Abstract: The finite-element formulation using the vector magnetic field and a penalty function to eliminate spurious modes was used for the analysis of unusual arrangements employing rectangular-core directional couplers with refractive indices chosen to match the characteristics of organic materials. The structure proposed is a fast electro-optic modulator/sensor that could be made by employing the electro-optic organic material 2-methyl-4-nitroaniline (MNA) without the need for special crystal growth methods. To verify the validity of the finite-element analysis, results for a coupling arrangement using materials of index close to 1.51 were compared with data from experimental measurements.

1 Introduction

The main purpose of this paper is to propose and analyse novel arrangements for making a fast electro-optic modulator utilising only- slab-sided organic electro-optic crystals and glass components. The material 2-methyl-4-nitroaniline (MNA) is chosen for a specific design example.

These structures require the use of optical glasses with high refractive indices, from which directional couplers might be made either by deposition of, for example, silicon oxynitride on to silicon wafers [1] or, in fibre form (with greater accuracy), from preforms made by cutting and polishing rectangular-sided slabs of optical glasses [2].

The analysis and design of rectangular-cored guiding structures can only be performed satisfactorily using finite-element (FE) methods. We begin with a brief discussion of the FE method used here and show, by comparison with available data, that this FE method is accurate enough for the purpose of establishing the feasibility of a design. It enables the analysis of the normal modes of the proposed asymmetric structures, without the necessity for the assumption of weak coupling between guides. Thus, by using an FE model of the coupler to calculate the propagation constants $\beta_1$ and $\beta_2$ of the ‘even-like’ and ‘odd-like’ fundamental ‘supermodes’ (normal modes) at wavenumber $k_0$ ($k_0 = 2\pi/\lambda_0$), the following quantities characterising a directional coupler are readily obtained, using the expressions given alongside each:

- **Coupling coefficient:**
  $$c = \min[\beta_1(k_0) - \beta_2(k_0)]/2$$

- **Phase mismatch:**
  $$m = \left\{[\beta_1(k_0) - \beta_2(k_0)]^2/4 - c^2\right\}^{1/2}$$

- **Coupling length:**
  $$L = \pi/[2(c^2 + m^2)^{1/2}]$$

- **Phase-matched coupling length:**
  $$L_{\text{max}} = \pi/2c$$

- **Normalised coupled power after distance $L$:**
  $$P_c(L_{\text{max}})/P(0) = (\pi/2)^2\text{sinc}^2[(\pi/2c)(c^2 + m^2)^{1/2}]$$

2 Description of finite-element method

The FE method used for the analysis employs the vector magnetic field $\mathbf{H}$ together with a penalty function term to eliminate spurious modes. The functional used is of the form

$$F(\mathbf{H}) = \int (\text{curl } \mathbf{H}^*)n^{-2}(\text{curl } \mathbf{H})dV + \rho \int \text{div } \mathbf{H}^*\text{div } \mathbf{H}dV - k_0^2 \int \mathbf{H}^*\mathbf{H}dV$$

where the refractive index $n$ is generally a real diagonal tensor of the form

$$n(x,y) = \begin{bmatrix}
n_x(x,y) & 0 & 0 \\
0 & n_y(x,y) & 0 \\
0 & 0 & n_z(x,y)
\end{bmatrix}$$

