approaches, Malvar's and Chan and Su's algorithms always give the minimum number of multiplications for all lengths shown. The present approach requires the same minimum number of multiplications for all lengths, however, it requires a smaller number of additions than that required by those two approaches.

Conclusions: A new algorithm is proposed in this Letter to realise a 2n-length discrete Hartley transform. This algorithm gives the lowest number of multiplications compared with other algorithms reported in the literature and requires a smaller number of additions compared with the other algorithms which require the same minimum number of multiplications.

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References

OPTICAL LAYOUT FOR SINGLE-TRANSVERSE-MODE OPERATION OF 2-D ARRAYS OF VERTICAL CAVITY SURFACE-EMITTING LASERS

An intracavity spatial frequency filter is proposed as a method of locking a 2-D array of vertical cavity surface-emitting lasers on a single, low-order transverse mode. Such a filter, which requires the interconnection of nearest-neighbour emitters, could be fabricated as a multilayer structure containing a number of multiplexed volume holographic optical elements.

The development of two-dimensional arrays of vertical cavity surface-emitting lasers should allow the generation of high optical power from a large active area. However, to reduce far-field beam divergence, lasing should be stabilised on a single, low-order transverse mode. Attention is therefore now being concentrated on 2-D arrays that are phase-locked by close packing. In this respect, the progress of 2-D surface-emitting arrays is following the pattern established by earlier linear array lasers. However, in such devices, simple codirectional coupling was shown to be an ineffective method of establishing the dominance of the lowest-order transverse mode. In fact, the least desirable situation often occurred, with lasing taking place predominantly on the highest-order mode.

This was due essentially to the lower level of gain in the interguide regions. A number of cures were attempted, including overpumping the interguide regions, and tailoring the gain distribution across the array. However, the best results were obtained by the insertion of an intracavity spatial frequency filter, in the Y-junction-coupled array laser. The selectivity of such a filter has recently been improved, through the introduction of non-nearest-neighbour coupling, in the Y-X-junction laser. It is therefore logical to consider immediately the insertion of an intracavity filter in surface-emitting array lasers. We show that such methods may be extended without difficulty to 2-D geometries, and provide similar mode selectivity.

Fig. 1 shows the proposed layout, in which it is assumed that the laser array is constructed as a composite cavity. The lowest level contains an array of surface emitters, which have one facet mirrored (M1), and the other AR coated. Above the AR coated facet, there is an interconnection network, and then a second mirror (M2). In the plan view, each surface emitter is shown as a shaded disc, and located at a point (n, m) in the array. The field amplitude of the wave leaving each emitter at the start of a round trip is a nm. Generally, the array might be rectangular, and limited to N × M emitters (as indicated by the dashed line), although other boundary shapes have been considered. Similarly, the combined feeds to the mirror M1 at the far end of the cavity are shown as unshaded discs. These occupy intermediate positions, so that the nth, mth feed is offset to the right and above the nth, mth emitter. The amplitude of the wave arriving at each feed is a nm. Emitter and feeds are nearest-neighbour connected, by a network oriented at 45° to the emitter array axes, and both the interconnect network and the feed array extend beyond the emitter array boundary.

Assuming that the array is entirely regular (so that all emitters are equally pumped, and equally separated in the n and m directions), that the mirror reflectivity is uniform, and that the interconnect network is symmetric, we may ignore the gain and phase change experienced on a round trip through the cavity, and concentrate instead on the moment selectivity provided by the spatial frequency filter.

The analysis is performed by assuming initially that the array is unbounded. If the splitters act as perfect four-way dividers, it is easy to show that

\[ b_{nm} = 1/4 \left( a_{n-1,m} + a_{n+1,m} + a_{n,m-1} + a_{n,m+1} \right) \]  

Extending this argument, we may show that the wave amplitudes \( a_{nm} \) leaving the emitters after an entire round trip are related to the initial values by

\[ A'_{nm} = 1/16 \left( a_{n-1,m-1} + a_{n+1,m+1} + a_{n-1,m+1} + a_{n+1,m-1} + a_{n+1,m} + a_{n-1,m} + 2a_{n,m+1} + a_{n,m-1} \right) \]  

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Eqn. 2 shows that each emitter is effectively coupled to its eight nearest neighbours, and to itself.

