UNIVERSITY OF LONDON
BSc/MSci EXAMINATION May 2007
for Internal Students of Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant Examination for the Associateship

SOLID STATE and ATOMIC PHYSICS

For Third-Year Physics Students
Tuesday 15th May 2007: 10.00 to 12.00

Answer ALL parts of Section A, ONE question from Section B and ONE question from Section C.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Write your CANDIDATE NUMBER clearly on each of the FOUR answer books provided.

If an electronic calculator is used, write its serial number in the box at the top right hand corner of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the horizontal box on the front cover of its corresponding answer book.

Hand in FOUR answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.
SECTION A

1. (i) The energy levels in hydrogen depend only on the \( n \) quantum number, but in multi-electron atoms they generally depend on both \( n \) and \( l \). Why? [2 marks]

(ii) Given an atom with total orbital angular momentum \( L \) and total spin angular momentum \( S \), what are the minimum and maximum values of the total angular momentum \( J \)? How many \( J \) values are possible? [2 marks]

(iii) Sodium, \( Z = 11 \), has the ground state configuration \([\text{Ne}]\,(3s)^1\) and is a “one-electron” atom. Magnesium, \( Z = 12 \), is a “two-electron” atom. Write down its ground state configuration and possible term values. [3 marks]

(iv) Sketch the Stark shift and weak field Zeeman shift energies of the ground state of Mg. Include a few words explaining each sketch. [3 marks]

[TOTAL 10 marks]
2. Write short notes on ALL of the following:-

(i) The effects of phonons on the electronic and optical properties of crystalline solids. [2 marks]

(ii) The *ensemble of oscillators* model of optical properties of a Solid. [3 marks]

(iii) Why magnetism is rare in Solids. [2 marks]

(iv) The theoretical relationship between drift and diffusion currents. [3 marks]

[TOTAL 10 marks]
SECTION B

3. (i) Explain the different levels of approximation for a multielectron atom in the context of calculating energy levels: a) **hydrogenic**, b) **configuration**, c) **term**, and d) **level**. As part of your answer briefly explain some of the physical effects which are treated in each case and use both He (Z=2) and Li (Z=3) to illustrate your answers.  

[11 marks]

(ii) What is a quantum defect? How is it defined?  

[2 marks]

(iii) Exchange symmetry in the ground state of He is certainly important, but much less important when discussing the energy levels of Li. Explain why this is the case.  

[4 marks]

(iv) What is the physical origin of spin-orbit coupling? Why can we treat it using the **L S** coupling approximation in alkali atoms? (Your answer should briefly explain the **L S** coupling approximation.)  

[3 marks]

[TOTAL 20 marks]
4. (i) Assume that the eigenvectors and eigenvalues have been found for a Hamiltonian \( \hat{H}_0 \), so that

\[
\hat{H}_0 |u_i^{(0)} \rangle = E_i^{(0)} |u_i^{(0)} \rangle.
\]

The energies \( E_i^{(0)} \) are non-degenerate. The system is subject to a perturbation \( \lambda \hat{H}_1 \). Starting from the first order Schrödinger equation

\[
\left( \hat{H}_0 + \lambda \hat{H}_1 \right) \left( |u_i^{(0)} \rangle + \lambda |u_i^{(1)} \rangle \right) = \left( E_i^{(0)} + \lambda E_i^{(1)} \right) \left( |u_i^{(0)} \rangle + \lambda |u_i^{(1)} \rangle \right)
\]

show that the first order energy shift is

\[
E_i^{(1)} = \langle u_i^{(0)} | \hat{H}_1 | u_i^{(0)} \rangle.
\]

[Hint: Express \( |u_i^{(1)} \rangle \) as a sum over \( |u_k^{(0)} \rangle \) and use orthogonality.] [10 marks]

