Equilibrium of Hinged and Hingeless Structures Rotated Using Surface Tension Forces

R. R. A. Sym

Abstract—Two different geometries for the rotation of microstructures by the surface tension force of molten solder are investigated theoretically and experimentally. The geometries are based on structures with and without a hinge acting to constrain the motion of a flap to pure rotation. The equilibrium geometry of a hinged structure is first found analytically by considering the surface energy of the solder. An analysis of hingeless structures is then performed that shows a hinge to be unnecessary under certain conditions. Macroscopic experiments performed using printed circuit board parts are then described; the results show that the behaviors of hinged and hingeless structures are similar. [126]

I. INTRODUCTION

As is well known, the force due to the weight of an object scales with volume. In contrast, the force due to the surface tension of a liquid scales with length, so that surface tension forces appear relatively more significant as the size of a structure is reduced [1]. One obvious demonstration of this effect is the structural collapse that can occur if conventional drying is used after wet chemical etching of small suspended beams [2], [3]. A similar advantageous scaling law is obtained with electrostatic forces, so these have been found to be an appropriate basis for both rotary [4], [5] and linear microactuators [6], [7].

Recently, we showed theoretically that surface tension can be exploited in a more advantageous way. In particular, we suggested that it can be used to rotate hinged joints (which have previously been demonstrated in a variety of different forms [8]–[10]) through a programmable angle, allowing the self-assembly of three-dimensional (3-D) microstructures [11], [12]. The first demonstrations of this technique are described in a companion paper [13]. However, the structures involved are rather different to those originally envisaged, in that they are based on hingeless joints. The purpose of this paper is to provide the theoretical background necessary to understand this modification.

We shall illustrate the method described in [11], [12] using the simple two-dimensional (2-D) geometry of Fig. 1. In Fig. 1(a), a flap (i.e., a generic part) of breadth b and depth d is attached to a substrate by a flexible hinge of thickness t. The gap l between the parts is assumed to be small compared to b. Above the hinge is a rectangular solder pad of width 2w and height h, which can wet the hinge material but not the surround. When the solder is melted, it will deform to reduce its surface energy. However, if all attitudes of the flap are considered, it can be shown that the surface energy is not at its minimum value when the flap is horizontal [Fig. 1(b)]; instead,
it is gradually reduced by a rotation of the flap [Fig. 1(c)], reaching a minimum only after the flap has rotated through an angle set by the initial volume of solder. If this volume is chosen correctly, the minimum is obtained after 90° rotation [Fig. 1(d)].

The angle-dependence of the surface energy implies the existence of a torque on the flap. When the size of the structure is sufficiently reduced, it was assumed that this torque can become large enough to overcome the weight of the flap and also to bend the flexible hinge. Using physical arguments, the torque was found as a function of the angles θ and ϕ [Fig. 1(c)]. For a given solder volume, the relation between these parameters was determined numerically, allowing the torque to be found as a function of θ alone. Finally, the stable angle was found as the angle at which the torque vanished.

Although the predictions of this model were interesting, the reliance on numerical analysis obscured many features of the problem. Furthermore, the neglect of the effects of gravity and of the hinge itself cast doubt on whether the method would work in practice. Despite this, we managed to demonstrate and film the rotation of millimeter-scale flaps [12]. These results confirmed the general validity of the theoretical assumptions. However, they differed in one crucially important respect: they were achieved using structures entirely lacking an integral hinge. Because omission of the hinge greatly simplifies the problem of devising a microstructure fabrication scheme, this advance allowed us to develop the processes for microstructure rotation referred to earlier [13].

In this paper, we address many of the points above. In Section II, we briefly describe the original model. In Section III, we review the approximations involved (particularly, neglect of the effects of gravity and of an integral hinge). In Section IV, we analyze the equilibrium of a hinged structure from a different viewpoint, based on considerations of minimum surface energy rather than torque balance. This approach allows the derivation of key analytic results for the geometry of the equilibrium state, the solder volume required for a given rotation, and fabrication tolerances. In Section V, we perform an analysis of hingeless structures that shows a hinge to be unnecessary. Contrary to expectation, the structure does not collapse; instead, surface tension acts to hold the parts together until equilibrium is reached. At this point, all forces vanish, and the structure becomes quasi-stable. However, we show that full stability can be achieved using a suitable restraint. Finally, in Section VI, we describe experiments with macroscopic, hingeless structures.

