Simple approximate theory for twin-guide Fabry–Perot laser amplifier switches

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Abstract. A simple modal analysis is presented of twin-stripe semiconductor Fabry–Perot laser amplifier switches that operate via simultaneous changes in gain and refractive index due to free carrier injection. The analysis of Adams [3], based on weak coupled mode theory, is first briefly reviewed, and compared with the modal method. An approximate solution is then derived, which shows good agreement with the full theory. Approximate resonance and lasing conditions are also found, and limiting values for switch crosstalk are estimated.

1. Introduction

There has recently been considerable interest in directional coupler-type switches based on twin-stripe travelling-wave laser amplifiers. These can be fabricated using both the GaAs/GaAlAs and InP/InGaAsP materials systems, and operate by the simultaneous changes in gain and refractive index induced by carrier injection [1, 2]. The presence of gain can compensate for insertion loss, and the particular switching mechanism allows the potential for either electrical or optical control.

To investigate the possibility of shortening the device, Adams has published an analysis of a similar twin-stripe Fabry–Perot structure, based on weak coupled mode theory [3]. Although this analysis is essentially complete as it stands, this device has been found to exhibit rather complicated response characteristics, which deserves further explanation. The purpose of this paper is to present an alternative description of electrically switched, linear Fabry–Perot devices. This is again derived from weak coupled mode theory, but is based on a modal approach, which emphasizes more fully the resonant behaviour.

The analysis of Adams [3] is first briefly reviewed, and compared with the modal method. An approximate solution is then derived, which shows extremely good agreement with the full theory for typical device parameters. Approximate resonance and lasing conditions are also found, and limiting values for switch crosstalk are estimated.

2. Modal analysis

The structure assumed for analysis is shown in figure 1. It consists of two coupled travelling-wave laser amplifiers that are oriented in the z-direction and run parallel for a distance d. At either end there are end-mirrors, forming a twin-guide resonator, but we shall begin by considering the response in the absence of mirrors. For simplicity, we adopt the most basic model, weak scalar coupled mode theory.
Although more accurate theories exist (for example [4–7]), the main features of the device behaviour are still contained in the approximate model, and it is felt that the additional complications introduced by strong coupled mode theory are not worthwhile at this stage.

We assume the two stripes are differentially pumped, so in isolation they would have propagation constants $\beta_1$ and $\beta_2$, and amplitude gains $g_1$ and $g_2$. Assuming both propagation constants are close to a reference value $\beta_0$, the following equations governing the variation of the mode amplitudes $A_1$ and $A_2$ in each guide may be derived by the normal rules of weak coupled mode theory:

$$\frac{d}{dz} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = -j \begin{bmatrix} \Delta \beta & \kappa \\ \kappa & -\Delta \beta \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix},$$

where $\kappa$ is the coupling coefficient, defined in the usual way, and the real and imaginary parts of the complex change in propagation constant $\Delta \beta$ are given by

$$\Delta \beta = (\beta_1 - \beta_2)/2; \quad \Delta \beta_0 = (g_1 - g_2)/2.$$  

We also follow Adams [3], and assume that the effect of pumping is to introduce linearly-related changes in $\Delta \beta_1$ and $\Delta \beta_0$, without changing $\kappa$. In this case, we may write:

$$\Delta \beta_1/\Delta \beta_0 = \alpha,$$

where $\alpha$ is a constant, the linewidth broadening factor, which is estimated to have the value $5$ for 1.5-μm-range InGaAsP/InP lasers [1, 3, 8].

Now, it is well known that equations similar to equation (1) can be written in the compact form $dA/dz = -jM A$, where $A$ is a two-element vector, and $M$ is a $2 \times 2$ coupling matrix. The output $A_4$ at $z = d$ may then be found for an input $A_0$ at $z = 0$ by direct integration, as

$$A_4 = \exp \{ -jMd \} A_0.$$  

The value of the matrix exponential is well known from previous work on passive couplers, e.g. [9], and was used to calculate the response of twin-guide amplifier switches [1]. It also formed the basis of Adams’ model of Fabry–Perot devices [3]. However, there are two problems with its use. First, some terms in the matrix
require numerical evaluation, since square roots of complex quantities are involved. Secondly, it is not appropriate for resonant devices, because it represents a combination of eigenmode solutions which should really be treated individually. Both features have tended to obscure the physical behaviour of Fabry–Perot switches. We will treat the two problems separately, and start by introducing an alternative eigenmode analysis.