(ii) For the interaction of an atom with a weak magnetic field \( B_z \), the perturbing Hamiltonian can be written as \( \hat{H}_{\text{mag}} = g_J \mu_B B_z \hat{J}_z \), where \( \hat{J}_z \) is the usual angular momentum operator, \( \mu_B \) is the Bohr magneton and \( g_J \) is given by

\[
g_J = \frac{3J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}
\]

(assuming we approximate the g-factor for a free electron to be \( g_e = 2 \) exactly). Consider the effect of this magnetic perturbation on electric dipole transitions between the ground state, \( 3s \) and first excited state, \( 3p \), in sodium. How many transitions are allowed between the \( J = 1/2 \) ground state and \( J = 1/2 \) excited state magnetic sublevels, assuming linearly polarized light? Sketch the transitions at \( B_z = 0 \) and at finite \( B_z \) and label the levels using appropriate spectroscopic notation. [6 marks]

(iii) How do these transition energies depend on \( B_z \)? [4 marks]

[TOTAL 20 marks]
SECTION C

5. (i) The electrical conductivity of intrinsic germanium at 300 K is found to be $2.2 \, (\Omega \text{m})^{-1}$. If the mobilities of the electrons and holes are $0.39 \, \text{m}^2 \, \text{V}^{-1} \, \text{sec}^{-1}$ and $0.19 \, \text{m}^2 \, \text{V}^{-1} \, \text{sec}^{-1}$ respectively, what is the intrinsic carrier concentration at 300 K? [5 marks]

(ii) A $10 \, \mu \text{W}$ beam of light of wavelength 650 nm falls on a germanium surface. Neglecting reflection processes and assuming that all the light is absorbed, estimate the number of additional electron-hole pairs generated per second. [4 marks]

(iii) If the beam is allowed to illuminate the surface continuously, what determines the steady state concentration of carriers in the germanium? [Germanium is an indirect band-gap material.] [3 marks]

(iv) The same beam illuminates a sample of gallium arsenide which has a direct band-gap of $1.42 \, \text{eV}$ at room temperature. The absorption coefficient of gallium arsenide is $10^7 \, \text{m}^{-1}$ and the sample is 200 nm thick. Neglecting reflection, calculate the fraction of this beam which is absorbed in the sample. [4 marks]

(v) If all electron hole pairs subsequently recombine radiatively, what power is dissipated in the Gallium Arsenide sample as heat? [4 marks]

[TOTAL 20 marks]
6. The *Shockley expression* for the current-voltage characteristics of a p-n junction reads

\[ I = I_0 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]. \]

(i) Describe the physical assumptions that have been made in deriving his expression. [4 marks]

A low-cost rectifying junction is formed between p-type silicon \((N_A = 3 \times 10^{23} \text{ m}^{-3})\) and n-type silicon \((N_D = 1 \times 10^{23} \text{ m}^{-3})\). Away from the junction the Fermi energy is separated from the appropriate band edges by an energy interval of 75 meV/50 meV in the n-type/p-type materials respectively. The \(I_0\) parameter is \(10^{-9}\) amps.

(ii) In the absence of an applied bias, sketch (approximately to scale) the energy profile of the conduction and valence band edges across the junction, clearly indicating the Fermi level position. [4 marks]

(iii) Calculate the built-in voltage, \(V_{bi}\), across the junction. [2 marks]

(iv) Calculate the fraction of the depletion zone thickness which lies to the n-type side of the junction between the two materials. [3 marks]

(v) The junction, when used as a short-circuited photovoltaic solar cell at 300 K, passes a current of 150 mA under illumination. Calculate the open-circuit voltage, \(V_{oc}\), the cell will develop when its contacts are subsequently disconnected. [3 marks]

(vi) Analysis of the solar cell’s performance reveals an efficiency of only 0.1%. Suggest reasons, and their possible remedies, for this poor performance. [2 marks]

(vii) A forward bias is now applied to the same diode with the hope of generating useful quantities of emitted light at the bandgap energy. Explain, giving your reasoning, whether or not this hope is realistic. [2 marks]

[Data:- \(E_G\) for silicon = 1.12 eV.]

[Total 20 marks]