PLANAR WAVEGUIDE INTEGRATED OPTICS

7.1 OVERVIEW OF PLANAR WAVEGUIDE COMPONENTS

We will now consider the range of devices that comprise the system of planar waveguide integrated optics. Since these operate on sheet beams (i.e., beams that are confined in one direction only), they are analogous to the devices used to manipulate free-space beams. Consequently, the major building blocks are refractive components (generally, lenses) and diffractive components (known as gratings). However, we must also consider the important question of input and output couplers, needed to connect the guided wave system to the outside world.

Before proceeding further, we must first ask what information is needed to understand the behaviour of an optical device. The general answer is that we will be happy if we can predict the directions and amplitudes of the output beams, for any given input. Some geometries are relatively simple, and yield this information quite readily - for example, in Chapter 5 we found the complete solution to the single interface problem. Others are less tractable, and force us to accept a partial solution. Many integrated optic devices fall into this 'difficult' category. The beam directions are easy to predict, but often the amplitudes can only be found after considerable mathematics. We will therefore concentrate on a simple method of finding the beam directions, based on the principle of phase matching.

7.2 PHASE MATCHING AT A SINGLE INTERFACE

We begin by revisiting the interface problem mentioned above. Figure 7.2-1 shows reflection and refraction at the interface between two semi-infinite media. Here a beam of wavelength \( \lambda_1 \) is incident at an angle \( \theta_1 \), and the two media have refractive indices \( n_1 \) and \( n_2 \), respectively. We assume that \( n_1 \) is greater than \( n_2 \), but that \( \theta_1 \) is less than the critical angle so that a propagating transmitted wave arises.

![Figure 7.2-1 Phase-matching at a single interface – transmission and reflection.](image)

The procedure in any phase-matching problem is the same. We start by identifying all the propagation constants of interest. Here there are only two, with values \( k_1 \) in Layer 1 and \( k_2 \) in Layer 2, given by:
We then consider the input wave. Assuming TE incidence, the electric field of the input beam only has a y-component. We know from past experience that this can be written in the form:

$$E_{yi} = E_i \exp\{-jk_0 n_1 [z \sin(\theta_1) - x \cos(\theta_1)]\}$$  

Now, however, we note that it could be written in shorthand form, as:

$$E_{yi} = E_i \exp(-jk_i \cdot r)$$  

Here $r$ is the radius vector, while $k_i$ (known as the propagation vector) has components:

$$k_i = -k_1 \cos(\theta_1) \hat{i} + k_1 \sin(\theta_1) \hat{k}$$

It is easy to demonstrate equivalence between the two forms, simply by evaluating the scalar product term in Equation 7.2-3.

Now, examining Equation 7.2-4, we see that the propagation vector is oriented parallel to the direction of travel of the wave, and has modulus:

$$|k_i| = \sqrt{(k_{ix}^2 + k_{iz}^2)} = k_1$$

Since we are only really interested in beam directions, this vector contains all the information we need - in fact, it corresponds roughly to the ray introduced in our earlier discussion of bulk optical devices. What properties does it have? Well, if one end of the vector is fixed, the locus of all possible positions of the other end in the $x$-$z$ plane must be a circle, of radius $k_i$. Figure 7.2-2 shows two equally valid representations of this circle. In three dimensions, the locus is a sphere. For the range of possible input beams in our interface problem, the locus is adequately represented by the upper half-circle in Figure 7.2-1. This can be drawn with a radius proportional to $k_i$ (or to $n_1$).

Similarly, we can represent the reflected beam by the vector $k_r$, and the transmitted beam by $k_t$. The moduli of these two vectors are $|k_r| = k_1$ and $|k_t| = k_2$, so the locus of the free end of $k_r$ is also defined by the upper half circle in Figure 7.2-1, while that of $k_t$ may be taken as the lower half-circle. Note that the radius of this second circle is proportional to $k_2$ (and thus to $n_2$), and is therefore smaller than the upper one, since $n_2 < n_1$. We will now use these vectors to interpret our previous results in a new way. Firstly, we know that the incident and reflected beams make equal angles with the interface normal, so the $z$-components of their wave vectors must be the same. We may therefore put:

$$k_{iz} = k_{rz}$$

Similarly, we know that the angle $\theta_2$ of the transmitted wave is governed by Snell's law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$. This requires that the $z$-component of the transmitted wave vector is also the same, so:

$$k_{iz} = k_{tz}$$

Together, these equations imply that the $z$-components of all three vectors are identical. This is consistent with our earlier result that all parts of the field travel at the same speed in the $z$-direction. However, it has given us a new way to find the beam directions.
We first draw circles of the correct relative radii, together with the propagation vector of the incident beam. We then add in the reflected beam vector, and compute the transmitted beam vector from the intersection with the lower circle of the interface normal through the end of the reflected wave vector. Of course, $\theta_1$ may be greater than the critical angle. In this case, total internal reflection occurs, and the dashed-line construction yields no intersection at all with the lower circle (as in Figure 7.2-3). Our interpretation of this event is that the field in Layer 2 cannot now consist of a propagating wave; instead, it must be a boundary wave of some kind, travelling parallel to the interface.

From previous experience, we recognise it to be an evanescent wave, with propagation vector $\beta$. The direction of $\beta$ must be parallel to the interface, and its magnitude is given by:

$$|\beta| = k_{iz} \tag{7.2-8}$$

The propagation vector $\beta$ is therefore defined by the revised dashed-line construction in Figure 7.2-3.
Figures 7.2-1 and 7.2-3 are both illustrations of the general principle of **phase-matching**, which requires that the components of all propagation vectors parallel to an interface must match at that interface. We shall now see how this can be used to analyse more complicated device geometries.

### 7.3 THE FTIR BEAMSPLITTER

We begin with the cube beamsplitter, which is also often known as the **frustrated total-internal-reflection** (or FTIR) beamsplitter. This normally consists of two symmetric 90° glass prisms, separated by a small, accurately-defined air gap. Figure 7.3-1 shows the device in use. Here, a beam input through the left-hand face of the upper prism is divided into two components: a reflected wave, emerging from the right-hand face of the upper prism, and a transmitted wave from the corresponding face of the lower prism. It turns out (as we shall see later) that the power splitting ratio between the two beams can be set by varying the gap width. Why should this behaviour occur?

![Figure 7.3-1 The FTIR beamsplitter.](image)

Well, the central region of the device can be considered as a stacked structure, consisting of three separate layers. The two outer layers correspond to the prisms, and have refractive index $n_1$, while the inner layer (the gap) has refractive index $n_2$. Since the device is a multilayer, it would seem reasonable that its operation can be explained in terms of the phase-matching principle. In fact, Figure 7.3-2 shows the required phase-matching diagram, and we will now explain its construction.

As before, we start by considering the fate of the incident wave. If we take $n_1 = 1.5$ and $n_2 = 1$ (i.e., glass prisms, separated by an air gap) then the critical angle at the upper prism/gap interface is $\theta_c = \sin^{-1}(1/1.5) = 41.8°$. If the input beam travels at the typical angle of $\theta_1 = 45°$, it will suffer total internal reflection. The upper part of the phase-matching diagram is therefore constructed exactly as for Figure 7.2-3; the incident and reflected wave-vectors are first drawn in, and the dashed line construction is used to show that an evanescent wave exists beneath the surface of the first prism.

