Answer THREE questions.

All questions carry equal marks.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Write your CANDIDATE NUMBER clearly on each of the THREE answer books provided.

If an electronic calculator is used, write its serial number in the box at the top right hand corner of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the horizontal box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.
SECTION A

1. (i) Stating your assumptions, show that the dry adiabatic lapse rate is given by:

\[ \Gamma_d = \frac{g}{c_p}, \]

where \( c_p \) is the specific heat at constant pressure of dry air and \( g \) is acceleration due to gravity. [5 marks]

(ii) Assuming dry air explain what is meant by (a) a stable, (b) an unstable and (c) a neutrally stable atmosphere. [3 marks]

(iii) For a stable atmosphere derive an expression for the frequency of oscillation of an air parcel displaced a small vertical distance from its equilibrium position (the buoyancy frequency) in terms of the temperature lapse rate. [6 marks]

(iv) A meteorological balloon is required to take measurements up to an altitude where the pressure and temperature are 10 hPa and 225 K respectively. The balloon is constructed of a non-stretch material which remains slack until the balloon reaches its peak altitude and has expanded to its full spherical shape. If the balloon contains helium (molecular weight 4 g mol\(^{-1}\)), the payload weighs 100 kg and the fabric is of thickness 25 \( \mu \)m and density 1000 kg m\(^{-3}\), what approximate radius of balloon is needed? [Universal gas constant \( R = 8.314 \text{J mol}^{-1} \text{K}^{-1}. \)] [6 marks]

[TOTAL 20 marks]
2. (i) The Clausius-Clapeyron expression for the pressure-temperature relationship at a phase change is given by:

\[
\frac{\partial p}{\partial T} \bigg|_{\text{phase change}} = \frac{\delta S}{\delta V},
\]

where \( \delta S \) and \( \delta V \) are the changes per unit mass, during a phase change of the system, in entropy and volume respectively. Show how this may be used to predict that the saturation vapour pressure of water, \( e_s(T) \), may be expressed as a function of temperature by \( e_s(T) = e_0 \exp \left(-\frac{L}{R_v T} \right) \) where \( L \) is the latent heat of vaporisation, \( R_v \) is the gas constant for water vapour and \( e_0 \) is a constant. \[6 \text{ marks}\]

(ii) Air at a temperature 20°C and pressure 1000 hPa has a dew point of 15°C. What is (a) its relative humidity and (b) its specific humidity (water vapour mass mixing ratio)?

[Take \( e_0 = 2.45 \times 10^9 \) hPa; \( L = 2.5 \times 10^6 \) J kg\(^{-1}\); universal gas constant \( R = 8.314 \) J mol\(^{-1}\) K\(^{-1}\); molecular weights of dry air and water are 29 and 18 g mol\(^{-1}\) respectively.] \[6 \text{ marks}\]

(iii) Air initially at sea level with a temperature 20°C and dew point 15°C is forced to rise over a mountain of height 1000 m. What are the temperature, dew point and relative humidity of the air when it has sunk to a level 200 m above sea level on the other side of the mountain?

[Take the dry adiabatic lapse rate to be 10 K km\(^{-1}\) and the saturated adiabatic lapse rate to be 7 K km\(^{-1}\).] \[8 \text{ marks}\]

[TOTAL 20 marks]
3. (i) Define the geopotential height, $Z$. Using the hydrostatic equation and the ideal gas law show that:

$$\frac{\partial Z}{\partial p} = - \frac{R_a T}{g_0 p}$$

where $R_a$ is the gas constant for dry air and $T$ and $p$ are temperature and pressure respectively. [3 marks]

(ii) In isobaric coordinates, and neglecting friction, the horizontal momentum equation can be written:

$$\frac{D\mathbf{V}}{Dt} = - f \mathbf{k} \times \mathbf{V} - g_0 \nabla_p Z,$$

where $\mathbf{V}$ is the horizontal velocity, $f$ the Coriolis parameter and $\mathbf{k}$ the unit vector in the vertical direction. Explain carefully what is represented by each of the three terms in this equation. [3 marks]

(iii) Discuss the circumstances under which a good approximation to $\mathbf{V}$ is given by:

$$\mathbf{V} = \frac{g_0}{f} \mathbf{k} \times \nabla_p Z.$$

[3 marks]

(iv) Explaining any assumptions you make, find an expression for the variation of $\mathbf{V}$ with $\ln p$ as a function of $\nabla_p T$. [3 marks]

(v) In a region near 60°N the geopotential height contours on a 500 hPa chart are oriented east-west and the spacing between adjacent contours (at 40 m intervals) is 200 km, with $Z$ decreasing towards the north. Calculate the speed and direction of the geostrophic wind. [3 marks]

