8.1 OPTICAL FIBRE TYPES

In this Chapter, we shall consider the properties of optical fibre, by far the most important type of waveguide for the transmission of information at optical frequencies. Optical fibre usually (though not always) has a circular symmetry, and is most often fabricated from very pure silica, or SiO$_2$. This has a refractive index of $n \approx 1.458$ at $\lambda_0 = 850$ nm. Figure 8.1-1 shows the typical geometry of a silica fibre; by selective doping, the centre of the fibre (which is known as the core) is arranged to have a higher refractive index than the surrounding region (the cladding), thus forming a waveguide. Useful dopants include germania (GeO$_2$) and phosphorus pentoxide (P$_2$O$_5$), both of which increase the refractive index of silica, and boric oxide (B$_2$O$_3$) and fluorine (F$_2$), which reduce it. Thus, a typical fibre might consist of a GeO$_2$:SiO$_2$ core, with a SiO$_2$ cladding. Alternatively, a pure SiO$_2$ core could be used, with a B$_2$O$_3$:SiO$_2$ cladding.

Other low-loss fibre materials that have been investigated include low melting-point silicate glasses (soda-lime silicates, germanosilicates and borosilicates) and halide crystals (TlBr, TlI, KCl, CsI and AgBr). Most recently, attention has been transferred to the fluoride glasses (mixtures of GdF$_3$, BaF$_2$, ZrF$_4$ and AlF$_3$), for reasons which will be discussed in the following section. However, whatever the exact composition of the fibre, the cladding is almost always surrounded by a protective plastic jacket, which acts as a strain relief and prevents the ingress of water.

Alternatively, plastic-coated silica fibres (or PCS fibres, which might consist of a natural quartz core clad in silicone resin) and all-plastic fibres (for example, consisting of a polystyrene core with a methyl methacrylate cladding) are often used. Although these are cheap, they suffer from considerably higher propagation loss than silica fibre ($\approx 10$ dB/km for PCS fibre, and $\approx 500$ dB/km for all-plastic fibre), and are therefore suitable only for transmission over short distances. For all-plastic fibres, the limit is normally tens of metres.

There are many variations on the basic geometry. Some are illustrated in Figure 8.1-2, which shows a number of different cylindrically symmetric fibre cross-sections, and the radial refractive index distribution defining them. The most important distinctions are as follows. Firstly, optical fibre is generally either single-moded or very multi-moded - the number of modes supported by a multi-moded fibre is normally very large (hundreds). Secondly, the core may be defined either by an abrupt increase in the refractive index, or by
a more gradual variation in index (which is typically parabolic). The former is known as step-index fibre, the latter as graded-index fibre.

![Figure 8.1-2 Optical fibre types.](image)

The dimensions of these fibre types differ quite considerably. Single-mode fibre has roughly the dimensions of a human hair, with a cladding diameter of the order of 125 µm, and a core diameter of 8 - 12 µm (depending on the operating wavelength). Multi-mode fibre is rather thicker: cladding diameters range from 125 - 400 µm, while the core diameters range from 50 - 200 µm. For field use, fibres are supplied in cable form. Cable styles vary widely, but a typical construction would consist of a plastic tube, strengthened with steel wires or Kevlar to resist tensile strain, which is filled with the fibre in a loose helical lay. A recent development has been the availability of fibre in the form of ribbon cable. This consists of a number (e.g. 8) of single-mode fibres held parallel in a flat ribbon by a plastic matrix. The individual fibres are often colour-coded, and may be separated out into individual strands, allowing multiple-fibre links to be constructed very simply.

### 8.2 LOSS IN SILICA AND FLUORIDE GLASS FIBRE

One of the most important features of optical fibre is its exceptionally low propagation loss, which is normally quoted in units of **decibels** (dB), according to:

\[
\text{loss} = -10 \log_{10}(P_{\text{out}}/P_{\text{in}}) \text{ dB}
\]

8.2-1

Thus, reduction of the power by 50% between input and output corresponds to 3 dB loss. Expressed in this way, loss varies linearly with distance, so fibre may be characterised by attenuation in dB/km. Current fibre performance has been achieved through a considerable research and development effort, which began in the mid-1960s. In fact, loss in silica-based fibre has been reduced quite staggeringly, from more than 1000 dB/km in 1966 to the present value of \( \approx 0.2 \text{ dB/km} \) (at \( \lambda_0 = 1.55 \mu\text{m} \)).

There are a number of fundamental mechanisms, which combine to set a lower limit on transmission loss. These are illustrated by Figure 8.2-1, which shows a typical plot of attenuation versus optical wavelength for silica fibre. At short wavelengths (\( \lambda_0 < 1.6 \mu\text{m} \)), attenuation is mainly dominated by scattering loss (rather than by absorption due to
electronic transitions at ultraviolet wavelengths, as might be expected). The effect itself is known as Rayleigh scattering; its origins lie in any small inhomogeneities and imperfections in the structure of the glass forming the fibre - for example, compositional fluctuations on solidification, trapped gas bubbles, dopants and so on. It can be shown that this type of scattering varies as $1/\lambda_0^4$, so it is responsible for the sharp rise in attenuation in Figure 8.2-1 at short wavelengths.

![Figure 8.2-1 Typical attenuation characteristics for silica-based optical fibre.](image)

At longer wavelengths, however, the attenuation is mainly caused by intrinsic absorption, arising from the excitation of lattice transitions at near-infrared wavelengths. Minimum attenuation is therefore obtained when the Rayleigh scattering and infrared absorption curves cross, which occurs at around $\lambda_0 = 1.55 \mu m$. If low loss is the main criterion, this is the most desirable operating point. Modern fibres approach the two envelopes of Rayleigh scattering and infrared absorption very closely. However, there may also be significant attenuation near a number of discrete wavelengths, as can be seen in Figure 8.2-1. This type of extrinsic absorption can be due either to the dopants themselves, or to additional impurities. The most significant absorption band lies at $\lambda_0 = 1.39 \mu m$, and is caused by the presence of residual hydroxyl (OH) ions, which also give rise to a number of smaller absorption peaks. These ions originate as water contamination, and must be removed by careful dehydration. If this is done, the ultimate limit of low-loss can be achieved almost exactly.

If different materials could be found that have their infrared absorption lines at longer wavelengths, the operating wavelength could be shifted in the same direction. The effect of Rayleigh scattering would then be reduced, and lower minimum loss obtained. This is the thinking behind current research on fluoride glass fibres. It is believed that such fibres could have an optimum attenuation as low as 0.001 dB/km, an incredibly small figure. Figure 8.2-2 shows theoretically predicted loss spectra for two different fluoride glass fibres, based on (a) $BaF_2-GdF_3-ZrF_4$ glass and (b) $CaF_2-BaF_2-YF_3-AlF_3$ glass. These suggest that improved performance should be available at slightly longer wavelengths, in the range $\lambda_0 = 2.5 - 3.5 \mu m$. Although this encouraging prediction has yet to be fulfilled (the current 'best' is only 0.7 dB/km), research is actively being continued.
8.3 STEP-INDEX OPTICAL FIBRES

We shall now consider the propagation of rays and guided modes in optical fibre, beginning with step-index fibre. Functionally, this operates in a similar way to the slab guide introduced in Chapter 6; the fibre is constructed from a core of radius $a$ and refractive index $n_1$, and a cladding of refractive index $n_2$ (such that $n_1 > n_2$), as shown in Figure 8.3-1a.

Figure 8.3-1 (a) The refractive index distribution of step-index fibre; (b) and (c) propagation of meridional and skew rays.

**Figure 8.2-2** Predicted loss spectra for two fluoride glass fibres (based on data by Shibata et al., Elect. Lett. 17, 775-777 (1981))
The guiding mechanism is again total internal reflection at the core/cladding interface. However, consideration of the possible paths that may be followed by rays travelling down the fibre reveals a new feature.

Two entirely different types of ray are now possible. **Meridional rays** follow pathways that can be drawn on a single plane (as shown in Figure 8.3-1b). These are directly analogous to the rays that propagate in planar slab guides. **Skew rays**, on the other hand, do not travel in the exact centre of the fibre, but propagate in an annular region near the outer edge of the core, following off-axis helical paths that effectively circle the fibre axis after a finite number of reflections. These have no such analogy in a slab model, and are best visualised in terms of their projection on the fibre cross-section (Figure 8.3-1c). The existence of such rays implies that the analysis of an optical fibre will be more difficult than a simple extension to cylindrical geometry would suggest.

**NUMERICAL APERTURE**

One of the most important parameters of a fibre is its **numerical aperture**, or N.A. This determines the acceptance cone of the fibre, and the angular spread of radiation it may emit. Despite the distinction between meridional and skew rays outlined above, a reasonable approximation to the NA may be obtained by considering only the former, as shown in Figure 8.3-2. The extent of the acceptance cone will be determined by the maximum angle \( \theta_{\text{emax}} \) that an external ray entering the end-face of the fibre may have, while still giving rise to totally-reflected rays within. This is limited by the onset of cutoff, so the largest angle \( \theta_{\text{imax}} \) allowed inside the fibre must be set by the critical angle, \( \theta_c \).