To construct the lower part of the diagram, we must consider the field in the gap more carefully. If the gap is large, it is reasonable to expect that the evanescent field below the upper prism will decay quickly enough for it to be unaffected by the presence of the lower interface. Typically, this occurs if the gap is wider than a few $\mu$m. In this case, the physical situation is exactly as for simple total internal reflection, and no power crosses either interface. There is then no transmitted beam.
If the gap is sufficiently small, however, a new phenomenon occurs. Remember that the field solutions for evanescent waves may be of two types, which either decay or grow away from an interface. Normal evanescent waves consist entirely of the former type, and we have discarded the latter as 'physically unrealistic' in the past, because they imply infinite field amplitudes at infinite distances from the interface. It turns out, however, that in order to satisfy the boundary conditions at the two interfaces here (which are only a finite distance apart), the field in the gap region must be constructed as a linear sum of the two solutions. It then consists of a rather more complicated boundary wave, which can have significant amplitude at the lower interface.

Figure 7.3-2 Phase-matching in the FTIR beamsplitter.

The waves generated by this hybrid field can be found by phase matching at both interfaces. We have already covered the procedure for the upper one. At the lower one, there is only one possibility - a propagating transmitted beam, travelling downwards at the angle $\theta_1$. This is found by a similar construction, matching the $z$-component of the propagation vector of the wave in Layer 3 with that of the boundary wave at the lower interface. The amplitude of the transmitted wave must clearly tend to unity as the gap tends to zero, because both interfaces will then disappear. In fact, it can be set to any desired value between unity and zero by choosing the gap correctly. Since the device is lossless, the power in the reflected beam must decrease as the transmitted beam increases. Generally, the gap is adjusted to equalised the two, whereupon the device acts as a 50 : 50 beamsplitter.

Rather surprisingly, therefore, power can be transferred across a gap region by a non-propagating, evanescent field, provided the gap is small enough. The process is often known as optical tunnelling. It explains the name of the FTIR splitter - the tunnelling effectively frustrates the total internal reflection that would normally occur at the upper interface. The gap required for significant tunnelling depends primarily on the decay rate of the evanescent fields. If $\theta_1$ is much larger than the critical angle, the fields decay very rapidly, and a very
small gap is needed. As \( \theta_1 \) approaches \( \theta_c \), the fields decay more slowly, and the gap may be increased. We will now show how optical tunnelling can be exploited for the excitation of the guided modes of a planar waveguide.

### 7.4 THE PRISM COUPLER

Planar guided modes can be excited by an external free-space beam using a **prism coupler**, shown in Figure 7.4-1. This consists of a prism of suitable refractive index, which is separated from the waveguide by a small gap. Typically, the prism index should be somewhat higher than that of the substrate, and the gap should be of the order of 1 \( \mu \text{m} \). This can be achieved by clamping the prism and guide together - dust particles then serve as spacers.

![Figure 7.4-1 The prism coupler.](image)

The input beam is again assumed to be a plane wave, which enters the prism from free-space at an angle \( \theta_1 \). If \( \theta_1 \) is large enough, this beam is total-internally-reflected at the prism/air-gap interface. Once again, therefore, the field beneath the prism is an exponentially-decaying evanescent wave, travelling parallel to the interface. The propagation constant of this field is given by:

\[
\beta = k_1 \sin(\theta_1)
\]  

This field can be used to excite a guided mode in the waveguide underneath, if \( \beta \) is matched to the propagation constant of the mode, as we now demonstrate.

The basic mechanism is illustrated by the phase-matching diagram shown in Figure 7.4-2. Here, the upper part of the diagram shows the excitation of a boundary wave beneath the prism. If the gap is very large, nothing further occurs. However, if it is small enough, the field solution in the gap must be modified as described for the FTIR splitter. This implies that there is again the possibility of power transfer across the two interfaces by optical tunnelling. If conditions are correct, this can give rise to a travelling wave in the waveguide layer. The propagation vector \( k_g \) of this wave is found using a similar construction - the intersection of the dashed line with a circle of radius proportional to \( n_3^3 \) gives \( k_g \).

By choosing the angle of incidence correctly, the angle \( \theta_2 \) can be made to correspond exactly with the direction of a downward-travelling, propagating wave in a zig-zag ray model of a guided mode. This wave will therefore be total-internally-reflected at the lower interface, since there is no intersection of the dashed line with the lowest circle (which has radius...
proportional to $n_1$). Naturally, such a reflection must give rise to an upward-travelling wave in the guide layer (with propagation vector $k_g'$), and an evanescent boundary wave in the substrate.

![Figure 7.4-2 Phase-matching in the prism coupler.](image)

We have now identified all the waves in Figure 7.4-2. Note, however, that the conditions for the constructions above to be valid are quite critical. The prism index must be higher than that of the gap material, so that total internal reflection does indeed occur at the upper interface. However, it must also be higher than that of the guide itself, so that the ray angles $\theta_2$ and $\theta_2'$ (which are close to 90° in a weak-index guide) are obtained at a more convenient external angle $\theta_1$ (say, $30^\circ < \theta_1 < 60^\circ$).

Figure 7.4-3 shows how the whole of the transverse field pattern might look at this point. Near the waveguide, the pattern looks very similar to that of an ordinary guided mode. In the substrate, it must be evanescent, while in the guide layer itself it must be a standing-wave pattern, composed of the sum of two zig-zag waves.

Certainly, there are differences in the gap region, where the field is a hybrid boundary wave, and in the prism itself, where it is again a standing wave. However, if the gap is not too close...
to zero, it is not inconceivable that the 'desired' portion of the field could be made to detach itself from the rest and propagate forward as a guided mode.

We can achieve this by removing the external excitation at a suitable value of $z$. Could we then be said to have launched a guided mode? The short answer is yes, but there are several provisos.

Firstly, to make a useful input coupler, the prism should be relatively small compared with the substrate length, so that the guided beam propagates out from under the prism to other devices. In this case, the input beam can no longer be a plane wave, but must be a bounded beam. The amplitude of the guided mode must therefore build up from zero at the left-hand end of the coupler, to some value at the right. Secondly, the power transfer process is bi-directional - a prism coupler can just as easily be used in reverse, to couple a guided mode out into free space. If we are not careful, therefore, the guided mode will start coupling back out of the coupler at its right-hand end.

Fortunately, it has been found that a halved prism coupler can convert a bounded beam into a guided mode with good efficiency. This provides a sharp termination to the coupling region, at a point when the guided mode has built up to a reasonable amplitude (Figure 7.4-4). If the beam reflected from the prism is now measured, it will be found to contain rather less than 100% of the input power. This is because a substantial fraction of the power has now been coupled into the guided mode, and the required energy must clearly be provided by the input beam.
So far, our discussion has concentrated on the situation that occurs when the input beam is incident at the correct angle $\theta_1 = \theta_p$, when it is phase-matched to the guided mode. What happens at other angles? If $\theta_1$ is substantially different from $\theta_p$, it turns out that the field amplitude inside the guide is drastically reduced from that shown in Figure 7.4-3. This implies that the guided mode is not excited nearly so well, and the prism reflectivity returns to close to 100%. For incidence angles near to $\theta_p$, the reflectivity falls, following the typical resonance curve shown in Figure 7.4-5.