(vi) At 750 hPa the geostrophic wind has the same magnitude but is from the direction 210°. What can you deduce about the horizontal gradient of the mean temperature of the 750–500 hPa layer? [The gas constant for dry air $R_a = 287 \text{ J kg}^{-1} \text{K}^{-1}$.] [5 marks]

[TOTAL 20 marks]
4. (i) State what is meant by (a) absolute, (b) relative and (c) planetary vorticity and how they are related. [4 marks]

(ii) Show that non-divergent horizontal flow can be expressed in terms of a streamfunction, $\Psi$, such that $u = -\frac{\partial \Psi}{\partial y}$ and $v = \frac{\partial \Psi}{\partial x}$, where $u$ and $v$ are, respectively, the zonal and meridional wind components and $(x, y)$ are the distances in the zonal and meridional directions. How is $\Psi$ related to relative vorticity? [4 marks]

(iii) Starting from the principle of conservation of absolute vorticity, and assuming small perturbations $(u', v')$ on a uniform zonal flow $\bar{u}$, deduce the Rossby wave equation:

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) \nabla^2 \Psi' + \beta \frac{\partial \Psi'}{\partial x} = 0,$$

where $\beta = \frac{df}{dy}$ and $f$ is the Coriolis parameter. [6 marks]

(iv) Find wave-like solutions to this equation and thus expressions for the phase and group velocities of Rossby waves. Why do these imply that stationary Rossby waves propagate energy downstream? [6 marks]

[TOTAL 20 marks]
5. (i) Monochromatic radiation, of wavelength $\lambda$, passes through a pure gas of mass absorption coefficient $\alpha_\lambda = 1 \text{ kg}^{-1} \text{ m}^2$. What fraction of the beam is absorbed in passing through a slab of gas containing 10 g m$^{-2}$? [You may neglect scattering of radiation.] [3 marks]

(ii) What mass per unit area of gas would the layer have to contain in order to absorb 50% of the incident radiation? [2 marks]

(iii) The gas is now mixed with dry air (assumed radiatively inactive) at a mass mixing ratio of $10^{-4}$. What horizontal path of this air mixture would absorb 50% of incident radiation at wavelength $\lambda$? [Take the density of the mixture to be $\rho_0 = 1 \text{ kg m}^{-3}$.] [2 marks]

(iv) What vertical path would be needed for 50% absorption? Comment on any difference with the result of part (iii). [Assume a density $\rho_0$ at the surface and a density scale height of 7 km.] [3 marks]

(v) Schwarzschild’s equation for the change in intensity, $I_\lambda$, of radiation crossing a slab of non-scattering gas of thickness $ds$ can be written:

$$dI_\lambda(s) = -(I_\lambda(s) - B_\lambda(s))\rho_0(s)\alpha_\lambda ds.$$ 

Carefully explain the terms in this equation. [4 marks]

(vi) By integrating Schwarzschild’s equation along a vertical path from the surface to the top of the atmosphere find an expression for the intensity of infrared radiation received by a satellite in terms of the surface temperature and the atmospheric temperature (Planck function) and transmittance profiles. [6 marks]

[TOTAL 20 marks]
6. (i) Describe the main factors that determine the Earth’s radiation budget. [4 marks]

(ii) Explain why an increase in atmospheric concentration of a ‘greenhouse’ gas only temporarily affects the radiation budget at the top of the atmosphere but permanently affects the radiation budget at the Earth’s surface. [2 marks]

(iii) What is meant by the radiative forcing of climate change? [2 marks]

(iv) How is it possible that an increase in concentration of only a few parts per million of carbon dioxide may significantly influence surface temperatures? [3 marks]

(v) The planetary equilibrium temperature, $T_E$, is defined by:

$$F_s = \sigma T_E^4,$$

where $F_s (= 240 \text{ W m}^{-2})$ is the global annually averaged absorbed solar radiation and $\sigma (= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})$ is the Stefan-Boltzman constant. Show that this expression leads to an estimate of the climate sensitivity parameter $\lambda = \partial T/\partial F$ of approximately $0.27 \text{ K (W m}^{-2})^{-1}$. [3 marks]

(vi) During the last glacial maximum (LGM) the planetary albedo was about 0.01 higher than the present value of 0.28. Assuming that there has been no change in incident solar irradiance what value of radiative forcing does this represent? How does this compare with greenhouse gas forcing which is estimated to have been $3.7 \text{ W m}^{-2}$ since the LGM? [3 marks]

(vii) Global mean surface air temperature during the LGM was about 5 K colder than at present. Using your radiative forcing values from part (vi) estimate a value of $\lambda$. Suggest reasons for the difference between this value and that derived in part (v). [3 marks]

[TOTAL 20 marks]