![Figure 8.3-2](image-url)  
**Figure 8.3-2** Geometry for calculation of the numerical aperture of a step-index fibre,

Simple trigonometry shows that \( \theta_{\text{imax}} = \pi/2 - \theta_c \). The internal and external angles \( \theta_{\text{imax}} \) and \( \theta_{\text{emax}} \) are, of course, related by Snell's law, so assuming that the fibre is surrounded by a medium of refractive index \( n_e \), we must have:

\[
 n_e \sin(\theta_{\text{emax}}) = n_1 \sin(\theta_{\text{imax}}) = n_1 \cos(\theta_c)
\]  
8.3-1

However, \( \theta_c \) is defined by the fibre core and cladding indices, as \( \theta_c = \sin^{-1}(n_2/n_1) \). After slight re-arrangement, we may then express Equation 8.3-1 as:

\[
 n_e \sin(\theta_{\text{emax}}) = \sqrt{(n_1^2 - n_2^2)}
\]  
8.3-2

According to the definition in Chapter 4, this is equal to the numerical aperture of the fibre. A large difference between core and cladding indices will therefore usually result in a high NA. Now, if the external medium is air, we know that \( n_e = 1 \). Using a small-angle
approximation, we may then put \( \sin(\theta_{\text{emax}}) = \theta_{\text{emax}} \). The solid angle \( \Omega \) of the acceptance cone is then:

\[
\Omega = \pi \theta_{\text{emax}}^2 = \pi (n_1^2 - n_2^2)
\]

8.3-3

If skew rays are considered as well, it can be shown that both the numerical aperture and the value of \( \Omega \) derived above are under-estimates, but they remain reasonable approximations.

EXACT MODES OF A STEP-INDEX FIBRE

Full analysis of a step-index fibre requires the solution of Maxwell's equations for a cylindrical geometry. Consequently, the fields are constructed from more abstract mathematical functions than were required for the slab guide (actually, Bessel functions), and the modes themselves are more complicated. For many of them, the axial field components \( E_z \) and \( H_z \) are both non-zero, so these modes are known as \textbf{hybrid} rather than TE or TM modes. Furthermore, due to the circular cross-section, the transverse fields have an angular variation as well as a radial one, so they require two identifying parameters instead of a single mode number. Because of these complications, we will omit the detailed analysis (to be found in many other texts) and present only the broad conclusions.

The modes of the step-index optical fibre are, in general, classified as \( \text{TE}_{0,\nu} \), \( \text{TM}_{0,\nu} \), \( \text{EH}_{\mu,\nu} \) or \( \text{HE}_{\mu,\nu} \) types. The last two are hybrid fields, which have \( E_z \) and \( H_z \) as the dominant axial component, respectively. The most important mode is the \( \text{HE}_{1,1} \) mode (roughly equivalent to the \( \text{TE}_0 \) and \( \text{TM}_0 \) modes of the slab guide) which cuts off at zero frequency. However, the \( \text{TE}_{0,1} \) and \( \text{TM}_{0,1} \) modes are also significant. These have the next lowest cutoff frequencies, and so their presence or absence may be used to define whether the fibre is multi-moded or not. Now, in Chapter 6 we showed that a parameter \( V = (k_0 h/2) \sqrt{(n_1^2 - n_2^2)} \) could be used to characterise a symmetric slab guide of thickness \( h \) and core and cladding indices \( n_1 \) and \( n_2 \). In the same way, a \( V \)-value may be defined for a step-index fibre of core radius \( a \) and similar indices, in the form:

\[
V = (k_0 a) \sqrt{(n_1^2 - n_2^2)}
\]

8.3-4

It turns out that the \( \text{TE}_{0,1} \) and \( \text{TM}_{0,1} \) modes cut off when \( V = 2.405 \). Thus, if the frequency is such that \( V < 2.405 \), only one mode will propagate and the fibre is single-moded. Assuming that \( n_1 = n_2 = n \), and \( n_1 - n_2 = \Delta n \) (which will be the case in weakly-guiding fibres) the minimum wavelength for single-mode operation is therefore:

\[
\lambda_0 = (\pi d/2.405) \sqrt{(2n \Delta n)}
\]

8.3-5

where \( d \) is the fibre diameter. For \( d = 8 \mu m \), \( n = 1.5 \), and \( \Delta n = 0.001 \), say, we require \( \lambda_0 > 0.57 \mu m \).

LP MODES

The hybrid modes have a complicated polarization, although the \( \text{HE}_{1,1} \) mode is almost plane-polarized. However, for weak guides, it can be shown that all the other modes can be grouped together in combinations, which are \textbf{degenerate} (which means they have the same propagation constant or phase velocity). A linear combination of these modes can then be used to construct a new set of nearly plane-polarized fields, which represent a considerable simplification over the exact ones. These modes are called 'linearly polarized', and are given new labels of the form \( \text{LP}_{\mu,\nu} \). Table 8.3-1 shows the constituents of some low-order LP...
modes. The particular case of the HE_{1,1} mode is the LP_{0,1} mode; as a rule, the LP_{1,1} mode is constructed from the HE_{2,v}, TM_{0,v} and TE_{0,v} modes, and the LP_{µ,v} mode from the HE_{µ+1,v} and EH_{µ-1,v} modes (for µ ≠ 0 or 1).

<table>
<thead>
<tr>
<th>LP mode</th>
<th>Exact mode constituents</th>
<th>LP mode</th>
<th>Exact mode constituents</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP_{0,1}</td>
<td>HE_{1,1}</td>
<td>LP_{0,2}</td>
<td>HE_{1,2}</td>
</tr>
<tr>
<td>LP_{1,1}</td>
<td>HE_{2,1}, TM_{0,1} &amp; TE_{0,1}</td>
<td>LP_{1,2}</td>
<td>HE_{2,2}, TM_{0,2} &amp; TE_{0,2}</td>
</tr>
<tr>
<td>LP_{2,1}</td>
<td>HE_{3,1} &amp; EH_{1,1}</td>
<td>LP_{2,2}</td>
<td>HE_{3,2} &amp; EH_{1,2}</td>
</tr>
</tbody>
</table>

Table 8.3-1  Constituents of some low-order LP modes

Figure 8.3-3 shows approximate representations of the intensity distributions of three low-order LP modes. The LP_{0,1} mode is circularly-symmetric, with a field distribution containing a single peak at the centre of the fibre core. The higher-order modes, on the other hand, have multilobed patterns, with 2µ maxima as measured in the circumferential direction, and ν maxima in the radial direction.

Figure 8.3-3  Approximate intensity distributions for the LP_{0,1}, LP_{2,1} and LP_{4,2} modes.

The dispersion diagram for the step-index fibre is of the general form shown in Figure 8.3-4. The main difference in relation to the slab guide is that higher-order modes start to propagate in much greater numbers as soon as the V-value exceeds the limit for single-mode operation.

Figure 8.3-4  General dispersion diagram for step-index optical fibre.
In fact, it can be shown that the number of modes propagating is given approximately by:

\[ N \approx \frac{V^2}{2} \]

provided \( N \) is large. A fibre with a \( V \)-value of 10 would therefore support \( \approx 50 \) modes.

**DESIGN EXAMPLE**

Using the approximate guidelines presented so far, we may compare the main features of single and multimode fibres. With the typical values of \( d = 8 \) \( \mu \text{m} \), \( n_1 = 1.5 \) and \( n_2 = 1.499 \) for the former, and \( d = 50 \) \( \mu \text{m} \), \( n_1 = 1.5 \) and \( n_2 = 1.49 \) for the latter, we obtain the parameters shown in Table 8.3-2 for a wavelength of \( \lambda_0 = 0.85 \) \( \mu \text{m} \). The high numerical aperture of the multimode fibre should be noted, together with the large number of modes that it supports.

<table>
<thead>
<tr>
<th>Fibre</th>
<th>( d ) (( \mu \text{m} ))</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>NA</th>
<th>V-value</th>
<th>No. of modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>8</td>
<td>1.5</td>
<td>1.499</td>
<td>0.055</td>
<td>1.62</td>
<td>1</td>
</tr>
<tr>
<td>#2</td>
<td>50</td>
<td>1.5</td>
<td>1.49</td>
<td>0.17</td>
<td>31.95</td>
<td>510</td>
</tr>
</tbody>
</table>

Table 8.3-2  Characteristic parameters of typical single- and multi-mode fibres.

**OTHER FIBRE MODES**

Just as in the slab guide, we may expect the guided modes of a fibre to be complemented by radiation modes, and that the full range of modes will exist in two sets, each corresponding to one of two possible polarizations. While this is true, there are two important new features. Firstly, the radiation modes are normally subdivided into fields that propagate inside and outside the cladding. The former are known as cladding modes, and are often a nuisance. They may be removed using a device known as a mode-stripper. In this, the plastic jacket is removed from a short section of the fibre, which is placed on a glass plate in a pool of index-matching liquid. Reflection at the edge of the cladding is then greatly reduced, and most of the unwanted light radiates into the glass plate. Secondly, since the fibre is cylindrically symmetric, the two sets of polarization modes are degenerate. A single-moded fibre therefore actually supports two modes of orthogonal polarization, which propagate at the same speed. As we will show later, this normally results in a random polarization fluctuation.

**8.4 PARABOLIC-INdex OPTICAL FIBRES**

We now consider parabolic-index fibre. Here, the possible ray trajectories are still more complicated. Because the refractive index distribution is now a continuous function of radius, rays no longer propagate as straight lines, with total internal reflection causing abrupt changes in direction. Instead, their paths are determined by a process of continual refraction. This has the effect of gradually bending the direction of any ray travelling away from the axis, until it turns towards the axis once again. For meridional rays, the result is a sinusoidal trajectory, oscillating on either side of the axis. Figure 8.4-1a shows two different ray paths in the same fibre. As we shall shortly demonstrate, each has almost exactly the same periodicity in the axial direction, although the turning points (e.g. \( z_a \) and \( z_b \)) lie at different radii. This has important consequences for signal transmission.
For skew rays, a similar process of continual refraction occurs. Consequently, the projection of a skew ray trajectory on the fibre cross-section has the appearance of a smooth, rotating orbit, as shown in Figure 8.4-1b. In fact, the ray path can again be shown to lie entirely between two coaxial cylinders. These are known as **caustic surfaces**, and represent the locii of all possible turning points.

Figure 8.4-1 (a) and (b) Trajectories of meridional and skew rays in a parabolic-index fibre.