and the value of the penalty-coefficient $\rho$ was set to $1/n_{el}^2$ (where $n_{el}$ is the lowest cladding index) to ensure the absence of spurious modes within the full range of guided modes [3]. The FE mesh is a generalisation of that used in [4] which, by using right-angled first-order elements, avoids the increased inaccuracy due to obtuse angles. Another, very important, advantage of this...
mesh for CAD purposes, is that it may be altered very easily between calculations, allowing both fairly rapid optimisation of the mesh shape and detailed changes to the waveguide geometry. Symmetry boundaries were treated as ‘magnetic’ or ‘electric walls’, while the rectangular outer boundary of the complete mesh was placed as far as possible from the waveguide core regions, as was consistent with the maximum size of the mesh (about 330 nodes and 570 elements) computable in a few hours on the 60-bit CDC computer that was used. The nodal numbering scheme was chosen to minimise the bandwidth of the matrices, while the NAG library routines [5] selected for the resulting eigenvalue problem exploited both the matrix symmetry and bandedness to reduce computing time. Typically the analysis used 2 Mbytes of memory, and 90% of the CPU time was spent on eigenvalue extraction. The roundoff error, estimated to be about $10^{-10}$, is insignificant. Subsequent transfer of the same FE programs to an SG1 workstation produced a comparable performance with an even larger mesh of up to 850 elements. The positive-definiteness of the eigenvalue problem (a consequence of the use of the penalty-function) enabled the optimisation of the mesh design by seeking the smallest eigenfrequency for a given propagation constant $\beta$.

Solution accuracy was estimated by calculating the HE11 mode of a circular guide with core and cladding refractive indices $n_{core} = 1.00488$ and $n_{clad} = 1.000$, respectively. The percentage error in the normalised propagation constant

$$B = \left[ (\beta/k_0)^2 - n_{clad}^2 \right] / \left[ n_{core}^2 - n_{clad}^2 \right]$$

(8)

varied from 1% to 10% as $B$ fell from 0.8 to 0.2. However, when modelling couplers, the discretisation errors (which are dominant in the FE method) can be shown, by changing the mesh size, to have identical signs and to be close in magnitude for both the ‘even-like’ and ‘odd-like’ supermodes. This is because: (i) their modal field patterns differ only over a small region of the mesh; (ii) in modelling both modes the mesh was chosen to have an identical shape; and (iii) local mesh refinement was employed to minimise discretisation errors. Because of this, the discretisation error associated with the difference $\beta_1 - \beta_2$ between the propagation constants of the two super-modes is less than the discretisation error of each propagation constant individually. The corresponding error in the phase-matched coupling length $L_{max}$ obtained from eqns. 4 and 1, is therefore considerably lower than might be expected from the magnitude of the possible errors in $\beta_1$ and $\beta_2$ themselves.

Assuming that the relative error is expected to vary roughly as $N^{-1/2}$, where $N$ is the number of nodes in the mesh [6], it was estimated by extrapolation that the error in $L_{max}$ (defined in eqn. 4) is less than 10% for practical values of $N$, despite the poor absolute accuracy of $\beta_1$ and $\beta_2$.

3 Comparison with some available data

As a demonstration that this approach gives reasonable accuracy for practical purposes, in the case of square-section guides, we consider the identical-squared-core directional coupler of Fig. 1. Table 1 gives the calculated variation of $L_{max}$ with intercore separation $s$. An experimental value (given in [7]) of $L_{max} = 22\text{mm}$ for $s = 5.5\mu\text{m}$ agrees well with the present calculations.

![Fig. 1 Cross-section of identical-square-cored directional coupler used to verify validity of finite-element analysis](image)

<table>
<thead>
<tr>
<th>$s, \mu\text{m}$</th>
<th>$L_{max}, \text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.9</td>
</tr>
<tr>
<td>2.0</td>
<td>3.4</td>
</tr>
<tr>
<td>3.0</td>
<td>5.8</td>
</tr>
<tr>
<td>4.0</td>
<td>10.1</td>
</tr>
<tr>
<td>5.0</td>
<td>17.5</td>
</tr>
<tr>
<td>5.5</td>
<td>22.9</td>
</tr>
<tr>
<td>6.0</td>
<td>30.2</td>
</tr>
<tr>
<td>7.0</td>
<td>53.3</td>
</tr>
<tr>
<td>8.0</td>
<td>94.9</td>
</tr>
</tbody>
</table>

We may also compare these results with those obtained from the following approximate formula extracted from [8]:

$$L_{max} = \pi\beta \left[ h^2 + p^2 \left( 1 + 2/pa \right) \exp(ps) \right]/4h^2p$$

(9)

for a coupler made up by two square cores $a \times a$ separated by a distance $s$, where $h^2 = n_{core}^2 k_0^2 - \beta_1^2$, $p^2 = \beta_2^2 - n_{clad}^2 k_0^2$, $\beta$ being the propagation constant. We found that this expression underestimates $L_{max}$ by as much as a factor of 2. However, eqn. 9 is mainly applicable to well confined modes ($pal2 \gg 1$), while in the present case $pal2 \approx 2$.

