Turbulence and Energy Transfer in Strongly-Stratified Flows

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Outline

• Define, discuss ‘stratified turbulence’
  – potentially relevant to strongly stable regions of atmosphere, oceans

• Some numerical simulations of ‘stratified turbulence’
  – direct numerical simulations – some evolving flows
  – large eddy simulations – forced flows

• Scaling arguments
  – possible ‘stratified turbulence’ inertial range

• Field data
  – mainly ocean results
Stratified Turbulence (Lilly, 1983)

- Controlling parameters
  - Reynolds number: \( R_\ell = u' \ell_H / \nu \)
    * \( u' \) – characteristic rms velocity
    * \( \ell_H \) – horizontal scale of energy-containing motions
  - Froude number: \( F_\ell = u' / N \ell_H \sim T_B / T_{FM} \)
  - Gradient Richardson Number: \( Ri = N^2 \left/ \left( \frac{\partial u}{\partial z} \right)^2 \right. \)

- Typically, for strongly stable atmospheric boundary layers, for the atmosphere near and above the tropopause, for much of the ocean, etc.
  - for \( \ell_H \sim 200 \text{ m} \), \( F_\ell < O(1) \), \( R_\ell \gg 1 \) (e.g., \( 10^8 \) or more)
  - \( Ro = u' / \Omega \ell_H \gg 1 \), no effect of rotation
Stratified Turbulence (cont’d)

- Definition of Stratified Turbulence
  - atmospheric/oceanic motions such that
    \[ F_\ell < O(1), \quad Ri \sim O(1), \quad R_\ell \gg 1 \]
  - contains both internal gravity waves and quasi-horizontal motions
    * potential vorticity is of importance
  - scaling arguments suggest that ‘classical’ turbulence will exist when
    \[ R_b \sim F_\ell^2 R_\ell \sim \epsilon/\nu N^2 > O(10) \]
    * \( R_b \) is called the ‘activity parameter’, ‘buoyancy Reynolds number’
Stratified Turbulence (cont’d)

\[ F_l^2 R_l = \epsilon / \nu N^2 = \text{constant} \]

- Stratified Turbulence
- Kolmogorov turbulence
- Laboratory turbulence
- Laminar?
- Numerical simulations
- Laminar
Laboratory Results – Stratified Turbulence

- Laboratory experiments, e.g., wake of sphere, wake of grid, jets (Flow Research, USC, ASU, Toulouse, Grenoble, Eindhoven, ...)
  - usually when turbulence is generated, $F_\ell \gg 1$
    * but flow decays, $F_\ell$ and $R_\ell$ both decay
  - when $F_\ell \leq \mathcal{O}(1)$,
    * development of quasi-horizontal vortices
    * simultaneous with propagating internal waves
  - but generally $R_\ell$ is low; $R_b$ is low; $R_i > \mathcal{O}(1)$
  - smaller-scale turbulence usually does not develop
  - scaling of full dynamics to geophysical turbulence unclear
Lin & Pao, 1979
Field Results

- Field experiments
  - usually an internal wave component
  - often meandering motions are observed
  - ‘classical’ turbulence is very intermittent, sporadic
  - effects of stratification ‘strong’ for $\ell > \ell_O \sim 1$ m (ocean)
    * where $\ell_O = (\epsilon/N^3)^{1/2}$, the Ozmidov scale
  - for the strongly stable atmosphere, $\epsilon \sim 5 \cdot 10^{-4}$ m$^2$/s$^3$, $\ell_O \sim 3$ m
  - component velocities highly non-isotropic
Theoretical Arguments – Stratified Turbulence

• Lilly (1983) used scaling arguments to suggest, for $F_\ell \leq O(1)$:
  – flows in ‘adjacent’ horizontal layers are somewhat decoupled
  – leads to increasing vertical shearing of horizontal flow
  – and to decreasing Richardson numbers

• Billant and Chomaz (1999)
  – induced velocities lead to strong vertical inhomogeneities and layering

• Even though strong, stable stratification,
  at high Reynolds numbers, both mechanisms lead to
  – smaller vertical scales continually developing
  – local instabilities and turbulence intermittently occurring
Questions – Stratified Turbulence

- What are the dynamics of turbulent motions when \( F_\ell \leq \mathcal{O}(1) \), especially with \( R_\ell \gg 1 \), \( F_\ell^2 R_\ell \gg \mathcal{O}(1) \)?
  - upscale or downscale transfer of energy?