**RAY TRAJECTORIES IN PARABOLIC-INDEX MEDIA**

Using some fairly simple scalar analysis, it is possible to calculate the trajectory of a ray in a graded-index guide, as we now show. We start by noting that the exact refractive index profile of the fibre is given by the full line in Figure 8.4-2. However, on the assumption that the light is confined mainly to the region of parabolic variation, we can disregard the index steps at the edge of the core and cladding, and model the index distribution by a continuous parabolic function of the form:

\[
n(r) = n_0 \sqrt{1 - \left(\frac{r}{r_0}\right)^2}
\]

where \(n_0\) is the refractive index at the centre of the fibre, and \(r_0\) is a characteristic radius. This profile is shown dashed in Figure 8.4-2; while it implies a negative refractive index when \(r > r_0\), we can assume that \(r_0\) is large enough for this to be unimportant for all practical purposes.

At this point, we shall adopt a one-dimensional model, ignoring the y-variation of the refractive index and constraining the trajectory to lie in the x - z plane. This will allow us to deduce many of the features of a two-dimensional guide in a qualitative way. Later on, we will check for agreement with the more complicated geometry. With this approximation, we may put \(r = x\) and \(r_0 = x_0\) in Equation 8.4-1. Now, the direction of the ray at any point \((x, z)\) may be then defined by a vector \(k\), such that \(|k|\) is equal to the local value of the propagation constant, \(k_0 n(x)\). Similarly, the local slope of the trajectory may be found as:
\[
dx/dz = k_x/k_z \tag{8.4-2}\]

where \(k_x\) and \(k_z\) are the \(x\)- and \(z\)-components of \(k\). However, if the entire trajectory corresponds to a guided mode, we may also set \(k_z = \beta\), where \(\beta\) is the modal propagation constant. Since \(k_x\) and \(k_z\) are related by \(k_x^2 + k_z^2 = |k|^2\), we obtain:

\[
k_x = \sqrt{(k_0^2 n_0^2 - \beta^2)} \tag{8.4-3}\]

Substituting from Equation 8.4-1, we then get:

\[
k_x = \sqrt{(k_0^2 n_0^2 - \beta^2) - k_0^2 n_0^2 (x/x_0)^2} \tag{8.4-4}\]

Equation 8.4-2 may be therefore written as:

\[
\beta^2 (dx/dz)^2 = (k_0^2 n_0^2 - \beta^2) - k_0^2 n_0^2 (x/x_0)^2 \tag{8.4-5}\]

We might guess that the solution of Equation 8.4-5 is indeed a sinusoidal trajectory, of the general form \(x = A \sin(Bz + \Psi)\), where \(\Psi\) is an arbitrary phase factor. We shall now try to verify this hypothesis, and simultaneously determine the unknown constants \(A\) and \(B\).

Performing the necessary differentiation, substituting into Equation 8.4-5, and rearranging the terms, we get:

\[
(k_0^2 n_0^2 - \beta^2) - (k_0^2 n_0^2 A^2/x_0^2) \sin^2(Bz + \Psi) - \beta^2 A^2 B^2 \cos^2(Bz + \Psi) = 0 \tag{8.4-6}\]

Now, the only way Equation 8.4-6 can be satisfied for all \(z\) is if the \(\sin^2\) and \(\cos^2\) terms can somehow be removed. This can be done if their coefficients are equal (since \(\sin^2(\theta) + \cos^2(\theta) = 1\)), which requires:

\[
k_0^2 n_0^2 A^2/x_0^2 = \beta^2 A^2 B^2 \quad \text{or} \quad B = k_0 n_0 / \beta x_0 \tag{8.4-7}\]

This places an immediate restriction on the value of \(B\), the spatial frequency of the periodic oscillation of the trajectory. Assuming that Equation 8.4-7 is indeed satisfied, Equation 8.4-6 reduces to:

\[
(k_0^2 n_0^2 - \beta^2) - (k_0^2 n_0^2 A^2/x_0^2) = 0 \tag{8.4-8}\]

so that the amplitude \(A\) of the oscillation is given by:

\[
A = x_0 \sqrt{(1 - \beta^2/k_0^2 n_0^2)} \tag{8.4-9}\]

Equation 8.4-9 shows that the smaller the value of \(\beta\) (i.e., the higher the order of the mode) the larger the amplitude. Our solution for the ray trajectory is therefore:

\[
x = x_0 \sqrt{(1 - \beta^2/k_0^2 n_0^2)} \sin((k_0 n_0 / \beta x_0)z + \Psi) \tag{8.4-10}\]

Now, for weak guides (when \(\beta \approx k_0 n_0\) for all modes), \(B \approx 1/x_0\). With this approximation, we can put:

\[
x = x_0 \sqrt{(1 - \beta^2/k_0^2 n_0^2)} \sin(z/x_0 + \Psi) \tag{8.4-11}\]
Equation 8.4-11 implies that the spatial frequency of the trajectory is virtually independent of $\beta$, i.e. the same for all modes. It turns out that the above analysis is modified somewhat in the case of a two-dimensional parabolic-index fibre, but only qualitatively so, providing comforting evidence for the shape of the ray paths in Figure 8.4-1. More importantly, it allows the use of a parabolic-index medium as a type of imaging device, known as a **GRIN-rod** lens (where the GRIN acronym stands for gradient-index).

**GRIN-ROD LENSES**

A GRIN-rod lens is a short length of glass rod of moderately large diameter (1 - 3 mm), which has the radial index variation of Equation 8.4-1. Formally, it is a section of highly multimoded fibre, but in practice no cladding is used. It is fabricated by ion exchange of thallium- or caesium-doped silica glass, in a bath of a molten salt. Typically, KNO$_3$ is used, at a temperature of 500$^\circ$C. During the exchange, sodium and either thallium or caesium ions diffuse out of the glass, and are replaced by potassium ions from the melt. The difference in polarizability of K$^+$ and Tl$^+$ ions then causes a reduction in the refractive index. Now, the local concentration of Tl$^+$ at the end of the process is determined by standard diffusion equations. Due to the cylindrically symmetric geometry, the concentration is higher at the centre of the rod than at the periphery, following a parabolic variation in between. The concentration of K$^+$ ions, on the other hand, is approximately uniform. The net result is a refractive index profile corresponding well to the ideal variation. GRIN-rod lenses are produced commercially by the Nippon Sheet Glass Co. of Japan, under the trade name 'SELFOC micro-lens'.

We shall again base the following argument on one-dimensional analysis. Firstly, we can guess that rays in a GRIN-rod lens will also follow periodic trajectories. The period $P$ of the oscillation is defined as the **pitch length**, given by:

$$P = 2\pi x_0$$

It is normal for the lens length to be chosen as some fraction of a pitch, close to either $P/2$ (a half-pitch lens) or $P/4$ (quarter-pitch). If this is done, a GRIN-rod lens may perform imaging operations, as we now demonstrate. Figure 8.4-3a shows a half-pitch lens, used to form an image of a point source of light located on-axis at A.

![Figure 8.4-3](image)

Figure 8.4-3  a) a 0.5P GRIN-rod lens, imaging a point source at A at A’; b) a 0.25P lens, used to image a point source at A at infinity, and c) a 0.23P lens, with working tolerance.
Naturally, this source will emit rays in all directions, but each will begin at the same point. Assuming that this is defined as \( x = 0, \ z = 0 \), the trajectories of all rays inside the lens have the form of Equation 8.4-11, with the phase factor \( \Psi = 0 \). Each sinusoidal trajectory will have a different amplitude, so that the slope of the trajectory at \( z = 0 \) matches the direction of an input ray. However, after travelling a distance \( z = P/2 \) through the lens, all rays will have returned to the axis once again, at the point \( A' \). Consequently, the lens may be considered to form a real image of \( A \) at \( A' \). Since an off-axis point object above \( A \) can be also shown to form a real point image displaced the same distance below \( A' \), the lens forms an inverted image of the entire input plane at the output end-face. Similar reasoning shows that a quarter-pitch lens forms an image of a point source at infinity, thus collimating a point emitter (Figure 8.4-3b).

For practical reasons, GRIN-rod lenses are normally made in lengths somewhat shorter than those above. For example, Figure 8.4-3c shows a 0.23P lens, which will collimate light from a point source slightly displaced from the lens, thus allowing a working tolerance for alignment and so on. This is particularly important, since dispersion of the lens material generally results in a wavelength-dependent value of \( P \). Without such a tolerance, operation at a wavelength for which \( P \) is shorter than the design value would require the source to be placed inside the lens.

GRIN-rod lenses have several advantages over conventional lenses. Since they are small, they are well-suited to use in micro-optic systems, and are often used to connect optical fibre components. Most importantly, their imaging properties do not stem from refraction at their input and output surfaces (which are planar); instead, redirection of rays is performed by the medium itself. This allows components to be fixed directly to the end faces with index-matching cement, reducing reflection losses and eliminating scratchable or movable interfaces. Consequently, the assembly can be highly rugged. Figure 8.4-4 shows a collection of GRIN-rod lenses.

![GRIN-rod lenses](image)

Figure 8.4-4. GRIN-rod lenses (photograph courtesy D.DeRose, NSG America Inc.).

The ray picture above is just one explanation for the imaging behaviour of a GRIN-rod lens, and an alternative view is provided by the transfer function model introduced in Chapter 4. Remember that the properties of a conventional lens stemmed from the parabolic variation of its optical thickness. The required variation was effected by controlling the thickness of a material with constant refractive index. In a GRIN-rod lens, much the same result is achieved by control of the radial distribution of refractive index, keeping the physical thickness constant. We shall provide one further explanation later on, in terms of guided modes.
THE EIGENVALUE EQUATION FOR A PARABOLIC-INDEX GUIDE

The solution for ray paths given in Equation 8.4-10 represents almost the entire story. However, to obtain a closed form solution, we need to know the allowed values of $\beta$. This requires the solution of an eigenvalue equation. To set up a suitable equation, we shall make use of the transverse resonance condition previously introduced for slab guides in Chapter 6, which required the total phase change accumulated in a round trip between the guide walls to be a whole number of multiples of $2\pi$. Remember that this consisted of two terms: the phase change incurred simply in propagating between the walls, and the phase shift resulting from reflection at the walls.