II. REVIEW OF THE ORIGINAL MODEL

We begin with a brief review of the analysis of a hinged structure originally presented in [11], based on the geometry of Fig. 1(c). However, we now use the alternative angular parameter ψ = π − θ to characterize the attitude of the flap; this change of variable simplifies the mathematics. The analysis was based on four assumptions: 1) that all forces except those due to surface tension can be ignored, 2) that, as a result, the free liquid surface will adopt a shape of minimum surface energy, 3) that there will be no pressure gradients inside the solder, and 4) that the solder is incompressible.

With these assumptions, an expression for the torque was obtained as follows. Assumption 3 suggests that the free boundary of the liquid solder will be cylindrical in form, with constant radius of curvature r. The curvature will give rise to an excess pressure $P_{ex}$ inside the molten solder, given by [14]

$$P_{ex} = \gamma / r$$

where γ is the surface tension coefficient of the solder.

The net torque on the flap was then found as the difference between a torque $T_{s} = \gamma w \cos(\alpha)$ per unit length (due to the surface tension force acting on the flap), and a torque $T_{p} = P_{ex} w^2 / 2$ in the opposite sense (due to the excess pressure). Since $\alpha = (\psi - \phi) / 2$ and $\tau = w \sin(\psi / 2) / \sin(\phi / 2)$, the net torque was then obtained as

$$T = \gamma w \cos[(\psi - \phi) / 2] - \sin(\phi / 2) / [2 \sin(\psi / 2)]$$

Note that (2) implies that the torque at $\psi = \pi$ (i.e., at the start of rotation) is given by

$$T_{s} = (\gamma w / 2) \sin(\phi / 2)$$

(3)

where $\phi_{s}$ is the value of $\phi$ at $\psi = \pi$. Consequently, the starting torque has a maximum value of $T_{s(\max)} = \gamma w / 2$, obtained with the volume of solder for which $\phi_{s} = \pi$ [11].

For lead-tin solder, the surface tension coefficient depends on alloy composition and temperature. For a mix of 95% lead and 5% tin, it is $\approx 0.445$ N/m over the range 300–340°C, i.e., just above its melting point [15]. For the representative land width of $w = 10$ μm, we then obtain $T_{s(\max)} = 2.225$ μNm per unit length. A hinge of length (say) 500 μm would then exert a maximum torque of 1.11 nNm.

The fourth assumption suggests that there should be no difference between the solder volumes before and after melting. Now, the initial volume of solder is $2 \omega w$ per unit length. After rotation through an arbitrary angle $\psi$, the volume per unit length may be found by geometry. Equating the results, the following relation between $\psi$ and $\phi$ was obtained [11]

$$\frac{\delta}{\sin^2(\psi / 2)} \{\phi - \sin(\phi)\} - \sin^2(\phi / 2) \{4 \eta - \sin(\psi)\} = 0$$

(4)

where $\eta = h / w$ is an important new parameter, the normalized height of the solder pad.

Equation (4) implies that there is a fixed relation between $\phi$ and $\psi$, so that the torque is really a function of $\psi$ only. In [11], numerical solution of (4) was used to calculate $T(\psi)$. Most importantly, it was found that the torque vanished at a particular equilibrium angle $\psi_{e}$, which depended on the solder volume as characterized by the parameter $\eta$. In the absence of other significant effects, it was therefore assumed that the flap would rotate to this angle, at which point the solder could be frozen into position. It was found that a final angle of $\psi_{e} = 90^\circ$ could be obtained for a normalized solder pad height of $\eta = (2 + \epsilon)^{1/8}$, close to the value for which the starting torque is maximum ($\eta = \pi / 4$).
III. Effect of Neglected Factors

Whether or not the flap will indeed rotate as predicted depends on the relative significance of effects omitted from the model. A number of static and dynamic effects have previously been considered in [12]; for completeness, we describe some of the major approximations here.