4 A directional coupler comprising an active strip-loaded slab waveguide and a square-section glass core

An optical directional coupler having the cross-section shown in Fig. 2 would provide a means of harnessing, to either glass fibres or silicon oxynitride-on-silicon waveguides, the strongly nonlinear electro-optic properties of crystalline slabs of, for example, organic substances in which optical waveguides are not readily formed. The difficulty of controlling waveguiding to exploit the very strong nonlinearities of such crystals is illustrated by the literature on this subject [9–11]. Here we present the results of an analysis of the properties of the proposed structure in order to explore the practicability of fabricating it.
The slab waveguide core is assumed here to be made from the organic crystal 2-methyl-4-nitroaniline (MNA) having a refractive index for the x-directed polarisation of \(n_x = 1.8 \pm 0.1\) \cite{1} at the Nd-YAG wavelength of 1.064\(\mu\)m. It is assumed to be a rectangular core with a very large aspect ratio, like the as-grown crystals of this material, and is placed in intimate contact with the plane surface of a composite glass structure. The loading strip, of index \(n_L\), mainly helps to sustain the coupling between the slab and the square core, through the concentration of the slab mode's field in the vicinity of the strip (Fig. 3).

![Cross-section of directional coupler analysed in this work](image)

**Fig. 2** Cross-section of directional coupler analysed in this work

\[
\begin{array}{c|c|c|c|c}
\text{slab core} & n_e & a & \text{loading strip} & n_L \\
\hline
\text{slab core} & n_e & a & \text{loading strip} & n_L \\
\hline
\end{array}
\]

**Table 2**: Phase-matched coupling length \(L_{\text{max}}\) against thickness \(t\) of slab core (normalised to phase-matching wavelength \(\lambda_0\)) for a coupler similar to that of Fig. 2

\[
L_{\text{max}} = 1.793, \quad \text{while in Fig. 5, the thickness } t \text{ is held at 5.51}\mu\text{m. The square core for both Figures is 2.9}\mu\text{m x 2.9}\mu\text{m.}
\]

**Fig. 4** Phase-matched coupling length \(L_{\text{max}}\) against thickness \(t\) of slab core (normalised to phase-matching wavelength \(\lambda_0\)) for a coupler similar to that of Fig. 2

\[
a = 2.9\mu\text{m}, n_L = 1.793
\]

**Fig. 5** Phase-matched coupling length \(L_{\text{max}}\) (at \(\lambda_0 = 1.064\mu\text{m}\)) against index \(n_L\) of loading strip for a coupler similar to that of Fig. 2

\[
a = 2.9\mu\text{m}, t = 5.51\mu\text{m}
\]

**Table 2**: Phase-matched coupling length \(L_{\text{max}}\) against thickness \(t\) of slab (normalised to phase-matching wavelength \(\lambda_0\)) for a coupler similar to that of Fig. 2 (with \(a = 2.9\mu\text{m}, n_L = 1.793\))

<table>
<thead>
<tr>
<th>(t, \mu\text{m})</th>
<th>(\lambda_0, \mu\text{m})</th>
<th>(t/\lambda_0)</th>
<th>(L_{\text{max}}, \text{mm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.03</td>
<td>1.220</td>
<td>1.618</td>
<td>0.56</td>
</tr>
<tr>
<td>2.90</td>
<td>1.084</td>
<td>2.726</td>
<td>0.93</td>
</tr>
<tr>
<td>3.77</td>
<td>0.976</td>
<td>3.862</td>
<td>1.65</td>
</tr>
<tr>
<td>4.64</td>
<td>0.909</td>
<td>5.107</td>
<td>3.04</td>
</tr>
</tbody>
</table>