Both the width and the depth of the resonance depend on several factors - the precise variation of the air-gap, the length of the prism, the shape of the input beam, and absorption losses within the system. However, measurement of the reflectivity minimum can be used to determine the phase-matching angle $\theta_p$, and hence the propagation constant $\beta$ of the guided mode (from Equation 7.4-1). Note that angles are generally measured external to the prism, and therefore correction must be made for refraction at its input face. Consequently, the prism angles and refractive index must be known accurately.

A prism coupler can also provide other information. For example, the number of minima in the reflectivity curve yields the number of modes supported by the guide. Knowledge of the
effective indices of all the modes may even be used to reconstruct the refractive index profile of the waveguide, using an inverse technique beyond the scope of this book. Waveguide propagation loss can also be established, using two prisms. One acts as an input coupler, while the other (a distance $z$ away from the first) is an output coupler. The power transferred through both prisms via the waveguide is then measured for different values of $z$. Any variation must clearly be due to propagation loss, since this depends on $z$. The prism coupler is therefore an extremely versatile characterisation tool.

**DESIGN EXAMPLE**

It is desired to characterise a weak planar guide ($\Delta n \approx 0.01$) formed on a substrate of refractive index 1.5. How should the coupling prism be chosen? Well, the effective index of the guide must lie in the range $1.5 < n_{\text{eff}} < 1.51$. The phase matching condition (Equation 7.4-1) can be written in the form:

$$n_{\text{eff}} = n_1 \sin(\theta_1)$$

7.4-2

For $\theta_1 \approx 60^\circ$, we obtain $n_1 \approx 1.7$. A prism of high-index glass would therefore prove suitable. It would be convenient to make this as a right-angle prism, with hypotenuse angles of 60° and 30°.

**7.5 PHASE MATCHING FOR GUIDED MODES**

Until now, we have only considered one-dimensional propagation of a mode, down the $z$-axis. However, a planar guided mode can travel in any direction in the $y$-$z$ plane. To proceed further, we need a more general description. This is provided by a vectorial notation similar to that introduced at the beginning of this Chapter. Assuming that the mode has a transverse field distribution $E(x)$ and propagation constant $\beta$, a suitable scalar solution might have the form:

$$E(x, y, z) = E(x) \exp(-j\beta \cdot \mathbf{r})$$

7.5-1

where:

$$|\beta| = \frac{2\pi n_{\text{eff}}}{\lambda_0}$$

7.5-2

Equation 7.5-1 still describes a mode with an $x$-dependent transverse field. However, its phase variation is defined by the vector $\beta$, which is assumed to lie in the $y$-$z$ plane. The orientation of $\beta$ gives the wave direction, while its magnitude describes the effective index of the mode. The locus of all possible propagation vectors in the $y$-$z$ plane is, once again, a circle, this time of radius $\beta$. However, if the guide supports more than one bound mode (say, $n$ of them), the loci of all possible modes of guided propagation are a set of concentric circles, of radii $\beta_1$, $\beta_2$ ... $\beta_n$. Figure 7.5-1 shows the circles for a three-moded guide, drawn with radii proportional to $n_{\text{eff}1}$, $n_{\text{eff}2}$ and $n_{\text{eff}3}$.

Just as in bulk optics, a guided mode can be reflected and refracted at a discontinuity. However, instead of the discontinuity being between two homogeneous media, it will generally be between two different types of waveguide. As discussed in Chapter 6, these could have different guide layer thicknesses, or the same thickness but different refractive indices, or different overlays, or be inclined at an angle to each other. Consequently, the propagation vectors in each guide may have different magnitudes, or lie in different planes.
Solution of the general boundary-matching problem at an interface is very complicated. To find a full solution, it is often necessary to assume the presence of both the full set of bound and radiation modes on either side of the interface, including both polarizations. This implies that mode conversion, polarization conversion and scattering into radiation modes can all occur at interfaces. The possible propagation directions are then governed by the phase matching principle.

Here, we will consider a simplified example involving a coplanar discontinuity in a three-moded guide. The interface is parallel to the y-axis, and we will restrict our attention to the bound modes. Figure 7.5-2 shows the phase-matching diagram, for the case when the lowest-order mode (which has effective index \( n_{\text{eff}1} \)) is incident obliquely from the left.

The diagram is constructed by first drawing circles of radii proportional to \( n_{\text{eff}1} \), \( n_{\text{eff}2} \), and \( n_{\text{eff}3} \) (using the relevant values on either side of the discontinuity). The propagation vector of the
input wave (labelled I) is then drawn in at the correct angle, and with length proportional to \( \text{n}_{\text{eff}} \). The component of this mode that is reflected must travel at an equal and opposite angle to the interface normal, allowing the vector \( R \) to be added. All other possible propagation vectors can then be found from the intersection of the dashed line - the normal to the interface passing through the end of \( R \) - with the two sets of circles. In this example, there are six possible vectors, corresponding to three transmitted waves (collectively labelled T) and three reflected waves (labelled R). The input beam therefore suffers reflection, refraction and mode conversion.

If the discontinuity is small, however, most of these effects can be neglected. From Chapter 6, we know that little mode conversion occurs at weak interfaces. The most important of the waves described above will therefore be the reflected component \( R \) and the transmitted component \( T \) that are of the same order as the input mode. A reasonable approximation to Figure 7.5-2 is therefore provided by the reduced diagram of Figure 7.5-3, which retains only the waves I, \( R \) and \( T \).

![Figure 7.5-3 Approximate phase matching diagram for the interface between two guides.](image)

Furthermore, unless the angle of incidence is very oblique, we might expect the amplitude to \( T \) to be much larger than that of \( R \), so that the main effect occurring is one of refraction of the input mode. We might guess that this will be governed by a modified Snell's law equation, of the form:

\[
\text{n}_{\text{eff}} \sin(\theta_1) = \text{n}_{\text{eff}2} \sin(\theta_2)
\]

where \( \text{n}_{\text{eff}} \) is the effective index of the lowest-order mode in waveguide 1, and \( \theta_1 \) is its angle of travel, and \( \text{n}_{\text{eff}} \) and \( \theta_2 \) are the corresponding values in waveguide 2. To illustrate this, Figure 7.5-4 shows the refraction of a real guided beam at an interface between two guides of different thickness.

To summarise, we may assume that if the interface is sufficiently weak, the input field will travel across it without losing too much power. Its transverse field distribution will not alter much, but its direction of propagation will change through refraction. This means that refractive optical components may operate by slight local modification of an otherwise uniform planar guide.
7.6 REFRACTIVE OPTICAL COMPONENTS

Refractive guided wave components can be subdivided into a number of classes. The simplest work through the sudden change in direction that takes place at an abrupt discontinuity, as described above. This allows guided wave lenses to be constructed; these are analogous to bulk optical lenses (except that in general they may only convert one cylindrical guided beam into another). For example, a simple plano-convex lens might be made by the deposition of high-index overlay material in a region bounded by a parabola and a straight line, as shown in Figure 7.6-1.