One important static effect is the gravitational torque $T_g$ per unit length due to the weight of the flap. For thin flaps (i.e., for $b \gg d$), $T_g$ is approximately

$$T_g = -\left(\rho_{fap} b^2 g / 2\right) \cos(\psi)$$

(5)

where $\rho_{fap}$ is the density of the flap material. $T_g$ has a maximum value $T_{g,\text{max}}$ when $\psi = \pi$; at this angle, it must be overcome by the starting torque $T_\pi$ if the flap is to rotate.

Assuming that the analysis will be largely unaffected provided $T_{\pi,\text{max}} \gg T_{g,\text{max}}$, we obtain

$$b \ll \sqrt{\left(\gamma w / \rho_{fap} g d\right)}.$$  

(6)

Assuming also that $w$ and $d$ are comparable (a reasonable assumption for structures made by surface micromachining), we obtain $b \ll 4.4$ mm for Si (for which $\rho_{fap} = 2.33 \times 10^3 \text{ kg/m}^3$). Now, a microstructural flap might have a maximum breadth of around 500 µm, roughly 10 times smaller than the figure just found. Even in this case, the gravitational torque will be around 100 times smaller than the surface tension torque, so that the neglect of gravitational effects should still be valid.

However, it should be noted that the net surface tension torque tends to zero at the equilibrium angle. For $\psi_e = 90^\circ$, $T_g$ also tends to zero at this point. For other final angles (or, if the whole structure is not horizontal to begin with), $T_g$ will no longer be zero at the equilibrium angle. Consequently, the assumption of negligible gravitational torque may break down at some point. However, the overall effect will be an error in the final angle, rather than significantly different behavior. We will estimate the error for an extreme case suggested by a reviewer of this paper. In this case, the whole structure is held vertical at the start of a 90° rotation, with the flap hanging down, so that the surface tension torque must counter an increasing gravitational torque as it attempts to rotate the flap into a horizontal attitude.

Because the rotated joint has a torsional stiffness $\partial T / \partial \psi$, it can counter a small applied torque by opening slightly. The effect of the gravitational torque (which now reaches a maximum value $T_{g,\text{max}}$ when the flap is horizontal) will therefore be to pull the hinge open by a small angle $\Delta \psi$, such that

$$\Delta \psi = T_{g,\text{max}} / (\partial T / \partial \psi).$$

(7)

The term $\partial T / \partial \psi$ can of course be found by differentiating (2). However, since this equation contains $\phi$, the result will contain $\partial \phi / \partial \psi$, which must be obtained by differentiating (4). Near the equilibrium angle of $\psi_e = 90^\circ$, the following result is obtained:

$$\partial T / \partial \psi = \gamma w (4 + \pi) / 4 \sqrt{2}.$$  

(8)

This result differs from the maximum starting torque $T_{\pi,\text{max}}$ by a factor $(4 + \pi) / 2 \sqrt{2} \approx 2.5$. Now, for the flap of breadth 500 µm discussed earlier, we showed that $T_{g,\text{max}} / T_{\pi,\text{max}} \approx 1/100$. Using this figure, we obtain $\Delta \psi \approx 1/250$, or around 0.25°. Even in this extreme case, the likely effect of the gravitational torque is therefore small.

Gravitational forces can also have an effect on the solder itself. For example, the geometrical analysis presented so far assumes that the free surface of the solder is cylindrical. This will no longer be the case if the solder deforms due to hydrostatic pressure. Now, the maximum likely hydrostatic pressure is $\approx \rho_{sol} g w$, where $\rho_{sol}$ is the density of the solder. Hydrostatic deformation will be negligible if $P_{xx}$ is much greater than this value. Assuming that $P_{xx} \approx \gamma / w$, we then require

$$w \ll \sqrt{\left(\gamma / \rho_{sol} g\right)}.$$  

(9)

Solder density depends approximately linearly on alloy composition [16], but for a mix of 95% lead and 5% tin it is $\approx 10.80 \times 10^3 \text{ kg/m}^3$. In this case, we obtain $w \ll 2$ mm. Given that the land width must be considerably less than the dimensions of typical microassemblies if the mechanism is to be useful, this figure is sufficiently large that hydrostatic deformation should be negligible in practical geometries.