**5 Results of the analysis**

Figs. 4 and 5 (and the corresponding Tables 2 and 3), illustrate the variation of the calculated coupling length \(L_{\text{max}}\) with, respectively, the thickness \(t\) of the slab core (normalised to the wavelength \(\lambda_0\) at which coupling occurs) and the index \(n_L\) of the loading strip. In Fig. 4,
Table 3: Phase-matched coupling length \( L_{\text{max}} \) against refractive index \( n_g \) of slab for a coupler similar to that of Fig. 2 (with \( a = 2.9 \mu m, t = 5.51 \mu m \)) at \( \lambda_0 = 1.064 \mu m \)

<table>
<thead>
<tr>
<th>( n_g )</th>
<th>( L_{\text{max}}, \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.789</td>
<td>14.90</td>
</tr>
<tr>
<td>1.791</td>
<td>8.35</td>
</tr>
<tr>
<td>1.793</td>
<td>4.91</td>
</tr>
<tr>
<td>1.795</td>
<td>2.43</td>
</tr>
</tbody>
</table>

This can be attributed to the fact that (with all other parameters remaining fixed) an increase in \( \lambda_0 \) effectively causes the peak of the slab field to move further away from the rectangular core, thus reducing the overlap of the coupled modes.

The results in Fig. 5 show an average rate of change with loading strip index \( n_L \),

\[
\frac{d(\ln L_{\text{max}})}{dn_L} \approx 300
\]

(11)

6 A possible design procedure

The choice of the coupler’s components is influenced by several practical considerations, of which the most important are now outlined. Since the slab core is a single crystal, its growth process determines the practical constraints on its thickness \( t \), while its refractive index \( n_g \) at a given wavelength is not freely variable other than to a small degree by changing the temperature or an externally applied electric field. Also, a practical value of the coupling length \( L_{\text{max}} \) must be produced, chosen here to be between 2 and 3 mm.

The present investigation is mainly aided by the data in Tables 2 and 3 (Figs. 4 and 5, respectively). During the design process, the slab thickness \( t \), the side of the square core \( a \) (equal to the side of the square loading strip) and the index \( n_L \) of the loading strip are considered as the variable parameters, while the other parameters characterising the coupler are considered fixed at the values given in Fig. 2. Thus, \( n_g \) is set to 1.793 (within the range of experimental error quoted in [1]). \( n_{sq} \) is fixed at 1.8, while the cladding indices \( n_{clad} \) and \( n_{sq} \) are set to 1.785 and 1.708, respectively.

Since the desired operating wavelength \( \lambda_0 \) is that of the Nd-YAG laser (1.064 \( \mu m \)), the slab thickness \( t \) must be close to 2.90 \( \mu m \) (second row of Table 2). In fact, \( t \) is set to a slightly higher value (3 \( \mu m \)) to anticipate (in connection with the lowering of \( n_L \) attempted below) a coupling length longer than 0.93 mm (the value derived for \( t = 2.90 \mu m \)). The sides of the square core and the loading strip are also set to 3 \( \mu m \) (equal to \( a \) as in Table 2).

The choice of the loading-strip index \( n_L \) is a difficult one, since it must simultaneously satisfy several constraints. It must be high enough to force the modal field of the slab to concentrate sufficiently on the vicinity of its centre. It must not, however, approach too closely the index of the slab, if the coupling to the square core is not to be weakened by shifting too much optical power away from that core and into the loading strip. Finally, \( n_L \) must be selected to produce a practical value of the coupling length \( L_{\text{max}} \), chosen between 2 and 3 mm. This is the most complicated requirement to satisfy, which is why it is considered last.

