power-added efficiency is given in Fig. 2. It can be seen that power-added efficiency is zero at unity power gain for all numerical values of basic efficiency.

5. Error characteristics

Both definitions of power-added efficiency take into account input RF power, but in different ways. It is assumed for the moment that the second definition is the most meaningful to a power amplifier designer. The error function, \(\Delta \eta_{\text{ADD}}\), can be taken as the difference in the results calculated using both definitions:

\[
\Delta \eta_{\text{ADD}} = \eta_{\text{ADD}}^{\#1} - \eta_{\text{ADD}}^{\#2}
\]

\[
\Delta \eta_{\text{ADD}} = \eta \times \left(\frac{G}{\eta + G} + \frac{1}{G} - 1\right) \times 100\%
\]  

(8)

The corresponding surface plot of this error function is shown in Fig. 3. It can be seen that the two definitions roughly agree with each other when power gain is greater than about 3. As the gain is reduced below approximately 3 the maximum error can increase rapidly, until the absolute maximum value of error is obtained at unity power gain. The unity gain error function, \(\Delta \eta_{\text{ADD}}(1)\), is given as follows:
Figure 2. Power-added efficiency characteristics using Definition 2.
Figure 3. Error characteristics in power-added efficiency calculations.
\[
\Delta \eta_{\text{ADD}}(1) = \frac{1}{1 + \frac{1}{\eta}} \times 100\% \tag{9}
\]

From equation (9) it can be seen that the absolute maximum theoretical error is 100%. This is achieved, in principle, when basic efficiency approaches infinity, i.e. if the input DC power consumption is zero. Consider two examples of three-port network. The first is a hypothetical network consisting of an ideal bias-Tee junction and a series DC-blocking capacitor inserted between the junction and the output RF port. With this network, the RF power gain will be unity and the basic efficiency will approach infinity. The calculated values of power-added efficiency are 100% and 0% using Definition 1 and Definition 2, respectively. The second example is an L-band, Class-A, power amplifier, having the following specifications (Robertson et al. 1993): \(P_{\text{DC}} = 50\, \text{W}; \, P_{\text{IN}} = 8\, \text{W}; \) and \(P_{\text{OUT}} = 18\, \text{W}.\) Therefore, the large-signal power gain \(G = 2.25\) and basic efficiency \(\eta = 36\%.\) Using equations (5) and (7), the amplifier can be said to have a power-added efficiency of either \(\eta_{\text{ADD}} = 31\%\) or 20\%, respectively. These results give a significant error of \(\Delta \eta_{\text{ADD}} = 11\%.\)

6. Discussion

For the specific case of a power amplifier, a meaningful definition of efficiency has to satisfy two basic requirements. The first is that it can not give a value in excess of 100%. Secondly, the true representation of efficiency must always give a value of zero when the power gain is unity, for all numerical values of basic efficiency. Therefore, all of the output RF power can then be said to have come from the input RF power.

It has been show that basic efficiency has no practical meaning when input RF power contributes to the output RF power. The first definition of power-added efficiency does not produce a value that exceeds 100% and can give a value of zero when the power gain is unity. However, this second requirement is only satisfied when the numerical value of basic efficiency is also zero. In other words, at unity gain, a zero power-added efficiency requires either zero output RF power or infinite input DC power, neither of which are practical. Therefore, the first definition effectively breaks down as the power amplifier approaches unity gain. Only the second definition satisfies both requirements.

7. Subsystem loss considerations

So far only the ideal, lossless, scenario has been considered. In practice, the complete RF power amplifier subsystem may have its gain stage embedded between two auxiliary passive networks. For example, with a cascaded topology, the input network could be a variable amplitude/phase stage and/or frequency equalization stage. The output network could be a power level sampling state and/or isolator. Alternatively, with a tandem topology, two identical gain stages could be employed to implement a balanced amplifier or push-pull amplifier. Here, the two auxiliary networks may be 90° hybrid couplers or 180° hybrid couplers, respectively, for a microwave design.