- What are the effects of strong, stable stratification on:
  - turbulence structure, decay rates, dispersion, mixing rates, etc.,
  - turbulence modeling issues?

- Do the results from laboratory and numerical experiments scale up to high Reynolds numbers characteristic of the atmosphere and oceans?
**Research Approach – Stratified Turbulence**

- Numerical simulation
  - solve the 3-D, time-dependent Navier-Stokes equations subject to the Boussinesq approximation
  - uniform stratification, no ambient shear
  - consider flows with $F_\ell \leq \mathcal{O}(1)$
    i. initial value problems; time evolving flows
      - initiate ‘late-stage’ turbulence for a range of $R_\ell, F_\ell$
        enables higher Reynolds number simulations
      - direct numerical simulation; no subgrid modeling
    ii. forced turbulence; statistically stationary flow
      - large eddy simulation; subgrid model

- Accompanying scaling analysis
Initial Value Problems

- Two specific flows considered
  - defined by initial conditions (initial value problems; not forced)
    * Taylor-Green flow + low-level, broad-banded noise, and
    * quasi-horizontal array of ‘Karman’-street vortices
  - for all cases $\rho = 0$ initially
  - for each case, exact same initial conditions, except for $F_\ell$ and $R_\ell$
    - $F_\ell = 4 \left( \frac{u'}{N \ell_H} \approx 0.6 \right)$, $200 \leq R_\ell \leq 9600$

- Both flows have some properties of the late-time vortices observed in the laboratory studies

- Discuss mainly the Taylor-Green results today; the ‘Karman’-street results are qualitatively consistent with these
Three-dimensional contour plots of the stream function for the case with $F_\ell = 4$, $R_\ell = 3200$ at $t = 0$ (left) and $t = 15$ (right).
Mean Square Shear $\left\langle \left( \frac{\partial u}{\partial z} \right)^2 \right\rangle_H$ versus $z$

Mean square vertical shearing of the horizontal velocity vs $z$. $F_\ell = 4$ and $R_\ell = 6400$ at different times.
Mean Square Shear \( \left\langle \left( \frac{\partial u}{\partial z} \right)^2 \right\rangle_H \) versus \( z \).

Mean square vertical shearing of the horizontal velocity vs \( z \).

\[ F_\ell = 4 \text{ and } t = 20 \text{ for } R_\ell = 800, 1600, 3200, 6400. \]
Mean square velocity vs $z$ at $t = 30$, $F_\ell = 4$, various $R_\ell$. 
Volume-Averaged Gradient Richardson Number versus $t$

\[
R_{iV} = \frac{N^2}{\left\langle \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right\rangle}
\]

where $N$ is the buoyancy frequency.
Local Gradient Richardson Number

Re=1600

\[ t=20 \]

Re=6400

\[ t=20 \]
Horizontal kinetic energy spectra at $t = 0, 10, 20, 30$; $F_{\ell} = 4$, $R_{\ell} = 6400$. 
Horizontal kinetic energy spectra at $t = 20$ for four different $R_\ell$ cases.
Scaled horizontal energy spectra, Lindborg (2005).
Horizontal kinetic energy spectra at $t = 18.5$ for $R_{\ell} = 9600$ case.
Implications