A further important static effect is the plastic deformation of the hinge material required to rotate the flap. Assuming simple plastic behavior (i.e., that the stress in the hinge material is roughly constant in the plastic regime), the torque required to deform the hinge is $T_\pi = \sigma_y Z_\pi$, where $\sigma_y$ is the yield stress and $Z_\pi$ is the plastic section modulus of the hinge. For a rectangular beam of thickness $t$, $Z_\pi = t^2 / 4$ per unit length [17]. Assuming that the analysis will be largely unaffected provided $T_{\pi,\text{max}} \gg T_\pi$, we then obtain the allowable hinge thickness as

$$t \ll \sqrt{2 \gamma w / \sigma_y}.$$  

(10)

Now, most ductile metals (e.g., Al, Au, Cu) have yield stresses of order $10^8 \text{ N/m}^2$. For $w \approx 10 \text{ µm}$, and 95% Pb: 5% Sn solder, we then obtain $t \ll 3000 \text{ Å}$, a very small figure. The assumption of negligible hinge deformation torque will therefore only be valid in structures with extremely thin hinges, which are likely to be difficult to fabricate. One possible solution is to segment the plastic section to reduce its overall length. However, since the only function of the hinge is to hold the structure together during solder melting, its complete omission would appear an attractive goal.

IV. Equilibrium of Hinged Structures

In this section, we consider an alternative method of analysis of a hinged structure, based on the consideration of surface energy rather than torque balance. This approach allows the derivation of a number of key results and can be generalized to allow the calculation of additional forces and torques. We begin with a simple proof of equivalence with the results of the previous section. For the geometry of Fig. 1, the surface energy $\sigma$ per unit length may be found as

$$\sigma = \gamma w \left( \phi \sin(\phi/2) / \sin(\phi/2) \right).$$  

(11)
The torque on the hinge may then be found by virtual work. If the hinge rotates by an angle $d\psi$, the free surface boundary will alter, by a length $d\sigma$. The amount of work done on the surface in the process is $dW = \gamma d\sigma$ per unit length of hinge. Now, although the excess pressure inside the liquid will also alter (due to the corresponding change in the radius of the free surface), no work is involved as the volume of liquid is fixed. Consideration of the work done on the surface is therefore sufficient. Assuming the work is carried out against a torque $T$, so that $dW = T d\psi$, the torque can be found as

$$T = \gamma \partial \sigma / \partial \psi.$$  \hfill (12)

The term $\partial \sigma / \partial \psi$ may be found by differentiating (11), obtaining $\partial \phi / \partial \psi$ from (4) as before. If this is done, the previous expression for the torque (2) is recovered exactly.

By considering surface energy rather than torque balance, it is possible to determine a new result—the overall equilibrium condition—in a relatively simple way. Equilibrium is reached when the surface energy $\sigma(\psi, \phi)$ is minimized. However, since the solder volume is fixed, this minimum must be found subject to the constraint $\phi(\psi, \phi) = 0$ previously given in (4). The problem may be solved using Lagrange's method of undetermined multipliers. We first construct a new function $\Sigma(\psi, \phi) = \sigma + \lambda \phi$, where $\lambda$ is an arbitrary multiplier. At any stationary points in $\Sigma, \partial \Sigma / \partial \phi = \partial \sigma / \partial \phi + \lambda \partial \phi / \partial \phi$ and $\partial \Sigma / \partial \phi = \partial \sigma / \partial \phi + \lambda \partial \phi / \partial \phi$ must both be zero. Combining these two equations, the multiplier can be eliminated to yield

$$\partial \sigma / \partial \phi (\partial \phi / \partial \psi) - (\partial \sigma / \partial \psi)(\partial \phi / \partial \phi) = 0.$$  \hfill (13)

Performing the necessary differentiation and some straightforward but tedious manipulation, (13) can be solved to yield an exceptionally simple relation between $\phi$ and $\psi$ at the equilibrium point

$$\psi_e = 2\psi_e.$$  \hfill (14)