The overall power insertion loss, \(\alpha,\) associated with the input and output networks (which includes transmission lines and connectors) are \(\alpha_i\) and \(\alpha_o,\) respectively.
Here, $\alpha$ has a maximum theoretical value of unity, which represents the lossless scenario. In general, the 3 dB directional couplers (whether 90° or 180° hybrids) are designed for equal power at their Coupled and Direct ports. Therefore, the corresponding level of insertion loss will be $\alpha [\text{dB}] = C[\text{dB}] + 3[\text{dB}]$, where $C[\text{dB}]$ is the measured level of equal power coupling.

The previous equations (1) to (9) now only apply to the gain stage. It can be shown that for the complete subsystem the corresponding parameters (represented by the suffix *) are given by the following equations:

$$P_{\text{Diss}} = \xi \cdot \left[ \left( P_{\text{DC}} + \frac{P_{\text{IN}}}{\alpha} \right) - \alpha \cdot P_{\text{OUT}} \right]$$

where

$$\xi = \begin{cases} 1 & \text{for a cascaded topology} \\ 2 & \text{for a tandem topology} \end{cases}$$

$$G^* = \alpha G_{\alpha}$$

$$\eta = \eta_{\alpha} \times 100\%$$

$$\eta_{\text{max}} = \frac{1}{\left( \alpha - 1 \right)} \times 100\%$$

$$\eta_{\text{ADD}}^{1*} = \eta - \frac{\alpha_{\text{DC}}}{\eta + \alpha G} \times 100\%$$

$$\eta_{\text{ADD}}^{2*} = \eta - \frac{1}{\alpha G} \times 100\%$$

$$\Delta \eta_{\text{ADD}}^{1*} = \eta \left( \frac{\alpha G_{\alpha} - 1}{\eta + \alpha G} - \alpha \right) \times 100\%$$

$$\Delta \eta_{\text{ADD}}^{2*} = \eta \left( \frac{1}{\alpha G_{\alpha}} \right) \times 100\%$$

As would be expected, it can be seen from the above expressions that as the auxiliary network losses increase the values of $G^*$, $\eta^*$ and $\eta_{\text{ADD}}^*$ decrease. As a result, the power gain of the gain stage must be at least $\alpha_{\text{DC}}\eta_{\text{DC}}$, otherwise the complete subsystem's power gain and true power-added efficiency will be less than 0 dB and 0%, respectively. Also, as the network losses increase, the values of $P_{\text{Diss}}$, $\eta_{\text{max}}$ and $\Delta \eta_{\text{ADD}}^*$ increase.

If the L-band power amplifier discussed in §.5 serves as the gain block (with $G = 2.25$ and $\eta = 36\%$ for a complete subsystem, the surface plot of the resulting error function is shown in Fig. 4. Here, the input and output auxiliary network losses vary independently, but are restricted in range so that the power gain and true power-added efficiency of the complete subsystem will always be greater than 0 dB and 0%, respectively. It can be seen in Fig. 4 that the errors in power-added effi-
Figure 4. Error characteristics for a complete subsystem with $G = 2.25$ and $\eta = 36\%$. 
ciency calculations increase rapidly as the auxiliary input network loss increases, but increase relatively slowly as the auxiliary output network loss increases. The maximum theoretical error is found when the complete subsystem is at unity gain. The new unity gain error function is given by (17). From (17), the absolute maximum theoretical error is obtained when the output network is lossless and the input network is at its most lossy value of $\alpha_i = 1/G$. Therefore, for the L-band power amplifier subsystem, the absolute maximum theoretical error in the calculated values of power-added efficiency is 26.5%, which would occur if there were no auxiliary output network and the auxiliary input network had an insertion loss of $-3.5$ dB.

8. Conclusions

Since both definitions exist in the open literature, it may be tempting to use the first one, as it can give under certain conditions greatly inflated values of power-added efficiency. For power amplifier applications, however, only the second definition is valid under all conditions. Therefore, for this specific application, the use of the first definition could give highly significant errors.

Finally, it has been found that the errors in power-added efficiency calculations, for a complete subsystem, increase rapidly as the auxiliary input network loss increases, but increase relatively slowly as the auxiliary output network loss increases.

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References