- Potential for stratified turbulence ‘inertial cascade’ for large $R_\ell$ (Riley and de Bruyn Kops, 2003; Lindborg, 2005)
  - if $F_\ell \ll 1$, with $\ell_i \sim u'/N$, then $\ell_H/\ell_i \sim \ell_H N/u' = 1/F_\ell$, so $\ell_H \gg \ell_i$
  - if $F_\ell \ll 1$, $R_\ell \gg 1$, $Ri \sim 1$
    * highly anisotropic ‘inertial’ subrange in the horizontal spectral dependence only on $\epsilon$, $\chi$ and $k$
    * $E_u(\kappa_H) = C_u \epsilon^{2/3} \kappa_H^{-5/3}$
    * $E_\theta(\kappa_H) = C_\theta \chi \epsilon^{-1/3} \kappa_H^{-5/3}$
    * $d_H^2(t) = C_d \epsilon t^3$ (patch size; possibly)

- Potential for Kolmogorov cascade
  - if $F_\ell R_\ell^{3/4} \gg 1$, then $\ell_i \gg \eta$
Shear Spectra – Ocean (Klymak, 2005)
Power spectra of temperature off the coast of San Diego (30°N, 124°W).
Displacement spectra – Ocean (Hollbrook & Fer, 2005)

Vertical displacement spectra from open ocean (squares) and near slope (dots)
Summary of Field Results

- Field experiments
  - 3-D turbulence is very intermittent, sporadic
  - Often observe in the oceans at scales $\ell_O < \ell_H < 100$'s m
    * horizontal spectra in velocity, temperature consistent with $\kappa_H^{-5/3}$
    * vertical spectra more consistent with $\kappa_V^{-3}$
  - not classical Kolmogorov-Oboukov-Corrsin spectra
    * highly nonisotropic
    * scales are much too large
    * influence of stable density stratification
  - consistent with numerical simulations, scaling arguments
Conclusions

• (At least) two types of dynamics are present – with $R_i$ initially large
  – horizontal growth of larger-scale, quasi-horizontal motions
  – continual decrease in vertical scales (as suggested by Lilly, 1983)
    * there is strong tendency for vertical shearing of the horizontal velocity to develop
    * this leads to local instabilities, ‘classical’ turbulence and mixing
    * this process occurring intermittently in space causes a downscale transfer of energy

• Both upscale and downscale spectral transfer of energy in the horizontal
  – spectral transfer is very nonistropic
Statistics of larger-scale motions relatively unaffected by changing $R_\ell$, if $R_\ell$ is large enough

- $u'$, $\epsilon$, $\chi$ and $\ell_H$ become approximately independent of $R_\ell$
  * $\epsilon \sim u'^3/\ell_H$, $\chi/\epsilon \approx 0.43$
  * $\lambda \sim R_\ell^{-1/2}$, $\left\langle \left( \frac{\partial u}{\partial z} \right)^2 \right\rangle \sim R_\ell$

- smaller-scale motions adjust to the larger-scale ones
- $w'$, $\rho'$ show more dependence on $R_\ell$
  * their statistics depend more on smaller-scale motions
Conclusions (cont’d)

- There are several important scales in this problem
  - horizontal, energy-containing scales continue to grow \((\ell_H)\)
  - instability scale \((\ell_i)\) behaves as: \(\ell_i/\ell_H \sim (u'/N\ell_H) \ll 1\) since \(Ri \sim 1\)
  - Ozmidov scale \(\ell_O\) behaves as: \(\ell_O/\ell_i \sim (u'/N\ell_H)^{1/2}\)
    * stratification effects ‘strong’ for \(\ell > \ell_O\)
  - Taylor scale \((\lambda)\):
    * decreases with time prior to appearance of ‘classical’ turbulence
    * behaves as: \(\lambda/\ell_H \sim (u'\ell_H/\nu)^{-1/2}\) after flow becomes turbulent
  - Kolmogorov scale \((\eta)\) behaves as: \(\eta/\ell_H \sim (u'\ell_H/\nu)^{-3/4}\)
  - Expect: \(\ell_H \gg \ell_i > \ell_O > \lambda > \eta\)
Conclusions (cont’d)

- Results suggest that, if the flows do not laminarize, they should approximately apply to geophysical flows
  - in laboratory experiments, numerical simulations
    * this could be a problem in the $F_\ell \leq O(1)$ range