Equation (14) now allows the equilibrium shape of the solder to be deduced. This shape is shown in Fig. 2(a) for $\psi_e < 90^\circ$. Here, the free boundary of the solder is an arc of a circle centered on point O and passing through the ends of the two lands (points A and B) and the hinge (point C). This geometry satisfies (14), since the angle subtended at the center (AOB) is twice the angle subtended at the circumference (ACB). Fig. 2(b) shows the corresponding shape for $\psi_e = 90^\circ$ and Fig. 2(c) shows the shape for $\psi_e > 90^\circ$. Note that the cross-sectional area of the solder increases with $\psi_e$, tending to infinity as $\psi_e$ tends to $180^\circ$. The value of $\eta$ needed for a given final angle can be found by substituting (14) into (4) to obtain

$$\eta(\psi_e) = \left\{ \sin^3(\psi_e) + (1 - \cos(\psi_e)) \right\} / \left\{ \psi_e - \sin(\psi_e) \cos(\psi_e) \right\} / 4 \sin^2(\psi_e).$$  \hfill (15)

This variation is shown in Fig. 3; it is in complete agreement with the numerical results previously obtained in [11]. For the special case of $\psi_e = \pi/2$, (15) reduces to the value previously found

$$\eta(\pi/2) = (2 + \pi)/8 = 0.6427.$$  \hfill (16)

Ninety-degree rotation can therefore be achieved with a solder pad of moderate aspect ratio.

Note that the analysis above says little about the geometry during rotation. In fact, for small $\psi_e$, the situation is more complex than we have implied because the free surface of the solder may become concave at some intermediate angles of rotation. However, it is trivial to show that the free surface cannot become concave at any angle for $\eta > 0.25$ (corresponding to $\psi_e > 50^\circ$). This range includes most final angles of practical interest.

The sensitivity of the final angle to errors in solder height can be found by differentiating (15). Usually, the absolute error $\Delta \psi_e$ caused by a fractional error $\Delta \eta/\eta$ is required. This is given by [12]

$$\Delta \psi_e = \left\{ \eta \psi_e / \partial \eta \right\} \times \Delta \eta / \eta.$$  \hfill (17)

Fig. 3 shows the variation of $\Delta \psi_e$ with $\psi_e$, for $\Delta \eta/\eta = 0.1$ (i.e. a relatively large fractional error of 10% in solder height). The errors in final angle are small, reaching a maximum of 4.35° at $\psi_e = 73^\circ$. For the particular case of $\psi_e = 90^\circ$, the sensitivity coefficient $\eta \psi_e / \partial \eta$ reduces to $(2 + \pi)/(4 + \pi) \approx 0.72$. For 10% fractional error in $\eta$, $\Delta \psi_e$ is then 0.072 rad, or 4.12°.

V. EQUILIBRIUM OF HINGELESS STRUCTURES

Using a similar approach, we now consider the equilibrium of a hingeless structure. To do this, we assume that there is now no constraint at the joint between the flaps, so that they may now be separated by a small gap $g$ [Fig. 4(a)]. The molten
solder will of course flow into this gap, so that it now has two free surfaces. However, if no pressure gradients are to exist, the radii of curvature of the two surfaces must be equal. We assume without justification that any energy minima will be found in a symmetric geometry, where the axis of symmetry is the line \( x - x' \).

The surface energy of the molten solder may again be found as a function of the geometric parameters \( \psi, \phi, r, \) and \( g \). However, the shape adopted is again subject to a constraint on the solder volume, which effectively implies that \( \sigma \) is a function of \( \psi \) and \( g \) only. Once again, the free surfaces may be concave (e.g., if \( g \) is large). Because of the complexity of the algebra, we present the initial results numerically.