First, we note that the matrix exponential solution above may be written in the form:

\[ A_4 = V \exp \{ -j\Gamma d \} V^T A_0 \]  

(5)

Here \( V \) is a \( 2 \times 2 \) matrix containing the normalized eigenvectors \( v_1 \) and \( v_2 \) of \( M \), arranged in columns, and \( \Gamma \) is a diagonal matrix of the eigenvalues \( \gamma_1 \) and \( \gamma_2 \). If we define a further vector \( C \) using:

\[ C = V^T A_0 \]  

(6)

then equation (5) provides an alternative model in which the modal amplitudes are represented as a summation of eigenmode solutions, each of the form \( v_i \exp \{ -j\gamma_id \} \), which are launched with initial amplitude coefficients \( c_i \). These can be found from the input boundary conditions.

The analysis above can also be applied to a device with end-mirrors. The approach we now follow is that previously used in the approximate analysis of coupled waveguide array lasers (for example [10–13]). We begin by defining propagation from \( z = 0 \) to \( z = d \) in terms of a transfer matrix \( Q \). This must include all propagation terms implicit in the coupled wave analysis, and may therefore be taken as

\[ Q = V \exp \{ -j\Omega d \} V^T, \]  

(7)

where

\[ \Omega = \Gamma + (\beta_0 + \Delta \beta_2 + jg_0)I; \]  

(8)

here \( g_0 \) is the average gain, and \( \beta_0 + \Delta \beta_2 \) is the average propagation constant. Next, we must specify the effect of the mirrors. This can be done using transmission and reflection matrices \( R_1 \) and \( T_1 \), and \( R_2 \) and \( T_2 \) (the former at the input end, the latter at the output). The total transmission matrix \( T \) can then be written as the infinite series. If this converges, it may be summed, to give

\[ T = T_2 Q[I - R_1 Q R_2 Q]^{-1} T_1. \]  

(9)

Now, a failure of the series to converge indicates the onset of lasing. This occurs when the inverse matrix above ceases to exist, or when

\[ |I - R_1 Q R_2 Q| = 0. \]  

(10)

To proceed further, we shall make some simplifying assumptions. We assume that the end-mirrors are identical, and put \( R_1 = R_3 = R \), and \( T_1 = T_3 \) and \( T_2 = T_3 \), such that \( R^2 + T_1 T_2 = 1 \). This implies that there will be no mixing of the eigenmodes on reflection, so that the cavity eigenmodes are the same as those of the twin-stripe system. If this is done, equation (9) reduces to

\[ T = (1 - R^2) Q[I - R^2 Q^2]^{-1}, \]  

(11)

which may then be transformed to

\[ T = (1 - R^2) V \exp \{ -j\Omega d \} (I - R^2 \exp \{ -j2\Omega d \})^{-1} V^T. \]  

(12)
Figure 2. Comparison between exact (full line) and approximate (dashed line) variations of $\gamma_{1d}$ and $\gamma_{2d}$ with $\Delta \beta_{r d}$, for $\kappa d = \pi/2$, and $a = -0.2$ (upper diagram) and $a = -1$ (lower diagram).
3. Approximate solution

To make further progress, we adopt the following new approximation to equation (18). For $|a|<1$ (corresponding to $|a|>1$, as will be likely in practice) we may use a binomial approximation, and put

$$\gamma_{1r} \approx \sqrt{(\kappa^2 + \Delta \beta_i^2)}; \quad \gamma_{11} \approx a \Delta \beta_i^2 / \gamma_{1r}.$$  \hspace{1cm} (19)

Using this, approximate variations (shown as dashed lines in figure 2) for $\gamma_{1r}d$ and $\gamma_{11}d$ may be computed. For the parameters used in the upper diagram there is hardly any difference at all from the exact variations. Surprisingly, the qualitative agreement is still very good in the lower diagram, where the approximation might be expected to break down. In fact, significant differences only seem to appear for still higher values of $a$. The approximation is therefore a very accurate one indeed.

This is important, since it allows estimates of a number of features of the device response to be made for a wide range of linewidth broadening factor. For example, we may compute approximate forms of the lasing conditions. Assuming that $\Delta \beta_i d = 0$, the longitudinal resonance conditions are the same for both modes, reducing to

$$\pm \frac{d}{\sqrt{(\kappa^2 + \Delta \beta_i^2)}} \approx p \pi.$$  \hspace{1cm} (20)

This can be represented graphically on the $\kappa d - \Delta \beta_i d$ plane as a series of concentric circles (figure 3). The value of $\Delta \beta_i d$ needed for a particular resonance can then be found from the intersection of a vertical line through the relevant value of $\kappa d$ with the circle concerned. Figure 3 shows the construction for $\kappa d = \pi/2$. The lowest-order resonance ($p = \pm 1$) occurs at

$$\Delta \beta_i d \approx \pm \sqrt{(\pi^2 - (\kappa d)^2)}.$$  \hspace{1cm} (21)

For $\kappa d = \pi/2$, for example, $\Delta \beta_i d \approx \pm \pi \sqrt{(3/4)} \approx \pm 2.72$. This should be compared with the exact value of $\pm 2.734$ given above; the qualitative agreement is excellent.