- Potential for stratified turbulence ‘inertial cascade’ (Lindborg; Riley and de Bruyn Kops)
  - if $F_\ell \ll 1$, then $\ell_H \gg \ell_i$
    * highly nonisotropic ‘inertial’ subrange
    * possible explanation for scaling range in field data
Temperature spectra – Ocean (Ewart, 1976)

Power spectra of temperature off the coast of Mexico (21°N, 110°W).
Power spectra of temperature near Cobb Seamount (47°N, 131°W).
Temperature spectra – Ocean (Ewart, 1976)

Power spectra of temperature near Hawaii (20°N, 156°W).
Zonal, meridional wind, and potential temperature (Nastrom and Gage, 1985)
Mean Square Patch Size – Ocean

Okubo, Deep-Sea Research, 1971
Spectra of Available Potential Energy – Ocean

Figure 4: Horizontal wave number spectra of available potential energy in the ocean, collected from different observations. Reproduced from Dugan et al. (1986). We have inserted a straight line representing a $k^{-5/3}$ curve.

Spectra of available potential energy in horizontal (Dugan et al., 1986)
Temperature Structure Function – Ocean

Voorhis and Perkins, Deep-Sea Research, 1966

Fig. 10. Temperature structure function along the eastward tow track.
Temperature Spectrum – Ocean

Lafond and Lafond, 1967, Marine Technical Society

Figure 18. Average power spectrum of 25 sections of isotherm depths in the main thermocline and in isotherm depths just below the main thermocline. The slope of $-5/3$ in the log-log relationship is shown for comparison.
Scaled horizontal kinetic energy spectra, Lindborg (2005)
Wave/Vortex Kinetic Energy Decomposition

Horizontal kinetic energy spectra (Riley & deBruynKops, 2003)
Taylor-Green Flow

- Initial velocity and density fields:

\[ \mathbf{v}(x, 0) = U \cos(\kappa z) \left[ \cos(\kappa x) \sin(\kappa y), -\sin(\kappa x) \cos(\kappa y), 0 \right] \]

\[ + \text{ broad-banded, low-level noise} \]

\[ \rho(x, 0) = 0 \]

- In all cases, exact same initial conditions, except for \( F_\ell \) and \( R_\ell \)

- For \( N = 0 \), flow develops into isotropic turbulence, with symmetries (without noise, Brachet et al., 1983)
Taylor-Green Flow (cont’d)

- Simulations discussed today: with $\ell = 1/\kappa$

\[ F_\ell = \frac{2\pi U}{N\ell} = 4, \quad R_\ell = 800, 1600, 3200, 6400 \]

\[ T_B = \frac{2\pi}{N} = 4, \quad T_A = \frac{\ell}{U} = 1 \]

- similar results for $F_\ell = 2$
- have now computer the range $200 \leq R_\ell \leq 9600$
- spans the range from laminar to very active turbulence
Mean square horizontal velocity vs $z$.

$F_{\ell} = 4$ and $R_{\ell} = 6400$ at $t = 0, 10, 20, 30$. 
Horizontal Kinetic Energy versus Time

Volume-averaged horizontal kinetic energy vs $t$, $F_\ell = 4$, various $R_\ell$. 
Volume-averaged vertical kinetic energy vs $t$, $F_\ell = 4$, various $R_\ell$. 
Volume-averaged potential energy vs $t$, $F_\ell = 4$, various $R_\ell$. 
Volume-averaged kinetic energy dissipation rate vs \( t, F_\ell = 4 \), various \( R_\ell \).
Volume-averaged potential energy dissipation rate vs $t$, $F_\ell = 4$, various $R_\ell$. 
Mixing Efficiency $\langle \chi \rangle / \langle \epsilon \rangle$ versus Time

Mixing efficiency vs $t$, $F_\ell = 4$, various $R_\ell$. 
colors: $w$ in xy plane $Lz/2$ at time $t = 182.8125$ s