Fig. 5 shows the normalized surface energy \( \sigma/\gamma w \) versus the normalized gap \( g/w \) for a number of different interim angles \( \psi_e = 90^\circ \). For \( \psi > 90^\circ \), the surface energy reduces monotonically with decreasing \( g \). Using a virtual work argument, it is simple to show that this implies the existence of a force \( F = \partial\sigma/\partial g \) acting in the \( \zeta \)-direction in Fig. 4(a), which would tend to close the gap. Furthermore, since \( \partial\sigma/\partial g \) is greater than zero for all \( g \), the gap closure force is always positive. For \( \psi = 90^\circ \), \( \partial\sigma/\partial g \) is positive for \( g > 0 \), but tends to zero at \( g = 0 \). This suggests that the gap closure force will vanish at \( \psi = 90^\circ \), \( g = 0 \) (which is of course the equilibrium condition for \( \psi_e = 90^\circ \)). For \( \psi < 90^\circ \), similar minima in the curves can be seen, but for nonzero gap values.

The gap closure force can be calculated analytically when \( g = 0 \). This can be done either using the virtual work argument, or by considering the forces acting. As shown in Fig. 4(b), these are a pressure \( p = \sigma/r \) per unit length distributed over the area of the wetted land, and surface forces \( \gamma \) per unit length acting on each end of the land. The lower of these forces was omitted in [11]; its omission was not relevant in the torque calculation performed therein, but is crucial in the argument that follows. Resolving forces, it is simple to show that, at \( g = 0 \), the gap closure force is

\[
F = \gamma \left( 1 + \cos(\phi/2) - \sin(\phi/2) \cot(\psi/2) \right). \tag{18}
\]

Similarly, by resolving forces in the orthogonal direction (i.e., along the line \( x - x' \)), it can be shown that the force \( F \) calculated above is the only force acting. In Fig. 6, we plot the normalized zero-gap closure force \( F/\gamma \) as a function of the interim angle \( \psi \), for different final angles \( \psi_e \). At \( \psi = 180^\circ \) (i.e., at the start of rotation from a horizontal position), the gap closure force is relatively large, decreasing monotonically to zero at \( \psi = \psi_e \).

The gap closure force at \( \psi = \pi \), i.e., at the start of rotation, is given by

\[
F_\pi = \gamma \left( 1 + \cos(\phi/2) \right) \tag{19}
\]
where $\psi_e$ is the value of $\psi$ at $\psi = \pi$. Fig. 7 shows the variation of $F_{\psi_e}/\gamma$ (and also of $T_{\psi_e}/\gamma\psi_e$) with $\psi_e$. We have previously pointed out that the starting torque is near its optimum value for $\psi_e = 90^\circ$; Fig. 7 now shows that the starting value of the gap closure force is near its midpoint for this final angle.

These results imply that, contrary to expectations, the lack of a hinge to hold the flaps together is not a disadvantage. Even though the solder might be expected to bulge out from the gap between the flaps, surface tension forces actually work to eliminate the gap, at least until equilibrium is reached. This is very advantageous, since it suggests that the hinge can be dispensed with. At a stroke, the large torque required to bend such a hinge can be eliminated, and any microfabrication process used to realize the structure can be simplified.

Why, however, should the zero-gap closure force vanish at $g = 0$ in Fig. 5? Similarly, why are there apparently other similar equilibrium points at nonzero values of $g$? These questions are best answered by Fig. 8(a), which shows the cross-section after $90^\circ$ rotation. Here we note that the free boundary length of the solder (shown white) is equal to the perimeter of the circle, less the circular boundaries of the shaded sectors 1 and 2. Similarly, its cross-sectional area is the area of the circle, less that of sectors 1 and 2. Consequently, the geometries of Fig. 8(a)-(c), where sector 2 is rotated through different angles, all have the same surface energy and cross-section. In the absence of a hinge constraint, all are equally valid minimum surface energy configurations.

Because of this, a hingeless joint will lose all torsional stiffness in the anticlockwise direction once the initial equilibrium condition at $g = 0$ is reached (in contrast to the hinged joint previously analysed). As a result, the flap will be free to “float” round on the solder to these alternative conditions (which have nonzero values of $g$), and will do so if driven by an anticlockwise torque (e.g. that due to the weight of the flap, however small). However, for $\psi_e > 90^\circ$, the gravitational torque will act in the clockwise direction. In this case, the structure will be inherently stable. The structure is quasi-stable only for $\psi_e \leq 90^\circ$; however, in this case, stability can be assured if a “stop” can be provided to prevent rotation past the design angle.

