Answer ALL parts of Section A, ONE question from Section B and ONE question from Section C.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Write your CANDIDATE NUMBER clearly on each of the FOUR answer books provided.

If an electronic calculator is used, write its serial number in the box at the top right hand corner of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the horizontal box on the front cover of its corresponding answer book.

Hand in FOUR answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.
SECTION A

1. (i) Describe how a laser can be used to correct for the eye defects of long-sightedness and short-sightedness. In your answer, state what laser is commonly used and explain why this laser is chosen. [3 marks]

(ii) A CO\textsubscript{2} laser (wavelength 10.6 \( \mu \)m) has a cavity with length 1 m, a partially reflecting plane mirror acting as output coupler and a high reflectivity back concave mirror with radius of curvature 4 m.

(a) What is the location of the minimum beam waist? [1 mark]

(b) Derive an expression for the minimum beam radius size \( w_0 \) of the fundamental mode of this type of cavity and hence calculate \( w_0 \) for the CO\textsubscript{2} laser. [3 marks]

(iii) In terms of dollars of sales, list the four main laser application areas and the four main types of lasers. [3 marks]

[TOTAL 10 marks]
2. (i) The coherence length for second harmonic generation (SHG) of a \( \lambda = 800 \text{ nm} \) laser in a nonlinear crystal is found to be 100 \( \mu \text{m} \). Calculate the wavevector mismatch, \( \Delta k \).

The refractive index of the crystal at 800 nm is 1.670. Assuming \( \Delta k = k(2\omega) - 2k(\omega) \), where the symbols have their usual meanings, determine the refractive index of the crystal at the second harmonic wavelength. What value would be required for the SHG to be phase-matched? [4 marks]

(ii) A difference frequency generation (DFG) scheme uses a *pump* field at \( \lambda = 532 \text{ nm} \) and a tunable *signal* field covering \( \lambda = 700–900 \text{ nm} \). Calculate the potential tuning range of the generated *idler* field. Name a practical factor that may limit the actual range achieved. [3 marks]

(iii) The refractive index of a glass at very low intensity is 1.54. What intensity is required to increase this by 1 part in 10^5, assuming the glass has a nonlinear refractive index of \( n_2 = 3 \times 10^{-16} \text{ cm}^2/\text{W} \) [Note: not SI units]? Which order of nonlinear susceptibility is responsible for this behaviour? [3 marks]

[TOTAL 10 marks]
3. A Q-switched Nd:YAG laser operating at 1064-nm with TEM\text{00}, M^2 = 1 spatial beam quality has average power 10W and an output pulse rate of 10kHz. It has output beam diameter of 6mm and is focussed by a lens with focal length 100mm onto a metal foil target.

(i) List advantages of using a laser for cutting or drilling compared to a mechanical tool. [4 marks]

(ii) What is the size of the focal spot? [4 marks]

(iii) Estimate a value for the accuracy to which the focus needs to be located at the work-piece. [2 marks]

(iv) What is the energy of a single pulse from the Q-switched laser? [2 marks]

(v) Energy per unit volume of 5 \times 10^9 \text{J/m}^3 is required to vaporise the metal foil material. The focused laser pulse is absorbed with 50\% efficiency. Calculate the thickness of foil that can be drilled with a single pulse. [6 marks]

(vi) If the laser had a spatial beam parameter M^2 = 2, how would that affect your answer to part (v)? [2 marks]

[TOTAL 20 marks]
4. A Nd:YAG laser with wavelength 1064 nm consists of a Nd:YAG rod (length 20 mm and radius 1 mm) placed in a laser cavity. The rod is uniformly excited throughout its volume by absorption of 80 W of diode laser pumping radiation at 808 nm and the Nd:YAG laser emits 30 W. The laser has a threshold pump power of 20 W of diode radiation.

(i) Plot a graph of laser output power versus diode pump power. What is the slope efficiency of the laser output power with respect to diode pumping power? [2 marks]

(ii) If the 80 W diode laser has operating current 80 A, a series resistance of 0.002 Ω, and band-gap energy 1.55 eV,
   (a) calculate the forward voltage across the diode at the operating current, [1 marks]
   (b) calculate the electrical-to-optical efficiency of the diode laser, [2 marks]
   (c) what is the electrical-to-optical efficiency of the Nd:YAG laser? [2 marks]

(iii) What is the heating power deposited into the Nd:YAG rod by the 80 W optical power of the diode pumping? [2 marks]

(iv) The steady-state heat diffusion equation, assuming just radial heat flow from the laser rod, is given by
   \[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = -\frac{Q}{\kappa} \]
   where \( T = T(r) \) is the temperature which is a function of the radial coordinate \( r \) relative to the rod central axis \( (r = 0) \); \( \kappa \) is the thermal conductivity of the rod and \( Q \) is the heating power per unit volume deposited by the diode pumping. Solve this equation to show that the temperature distribution across the rod is given by
   \[ T(r) = T(0) - \frac{Q}{4\kappa} r^2. \] [4 marks]

(v) Explain why this temperature distribution leads to the rod exhibiting a lensing action. [2 marks]

(vi) The focal length \( f \) of the thermally-induced rod lens is given by the expression
   \[ \frac{1}{f} = \frac{dn}{dT} \frac{Q \ell}{2\kappa}, \]
   where \( \ell \) is the rod length and \( \frac{dn}{dT} \) is the refractive index change with temperature. Calculate the focal length for the Nd:YAG rod. [You should take \( \frac{dn}{dT} = 7.3 \times 10^{-6} \, \text{K}^{-1} \) and \( \kappa = 14 \, \text{Wm}^{-1} \, \text{K}^{-1} \).] [3 marks]

(vii) Apart from thermally-induced lensing, what other major optical problem occurs in the Nd:YAG laser rod due to its temperature distribution under pumped conditions? [2 marks]

[TOTAL 20 marks]
5. For Type I phase-matching \( (\omega + \omega \rightarrow 2\omega) \) of second harmonic generation (SHG), the phase-matching condition is

\[
k_o (\omega) + k_o (\omega) = k_e (2\omega, \theta_I)
\]

where the symbols have their usual meanings.

