Energy flux to subgrid scales
as obtained from particle tracking

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Motivation

$\hat{A}_{ij}$ properties


LES context


What can 3D-PTV contribute?

so far: HPIV, 2D PIV, DNS
Content of this presentation

• 3D particle tracking
• filtered derivatives
• energy flux how to decompose/represent it
• energy flux from PTV & nonlinear model
• correction for nonlinear model
• conclusions
Main idea of 3D-PTV

to follow a 3D (!) particle position as opposed to 2D PIV!

List of technical aspects

- flow tracers
- illumination
- cameras
- observation volume
- camera callibration
- particle detection
- from 2D to 3D positions
- particle tracking
Flow tracers

high tech, accurate, expensive:

low tech, accurate, cheap:

Idea: Søren Ott & Jakob Mann, Risø, Denmark
fly ash $\rightarrow$ sieving $\rightarrow$ Ø50-60µm
Illumination

LED array, TU/e

Lorenzo del Castello, Herman Clercx

trend towards smarter solutions
Fast digital cameras

- pixel: 500x500
  - frame rate: 50Hz
- pixel: 1000x1000
  - frame rate: 5000Hz
  - or
- pixel: 250x250
  - frame rate: 80’000Hz

Data storage is main bottleneck
Camera callibration

• teach the cameras with known grid points
• problem: how to have space filling target?
• solution in part: callibration on flow tracers
From 2D to 3D position

callibration and 2D position accuracy, seeding density, etc.
Tracking through consecutive images

Tracking criteria:
particle must not travel further than their typical spacing

codes available at www.3dptv.schtuff.com
Many dependencies, many choices…

- flow speed
- trackability
- field of view
- depth of view
- optical working distance
- camera recording rate
- illumination
- particle diameter
- camera pixel resolution
- number of tracer particles
- flow scales one would like to resolve
- particlediameter
- number oftracer particles
- trackability
- flow speed
Final output is the start for analysis

if all goes well, one can finally start ’learning’ about the flow
Velocity derivatives

differentiate convoluted velocity field to get velocity derivatives

challenge to get HIGH SEEDING DENSITY

\[ \tilde{u}(x) \approx \frac{4\pi(\Delta/2)^3}{3n} \sum_{x' \in B_\Delta(x)} \rho_\Delta(x - x') u(x') \]

\[ \rho_\Delta(r) = \begin{cases} 
\frac{15((\Delta/2)^2 - r^2)}{8\pi(\Delta/2)^3} & \text{for } r \leq \Delta/2 \\
0 & \text{for } r > \Delta/2 
\end{cases} \]

\[ \tilde{A}_{ij}(x) \approx \frac{20}{(n - 1)\Delta^2} \sum_{x' \in B_\Delta(x)} (x'_j - x_j) u_i(x') \]

B. Lüthi ETH, Søren Ott Risø, Jacob Berg, Jakob Mann
Particle seeding, scales?

How dense can we track??

How fast can we record??

\[ \int_V \quad \frac{d^3 x'}{\Delta} \]

current seeding range

\[ \eta \quad L \]

current \( \text{Re}_\lambda \) : 170, \( L/\eta \sim 200 \)
Velocity gradients
Velocity gradients
Self amplification

(a) >12 points

\( \Delta\eta = 100 \)

\( \Delta\eta = 300 \)

(b) 4 points

\( \Delta\eta = 300 \)

\( \Delta\eta = 100 \)
Structure
RQ invariant maps
Figure 3. Streamfunction contours of the average velocity field and mean streamwise velocity profiles at ten downstream positions ($\text{Re}_{\infty,b} \approx 2800$). Positive streamfunction values appear in red, negative values in blue.

Edges of mean flow recirculation zones. Present LES, DNS by Peller & Manhart.¹

Jörg Ziefle, Kleiser LES group ETH
Definition of SGS TKE production rate\(^1\) or 'energy flux’

\[ \tilde{u}_{i,j} = 0, \]

\[ \dot{u}_i + (\tilde{u}_i \tilde{u}_j)_j = -\tilde{p}_j/\rho - \tau_{ij,j} + \nu \tilde{u}_{i,ij} \]

\[ \tau_{ij} = u_i u_j - \tilde{u}_i \tilde{u}_j \]

\[ \tau_{ij}^* = \tau_{ij} - \frac{\delta_{ij}}{3} \tau_{kk} \]

\[ P_r = -\tau_{ij}^* \tilde{S}_{ij} \]

also referred to as:

- energy flux
- SGS dissipation

Role of PTV for LES?