For $90^\circ$ rotation, it should be possible to form a stop as shown in Fig. 9(a). Here, two movable flaps A and B are linked by solder pads to a fixed part C. In addition, solder pads of half the normal width are located on adjacent edges of A and B. When the structure is melted, the movable flaps should both rotate upright. As they near their equilibrium position, the half-width pads should merge [Fig. 9(b)]. The resulting fillet will form a third hingeless joint between A and B, and (since its total solder volume is now the same as that of the other joints) it should reach equilibrium when a $90^\circ$ corner joint has been formed between flaps A and B. The gap closure force of this joint should provide the stabilizing force needed to prevent further rotation of the other two joints.

VI. EXPERIMENTS WITH MACROSCOPIC HINGELESS STRUCTURES

While the aim of this work has been to develop processes applicable to microstructures, proof-of-principle can be performed with much larger structures [12]. Such experiments require only very simple materials and apparatus: Lightweight copper-clad printed circuit board (PCB), solder, patterning apparatus, and a hotplate. In order to demonstrate the rotation of hingeless structures, we used flexible PCB consisting of a 35-μm-thick copper conductive layer on a 100-μm-thick polyimide backing. After patterning and etching to form 0.5-mm-wide copper tracks, the material was cut into strips, with a track at one edge and a region of bare polyimide at the other. Patterned strips were then used as both fixed and moving parts; the former were merely cut slightly wider than the latter (which were ±2 mm wide).

Initially, the strips were straightened and placed apart on a temperature-controlled hotplate set to 220°C. Fluxed solder was then used to tin the tracks and build up a uniform solder
thickness and the strips were cooled and deflued. The process was then repeated, this time with the strips arranged in pairs with their tracks adjacent. Additional solder was used to bridge any gap between the tracks and build up a solder volume corresponding as far as possible to that required for 90° rotation. At this stage, adhesion caused by burnt flux typically prevented rotation. The strips were then cooled and deflued again, and cut to 10-mm lengths. Parts prepared in this manner normally rotated as soon as they were reheated to 220°. If they did not rotate immediately, a small quantity of solder was added. Since the starting torque is a strong function of solder volume [11], this normally provided sufficient additional torque to overcome any residual flux adhesion.

Fig. 10 shows four consecutive stills from a video of rotation. The stills are 60 frames apart, and the frame rate is 25 per second. The time between the first and last still is therefore 7.2 seconds. In the first still (at 12:02), a small quantity of solder is added to a prepared pair of parts to trigger rotation. In the second (at 12:05), third (at 12:07), and fourth (at 12:09) stills, the part furthest from the camera rotates to angle $\psi_c \approx 100^\circ$. In this example, the quantity of solder is therefore too high for 90° rotation. However, despite the short overall length of the parts and the lack of an integral hinge, the solder shape is essentially that predicted by the previous analysis. The assembly was stable for at least 5 minutes with the solder molten and was then solidified by cooling. The ability of structures of this size to rotate and remain stable suggests that the theoretical basis of the method is sound.

VII. CONCLUSION

We have shown that the behavior of hingeless structures designed to rotate microfabricated parts during self-assembly operations powered by surface tension force is essentially similar to that of similar hinged structures previously described. The results of our new analysis suggest that a hinge—possibly the most difficult part to manufacture in a microfabrication process—may be unnecessary. Contrary to expectations, surface tension forces act to hold the structure together until the equilibrium geometry is reached. At this point, the structure becomes quasi-stable, but it appears relatively simple to provide suitable stabilizing forces in a 3-D geometry of only moderately increased complexity. Initial experiments with millimeter-scale structures have confirmed the general validity of these conclusions, which have now been adopted in the design of micron-scale structures used for the first experimental demonstration of a corresponding integrated self-assembly process [13].

ACKNOWLEDGMENT

The author is grateful to Dr. E. Yeatman for many useful discussions, and for the assistance of C. Grimshaw with video photography.
REFERENCES


R. R. A. Symns, for a photograph and biography, see this issue, page 176.