In the one-dimensional parabolic-index guide, propagation between any two consecutive turning points (e.g. $z_a$ and $z_b$) clearly corresponds to half of one round trip. The first term may be therefore be found by integrating the x-component of the propagation constant over the transverse distance between these points, as:

$$2 \int_{x_a}^{x_b} k_x \, dx = 2 \int_{z_a}^{z_b} k_x \, (dx/dz) \, dz$$  \hspace{1cm} 8.4-13

However, using Equation 8.4-2, this may be transformed to:

$$2 \int_{x_a}^{x_b} k_x \, dx = (2/\beta) \, \int_{z_a}^{z_b} k_x^2 \, dz$$  \hspace{1cm} 8.4-14

Substituting our solution for the trajectory (Equation 8.4-10) into Equation 8.4-4, and inserting the result into Equation 8.4-14, we then get:

$$2 \int_{x_a}^{x_b} k_x \, dx = (2/\beta) \left( k_0^2 n_0^2 - \beta^2 \right) \int_{z_a}^{z_b} \cos^2 \left\{ \left( k_0 n_0 / \beta x_0 \right) z + \Psi \right\} \, dz$$  \hspace{1cm} 8.4-15

Now, since $z_a$ and $z_b$ are separated by half a cycle, we must have $(k_0 n_0 / \beta x_0) (z_b - z_a) = \pi$.

Integrating over this distance, we get:

$$2 \int_{x_a}^{x_b} k_x \, dx = x_0 \pi \left( k_0 n_0 - \beta^2 / k_0 n_0 \right)$$  \hspace{1cm} 8.4-16

The second term - the phase change experienced on reflection - is more difficult to deal with. After all, if the ray path is determined by continuous refraction, total internal reflection is not needed to change the direction of the ray. Nonetheless, the sign of $k_x$ does actually change at each of the turning points, which implies that some form of reflection must be occurring there. To assess the consequences, we need to return to Chapter 5, where we found general expressions for the reflection coefficients for TE- and TM-polarized light at a dielectric interface. In each case, it can be shown that when the index step tends to zero and the angle of incidence approaches 90°, the reflection coefficient tends to $\exp(j\pi/2)$.

Consequently, there is an additional phase shift of $\phi_a = -\pi/2$ at each of the turning points.

Including all contributions, the transverse resonance condition can therefore be written:

$$2 \int_{x_a}^{x_b} k_x \, dx + 2\phi_a = 2\nu \pi \quad (\nu = 1, 2, \ldots) $$  \hspace{1cm} 8.4-17

Inserting the values found above, we then get:

$$x_0 \pi \left( k_0 n_0 - \beta^2 / k_0 n_0 \right) - \pi = 2\nu \pi$$  \hspace{1cm} 8.4-18

which yields the following allowed values for the propagation constant:
\[ \beta_v = k_0 n_0 \sqrt{1 - \frac{2(2v + 1)}{k_0 n_0 x_0}} \] 8.4-19

If the second term in the square root above is small (which implies weak guidance, and therefore paraxial ray paths), we may approximate Equation 8.4-19 as:

\[ \beta_v \approx k_0 n_0 - \frac{(2v + 1)}{2x_0} \] 8.4-20

Since \( k_0 = \frac{\omega}{c} \), the dispersion characteristics of parabolic-index guide consist of a set of parallel straight lines, as shown in Figure 8.4-5. This has extremely important consequences for signal transmission in fibres, which we shall discuss later on. Notice that we have deliberately obscured details of the cutoff conditions, when ray paths must stray out to the edge of the core, since these are not modelled realistically using the approximate index profile we have assumed.

![Figure 8.4-5 Dispersion characteristics of a one-dimensional parabolic-index guide.](image)

In the mean time, we simply note the following result, which is relevant to image transmission through a GRIN-rod lens. As we saw in Chapter 6, a general one-dimensional, forward-travelling field \( E_{in}(x) \) specified at an input plane \( z = 0 \) may be expanded in terms of guided modes as:

\[ E_{in}(x) = \sum_v a_v E_v(x) \] 8.4-21

where \( a_v \) is the amplitude of the \( v \)th mode, and \( E_v(x) \) is its transverse field distribution. After propagating a distance \( z \), this field will be modified to:

\[ E_{out}(x) = \sum_v a_v E_v(x) \exp(-j\beta_v z) \] 8.4-22

Inserting the propagation constants specified by Equation 8.4-20 into the above, we get:

\[ E_{out}(x) = \exp(-j\psi) \sum_v a_v E_v(x) \exp(jvz/x_0) \] 8.4-23

where \( \psi = (k_0 n_0 - 1/2x_0)z \). We now note that if \( z = 2\pi x_0 \) (i.e., if \( z = \Phi \)), Equation 8.4-23 reduces to:

\[ E_{out}(x) = \exp(-j\psi) \sum_v a_v E_v(x) \] 8.4-24
At this point, $E_{out}$ is equal to $E_{in}$ multiplied by an unimportant phase factor. Propagation through one pitch length will therefore leave an image effectively unchanged, as we would expect from the ray model of a GRIN-rod lens described earlier. We can interpret the formation of an inverted real image by a half-pitch lens in a similar way.

**MODAL ANALYSIS OF PARABOLIC-INDEX FIBRE**

We shall now show how scalar theory may be used to determine the actual guided modes in a two-dimensional parabolic-index fibre. We base the analysis on the TEM model introduced in Chapter 6, which assumes that the index changes forming the guide are weak. For the refractive index distribution of Equation 8.4-1, the scalar waveguide equation we must solve is:

$$\nabla^2 E(x, y) + \left\{ n_0^2 k_0^2 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] - \beta^2 \right\} E(x, y) = 0$$

8.4-25

Where $E(x, y)$ is the transverse field and $\beta$ is the propagation constant. Since the fibre is cylindrical, we shall begin by looking for a solution that varies only in the radial direction. In this case, we may put $E(x, y) = E(r)$, and evaluate its derivative with respect to $x$ as:

$$\frac{\partial E}{\partial x} = dE/dr \quad \frac{\partial r}{\partial x} = \left( \frac{x}{r} \right) dE/dr$$

8.4-26

Similarly, its second derivative with respect to $x$ is given by:

$$\frac{\partial^2 E}{\partial x^2} = \left[ \frac{1}{r} - \frac{x^2}{r^3} \right] dE/dr + \left( \frac{x^2}{r^2} \right) \frac{d^2 E}{dr^2}$$

8.4-27

$\frac{\partial^2 E}{\partial y^2}$ may be evaluated in the same way, so we obtain:

$$\nabla^2 E = \left( \frac{1}{r} \right) dE/dr + \frac{d^2 E}{dr^2}$$

8.4-28

Equation 8.4-28 may now be substituted into Equation 8.4-25, to give:

$$\frac{d^2 E}{dr^2} + \left( \frac{1}{r} \right) \frac{dE}{dr} + \left\{ n_0^2 k_0^2 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] - \beta^2 \right\} E = 0$$

8.4-29

At this point, we shall make a guess at the solution. Somewhat arbitrarily, we assume that the transverse field is given by a Gaussian distribution, of the form:

$$E(r) = E_0 \exp\left( -\frac{r^2}{a^2} \right)$$

8.4-30

where $E_0$ and $a$ are constants. Substituting into Equation 8.4-29, and removing the common terms, we get:

$$r^2 \left\{ \frac{4}{a^4} - n_0^2 k_0^2 \frac{r^2}{r_0^2} \right\} + \left\{ n_0^2 k_0^2 - \frac{4}{a^2} - \beta^2 \right\} = 0$$

8.4-31

where we have grouped the terms into two blocks, one dependent on $r^2$, the other a constant. Now, in order to satisfy Equation 8.4-31 for all $r$, the two blocks must be zero, independently. The former condition leads to:

$$a = \sqrt{2r_0/n_0 k_0}$$

8.4-32

While the latter requires:

$$\beta^2 = n_0^2 k_0^2 - \frac{4}{a^2} \quad \text{or} \quad \beta = n_0 k_0 \sqrt{\left\{ 1 - \frac{2/(n_0 k_0 r_0)}{a^2} \right\}}$$

8.4-33
This shows that the Gaussian transverse field we have assumed is indeed a possible mode of the parabolic-index fibre. Since its phase-front is plane, this mode will radiate into free space from an end-face of the fibre as a Gaussian beam; the beam waist (which has radius a) will be located at the fibre end, and behaviour in the far-field will be as described earlier in Chapter 4.