Now, eigenmode solutions will be of the form $a_{n,m} = r_1 a_{n,m}$, where $\gamma$ represents the round-trip transmission of the spatial frequency filter. If we seek a separated periodic solution, in the form $a_{n,m} = a \exp(\frac{-r_1}{n}) \exp(\frac{-r_1}{m})$, where $k_n$ and $k_m$ are spatial frequencies in the $n$ and $m$ directions, then substitution into eqn. 2 yields the following relation between $\gamma$, $k_n$, and $k_m$:

$$\gamma = \frac{1}{4} \left[ 1 + \cos k_n \right] \left[ 1 + \cos k_m \right]$$

This relation is shown schematically in the contour map of Fig. 2. For $0 \leq k_n, k_m \leq \pi$, it peaks at $k_n = k_m = 0$, and decreases steadily as the spatial frequency in either direction rises.

![Fig. 2 Contours of equal $\gamma$ on $k_n - k_m$ plane](image)

The eigenmodes of a finite rectangular array (e.g. one having $1 \leq n \leq N$ and $1 \leq m \leq M$) may be constructed from the solution above, using periodic boundary conditions. A total of $N \times M$ distinct modes is possible. Each is defined by two indices $n$ and $m$, such that $1 \leq n \leq N$ and $1 \leq m \leq M$. The transverse amplitude distribution of each mode has the form

$$a_{n,m} = r \sin(k_n n) \sin(k_m m)$$

with the relevant spatial frequencies in the $n$ and $m$ directions being given by

$$k_n = n \pi / N + 1 \quad k_m = m \pi / M + 1$$

Clearly, the mode with the smallest round-trip loss is that having $r = s = 1$, which has the most uniform amplitude distribution. This will have the lowest threshold, and so will lase preferentially. For example, for a $2 \times 2$ array, $\gamma_1 = 0.5625$, $\gamma_2 = 0.1875$, and $\gamma_3 = 0.0625$. The threshold of the lowest-order mode should therefore be a factor of three lower than that of any other mode.

The entire device could be constructed as a multilayer structure, using free-space interconnects (Fig. 3). The lowest level contains the emitter array and the mirror $M_1$. The output from each emitter is collimated by a second layer, containing a planar array of integrated microlenses. Each of the resulting vertical, parallel beams is then split four ways, by a third layer containing four superimposed volume holographic transmission gratings. These are of equal strength, and recorded so that there is no phase shift between the four outputs. (In fact, beam collimation could also be performed by this VHOE). After propagation through a suitable thickness of spacer, the beams are recombined by a second VHOE, identical to the first. The final layer is a reflector, the mirror $M_2$, with outputs being taken from either $M_1$ or $M_2$. A non-nearest-neighbour interconnection analogous to that used in the Y-X junction array could be implemented by the addition of further VHOE layers. Assessment of the performance of the holographic splitter requires a vectorial analysis of the problem of four superimposed gratings. Using standard first-order coupled wave theory, we have determined that the input eigenpolarisations of the splitter are oriented along the diagonal axes of the array, not along the $n$, $m$ axes. This implies that the polarization modes of the array will also be so aligned. Furthermore, with equal-strength gratings, neither polarisation yields an equal four-way split, due to the well-known dependence of diffraction efficiency on polarisation. This modifies the eigenvalue equation (eqn. 4) and greatly complicates determination of the eigenmodes. However, for small angles of deflection, and for low-order modes, the mode shapes are still approximately as given by eqn. 5. The array might then be locked onto the $n$ or $m$ axes (i.e. onto a linear superposition of the two polarisation eigenmodes) either by control of the cross-sectional shape of the emitters or by using a polarisation-dependent reflector (such as a submicron wire grid) on the upper layer. Most usefully, the splitter may be 100% efficient at Bragg incidence.

**References**


**NONLINEAR BEAMFORMING**

**Indexing term:** Antennas, Antenna theory

A totally new concept for beamforming is introduced, which is based on using a nonlinear model. Simulation results are presented to show that the technique is effective and robust, and, therefore, has a great deal of promise.

**Introduction:** For decades, a considerable amount of research has been devoted to digital beamforming, wherein array signal processing techniques have been conceived, developed, tested...