If $\Delta \beta_i d \neq 0$, equation (20) modifies to

$$\pm d / (\kappa^2 + \Delta \beta_i^2) \approx p \pi - \Delta \beta_i d.$$  \hspace{1cm} (22)

![Figure 3. Approximate loci of resonances on the $\kappa d - \Delta \beta_i d$ plane, for $\Delta \beta_i d = 0$.](image-url)
This implies that the resonance conditions for the two characteristic modes are now different. The resonance locii are still circles on the $\kappa d-\Delta \beta d$ plane. However, for a small value of $\Delta \beta d$ (i.e. $<\pi$), they are symmetrically split about the values given by equation (20) (figure 4). There should, therefore, be double the number of resonances appearing in any switch response. However, these will not be equally significant, since one mode has lower gain.

Similarly, we may estimate the threshold gains. Using equation (19), the lasing condition reduces to

$$R^2 \exp \left[2\{g_d d + a\Delta \beta d^2 \pm \sqrt{\kappa^2 + \Delta \beta^2}\}\right] \geq 1.$$  \hspace{1cm} (23)

Assuming that $\Delta \beta d = 0$, and combining this with the resonance condition, we obtain for the lowest-order resonances ($p = \pm 1$):

$$R^2 \exp \left[2\{g_d d \pm a\pi[1 - (\kappa d/\pi)^2]\}\right] \geq 1.$$  \hspace{1cm} (24)

In order to avoid lasing of either mode, with an arbitrary value of $a$, we therefore require

$$g_d d < \ln(1/R) - |a|\pi[1 - (\kappa d/\pi)^2]$$  \hspace{1cm} (25)

The threshold gain is therefore lower than that of a single-stripe device (for which we would expect $g_d d_{\text{single}} < \ln(1/R)$). For example, for $\kappa d = \pi/2$, equation (25) reduces to

$$g_d d < \ln(1/R) - |a|3\pi/4.$$  \hspace{1cm} (26)

Consequently, a number of the theoretical switch curves presented by Adams [3] (even some of those for which $g_d d < g_d d_{\text{single}}$) actually correspond to devices operating beyond threshold. This, of course, is not immediately apparent from simple evaluation of equation (12), which provides no indication that it may represent a non-convergent series.
Figure 5. Exact (full line) and approximate (dashed line) switch characteristics for the parameters $\kappa d = \pi/2$, $g_d = 0.95$, $R = 0$ and $\Delta \beta_r d = 0$, for $a = 0.2$ (upper diagram) and $a = -0.6$ (lower diagram).
Figure 6. Modulus squared of the two modal launch coefficients for the same parameters used to compute the upper diagram in figure 5.

Figure 7. Exact (full line) and approximate (dashed line) switch characteristics for the parameters $\kappa d=\pi/2$, $g_0d=0.95$, $R=0.2236$, $\Delta\beta'd=0$ and $a=-0.2$. 
for these parameters as $g_0 d \approx 10.26$. However, we note that the output in the cross-
state (point A) is lower than in figure 5, while the bar states (B and B') are higher. 
Unfortunately, crosstalk in the bar states is considerably worse. This feature can also 
be traced to modal effects, as we now show.

We have mentioned that the resonance conditions for the two modes should be 
identical, if $D\beta_d d = 0$. However, the contribution of the high-gain mode should 
greatly outweigh that of the low-gain one at resonance. Figure 8 shows a comparison 
between the exact response calculated for $P_1$ in figure 7 and the equivalent results 
found by entirely neglecting the low-gain mode. This shows that the viewpoint 
above is indeed quite accurate near B. Near B', there is significant difference between 
the curves. This is because the high-gain mode is not launched in sufficient 
proportion to dominate the response, as we saw in figure 6.

To demonstrate the effect of a wavelength shift, we show in figure 9 the exact 
variation of $P_1$ for the same parameters as before, but with three different values of 
$D\beta_d d$ (-0.5, 0 and +0.5). From the previous section, we would expect each 
resonance to split into two peaks when $D\beta_d d \neq 0$, located on either side of the single 
resonance observed for $D\beta_d d = 0$. This does not appear to be the case; the clearest 
features (on the left-hand side of the figure) still seem to consist of single peaks. 
Although (as expected) these move sideways as $D\beta_d d$ alters, they also change in 
amplitude. The explanation is that these features still reflect mainly the behaviour of 
the dominant, high-gain characteristic mode, which causes the appearance of a 
singly-resonant system for negative $D\beta_d d$. The changes in amplitude occur because 
the value of $D\beta_d d$ needed to achieve resonance for this single mode varies with $D\beta_d d$, 
which causes a corresponding alteration in modal gain at resonance. The simplest
Figure 9. Switch characteristics for the parameters $\kappa d = \pi/2$, $a = -0.2$, $g_{pd} = 0.8$, $R = 0.2236$ and $\Delta \beta_d = -0.5, 0$ and $+0.5$.