Using similar methods, but this time adopting rectangular coordinates, it is possible to show that the higher-order guided modal solutions are all Hermite-Gaussian modes, with transverse field variations of the form:

$$E_{\mu,\nu}(x, y) = H_{\mu}(\sqrt{2}x/a) H_{\nu}(\sqrt{2}y/a) \exp\left\{-\frac{(x^2 + y^2)}{a^2}\right\}$$

where $\mu$ and $\nu$ are two mode numbers, and $H_{\mu}(\zeta)$ is a Hermite polynomial of order $\mu$, defined as satisfying the differential equation:

$$\frac{d^2 H_{\mu}(\zeta)}{d\zeta^2} - 2\zeta \frac{dH_{\mu}(\zeta)}{d\zeta} + 2\mu H_{\mu}(\zeta) = 0$$

while the propagation constants $\beta_{\mu,\nu}$ are given by:

$$\beta_{\mu,\nu} = n_0 k_0 \sqrt{1 - 2(\mu + \nu + 1)/(n_0 k_0 r_0)}$$

The first few Hermite polynomials are as given in Table 8.4-1. From this, it should be clear that the Gaussian solution above (for which $H_{\mu}(\sqrt{2}x/a) = H_{\nu}(\sqrt{2}y/a) = 1$) is the lowest-order mode, with $\mu = \nu = 0$. This is confirmed by the agreement of Equations 8.4-33 and 8.4-36. Although the latter differs slightly from Equation 8.4-19, found using our earlier ray model, it still has the same general form. Consequently, we may deduce that the general characteristics of a parabolic-index fibre will indeed be qualitatively similar to those for a one-dimensional parabolic-index guide.

$$\begin{array}{|c|c|}
\hline
H_0(\zeta) = 1 & H_3(\zeta) = -12\zeta + 8\zeta^3 \\
H_1(\zeta) = 2\zeta & H_4(\zeta) = 12 - 48\zeta^2 + 16\zeta^4 \\
H_2(\zeta) = -2 + 4\zeta^2 & H_5(\zeta) = 120\zeta - 160\zeta^3 + 32\zeta^5 \\
\hline
\end{array}$$

Table 8.4-1. Some low-order Hermite polynomials.

8.5 SIGNAL DISPERSION

We shall now consider a number of aspects of signal propagation in optical fibre, beginning with dispersion. In Chapter 6, we showed that there are generally two contributions to dispersion in waveguides: intermodal and intramodal dispersion. The latter can be further subdivided into contributions from material and waveguide dispersion. We shall consider them separately, and examine the dominant effects in both step- and graded-index fibre. First, we reiterate that all forms of dispersion arise from variations in the group velocity over the signal bandwidth. Intermodal dispersion results from differences in $v_g$ between different modes, and is therefore important only in multimode fibre. However, if it is present, it normally dominates all other contributions.

Clearly, to evaluate the group velocities of the relevant modes, we must have already made an accurate calculation of the dispersion diagram. Since we have not done this for step-index fibre, we must use an approximation. Our argument runs like this. Modes that are far from cutoff will have most of their energy contained inside the core, so their group velocity
will be close to that of the core material. Ignoring all other forms of dispersion, this will be roughly equal to the phase velocity in a medium of refractive index $n_1$, namely $c/n_1$. Similarly, modes that are close to cutoff will have their energy mostly propagating in the cladding, so that their group velocity will be roughly $c/n_2$. In a distance $L$, the time-spread of a signal due to intermodal dispersion is then:

$$\Delta t = L \left\{ 1/v_{g_{\text{min}}} - 1/v_{g_{\text{max}}} \right\} \approx \left( L/c \right) (n_1 - n_2) \quad 8.5-1$$

Assuming the typical value of $n_1 - n_2 = 0.01$, we obtain $\Delta t/L \approx 33$ ns/km. Now, in a digital communications system, pulses of duration $T$ are normally sent separated by gaps of $T$. Intersymbol interference will occur when consecutive pulses start to overlap, i.e. when $\Delta t \approx T$. Consequently, intermodal dispersion limits the maximum bit rate $B$ to:

$$B = 1/(2\Delta t) \quad 8.5-2$$

This is normally expressed in terms of a bit-rate : length product $BL$, given by:

$$BL \approx c/[2(n_1 - n_2)] \quad 8.5-3$$

For the fibre above, we obtain $BL \approx 15$ Mbit/s km, implying that data may be sent at 15 MBit/s over a distance of 1 km, or at 1.5 Mbit/s over 10 km. The small value of $BL$ renders this fibre impractical for long-distance, high bit-rate communications.

A considerable improvement is offered by parabolic index fibre, for which we have already calculated the dispersion characteristics. Assuming weak guidance, Equation 8.4-36 may be differentiated to give $d\omega/d\beta_{\mu,\nu} \approx c/n_0$. This shows that the group velocities of all modes are approximately equal, a highly advantageous feature. The explanation is found in the ray paths of Figure 8.4-1a. Trajectories with small amplitudes of oscillation obviously travel through shorter physical distances than do those with large amplitude. However, due to the parabolic index variation, paraxial rays spend more time in regions of high index, and therefore travel more slowly. The two effects - decreased physical distance, and increased refractive index - balance out, so that the transit times of all rays are roughly equalised. Consequently, a drastic reduction in intermodal dispersion is obtained, allowing a considerable increase in the bit-rate : length product, typically to 1 Gbit/s km. For this reason, multimode parabolic-index fibre has been very successful, allowing transmission at respectable data rates over short distances. However, for higher rates and longer distances, it is essential to eliminate intermodal effects entirely by using single-mode fibre. The major limitations on bandwidth then have different origins.

In Chapter 2, we showed that the broadening of a signal due to variations in the propagation constant with frequency could be written as $\Delta t = L \Delta \omega d^2 k/d\omega^2$, where $\Delta \omega$ is the signal bandwidth. Although this was originally derived for plane waves, we would expect that the following analogous expression would describe similar effects caused by variations in the $\beta$-value of a single guided mode:

$$\Delta t \approx L \Delta \omega d^2 \beta/d\omega^2 \quad 8.5-4$$

This phenomenon is known as **intramodal dispersion**. In general, it consists of contributions from material and waveguide dispersion, as we now show. Firstly, we recall from Chapter 2 that dispersion in plane waves could be related to variations in the refractive index with wavelength, according to $\Delta t = -(L\lambda_0 \Delta \lambda/c) d^2 n/d\lambda^2$, where $\Delta \lambda$ is the wavelength range of the signal. For a guided mode, we simply replace $n$ by $n_{\text{eff}}$, to obtain:
\[ \Delta t = -(L \Delta \lambda_0 / c) \frac{d^2 n_{\text{eff}}}{d \lambda_0^2} \]

Consequently, intramodal dispersion follows from variations in the effective index with wavelength. These will arise if the constituents of the core or the cladding display any wavelength-dependence, i.e. through material dispersion. Now, we have already plotted the dispersion characteristic of pure silica in Chapter 3, and noted that dispersion goes to zero at \( \lambda_0 \approx 1.27 \, \mu m \). This would be the ideal operating point if signal distortion were the only criterion, but (as we showed earlier) the optimum wavelength for transmission is around \( \lambda_0 = 1.55 \, \mu m \). However, several methods may be used to move the point of zero dispersion to longer wavelengths. Firstly, the addition of some dopants (e.g. GeO\(_2\)) to silica shifts the point in the required direction. Secondly, there is a further contribution to dispersion that we have not yet included: variations in effective index arise simply from the changes in angle between any guided ray and the fibre axis that must occur with changes in wavelength. This is known as waveguide dispersion, and is independent of material effects.

In step-index fibre, particular parameter choices can lead to waveguide dispersion of opposite sign to the material dispersion, resulting in cancellation of the total dispersion at the required wavelength. Unfortunately, this type of dispersion-shifted fibre normally has a rather small core, leading to problems with jointing. However, by using a more complicated refractive index profile (e.g. a W-shaped profile) it is possible to achieve similar cancellation using cores of larger cross-section.

### 8.6 MODE CONVERSION IN FIBRES

In our analysis, we have concentrated on infinite, perfect guides. The exception has been a discussion of tapers in Chapter 6, where we showed that variations in guide shape can induce mode conversion. Now, although fibre is nominally made with a uniform cross-section, there will inevitably be small irregularities (e.g., at the interface between core and cladding, as in Figure 8.6-1a). Any change in ray direction resulting from scattering by such an irregularity then corresponds to mode conversion. Slight changes in direction (Figure 8.6-1b) lead to similar effects. Over the large distances involved in optical fibre transmission, even a small amount of intermodal scattering can lead to a significant transfer of power between modes. This leads to a range of consequences, which assume differing importance in the various fibre types. We shall begin by discussing the effects in multimode fibre.

![Figure 8.6-1 Mode conversion, at (a) a small irregularity, and (b) a change in direction](image)

Firstly, it might be thought that intermodal dispersion can be avoided by launching only a single mode. However, since the processes above lead to a gradual transfer of energy to higher-order modes, multimode operation must occur after a finite distance. Intermodal dispersion is therefore inevitable. Secondly, it is likely that scattering will lead to some rays striking the core/cladding interface at angles lower than the critical angle. These will not be reflected, leading to energy loss through the excitation of radiation modes. This is known as...
**microbending loss** if it is caused by small changes in guide direction (or as **bend loss**, if the changes are large). High-order modes, which strike the interface at angles closer to $\theta_c$, are more susceptible to bend loss. Now, it is likely that intermodal scattering will sooner or later lead to the excitation of modes near to cutoff, which are very sensitive to radiation loss. Consequently, multimode guides show an increase in bend loss with distance. One way to alleviate this is to decrease the critical angle, which requires a large index difference between core and cladding. However, in step-index fibre, this increases intermodal dispersion.

As we have mentioned, intermodal dispersion is not a problem in single-mode guides. Nonetheless, intermodal **scattering** still leads to undesirable effects. Bend loss remains, although this can be be held to acceptable levels by maximising the index change forming the guide, and ensuring that the bend radius always exceeds a specified minimum value. We shall discuss this point more fully in Chapter 9. Here, we will concentrate on an alternative phenomenon: **polarization fluctuation.** This occurs in circularly symmetric fibre, due to random perturbations acting along the length of the fibre.