The selection of \( n_L \) is aided by the data in Figs. 4 and 5 (Tables 2 and 3, respectively), even though these refer to different cases. The coupler dimensions given in Fig. 2 (\( \lambda_g = 2.819 \)) lie close to those of the second row of Table 2 (Fig. 4), though the corresponding coupling length of 0.93 mm is shorter than desired. However, Fig. 4 is for the case \( n_L = n_{sq} = 1.793 \), so that a lowering of \( n_L \) both satisfies the requirement that \( n_L < n_{sq} \) and, according to Fig. 5 (Table 3), should raise the coupling length \( L_{\text{max}} \). Indeed, a trial with \( n_L = 1.789 \) (and \( t = a = 3 \mu m \)) produces the value \( L_{\text{max}} = 2.43 \mu m \) within the desired range 2–3 mm.

The final consideration in designing the coupler is to ensure that higher-order supermodes play no part in the directional coupling process. This is ensured if the symmetry of the higher-order modes is such that no linear superposition of them approximates to the fundamental mode of the isolated square core alone. This can be achieved as long as the number of supermodes is small as can be seen from the shapes of the supermodes in Fig. 6.

![Fig. 6 Schematic diagram illustrating distribution of modal fields of guided supermodes for coupler of Fig. 2](image-url)

Numbers included denote maximum values of field

![Fig. 7 Variation of dominant component of two fundamental supermodes over cross-section of coupler of Fig. 2](image-url)

\( x \) : variation, ‘even-like’ supermode
\( y = a \) : along centre of slab
\( y = 0 \) : along centre of strip
\( x = \alpha r \) : along centre of square

The variation of the dominant component of the two fundamental supermodes over the coupler’s cross-section is depicted in Fig. 7–9.
7 An application of the coupler: an intensity modulator/sensor

The coupling structure under consideration is usable in either an intensity modulator or a sensing element by exploiting the sensitivity of the coupling to changes in the index $n_s$ of the slab. A schematic configuration is shown in Fig. 10. The calculated variation of power $P_{\text{out, sq}}$ left in the square core at the end of a device, with a coupling region exactly one coupling length long, is shown in Fig. 11 as a function of the slab index $n_s$. The index change needed to produce a modulation depth ($P_{\text{out, sq}}/P_0$ from about 0.95 to about 0.05) is $dn = 4.5 \times 10^{-4}$. This change can be achieved in MNA (for which $n_s = 1.8 \pm 0.1$ and $r_{11} = 67 \pm 25 \times 10^{-12}$ m/V at the Nd-YAG wavelength [12]) using a field strength $E = 2dn/n_s^3 r_{11}$ (12)

of about $2.3$ V/μm.

The field strength required could be lowered by increasing the sensitivity to refractive index, which, however, would be at the expense of the optical 3dB bandwidth of the structure. This, for the structure under consideration, was calculated to be 40nm (Fig. 12).

The electrodes of such an arrangement may be fabricated readily by depositing them as patterned thin films onto the surface of either the glass or the crystalline slab.
The aim of this paper was to explore the practicability of fabricating novel structures employing rectangular-core directional couplers with refractive indices chosen to match the characteristics of organic materials. The structure proposed is a fast electro-optic modulator employing a directional coupler consisting of a glass square core of high refractive index (about 1.8) and a strip-loaded crystalline slab made from the material 2-methyl-4-nitroaniline (MNA) that has a refractive index for the x-directed polarisation of $n_x = 1.8 \pm 0.1$ at the Nd:YAG wavelength of 1.064 $\mu$m. By using a finite-element formulation employing the vector magnetic field and a proper penalty function to eliminate spurious (nonphysical) modes, it was possible to analyse the above directional coupler and propose a specific design with phase-matched coupling length $L_{\text{max}} = 2.43$ mm. This design could be the basic element of an electro-optic intensity modulator/sensor capable of producing a 20:1 modulation depth ($P_{\text{out, max}} / P_{\text{in}}$) from about 0.95 to about 0.05) with a $dn = 4.5 \times 10^{-4}$ achievable by a field strength of about 2.3 V/$\mu$m.

9 Acknowledgments

The authors wish to thank the SERC and the Imperial College Computer Centre for the provision of computing facilities, and the Greek State Scholarship Foundation (IKY) for the provision of maintenance grant for one of us (G.P.). Part of this work is based on the thesis [13] submitted to the University of London for the PhD degree.

10 References