• holographic PIV, Tao et al. 2002, van der Bos et al. 2002
• sonic anemometer array, Higgins et al. 2003
• 3D-PTV

• check SGS modeling assumptions
• check is possible in real flows
Energy flux from 3D-PTV

mean flux \sim 0.7 \varepsilon

Tao et al. 2002:
mean flux > 2 \varepsilon

how to trust flux?
analysis and sources of error?
Eigenvalues of $\tau_{ij}$

Chumakov 2006 JFM also introduces $q^*$
Alignment of $\tau_{ij}$ to $s_{ij}$ (Tao et al. 2002)

causes backscatter →

Smagorinsky case
Contributions to flux

most important $\tau_1$ least important $\tau_2$
Alignment of $\tau_{ij}$ to $s_{ij}$

- $\tau_1$, $\tau_3$ most relevant
- $\tau_1$ aligned with $\lambda_3$
- $\tau_2$, $\tau_3$ perpendicular to $\lambda_3$
Alignment of $\tau_{ij}$ to $s_{ij}$
Flux in terms of RQ

van der Bos et al. 2002, Physics of Fluids 14(7) holographic PIV
Flux in terms of RQ

\[ \frac{\langle \tau_{ij} S_{ij} \rangle}{2\nu \langle s^2 \rangle} \cdot p(R, Q) \]
Flux in terms of RQ

\[ \frac{\langle \tau_{ij} S_{ij} \rangle}{(2\nu \langle s^2 \rangle)} \]

PTV

DNS

public data from Biferale, Boffetta, Toschi etc.
Smagorinsky, nonlinear, mixed, ...

Scalar eddy viscosity:
- related to strain
- no backscatter possible
- stable

Tensor eddy viscosity:
- related to strain and vorticity production
- allows for ’backscatter’
- is unstable

$$\tau_{ij}^{\text{smag},d} = -2c_s^2 \Delta^2 |\tilde{S}| \tilde{S}_{ij},$$

$$\tau_{ij}^{nl} = c_{nl} \Delta^2 \tilde{A}_{ki} \tilde{A}_{kj},$$

$$\tau_{ij}^{\text{mix}} = c_{nl-m} \Delta^2 \tilde{A}_{ki} \tilde{A}_{kj} - 2c_{s-m}^2 \Delta^2 |\tilde{S}| \tilde{S}_{ij}$$
Validation: Eigenvalues of $\tau_{ij}$

nonlinear model:
- overestimates large $\tau_1$
- underestimates $\tau_2$ and $\tau_3$
Validation: Alignment of $\tau_{ij}$ to $s_{ij}$
Validation: Alignment of $\tau_{ij}$ to $s_{ij}$

nonlinear model:
- too deterministic
- too little $\tau_1 \lambda_3$ alignment
- too much $\tau_2 \lambda_1$ alignment
Flux error in terms of RQ
Flux error in terms of RQ

\[
\frac{\langle \text{flux error nl} \rangle}{2 \nu \langle s^2 \rangle} \cdot p(R, Q)
\]
Possible correction for nonlin. model

2 rotations to get $\tau_1$ 'right'

1 rotation around $\tau_1$

blue on red $\rightarrow$ more backscatter
blue on blue $\rightarrow$ more energy flux to small scales

green on red $\rightarrow$ more backscatter
green on green $\rightarrow$ more Smagorinsky
$\rightarrow$ more energy flux to small scales
Fig. 3. Normalized prediction error density $\Delta \Pi / \langle 2 \nu s^2 \rangle \cdot p(R, Q)$ for a) the nonlinear model and b) for the corrected nonlinear model. Yellow to red colours denote over prediction of back-scattering or to weak energy flux from large to small scales, and light to dark blue colors show where energy flux from large to small scales is too strong.
Conclusion

• particle tracking can access (LES) energy flux  
• can be used to study SGS models  
• e.g. the ’nonlinear model’  
• we find systematic misalignment  
• ’corrected model’ has less flux error  

• need&possibility for more specific experiments