Now, it can be shown that energy may be coupled most effectively between any two modes by an external perturbation if it is a periodic one, with a spatial frequency that phase-matches the modes. We have already met one example of this in Chapter 7 (the phase matching of beams travelling in different directions by an optically thick grating) and we will encounter some others in Chapter 10. The general idea is that two codirectional modes with propagation constants $\beta_{\nu}$ and $\beta_{\mu}$ can be phase-matched by a perturbation with a spatial frequency $K$ such that:

$$K = \frac{2\pi}{\Lambda} = |\beta_{\nu} - \beta_{\mu}|$$

8.6-1

Any random perturbation along the length of a fibre may be resolved into a set of Fourier components, each corresponding to a given spatial frequency of perturbation. Although the exact frequency spectrum cannot be specified, we can make reach some broad conclusions as to its general form.

i) It will be difficult to avoid a component at zero spatial frequency, since this represents the 'average' level of perturbation.

ii) The spectrum will fall off rapidly at higher frequencies, because anything else would require rapid variations in the fibre with distance; (iii) the spectrum will not be constant, because the most common disturbances (temperature changes and vibration) are inherently time varying.

As we have mentioned before, even a single-moded fibre actually supports two modes. These have orthogonal polarizations, and are degenerate in circularly symmetric fibre. Consequently, the zero- spatial-frequency component above can phase-match them. Random coupling between the two then leads to a rotation of the polarization. This is a problem in interferometric systems, where the polarization of the interfering beams must be held parallel. Clearly, these require a different type of fibre, which holds the plane of polarization. Naturally enough, this is known as **polarization- preserving fibre**. Its improved performance is obtained by adopting a non-circularly-symmetric core geometry, which eliminates the degeneracy between the two polarization modes. Since these now have different propagation constants (say, $\beta_x$ and $\beta_y$) they are no longer phase-matched by the zero- spatial- frequency component of any external perturbation. In fact, the larger the difference between $\beta_x$ and $\beta_y$ the better, since this will result in a larger phase-matching frequency (which will have a smaller component in the spectrum of a typical perturbation).
The effectiveness of polarization-preserving fibre is normally measured in terms of the beat length \( \Lambda \) between the two modes, defined as:

\[
\Lambda = \frac{2\pi}{|\beta_x - \beta_y|}
\]

Values of \( \Lambda \) in available fibre are generally of the order of millimetres.

It will be appreciated that a difference in propagation constant between two polarization modes is just another manifestation of the familiar phenomenon of birefringence. Polarization-preserving fibre is therefore also known as high-birefringence or hi-bi fibre. There are two basic methods of inducing the required effect: the first operates by modifying the physical symmetry of the fibre cross-section, while the latter works by removing the isotropy of the fibre material. In each case, preferred axes are imposed on the overall geometry, which become the polarization axes of the modified fibre.

To obtain form-induced birefringence, the core is made non-circular, usually by adopting an elliptical cross-section. The two preferred polarizations are then aligned parallel to the semi-major axes of the ellipse. So that these may readily be identified, a flat is often ground on the cladding, parallel to one axis (Figure 8.6-2a). This also improves the accessibility of the core, which is often advantageous (for example, in the directional coupler structures that will be discussed in Chapter 10). For obvious reasons, this type of fibre is known as D-fibre.

Alternatively, in stress-induced birefringence, the circular shape of the core is retained, but an anisotropic stress is applied to it. The most effective method uses the segmented construction shown in Figure 8.6-2b. Here an annular region of the cladding is divided into quadrants, with alternating segments made of glasses with different thermal expansion coefficients. Silica (with expansion coefficient \( \alpha_1 \)) is used for most of the cladding, while the shaded areas consist of \( \text{B}_2\text{O}_3 \)-doped silica (which has a higher coefficient \( \alpha_2 \)). During the cooling stages of the fibre fabrication process, differential contraction causes a large uniaxial stress to be applied to the core, which in turn induces birefringence in the core material (\( \text{GeO}_2 \)-doped silica) through the photoelastic effect. This type of fibre is known as bow-tie fibre (from the shape of the stress-producing sectors) and offers beat lengths as short as 0.5 mm at \( \lambda_0 = 0.633 \mu \text{m} \).

8.7 COUPLING TO FIBRES

We will now examine some of the technology required to connect optical fibre and other guided wave components in complete systems. This is important, because it is only the
availability of low-loss, low-cost joints and splices that has allowed the full potential of optical fibre to be realised outside the laboratory. Two aspects are involved: the coupling of free-space beams into guided-wave components, and the direct connection of waveguides. Both may be tackled using a general method known as transverse coupling, and analysed using similar mathematical apparatus. We shall concentrate on the former in this section, leaving the latter to the next.

The technique of coupling free-space beams into guided modes by transverse excitation is known as end-fire coupling. For multimode fibres, the process is easy to visualise. In a ray model, each guided mode is defined by a characteristic angle of propagation relative to the fibre axis. Ignoring reflections, a field comprising an arbitrary collection of modes will then exit through an end-face of the fibre as a cone of light, whose apex angle is set by the numerical aperture defined earlier. Reversing this argument, we would expect that a cone of external radiation impinging on a fibre end-face will excite a random set of guided modes, which will then propagate down the fibre. The required external field can be generated very simply by a lens, leading to the typical arrangement shown in Figure 8.7-1.

![Figure 8.7-1 End-fire coupling into a multimode fibre.](image)

Notice that this argument contains no details of the proportions in which the modes are excited. However, we can deduce that it will be unproductive to use a beam whose numerical aperture is much greater than that of the fibre, because a considerable fraction of the input power must then be wasted on the excitation of cladding modes. Equally, we can guess that a low-NA input field will launch a greater proportion of low-order modes, which have shallower ray angles. A simple design approach would therefore be to match the numerical aperture of the input to that of the fibre. For example, fibre #2 in Table 8.3-2 has an NA of 0.17. Assuming that the parallel beam has a diameter of 1 mm, the required focal length of the lens is approximately $0.5/0.17 \approx 3$ mm.

Another feature ignored by the model above is the relative positions of the components. For example, if the fibre is placed a considerable distance from the focus of the lens, we would expect low coupling efficiency. To include this aspect, we must consider the excitation of individual modes rather more carefully. This can be done by examining the fields at the boundary between the free-space and waveguide systems in more detail, as in Figure 8.7-2. On the left, we have the input wave. Near the focus, this can be specified by a transverse field $E_{in}$ which travels almost entirely in the axial direction. Exactly at the focal plane, $E_{in}$ will be a Gaussian function if the input beam is itself Gaussian. The particular mode of interest (say, the $\nu$th mode, whose transverse field is $E_{\nu}$) is on the right. According to the analysis of Chapter 6, the excitation efficiency of the $\nu$th mode may then be found in terms of overlap integrals, as:

$$\eta = \left| \langle E_{in}, E_{\nu} \rangle \right|^2 / \{ \langle E_{in}, E_{in} \rangle > \langle E_{\nu}, E_{\nu} \rangle \}$$

8.7-1
Consequently, the main feature determining $\eta$ is the overlap between $E_m$ and $E_\nu$. For high efficiency, the two fields should coincide spatially as far as possible. Given that the field diameter will be of the order of $\mu$m in single-mode fibre, this implies stringent alignment tolerance. Furthermore, the fields should match each other closely in shape and size. We shall investigate this using an example.

![Figure 8.7-2 Geometry for modal analysis of end-fire coupling.](image)

**DESIGN EXAMPLE**

A typical end-fire coupling operation might involve the launching of light from a He-Ne laser into a single-moded parabolic-index fibre using a microscope objective. We can estimate the dependence of coupling efficiency on the focal spot size, as follows. Firstly, from the results of Chapter 4, we can assume that since this type of laser emits a Gaussian beam, the phase-front at the focus of the lens will be planar, with the diffraction-limited amplitude distribution $E_L(r) = A_L \exp(-r^2/w_0^2)$. Similarly, we may take the guided mode to have a Gaussian transverse field, given by $E_1(r) = A_1 \exp(-r^2/a^2)$. The fibre is assumed to be aligned exactly on-axis, with its end face at the focal plane. In this case, the overlap integrals are best performed in radial coordinates. If this is done, we obtain:

\[
\begin{align*}
\langle E_L, E_L \rangle &= \int_0^\infty 2\pi r E_L^2(r) \, dr = (w_0^2 \pi/2) A_L^2 \\
\langle E_1, E_1 \rangle &= \int_0^\infty 2\pi r E_1^2(r) \, dr = (a^2 \pi/2) A_1^2 \\
\langle E_L, E_1 \rangle &= \int_0^\infty 2\pi r E_L(r) E_1(r) \, dr = \{w_0^2 a^2 \pi/(w_0^2 + a^2)\} A_L A_1
\end{align*}
\]

Substitution into Equation 8.7-1 then yields:

\[\eta = 4w_0^2 a^2 \{w_0^2 + a^2\}^2 \]

8.7-3

By differentiation, it can be shown that maximum coupling efficiency is obtained when the fields are exactly matched in size, so that $w_0 = a$; simple substitution then shows that $\eta_{\text{max}} = 100\%$. However, $\eta$ falls rapidly away from this condition; for example, when $w_0 = 2a$, we obtain $\eta = 64\%$. Although this figure might still seem reasonable, it actually corresponds to an insertion loss of $\approx 2 \text{ dB}$, equivalent to the attenuation incurred in propagating through about 10 km of low-loss fibre!

**COUPLING BETWEEN LASER DIODES AND OPTICAL FIBRE**

End-fire coupling via a lens may be used with other types of laser. For example, laser diodes (to be covered in Chapter 12) emit radiation from a narrow stripe, so their output is
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highly divergent, with a numerical aperture that is normally much larger than that of single-mode fibre. However, a lens may be used to collect the output, and redirect it towards the fibre in a converging beam of reduced NA. This process is equivalent to the formation of a magnified image of the lasing stripe on the fibre end. In practice, highly compact optical arrangements are often used. Figure 8.7-3 shows three examples. In Figure 8.7-3a, a separate GRIN-rod lens is used between the laser diode (LD) and the fibre. In Figure 8.7-3b, a hemispherical glass microlens is attached to the end of the fibre, which has been tapered down to a reduced diameter. Finally, in Figure 8.7-3c, the end of a tapered fibre has been melted to form a lens directly in the fibre material.