(i) Sketch a typical setup for Type I phase-matching taking care to show the phase-matching angle and the relative polarisations of the fundamental and second harmonic fields, assuming input and output beams are perpendicular to the parallel crystal surfaces. [4 marks]

(ii) Write down an expression for the wavevector mismatch \( \Delta k_I \) for Type I SHG phase-matching in terms of \( n_e(2\omega, \theta) \) and \( n_o(\omega) \), where \( n_o \) and \( n_e \) are the ordinary and extraordinary refractive indices, respectively. [3 marks]

(iii) Given that

\[
n_e(2\omega, \theta) = \left[ \frac{\cos^2 \theta}{\bar{n}_e^2(2\omega)} + \frac{\sin^2 \theta}{\bar{n}_e^2(2\omega)} \right]^{-\frac{1}{2}}
\]

where \( \bar{n}_e(2\omega) \) is the principal value of the extraordinary refractive index, show that for SHG of a \( \lambda = 1.064 \mu m \) laser in a beta borium borate (BBO) crystal the Type I phase-matching angle at a temperature \( T = 25^\circ \text{C} \) is \( \theta_I \approx 22.87^\circ \).

[Note: You are not required to derive an expression for \( \theta_I \).]

For BBO at \( T = 25^\circ \text{C} \), \( n_o(\omega) = 1.65510, n_o(2\omega) = 1.67490, \bar{n}_e(\omega) = 1.54250 \) and \( \bar{n}_e(2\omega) = 1.55550 \). [5 marks]

For BBO, \( n_o \) and \( \bar{n}_e \) vary with temperature, \( T \), according to the following relations:

\[
\frac{dn_o}{dT} = -9.3 \times 10^{-6}/^\circ \text{C}
\]

\[
\frac{dn_o}{dT} = -16.6 \times 10^{-6}/^\circ \text{C}
\]

which can be assumed to be valid at both \( \omega \) and \( 2\omega \).

SHG of a \( \lambda = 1.064 \mu m \) laser in a 1 cm long BBO crystal is phase-matched at 25°C with \( \theta_I = 22.87^\circ \). Although Type I phasematching in BBO is quite insensitive to small temperature fluctuations, a serious malfunction of a temperature controller causes the crystal temperature to increase by 50°C!

(iv) Determine the wavevector mismatch due to this temperature increase. [You will need to carry 5 decimal points in your calculations]. What is the corresponding coherence length? [5 marks]

(v) By at least what factor will the SH intensity be reduced? You may assume that the SHG was unsaturated before the temperature increase. [3 marks]
6. (i) A medium has a nonlinear polarisation response to an intense laser field 
\[ E = E_0 \cos(\omega t) \] given by 
\[ P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(3)} E^3 \]
where the symbols have their usual meanings. Show how this leads to an intensity-dependent refractive index. You may use \( \cos^3 x = \frac{3}{4} \cos x + \frac{1}{4} \cos(3x) \). [5 marks]

(ii) Explain briefly why an intensity-dependent refractive index is possible in a gaseous medium, but second harmonic generation is not. [2 marks]

An intense laser pulse (shown in figure below) has an intensity envelope given by
\[ I(t) = I_0 \left( 1 - \frac{t^2}{a^2} \right) \quad \text{for} \quad -a \leq t \leq a \]
\[ I(t) = 0 \quad \text{for} \quad t < -a, t > a , \]
where \( t \) is the time.

![Intensity envelope of a laser pulse](image)

The pulse propagates through a medium with an intensity-dependent refractive index.

(iii) By considering the phase of the pulse \( \phi(t) = \omega_0 t - k z \), show that its instantaneous frequency after propagating a distance \( L \) in the medium is given by
\[ \omega(t) = \omega_0 + \frac{2n_2 I_0 \omega_0 L}{ca^2} t \]
where \( \omega_0 \) is the central frequency of the pulse, \( n_2 \) is the nonlinear refractive index of the medium and \( c \) is the speed of light. [5 marks]

(iv) Sketch \( \omega(t) - \omega_0 \) as a function of time. What kind of frequency chirp does the pulse have? Assuming a negligible initial bandwidth, show that the bandwidth of the pulse is broadened by an amount
\[ \Delta \omega = \frac{4n_2 I_0 \omega_0 L}{ca} . \] [4 marks]

(v) Calculate \( \Delta \omega \) and the corresponding \( \Delta \lambda \) for a laser pulse with \( \lambda_0 = 800 \text{ nm} \) and a peak intensity of \( 5 \times 10^{10} \text{ W/cm}^2 \) [Note: not SI units] propagating through 5 cm of a medium with \( n_2 = 4.70 \times 10^{-16} \text{ cm}^2/\text{W} \). The “duration” of the pulse (= 2a) is \( 500 \times 10^{-15} \text{ s} \). [4 marks]

[TOTAL 20 marks]