Figure 10. Switch characteristics for the parameters $\kappa d = \pi/2$, $a = 0$, $g_{pd} = 1.4$, $R = 0.2236$ and $\Delta \beta_d = +0.5$. 
way to verify that split resonances can actually occur is to equalize the two
supermode gains. Figure 10 shows exact results for $\kappa d = 1.57$, $a = 0$, $g_0 d = 1.4$, $R = 0.2236$ and $\Delta \beta_d = +0.5$. There are now clearly four resonance peaks, as
expected. The differences in height of the resonances now follow mainly from
differences in the modal input coefficients, as calculated for figure 6.

5. Limiting crosstalk performance

We may use the approximate solution to estimate crosstalk figures. Now, we have
shown that at point B in figure 7, the response is dominated by the high-gain mode.
In this case, we may put $A_2 \approx w_2 f_2$. From equations (27) and (28) we then find the
ratio of the two outputs to be independent of the value of the Airy function $f_1$.
Neglecting the small quadrature components in equation (28), we get

$$(P_1/P_2)_{B} \approx (\gamma_2 + \Delta \beta_d)/\kappa)^2.$$  (29)

Now, at the left-hand resonance, we have $\gamma_2, d \approx -\pi$, and $\Delta \beta_d = -\sqrt{[\pi^2 - (\kappa d)^2]}$.
Hence

$$(P_1/P_2)_{B} \approx (\pi + \sqrt{[\pi^2 - (\kappa d)^2]}/\kappa)^2.$$  (30)

For $\kappa d = \pi/2$, this yields $(P_1/P_2)_{B} = [2(1 + \sqrt{[3/4]})]^2 \approx 14$, or $\approx 11.5$ dB. We may use
a similar argument to compute the crosstalk at B', provided the gain is sufficiently
high that the high-gain mode is actually dominant. In this case, we have $\gamma_2', d \approx -\pi$, and $\Delta \beta_d = +\sqrt{[\pi^2 - (\kappa d)^2]}$, so that

$$(P_1/P_2)_{B'} \approx (\pi - \sqrt{[\pi^2 - (\kappa d)^2]}/\kappa)^2.$$  (31)

Comparing equations (30) and (31), it is clear that $(P_1/P_2)_{B} \times (P_1/P_2)_{B'} = 1$, so that
$(P_1/P_2)_{B} \approx 1/14$. The crosstalk is therefore roughly 11.5 dB once again. This
argument shows that we will have equal crosstalk in the two bar-states, with the
relatively poor limiting values given above, merely provided we are sufficiently close
to resonance. In figure 7, the actual values are $\approx 12$ dB (B) and $\approx 4.5$ dB (B').
The worse agreement in the latter case is due to the low value of $g_0 d (0.95)$, which places
the device too far from resonance ($g_0 d = 1.026$). For $g_0 d = 1$, on the other hand, the
value at B remains approximately unchanged, while that at the B' improves to about
9 dB. However, this is achieved only by operating very near to threshold.

Equation (30) also suggests that an improvement in bar-state crosstalk will be
obtained if $\kappa d$ is lowered from $\pi/2$. This is indeed true, but the cross-state
performance then degrades very rapidly, and the optimal improvement (found by
equalizing crosstalk in the two states) is very slight.

6. Conclusions

The performance of twin-stripe Fabry–Perot laser amplifier switches has been
examined using an approximate modal analysis, based on weak coupled mode
theory. This shows very good agreement with earlier work [3], but contains analytic
approximations which allow a number of new conclusions to be drawn. First, the
conditions for lasing have been identified. Secondly, it has been shown that the F–P
switching characteristic is fundamentally different to that of the equivalent
travelling-wave device, because its response is dominated by the resonant behaviour
of a single characteristic mode rather than by beating between two such modes.
Thirdly, the limiting crosstalk performance has been estimated and found to be
poor. In view of this, it appears unlikely that the deliberate incorporation of mirror
feedback will improve the performance of travelling-wave devices, and accidental feedback from uncoated facets should probably also be avoided. Although the theory used was highly simplified, it is likely that the inclusion of strong coupling effects will reinforce this view [19].

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References