![Figure 8.7-3](image_url)

**Figure 8.7-3** Laser diode-to-fibre connection schemes: a) using a GRIN-rod lens, b) using a microlens attached to the fibre, and c) using a tapered, lensed fibre.

**LENSED FIBRE CONNECTORS**

The end-fire coupling principle can be used to construct a demountable connector known as a **lensed fibre connector**. Figure 8.7-4a shows the basic idea.

![Figure 8.7-4](image_url)

**Figure 8.7-4**  (a) principle of lensed fibre connector; (b) and (c): connectors based on spherical microlenses and GRIN-rod lenses (after A.Nicia, Elect. Lett. 14, 511 (1978)).

Each half of the connector consists of one prepared fibre end and one lens, which has a numerical aperture matched to that of the fibre. The left-hand lens is used to collimate the output from fibre #1, while the right-hand lens focusses this beam down onto the input of
fibre #2. Because the expansion-contraction process results in a near-plane beam of relatively large diameter passing between the two connector halves, errors on longitudinal alignment have virtually no effect, and the significance of errors in lateral position is drastically reduced. Naturally, extremely small and cheap lenses are required. Figures 8.7-4b and 8.7-4c show two possibilities; in the former, small spherical glass balls are used, while the latter is based in GRIN-rod lenses.

8.8 FIBRE INTERCONNECTS

We now consider the direct interconnection of fibre components. In this case, the transverse coupling procedure is known as **butt coupling**. The basic idea is very simple; the two components to be joined have their end-faces prepared accurately orthogonal to their axes of propagation. The axes are then aligned, and the components are simply butted together. Permanent attachment may then be achieved by gluing or by fusion splicing (to be described later). However, this process is naturally subject to error.

Figure 8.8-1 shows examples of the difficulties that may arise in connecting two fibres. The fibres could be displaced from their ideal relative positions, by (a) a short axial distance $\Delta z$, (b) a small transverse distance $\Delta x$, or (c) a rotational misalignment $\Delta \theta$. All such errors result in reduced coupling efficiency. The effects may be calculated in terms of the overlap between the field on the left-hand side of the joint and that on the right, using an expression similar to Equation 8.7-1.

![Figure 8.8-1 Misalignment of a fibre joint in a) axial, b) lateral and c) rotational orientation.](image)

Alternatively, they may be measured directly. For example, Figure 8.8-2 shows experimental measurements for the variation in coupling efficiency with off-axis misalignment $\Delta x$, for joints between identical single-mode fibres. Two sets of data are shown:

Fibre #1 has a diameter of 8.6 $\mu$m, a $V$-value 2.41 and a numerical aperture of 0.056, while Fibre #2 has $d = 4.3$ $\mu$m, $V = 2.18$ and $NA = 0.102$. In each case, efficiency falls drastically for offsets measured in $\mu$m, although the larger- diameter fibre is more tolerant. The only answer is stringent alignment accuracy.
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Figure 8.8-2  Experimental variation of coupling efficiency with transverse displacement in butt-coupled joints (after W.A.Gambling et al., Elect. Lett. 14, 54-55 (1978)).

FIBRE END PREPARATION

The problems involved in achieving the necessary alignment accuracy are enormous. To begin with, the ends of the components to be joined must be smooth, flat and accurately orthogonal to their axis of propagation. Two basic methods are used to achieve this: polishing and cleaving. In the former, the fibre is mounted in a chuck, allowing one end to be held perpendicular to a rotating lap. The end-face is first ground flat, using an abrasive such as carborundum or aluminium oxide. Several grinding stages are needed, using abrasive of successively finer quality. Finally, the end is polished using diamond grit, to remove as far as possible the scratches induced by grinding. Alternatively, a chemical polish such as syton (colloidal silica) may be used. Although the procedure appears laborious, automatic machines exist that may polish a number of fibres simultaneously.

In the latter, a free end is first stripped of its protective jacket for a few inches, using a chemical stripper. It is then placed in a cleaving tool, as shown in Figure 8.8-3. This usually consists of four main components: two clamps, which hold the fibre at points a few centimetres apart, a curved anvil, which is used to tension the fibre, and a diamond scoring blade.

Figure 8.8-3  Cleaving an optical fibre.

The procedure is to score the surface of the fibre with the diamond, inducing a crack. This is developed right through the cross-section, by moving the anvil so that the tension in the fibre is increased past a critical point. The tip of the crack travels at the speed of sound, and,
provided the tool is correctly adjusted, can leave a smooth, hackle- free surface orthogonal to the axis to within 0.5°. Greater control of this angle may be obtained by using a focussed ultrasonic wave to supply the energy needed to propagate the crack.

FUSION SPLICING

Permanent splices between identical fibres may be made by a fused joint, using either an electric arc or an oxyhydrogen flame. The process is known as fusion splicing, and the basic steps involved are illustrated in Figure 8.8-4. The two ends to be spliced are first cleaved, and clamped into the splicing machine (8.8-4a). This normally contains a set of precision piezo-electric micromanipulators, which allow the two ends to be aligned under a microscope (8.8-4b). To improve the alignment, the transmission through this interim butt-joint may be optimised. For example, light may be launched into fibre #1 by injection through a bend, and detected by radiation from a bend in fibre #2.

![Diagram of fusion splicing steps](image)

The position of the stages may then be adjusted under the control of a microprocessor, which is driven by a hill-climbing algorithm programmed to maximise the transmitted power. When the alignment is satisfactory, the two fibres are separated axially by a short distance (8.8-4c), and the electric arc is used to pre-fuse their tips. Surface tension causes each one to melt into an accurate hemisphere (8.8-4d), removing any irregularities caused by cleaving. The two fibres are then fed together, and the electric arc is used to perform the final fusion (Figure 8.8-4e). In this stage, surface tension forces once again perform a useful function, pulling the fibres into improved transverse alignment (8.8-4f).

Modern fusion splicing machines are semi-automatic, and routinely achieve splices with extremely low loss (≈ 0.1 dB); Figure 8.8-4g shows a typical commercial splicing machine. Splicers have also been developed for jointing fibre ribbon cable. This is a highly demanding task, requiring the fabrication of joints between closely-spaced fibres. Besides the constraints of space, each new fusion operation must not disturb the previous one.
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Figure 8.8-4 g) an automatic fusion splicer, the BICCOTEST 4100 (photograph courtesy D.Henderson, Biccotest Ltd.).

After completion of the splice, the unjacketed fibre is rather fragile, and some kind of reinforcement is normally used. A typical reinforcement kit consists of short sections of meltable ethylene vinyl acetate (EVA) tube and steel rod, held in a length of polyethylene (PE) heat-shrink tubing. The components are placed over the splice and heated, so that the EVA melts onto the fibre. Simultaneously, the PE tubing is collapsed, to hold the steel rod firmly against the EVA-coated splice (Figure 8.8-5).

![Diagram of fibre splice reinforcement](image)

Figure 8.8-5 Fibre splice reinforcement (after M.Miyauchi, Elect. Lett. 17, 907 (1981))

MECHANICAL SPLICES

For less demanding applications, mechanical splices are often used. In this case, the two fibres are placed in a precision assembly, which is secured with epoxy. Figure 8.8-6 shows one example, based on a preferentially-etched Si substrate. This has been lithographically patterned and processed in an anisotropic etch (such as ethylene diamine pyrocatechol, or EDP). Crystal planes with a higher density of atoms are more resistant to attack by such a mixture, so the etching follows these planes. For a suitable crystal orientation, the result is a pattern of deep etched grooves with sloping side walls (V-grooves), which act as a precision kinematic mount for cylindrical fibre, locating it accurately in the transverse plane. Two fibres are therefore self-aligned simply by placing them both in a groove and butting them together. The whole assembly is then glued together under pressure, or held mechanically if the splice is to be reusable. For mass-production, plastic replicas may be made of the etched substrates. In the first stage, a negative copy is made of the grooved
surface by nickel electroforming. This 'master' is then used to produce large numbers of positive copies by precision moulding.

![Figure 8.8-6](image)

**Figure 8.8-6** A self-aligned mechanical splice based on preferentially-etched silicon.

**DEMOUNTABLE FIBRE CONNECTORS**

Reusable splices have only limited lifetime, and proper demountable connectors are often required. We have already described lensed connectors; **butt-jointed connectors** form the other main type. In single-fibre connectors, each fibre end is normally encased in a precision ferrule, which increases the effective diameter and allows accurate coaxial alignment of the two ends by a mating sleeve. The ferrules may be cylindrical (in which case a cylindrical sleeve is used), but some misalignment inevitably follows from the clearance between the ferrules and the sleeve. Errors also follow from any lack of concentricity between each ferrule and its fibre, but these may be reduced by using a core-centering lathe for assembly.

Connectors of this type are suitable for multimode fibres. For higher precision, conical ferrules are used. Insertion of the ferrule into a tapered sleeve then allows the virtual elimination of machining tolerance, so that dust particles set the final limit on the transverse alignment. This is the principle underlying the operation of the **biconical taper connector**, which is often used for single-mode fibres. Figure 8.8-7 shows a number of different fibre connector plugs.