DESIGN EXAMPLE

We will now try to estimate the factors governing the performance of overlay lenses. Previously, we obtained the following formula for the focal length of a bulk-optic lens, made of material of refractive index $n_L$ and with two spherical surfaces of radii $r_1$ and $r_2$: 
\[ 1/f = (n_L - 1) \left( 1/r_1 + 1/r_2 \right) \]

In this case, the surrounding medium is air (which has unity refractive index). More generally, we might expect that for a surround of index \( n_M \), the formula above modifies to:

\[ 1/f = \left( n_L / n_M - 1 \right) \left( 1/r_1 + 1/r_2 \right) \]

Now, if a guided wave lens also works by refraction, we would expect its operation to be controlled by the values of the effective index inside and outside the lens. If these are \( n_{\text{effL}} \) and \( n_{\text{effM}} \), the corresponding formula is:

\[ 1/f = \left( n_{\text{effL}} / n_{\text{effM}} - 1 \right) \left( 1/r_1 + 1/r_2 \right) \]

where \( r_1 \) and \( r_2 \) are now the radii of curvature of the two circular boundaries of the high-index overlay.

We can now directly compare the lens shapes required to perform the same task in bulk and guided wave optics. Suppose we wish to design a plano-convex lens, with a given focal length \( f \). In this case, we may take \( r_1 \) to be infinite, so that \( r_2 \) is given by:

\[ r_2 = \left( n_L / n_M - 1 \right) f \quad (\text{in bulk optics}) \]
\[ r_2 = \left( n_{\text{effL}} / n_{\text{effM}} - 1 \right) f \quad (\text{in guided wave optics}) \]

Typically, \( n_L / n_M - 1 \) will be around 1.5 - 1 = 0.5 in the bulk device. What is the corresponding value of \( n_{\text{effL}} / n_{\text{effM}} - 1 \)? With a substrate of refractive index \( n_s = 1.5 \), we might expect to be able to form guides with effective indices \( n_{\text{effM}} = 1.6 \) using a high-index guiding layer. Now, the discontinuities forming the lens must be weak, to prevent scattering to radiation modes. This implies that \( n_{\text{effL}} - n_{\text{effM}} \) must be much smaller than \( n_{\text{effM}} - n_s \). An upper limit might be \( n_{\text{effL}} - n_{\text{effM}} = 0.02 \). We then find that \( n_{\text{effL}} / n_{\text{effM}} - 1 \) is 0.02/1.6 = 0.0125. Substituting these values into Equation 7.6-4, we conclude that the guided wave lens has a radius of curvature 0.5/0.0125 = 40 times smaller than the bulk device.

If we now take the focal length of the guided wave lens to be 5 cm (the maximum that will fit on a chip of reasonable dimensions), we find that \( r_2 = 0.63 \) mm. This implies that the largest possible aperture of the lens (2\( r_2 \)) is only 1.26 mm, and to achieve this the lens must differ in shape quite considerably from the ideal thin-lens assumed in the calculation - in fact, it must be semicircular. Consequently, the lens will have a very low numerical aperture, and will also produce a very aberrated focus.

**FRESNEL, LUNEBERG AND GEODESIC LENSES**

There are several solutions to this problem. Remember that the transfer function of a lens is given by:

\[ \tau_L = \tau_s \exp(jk_0 R^2/2f) \]

Where \( \tau \) is a constant, and \( R \) is the distance from the axis. The focusing property of the lens can thus be ascribed to the parabolic variation of phase-shift it imparts. In guided wave optics, \( \tau_L \) must have a similar form. Now, any phase shift of \( 2\pi v \) radians (where \( v \) is an integer) can be disregarded, since all these are equivalent to no shift at all. We can therefore safely remove any parts of the overlay that simply contribute shifts of \( 2\pi v \), and still obtain a parabolic phase variation. The modified overlay pattern shown in Figure 7.6-2, where each
step in thickness is a step of $2\pi$ in phase, is therefore equivalent to a plano-convex lens; however, it is much thinner. This is known as a Fresnel lens.

Though the Fresnel lens is useful, it does not provide an answer to the underlying problem: the weak change in effective index possible at an abrupt interface leads to a requirement for small radii of curvature of the lens boundaries. The answer is to use larger changes in effective index, which are applied gradually to prevent mode conversion. We cannot analyse such lenses here, but merely state that there are two common types. Both are circularly symmetric, and can therefore nominally focus a plane guided wave with aberrations that are independent of the angle of incidence of the beam. In fact, they are two-dimensional forms of the Maxwell fish-eye or Luneburg lens in bulk optics.

![Fresnel overlay lens](image)

**Figure 7.6-2** Fresnel overlay lens.

The first is the overlay Luneberg lens, shown in Figure 7.6-3. As before, the lens is formed from a high-index overlay, but this time with a graded, circularly-symmetric thickness profile. For some particular overlay profiles, focussing is achieved through continuous refraction of the input wave, and the ray paths inside the lens are therefore curved. The desired overlay profile may be approximated by sputtering material through a circular aperture.

![Overlay Luneberg lens](image)

**Figure 7-6-3 Overlay Luneberg lens.**

An equivalent lens can also be formed without using an overlay, as follows. Before fabrication of the guide, a circularly-symmetric depression (generally spherical) is precision-turned in the substrate using a diamond lathe. Usually, the edges of the depression are also rounded, to provide a gradual input transition that reduces scattering. The guide is then fabricated, with uniform thickness, to follow the topology of the depression. The slow change in local orientation of the guide then causes continuous
refraction, so that rays follow a geodesic path (the shortest path, in accordance with Fermat's principle) through the lens, which is consequently known as a **Geodesic lens**.

Figure 7.6-4 shows a geodesic lens. Note that the optical path through the centre of the lens is larger than that at the edges, because of the increased distance that must be travelled in crossing the deepest parts of the depression. This retards the centre of the wavefront, so that a converging output is obtained.

![Geodesic lens](Figure_7.6-4_Geodesic_lens.png)

Figure 7.6-4  Geodesic lens.

Figure 7.6-5 shows ray paths through a real geodesic lens, which are made visible through the use of a fluorescing dye. Clearly, it is impossible to manufacture a diverging lens using the geodesic technique. However, we note that two waveguide lenses, with differing focal lengths, can be used as a telescope beam expander (previously described in Chapter 4). This arrangement is often used to expand a beam coupled directly into the guide from a laser to a more useful cross-section.

![Ray paths in a geodesic lens](Figure_7.6-5_Ray_paths_in_a_geodesic_lens_photo_courtesy_Y_Okamura_Osaka_University.png)

Figure 7.6-5  Ray paths in a geodesic lens (photo courtesy Y.Okamura, Osaka University).

### 7.7 GRATINGS

A further important class of components operate by diffraction rather than refraction. These are periodic structures, known as **gratings**. In many ways, they are ideal guided wave components, and they may be analysed using an extension of the phase matching principle. We will show how this is done first in bulk optics, and then in guided wave optics.
CHAPTER
SEVEN

OPTICALLY THIN GRATINGS

To investigate the operation of a grating, we need a model. Figure 7.7-1 shows the simplest possible representation of a periodic structure, which is made from an infinite array of point scatterers spaced at equal distances $\Lambda_y$ along the y-axis. We can work out what happens when a light-wave strikes it using qualitative arguments. The wave is infinite, plane, and of free-space wavelength $\lambda_0$; it travels at an angle $\theta_0$ to the z-axis, and the refractive index everywhere is n, so the effective wavelength is $\lambda_0/n$.

Allowing each point to scatter isotropically and independently, we might initially expect light to be transmitted through the grating in all directions - in other words, as a spectrum of all possible plane waves. However, this argument completely ignores the periodicity of the structure. Further thought reveals that a plane wave output is actually possible only when the scattered components from any two adjacent points (e.g. A and D) add up exactly in-phase. If this is not the case, the net contribution from all the scatterers will average to zero in the far field of the grating (see, for example, Question 4.6).

This type of constructive interference can only occur when the difference between the path lengths AB and CD in Figure 7.7-1 is a whole number of wavelengths. We can calculate the relevant path difference for an output angle $\theta_L$ as:

$$AB - CD = \Lambda_y [\sin(\theta_L) - \sin(\theta_0)]$$  \hspace{1cm} 7.7-1

For constructive interference, we require $AB - CD = L \lambda_0/n$, where L is an integer, so that:

$$\sin(\theta_L) = \sin(\theta_0) + L \lambda_0 / n \Lambda_y$$  \hspace{1cm} 7.7-2

Equation 7.7-2 implies that constructive interference only occurs at a set of discrete angles $\theta_L$. The action of the grating is thus to split the input beam up into a number of plane waves, travelling in different directions. These are known as diffraction orders; associated with each is an index L, and the solution of Equation 7.7-2 gives the direction of the $L^{th}$ order. Since the equation contains $\lambda_0$, the angles depend on the wavelength of the optical beam, so gratings are intrinsically dispersive and will produce a series of output spectra from a polychromatic input. Equation 7.7-2 also shows that the diffraction orders will be more...
widely separated the closer the spacing of the scatterers, i.e. as $\Lambda_y$ decreases. To get a useful separation and dispersive power, $\Lambda_y$ should be of the same order as $\lambda_0$.

We cannot estimate the intensities of the individual diffraction orders from this argument, but we would expect to see many of them, of roughly equal intensity. This is often a nuisance, as we may be interested in just one order. Equally, we would not expect the intensities to depend much on the wavelength or the angle. This type of grating is therefore unselective, favouring no particular incidence condition or diffraction order. It is usually known as an \textbf{optically thin} grating, and the multiwave diffraction regime is called the \textbf{Raman-Nath} regime, after its original investigators in the field of ultrasonics (Sir C.V. Raman and N.S.N. Nath).

Before proceeding further, we note that the results above may be interpreted geometrically, using the modified phase-matching diagram shown in Figure 7.7-2. This is rather different to the diagrams we have used so far, and deserves a reasonably thorough explanation. First, an important new parameter, known as the \textbf{grating vector}, is defined. For historical reasons, this is labelled $K$. It is oriented in the direction of the periodicity of the grating (i.e. in the $y$-direction in this case), and has modulus:

\[
|K| = \frac{2\pi}{\Lambda_y}
\]

Next, the usual circular locus for the propagation vectors is drawn, with radius proportional to $k = n\lambda_0$, and the propagation vector of the input wave is added. To avoid confusion with the grating vector, this is labelled $\rho_0$. The propagation vector $\rho_L$ of the $L^{th}$ diffraction order is then found by first adding $L$ times the grating vector to $\rho_0$, and then finding the intersection with the circular locus of the dashed line, drawn normal to the grating boundary through the tip of this new vector.

Simple geometry then shows that:

\[
\rho_{Ly} = \rho_{0y} + L|K|
\]

so that:

\[
nk_0 \sin(\theta_L) = nk_0 \sin(\theta_0) + L|K|
\]
Apart from a slight rearrangement, it should be clear that Equation 7.7-6 is identical to 7.7-2. The modified phase-matching diagram, which is known as a \textit{K-vector diagram}, is therefore a useful way to predict the directions of diffraction orders geometrically. Often, the dashed-line construction is omitted for clarity, being implicitly understood, and the wave vectors are taken simply as:

$$\rho'_L = \rho_0 + L \mathbf{k}$$  \hspace{1cm} 7.7-7

This relation is termed \textit{K-vector closure}. Figure 7.7-3 shows how this modified construction can be used to predict the wave directions for an optically thin grating. In this case, $\Lambda_y$ is large (so $|\mathbf{k}|$ is small) and the output consists of many diffraction orders, all travelling roughly parallel to the z-axis. The correction in wave direction that results from completing the construction is then only a small one.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7.7-3}
\caption{Reduced phase matching diagram for higher diffraction orders in the Raman-Nath diffraction regime.}
\end{figure}

\section*{Optically Thick Gratings}

To remove the unwanted diffraction orders, we need to introduce some kind of inherent selectivity into the grating. Figure 7.7-4 shows a suitable structure (known as an \textit{optically thick grating}) which now has a finite thickness, and consists of fringes or extended scatterers instead of points. These are slanted at an angle $\phi$ to the z-axis and are spaced at distances $\Lambda$ apart, but the y-component of their spacing is still $\Lambda_y = \Lambda / \cos(\phi)$.

Again we ask what happens when the grating is illuminated, by the same wave as before. Equation 7.7-2 must still be valid, because the y-period of the grating is the same. However, the scattering is now distributed in the z-direction. This constrains the allowed diffraction angles even more. Ignoring multiple scattering, we argue that constructive interference only occurs when all scattered components add in-phase. This must be true not only for components scattered by different fringes, but also for contributions from all points along the same fringe. Path lengths like EF and HG in Figure 7.7-4 must therefore be equal. This restricts the diffraction angles additionally to:

$$\theta_L = \theta_0 \ \text{or} \ \theta_L = 2\phi - \theta_0$$  \hspace{1cm} 7.7-8
The fringes then act as partial mirrors, allowing only transmission or reflection.

Figure 7.7-4 An optically thick grating.

Now, if $\theta_L = \theta_0$, the only solution to Equation 7.7-2 is $L = 0$, so the only allowed wave is the 0th diffraction order (the input wave). However, if $\theta_L = 2\phi - \theta_0$, we get:

$$\sin(2\phi - \theta_0) - \sin(\theta_0) = 2\sin(\phi - \theta_0)\cos(\phi)$$

$$= \frac{L\lambda_0}{n}\Lambda$$

Or:

$$2\Lambda \sin(\phi - \theta_0) = \frac{L\lambda_0}{n}\Lambda$$

This implies that other diffraction orders can exist, but only at certain specific angles. The -1th order, for example, is allowed at the first Bragg angle, which is defined by Bragg's law:

$$2\Lambda \sin(\theta_0 - \phi) = \frac{\lambda_0}{n}\Lambda$$

Other waves are allowed at higher Bragg angles, but generally the first diffraction order is the most useful. Combining the two conditions above, we can show that up to two waves can exist at once. For incidence exactly at the Bragg angle, the two permitted waves are the 0th (the input) and -1th orders. Away from this angle, the input wave travels through the grating unaltered. This regime of diffraction is known as the two-wave or Bragg regime, and the modified structure has therefore introduced the desired selectivity into the diffraction process.