Clearly, any approach involving discrete ferrules is impractical for the multiple joints required in ribbon fibre connectors. Most often, these are based on etched silicon substrates, using an extension of the principle previously described for mechanical splicing. For, example, the two ribbon fibre ends may be mounted on precision-etched baseplates, so that each fibre in the array is accurately fixed in a V-groove. On the male half of the connector, two larger grooves are used to locate a pair of precision steel pins, which mate with corresponding grooves on the female half. A joint may then be made simply by aligning the pins and sliding the two connector halves together. Figure 8.8-8 shows a typical multifibre connector based on similar principles.
8.9 Fibre-based components

We now consider a number of simple passive components constructed from optical fibre, for use in multimode and singlemode systems. In the former, devices are often based around GRIN-rod lenses and bulk-optic beamsplitters. Figure 8.9-1 shows two examples. The first is a 3 dB power splitter, which divides an input beam equally between two outputs (Figure 8.9-1a). The beam enters from the left-hand fibre, and is collimated by a 1/4-pitch GRIN-rod lens. The resulting parallel beam is then passed into a frustrated-total-internal-reflection beamsplitter, of the type described in Chapter 7. This divides the beam equally into two components, which are end-fire coupled into output fibres using further 1/4-pitch GRIN-rod lenses. With the addition of a fourth unused port, the device is symmetric, and may act as a bidirectional 2 x 2 splitter.

The second component (Figure 8.9-1b) is a demultiplexer, used to separate or combine light of two different wavelengths in a wavelength-division-multiplexed communications.
system. Just as in the previous example, a bulk-optic component (in this case, a multilayer filter, similar to the Bragg reflection gratings discussed in Chapter 7) is sandwiched between 1/4-pitch GRIN-rod lenses. However, only two lenses are used, and the input and output fibres are not located on-axis. Light of wavelength $\lambda_1$ is therefore collimated by the left-hand lens as an off-axis parallel beam. This particular wavelength is reflected by the filter, so the parallel beam is returned off-axis by an equal and opposite angle, and is then end-fire coupled into the left-hand output fibre. Light of a wavelength outside the reflective band of the filter (say, at $\lambda_2$) passes straight through, and is end-fire coupled into the right-hand output. Of course, the operation of the device could be reversed, so that two inputs at $\lambda_1$ and $\lambda_2$ are combined into a single output.

![Diagram](image1)

**Figure 8.9-1** Multimode fibre components based on GRIN-rod lenses: (a) beamsplitter, and (b) wavelength demultiplexer (after S.Sugimoto et al., Elect. Lett. 14, 15-17 (1978)).

Single-mode fibre components are rather different. Because of the stringent alignment tolerances, end-fire coupling operations are inherently lossy; in fact, the best results are normally obtained by sticking to single-mode operation throughout. A component that is functionally equivalent to the 3 dB splitter above is known as a **fused tapered coupler**. To fabricate the device, two sections of single-mode fibre are first stripped of their protective jackets, placed parallel, twisted together and clamped by their free ends. Flame heating is then applied to the twisted section, so that the fibres gradually melt into one another. At the same time, the clamped ends are gradually drawn apart, so that the central section is tapered. As each fibre is thinned, it approaches cutoff, and light propagating in either core is spread further into the cladding.

![Diagram](image2)

**Figure 8.9-2** Cross-section of a fused tapered coupler (photograph courtesy D.Mortimore, British Telecom Research Laboratories).
Since the fibres are melting together, the cores approach each other, so the modal fields in the two fibres start to overlap. There is then a possibility for power to be transferred from one fibre to the other. In this case, the mechanism is complicated, and we will simply assume that power exchange takes place. However, in Chapter 10 we will examine a similar device - the **directional coupler** - where a reasonable analytic explanation of the phenomenon can be given. The process is sufficiently controllable that it can be terminated when any desired splitting ratio is achieved. After encapsulation of the fragile central section, the device is complete. Figure 8.9-2 shows the cross-section of a typical coupler.

More complicated components may be constructed using similar techniques. **Polarization-preserving couplers** can be made by fusing together two high-birefringence fibres - for example, two D-fibres, arranged with their flat surfaces in contact. Similarly, larger numbers of fibres may be fused together, to make 1 x N or N x N power splitters (where N might lie in the range 3 - 7). When N becomes much larger, the monolithic approach is impractical, and it is simpler to synthesise the required structure by splicing together a number of smaller devices in an array. For example, Figure 8.9-3 shows an 8 x 8 **star coupler**, built up from twelve 2 x 2 fused tapered couplers. Note that there is exactly one pathway from each input to each output. Furthermore, all paths pass through the same number of 2 x 2 couplers (three), and have the same transmission through each (50%). Consequently, light incident on any of the eight input ports is equally divided among all the outputs.

![Figure 8.9-3](image)

We now introduce an entirely different single-mode fibre device: a **polarization controller**. We have already noted the phenomenon of polarization fluctuation in circularly-symmetric fibre. In many situations, it is necessary to adjust the output polarization from a fibre, before coupling into another component (which may be polarization-sensitive). Now, in bulk optics, it can be shown that a plane wave of arbitrary polarization may be transformed into any desired output polarization state using two quarter-wave plates. We recall from Chapter 3 that these are slabs of birefringent material, with a thickness chosen to give a $\pi/2$ phase difference between the two orthogonal polarization components. An in-line single-mode polarization controller can perform a similar trick, using the fibre-optic equivalent of a quarter-wave plate.

Figure 8.9-4a shows the principle. The fibre is formed into a coil, of radius $R$ and length $L$. If $R$ is sufficiently small, bending the fibre in this way results in a uniaxial stress in the core, acting towards the centre of the loop. As we saw in bow-tie fibre, this causes stress-induced birefringence, and $L$ is simply chosen so that a $\pi/2$ phase shift is obtained between the two polarization modes in travelling round the loop. It is also necessary to be able to
achieve an effect analogous to the rotation of a wave plate. This is done by fixing the input and output of the coil (points A' and B'). The entire coil is then rotated about the line A' - B', twisting the sections A' - A and B - B'. This can induce the necessary rotation of the plane of polarization with respect to the principal axes of the coil. Two coils are needed in a complete polarization controller, which is then as shown in Figure 8.9-4b.

Figure 8.9-4 Single-mode fibre fractional wave devices: a) basic principle, and b) polarization controller.
PROBLEMS

8.1. Attenuation in silica glass is approximately 4 dB/km at $\lambda_0 = 0.65 \mu$m. What is the fraction of power transmitted through 10 km of silica-based fibre? Assuming that the main contribution to loss arises from Rayleigh scattering, estimate the attenuation at $\lambda_0 = 1.3 \mu$m. What is the fraction of power transmitted through the same length of fibre at this wavelength?
[0.01%; 0.25 dB/km; 56.2%]

8.2. A multimode fibre, with a numerical aperture of 0.2, supports approximately 1000 modes at a wavelength of 0.85 \mu m. Estimate its diameter.
[60.5 \mu m]

8.3. Using (i) a one-dimensional ray-optic argument, and (ii) a modal argument, explain the formation of an inverted real image by a half-pitch GRIN-rod lens.

8.4. Show that the reflection coefficients for TE- and TM-polarized light at a dielectric interface both tend to exp(j$\pi$/2), when the refractive index step tends to zero and the angle of incidence approaches 90°.

8.5. Calculate the half-width of the lowest-order mode of a parabolic-index fibre, assuming that the peak refractive index of the fibre is $n_p = 1.5$, the characteristic radius is $r_0 = 25 \mu$m and the wavelength is $\lambda_0 = 0.85 \mu$m. What will be the half-angle of divergence of the far-field radiation pattern of this mode? (Hint: refer to Equation 4.6-18.)
[2.1 \mu m; 7.4°]

8.6. The waveguide equation for an idealised model of a parabolic-index fibre may be taken as:

$$ \nabla^2 E_{\mu,\nu}(x, y) + \{n_0^2 k_0^2 [1 - (r/r_0)^2] - \beta_{\mu,\nu}^2 \} E_{\mu,\nu}(x, y) = 0. $$

Show by direct substitution that permissible solutions for the transverse fields of higher-order guided modes are given by:

$$ E_{\mu,\nu}(x, y) = H_\mu(\sqrt{2}x/a) H_\nu(\sqrt{2}y/a) \exp\{- (x^2 + y^2)/a^2 \} $$

where $H_\mu(\zeta)$ is a Hermite polynomial of order $\mu$ and $a = \sqrt{(2r_0/n_0 k_0)}$, and that the corresponding propagation constants $\beta_{\mu,\nu}$ are given by

$$ \beta_{\mu,\nu} = n_0 k_0 \sqrt{1 - 2(\mu + \nu + 1)/ (n_0 k_0 r_0)}. $$

8.7. Show that the first few low-order Hermite polynomials given in Table 8.4-1 do indeed satisfy the governing differential equation:

$$ d^2 H_\mu(\zeta)/d\zeta^2 - 2\zeta dH_\mu(\zeta)/d\zeta + 2\mu H_\mu(\zeta) = 0. $$

Sketch the variation of the transverse fields

$$ E_{\mu,\nu}(x, y) = H_\mu(\sqrt{2}x/a) H_\nu(\sqrt{2}y/a) \exp\{- (x^2 + y^2)/a^2 \} $$

along the line $y = 0$. Under what conditions will the solution prove inaccurate?
8.8. What properties are the most desirable in optical fibre that is to be used in a long-distance telecommunications system? You should discuss loss, dispersion, ease of jointing, and the availability of other systems components such as sources and detectors.

8.9. The theoretical expression for the end-fire coupling efficiency between a Gaussian beam of characteristic radius $w_0$ and the lowest-order mode of a parabolic-index fibre (which also has a Gaussian transverse field, of radius $a$) is given by:

$$\eta = \frac{4w_0^2a^2}{(w_0^2 + a^2)^2}.$$  

Sketch the variation of $\eta$ with the parameter $\alpha = \log(w_0/a)$. Prove that the variation is symmetric about the point $\alpha = 0$, when maximum efficiency of 100% is obtained.

8.10. Sketch the layout of a 16 x 16 star coupler, based on an array of 2 x 2 fused tapered couplers.
SUGGESTIONS FOR FURTHER READING

Mortimore D.B. "Low-loss 8 x 8 single-mode star coupler" Elect. Lett. 18, 82-84 (1982)