In practice, the size of the grating will be limited and conditions 7.7-2 and 7.7-11 relax accordingly. Neither the grating nor the input wave can be infinite, so the wave directions will be less well defined. This affects the resolution of the grating, but provided it is many optical wavelengths wide our arguments should be valid. Finite thickness is more important. Depending on its parameters, a grating may either act like the optically thin grating in Figure 7.7-1, and produce many diffraction orders, or more like the optically thick grating in Figure 7.7-4 and give only two. A reasonable guide is that two-wave behaviour occurs whenever the input wave has to cross many fringe planes before emerging from the grating.
We can work out the value of $\Lambda$ needed to satisfy Bragg's Law (Equation 7.7-11) for some typical values. Assuming that $\lambda_0 = 0.5145$ μm (a green argon-ion laser line) and $n = 1.6$, that the grating is unslanted (so $\phi = 0$) and that the Bragg angle is $\theta_0 = 45^\circ$, we get $\Lambda = 0.23$ μm. This is an extremely small figure, which implies a grating with a large deflection angle will be hard to make.

THE BANDWIDTH OF OPTICALLY THICK GRATINGS

Even in the two-wave regime, restricted thickness will result in a finite angular and wavelength range over which significant diffraction occurs. The bandwidth can be estimated by assuming that the efficiency of the first diffraction order $(L = -1)$ will be zero when there is a whole wavelength difference between contributions scattered from either end of a fringe. We can work out the change in optical wavelength needed for this to occur as follows, assuming an unslanted grating of thickness $d$. The path difference between the two components is:

$$\Phi = d \left( \cos(\theta_0) - \cos(\theta_{-1}) \right)$$  \hspace{1cm} 7.7-12

When the Bragg condition is satisfied, we must have $\theta_{-1} = -\theta_0$, so the path difference is zero. Now, the change in path difference $\Delta \Phi$ accompanying a change in wavelength $\Delta \lambda_0$ is given by:

$$\Delta \Phi = (d \Phi/d\lambda_0) \Delta \lambda_0$$  \hspace{1cm} 7.7-13

We may evaluate Equation 7.7-13 by first expanding it using the chain rule for differentiation, as:

$$\Delta \Phi = (d \Phi/d\theta_{-1}) (d \theta_{-1}/d\lambda_0) \Delta \lambda_0$$  \hspace{1cm} 7.7-14

d$\Phi/d\theta_{-1}$ and $d \theta_{-1}/d\lambda_0$ may then be found by differentiating Equations 7.7-12 and 7.7-2 in turn, to get:

$$d \Phi/d\theta_{-1} = d \sin(\theta_{-1}) \quad \text{and} \quad d \theta_{-1}/d\lambda_0 = -1 / \{ n \Lambda \cos(\theta_{-1}) \}$$  \hspace{1cm} 7.7-15

Substituting these expressions into Equation 7.7-14, and putting $\theta_{-1} \approx -\theta_0$, we then find that:

$$\Delta \Phi = d \Delta \lambda_0 \sin(\theta_0) / \{ n \Lambda \cos(\theta_0) \}$$  \hspace{1cm} 7.7-16

Now, according to our criterion, the efficiency is zero when $\Delta \Phi = \lambda_0/n$, so that:

$$\Delta \lambda_0/\lambda_0 = (\Lambda/d) \cot(\theta_0)$$  \hspace{1cm} 7.7-17

This important result implies that the bandwidth of an optically thick grating is inversely proportional to its thickness, so that the thicker the grating, the smaller the range of wavelengths over which it will diffract light efficiently.

DESIGN EXAMPLE

For the example used before, namely $\lambda_0 = 0.5145$ μm, $\theta_0 = 45^\circ$ and $\Lambda = 0.23$ μm, and a moderate thickness of $d = 10$ μm, we get $\Delta \lambda \approx 12$ nm. For a thicker grating, with $d = 1$ mm,
this reduces to $\Delta \lambda = 12 \text{ Å}$. An optically thick grating can therefore act as an extremely narrow-band wavelength filter, and it is equally simple to show that its angular selectivity must be correspondingly high.

OPTICALLY THICK PHASE GRATINGS

In addition to high selectivity, we will generally also require high diffraction efficiency. It turns out that an optically thick grating can be very efficient indeed, provided the structure is lossless, and the fringe planes are formed by periodic modulation of the refractive index of the medium. The desired structure is therefore an optically thick phase grating. We cannot calculate the exact response characteristic for such a grating using the information above (though we will see how this is done for one particular geometry in Chapter 10). However, using coupled wave theory, it can be shown that the response characteristic is as in Figure 7.7-5.

![Figure 7.7-5 Filter response of a high efficiency, optically thick grating.]

This figure shows how the transmitted and diffracted beam intensities vary, as either the angle of incidence or the wavelength of the input beam is changed. At Bragg incidence, when $\theta_0 = \theta_B$ (or $\lambda_0 = \lambda_B$), the diffracted beam is excited efficiently. There is then a strong depletion of the transmitted beam, which may reach 100% if the grating is correctly designed (as in the example above). As $\theta_0$ (or $\lambda_0$) is changed, the diffracted beam intensity falls. The transmitted intensity shows a corresponding rise, and both curves have a typical 'filter' characteristic. It turns out that the expression for the bandwidth calculated above gives surprisingly good agreement with the predictions of coupled wave theory.

K-VECTOR CLOSURE IN AN OPTICALLY THICK GRATING

The behaviour of a slanted, optically thick grating can also be predicted, using a modified K-vector closure diagram. To cope with slanted fringes, we simply introduce a more general grating vector $\mathbf{K}$. This is oriented normal to the plane of the fringes, and its magnitude is:

$$|\mathbf{K}| = \frac{2\pi}{\Lambda}$$

7.7-18

The diagram is then constructed as before. For example, Figure 7.7-6 shows the diagram for a slanted grating, replayed at the Bragg angle. The input wave vector $\mathbf{p}_0$ is first drawn in. The corresponding vectors $\mathbf{p}_\pm$ for the diffraction orders are then computed from Equation 7.7-7. When this is done, it is found that the end of one such vector, $\mathbf{p}_-1'$, lies exactly on the circle, so that $\mathbf{p}_-1' = \mathbf{p}_0$. This particular circumstance only arises exactly at Bragg incidence, and the -1th order is then said to be phase matched by the grating. This order can then be
excited efficiently, because all the components scattered in the direction concerned will add in-phase. The ends of other propagation vectors (e.g. $p_{-2}$) lie much further away from the circle. This suggests that the $-2^{th}$ order is far from being phase-matched, and will not be excited efficiently. For other diffraction orders (e.g. the $+1^{th}$), the dashed line construction provides no intersection with the circle at all. These waves cannot exist as propagating waves, and are therefore cut off. Thus, according to the diagram, only the two waves defined by the bold vectors are important, and we may reduce the $K$-vector construction accordingly. We will do so in the discussion below, which is devoted to the use of gratings in guided wave optics.

Figure 7.7-6  Reduced phase matching diagram for a slanted, thick grating in the Bragg diffraction regime.

7.8 GRATINGS IN GUIDED WAVE OPTICS

A wide variety of methods can be used to construct a phase grating in guided wave optics. All that is required is that the effective index be varied periodically. Figure 7.8-1 shows one example, where the surface of a planar guide has been corrugated, by etching through a patterned mask. The resulting changes in thickness of the guide layer then give rise to the necessary variation in effective index.

Figure 7.8-1  A corrugated grating.

TRANSMISSION GRATINGS

Figure 7.8-2 shows how such a corrugation might be used as a transmission grating. This is assumed to be optically thick, and therefore operating in the two-wave diffraction regime. Figure 7.8-2a shows a plan view of the waveguide; the input beam is incident from the left, and gives rise to two transmitted waves, the $0^{th}$ and $-1^{th}$ diffraction orders, on the right. Figure 7.8-2b shows the $K$-vector diagram, for incidence at the Bragg angle. Note that
only two propagation vectors are included in the construction, and that the grating vector (which is normal to the horizontal fringes in Figure 7.8-2a) is oriented vertically. This demonstrates an important point. A large angular deflection of the input beam has been achieved, using a relatively small perturbation to the effective index of the guide, because the device operates by diffraction rather than refraction. Since small index changes are the norm in guided wave optics, gratings are ideal waveguide components in this respect.

Figure 7.8-2 Transmission grating: a) schematic and b) K-vector diagram.

GRATING LENSES

In fact, diffractive devices can act as more effective waveguide lenses than the overlay components discussed earlier, because larger beam deflection allows a higher numerical aperture. Figure 7.8-3 shows one example, the chirped grating lens. Here a slow variation in the grating period produces a corresponding change in the deflection angle over the lens aperture, which results in an approximate focussing action. A linear chirp (i.e., a linear variation in period) is suitable for focussing an off-axis plane input beam, as shown. More generally, the grating fringes must be curved, and the local fringe orientation and spacing is found by local application of the Bragg condition.

Figure 7-28 The chirped grating lens.

Grating lenses suffer from two main disadvantages. Firstly, because of their inherent dispersion, the focal spot moves as the optical wavelength is altered, and can become highly aberrated. Secondly, their angular selectivity results in a greatly restricted field of view. However, these can be unimportant (for example, in a telescope beam expander, when a fixed angle and wavelength are used).
CHAPTER SEVEN

REFLECTION GRATINGS

Figure 7.8-4a shows a plan view of an alternative structure. This is a reflection grating, formed using fringes that are now oriented parallel to the input boundary. The input wave is incident as before, but this time the two diffraction orders emerge from different sides of the grating. The 0th order is transmitted, but the -1st order is now a reflected wave. Figure 7.8-4b shows the K-vector diagram, again for incidence at the Bragg angle. Note that the grating vector is now oriented horizontally, and that its magnitude is rather larger than that in the transmission grating above. Reflection gratings therefore typically require much smaller grating periods.

'TWO-DIMENSIONAL' GRATINGS

It is of course unnecessary to consider grating boundary shapes to be fixed in the form of the simple slab geometries considered so far. Figure 7.8-5 shows a grating whose fringes are oriented at 45° to the edges of a boundary, which has the form of a long, thin rectangle. Here, a relatively narrow input beam can be diffracted as a much wider output beam. The device therefore functions as a compact beam expander; which could replace an expansion telescope. However, the grating strength must be carefully tailored along its length, to ensure a uniform amplitude distribution in the output beam.
ELECTRO- AND ACOUSTO-OPTIC GRATINGS

Equally, it is not necessary to consider the grating to be a static entity. There are two common forms of electrically-switchable grating, which operate using combinations of the material effects discussed in Chapter 3. The first is the **electro-optic grating**, shown in Figure 7.8-6a. Here a periodic metal structure, known as an **interdigital electrode**, is placed on the surface of the guide that is assumed to have been fabricated in an electro-optic material (e.g., LiNbO$_3$ or LiTaO$_3$). With no voltage applied to the electrodes, there is no grating. However, when a static voltage is applied, the 'finger' voltages alternate in sign. This results in a periodic variation in electric field beneath the electrodes (Figure 7.8-6b), which induces a corresponding variation in index in the guide through the electro-optic effect. The grating may therefore be switched on and off at will.

However, the beam deflection angle (which is determined by the electrode pitch) is fixed. Usually, it is rather small, since the lithographic process used to define the electrodes is limited to a linewidth of $\approx 0.5 - 1 \mu m$, which yields a period of $\Lambda \approx 2 - 4 \mu m$. Despite this, the electrodes may be sufficiently long that the device acts as a volume grating, so the diffraction efficiency can be high. 98% efficiency has been achieved with Ti-indiffused guides on LiTaO$_3$ substrates. The electro-optic grating may therefore act as an effective modulator. Its speed is limited to about 1 GHz by the capacitance of the electrodes.

The second is the **acousto-optic grating**, shown in Figure 7.8-7. This is a more versatile device, which can steer a beam as well as modulating it. Again, a periodic electrode structure is placed on the surface of the guide, which is assumed this time to have been fabricated in a piezo-electric, acousto-optic material. LiNbO$_3$ is again suitable, but non-piezo electric materials (e.g. Si) can also be used if a layer of piezo-electric material (typically ZnO) is deposited between the electrodes and the substrate.

This electrode structure, which is known as an **interdigital transducer**, does not itself act as the grating. However, when it is excited by an A.C. radio-frequency voltage, it creates a time-varying, spatially-periodic electric field in the material beneath, which in turn creates a similar pattern of stress through the piezo-electric effect. Now, this type of stress distribution effectively corresponds to a pattern of standing acoustic waves, which can of course be decomposed into the sum of two travelling waves. Consequently, the net result is

---

**Figure 7.8-6**  The electro-optic grating: a) basic principle, and b) electric field distribution.
the excitation of two acoustic waves, which emerge from beneath the transducer, travelling in opposite directions. Since the waves propagate near the surface of the material, they are known as surface acoustic waves (or SAWs). Typically, only one wave is required, and the other is removed using a surface absorber. The remaining wave creates a moving variation in effective index, through the acousto-optic effect, and it is this variation which acts as the grating. Once again, the grating may be switched on and off, so the device can act as a modulator. This time, speed is mainly limited by excessive acoustic propagation losses at frequencies above \( \approx 1 \text{ GHz} \).

**Figure 7.8-7** The acousto-optic grating.

Most importantly, the grating period may be varied over a limited range (depending on the transducer bandwidth) by changing the RF frequency. This alters the diffraction angle, so acousto-optic gratings may also be used as beam deflectors. Diffraction may again take place in the Bragg regime at high frequencies, but the Raman-Nath regime is often used to minimise the efficiency variations that occur when the frequency is altered. One further important difference from the electro-optic grating is that the diffraction orders are all frequency-shifted (by integer multiples of the acoustic frequency) through the Doppler effect. The acousto-optic grating may therefore also be used as a frequency modulator.

**DESIGN EXAMPLE**

We may estimate the parameters of a guided-wave acousto-optic beam deflector formed on a lithium niobate substrate as follows. For LiNbO\(_3\), the acoustic velocity is 6.57 \( \times 10^3 \text{ m/s} \) and the refractive index is \( n_e = 2.2 \). Hence, at a typical operating frequency of 500 MHz, the acoustic wavelength is \( 6.57 \times 10^3 / 500 \times 10^6 = 1.31 \times 10^{-3} \text{ m} \), or 13.1 \( \mu \text{m} \). The period \( \Lambda \) of the interdigital transducer must have the same value, so assuming a mark-to-space ratio of unity, each finger electrode must be \( 13.1 / 4 = 3.275 \mu \text{m} \) wide, a feature size well within the capacity of photolithography. Assuming an optical wavelength of 0.633 \( \mu \text{m} \) (red light from a He-Ne laser), and normal incidence, the angle of deflection of the first diffraction order will be \( \theta_1 = \sin^{-1}(\lambda_0 / n \Lambda) = \sin^{-1}(0.633 / 2.2 \times 13.1) \approx 1.25^\circ \).

**MODE CONVERSION BY A GRATING**

Finally, gratings are not restricted to interactions involving the phase matching of two modes of the same type. Consider, for example, a single-moded guide. We have seen that this can...
support two polarization modes; consequently, even in this case there are two possible loci for propagation vectors. Figure 7.8-8a shows the K-vector diagram illustrating the conversion of a TE-moded input into a TE-moded diffracted beam. Here, both $\rho_0$ and $\rho_\perp$ lie on the full-line circle, the TE-mode locus. However, an equally valid possibility is the diffraction of a TM-moded input into a TM-moded output. This is shown by the alternative construction, where $\rho_0$ and $\rho_\perp$ lie on the dashed-line TM-mode locus. Because of the difference in radii of the circles, the Bragg angles for TE-TE and TM-TM diffraction are slightly different. One further possibility exists: a TE-moded input may be diffracted as a TM-moded output. The phase-matching condition for this case is shown in Figure 7.8-8b; once again, the Bragg angle involved is different. In this case, mode-conversion accompanies diffraction. In designing a grating device, therefore, care is needed to ensure that only the desired interaction occurs.

![Figure 7.8-8](image)

**Figure 7.8-8** K-vector diagrams for a) TE-TE and TM-TM diffraction, and b) TE-TM mode conversion.

**GRATING COUPLERS**

A grating may even be used to phase-match guided and radiation modes, forming an input/output coupler similar to the prism coupler discussed earlier. This device is known as the **grating coupler**, and its principle of operation is shown in Figure 7.8-9.

![Figure 7.8-9](image)

**Figure 7.8-9** The grating coupler.
Here a plane input beam is incident from free space on a corrugated grating, which is fabricated on the surface of a planar waveguide. Figure 7.8-10 shows the corresponding phase-matching diagram. If the angle of incidence $\theta_1$ is correct, the $z$-component of the first diffraction order of the grating will equal the $z$-component of the propagation vector of the guided mode. This occurs when:

$$k_{iz} + \beta = \beta$$  \hspace{1cm} 7.8-1

If this is the case, power will be transferred to the guided beam, much as in the prism coupler.

![Figure 7.8-10 Phase-matching in the grating coupler.](image)

Although prism and grating couplers are functionally similar, the latter have several important advantages. Firstly, they are flat, rugged, and fully integrated with the waveguide. Secondly, it is possible to design grating couplers with a varying periodicity, which can perform multiple functions - for example, they can couple a guided beam into free space, and focus it at the same time. Figure 7.8-11 shows the *chirped grating coupler*, which contains a grating with linearly varying period.

![Figure 7.8-11 The chirped grating coupler.](image)

A guided mode incident on such a grating will be diffracted into a free-space beam, whose direction of propagation depends on the local grating period. In this case, the output will be
a cylindrical wave converging on a line focus above the guide. However, by simultaneously chirping the grating period and curving the grating fringes, an output can be generated that converges onto a point focus. This is important in one of the applications discussed in Chapter 14, the integrated optic disc pickup.
PROBLEMS

7.1. The figure below shows the cross-section of an FTIR beamsplitter, formed from two layers of dielectric of refractive index \(n_1\), separated by a layer of refractive index \(n_2\) and thickness \(h\). A plane wave is incident on the upper interface as shown. Assuming TE incidence, and that \(\theta_1 > \sin^{-1}(n_2/n_1)\), write down general expressions for the electric field in all three layers. What boundary conditions are satisfied by these expressions?

7.2. Repeat the analysis of Question 7.1, assuming a solution in modal form. Show that, provided \(h\) is large enough, the amplitude of the transmitted wave decreases exponentially with \(h\).

7.3. A high-index glass prism (\(n = 1.7\)) is used at 1.5 \(\mu m\) wavelength to characterise a planar guide formed by ion exchange in glass. Assuming that the guide supports three modes, with effective indices of 1.515, 1.530 and 1.545, respectively, sketch the variation with launch angle that you would expect in prism reflectivity. What are the propagation constants of the three modes?

7.4. A step-change in thickness is etched into a planar guide. On the left of the step, the guide is two-moded (with effective indices for the two modes of 1.504 and 1.505, respectively). On the right, it is single-moded, with effective index 1.5025. The lowest-order mode is incident from the left at an angle \(\theta_1\). Sketch the phase matching diagrams you would expect for (a) \(\theta_1 \approx 0^\circ\), and (b) \(\theta_1 = 87.5^\circ\), and interpret your results.

7.5. Find the most compact arrangement you can for a x5 telescope beam expander, assuming that it is to be fabricated on a planar waveguide using step-index overlay lenses. The input beam width is 0.5 mm, the effective index of the guide is 1.6, and the maximum change in effective index that can be obtained using the overlay without inducing mode conversion is 0.02.

7.6. Calculate the Bragg angle for an unslanted transmission grating of period \(\Lambda = 1.0 \mu m\), in a material of refractive index 1.6. The optical wavelength is 1.5 \(\mu m\). Estimate the spectral bandwidth of the grating, assuming that it is optically thick, with a physical thickness of 50 \(\mu m\). How many fringe planes are crossed by the input beam, before it emerges from the grating? [27.95°; 56.5 nm; \(\approx 27\)]

7.7. Show that the angular bandwidth of an unslanted transmission grating of period \(\Lambda\) and thickness \(d\) is given by \(\Delta\theta_0 \approx \Lambda/d\). Hence calculate the bandwidth of a grating of...
thickness 20 µm in a medium of refractive index 1.6, assuming that the Bragg angle is 30° at $\lambda_0 = 0.633 \mu m$.

[1.13o]

7.8 Show that Bragg's law may be written in terms of the grating vector and the propagation constant as $2k \sin(\theta_0 - \phi) = |K|$. Interpret this geometrically using a phase-matching diagram.

7.9 Sketch the K-vector diagrams you would expect for the following geometries: (a) an unslanted transmission grating of thickness 5 µm and period 5 µm, for normal incidence, and (b) an unslanted reflection grating of thickness 5 µm and period 0.5 µm, for incidence at the Bragg angle. The refractive index of the medium is 1.6, and the optical wavelength is 0.633 µm.

7.10 Discuss the advantages and disadvantages of Bragg grating lenses, as compared with other types of waveguide lens. You should consider size, bandwidth, and ease of fabrication.
SUGGESTIONS FOR FURTHER READING

Ohmachi Y. "Acousto-optic TE_0-TM_0 mode conversion in a thin film of amorphous tellurium dioxide" Elect. Lett. 9, 539-541 (1973)