Turbulent Rotating Rayleigh-Bénard Convection: DNS and SPIV Measurements

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First IMS Turbulence Workshop
Interscale energy transfers in various turbulent flows
March 26-28, 2007, Imperial College London, UK
Convection and Rotation

In geophysical/astrophysical flow settings

www.NOAA.org

www.PSC.edu
Influence of background rotation

Taylor–Proudman theorem $\leftrightarrow$ no vertical variation of velocity under geostrophic conditions

“Taylor column” above object dragged through a rotating fluid
Spin-up of plumes

Converging flow near walls $\rightarrow$ plumes with cyclonic vorticity
Outline

- Direct Numerical Simulations

  Simplified geometry, basic effects of rotation on RB convection

- Laboratory Experiments
DNS: computational domain

periodic

\[ \Omega \]

no-slip; \( T = 0 \)

no-slip; \( T = 1 \)

128 gridpts

64/128 gridpts

128 gridpts

2

1

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DNS: equations

Navier-Stokes and heat equations in Boussinesq approximation with incompressibility:

\[
\frac{Du}{Dt} + \sqrt{\frac{\sigma Ta}{Ra}} \mathbf{z} \times \mathbf{u} = -\nabla p + T\mathbf{z} + \sqrt{\frac{\sigma}{Ra}} \nabla^2 \mathbf{u},
\]

\[
\frac{DT}{Dt} = \frac{1}{\sqrt{\sigma Ra}} \nabla^2 T,
\]

\[
\nabla \cdot \mathbf{u} = 0,
\]

Rayleigh: \( Ra = \frac{g\alpha \Delta T H^3}{\nu \kappa} \)  
Taylor: \( Ta = \left( \frac{2\Omega H^2}{\nu} \right)^2 \)  
Prandtl: \( \sigma = \frac{\nu}{\kappa} \).

Buoyancy/Coriolis ratio \( \rightarrow \) Rossby number: \( Ro = \sqrt{\frac{Ra}{\sigma Ta}} \).
Simulation values

Two series:

- \( \sigma = 1, \ Ra = 2.5 \times 10^6 \)
  - \( Ta = 0 \ldots 10^8 \)
  - \( Ro = \infty \ldots 0.16 \)

- \( \sigma = 1, \ Ra = 2.5 \times 10^7 \)
  - \( Ta = 1.6 \times 10^6 \ldots 2.3 \times 10^8 \)
  - \( Ro = 4.0 \ldots 0.33 \)
Temperature isosurfaces

$Ra = 2.5 \times 10^6$

$Ro = 1.33$

$Ro = 0.5$

Strong rotation (lower $Ro$) gives columnar flow structuring
**Vertical-velocity skewness**

\[ S_w = \frac{\langle w^3 \rangle}{\langle w^2 \rangle^{3/2}} \]

Indicates area fraction of horizontal cross-sections containing upward/downward motion.

**\( S_w > 0 \):** Fraction of cross-section containing upward motion smaller than fraction containing downward motion.

Quantification of localization.
Vertical-velocity skewness for $Ra = 2.5 \times 10^6$

`Switch' of $S_w$ points to change in flow structuring near wall; extremum around $Ro = 0.75$
Cross-sections on top of lower viscous BL

$w$  $\omega_z$  $T$

$Ro = \infty$

- Vertical motion in sheet-like structures
- No clear relation with vorticity field
Cross-sections on top of lower viscous BL

$Ro = 0.5$

- Vertical transport inside vortical columns fed by Ekman pumping
- Definite correlation of $w$, $\omega_z$ and $T$
Horizontal and vertical rms velocities

Rotation lowers both horizontal and vertical rms velocities
Average and rms temperatures

Mean temperature gradient over bulk; rms temperature increases, then collapses
Boundary layer thicknesses \((Ra = 2.5 \times 10^6)\)

\[ \lambda_v = \text{viscous BL} \]

\[ \lambda_\theta = \text{thermal BL} \]

BL thickness

= position at which rms value is maximal

\[ \sim Ta^{-1/4} \]
Heat transfer

Nusselt number calculated as:

\[ \text{Nu} = \frac{\partial \langle T \rangle}{\partial z} \bigg|_{\text{wall}} \]

\[ \text{O, X } Ra = 2.5 \times 10^6 \]
\[ \text{◊, + } Ra = 2.5 \times 10^7 \]

Rossby \textit{J. Fluid Mech.} \textbf{36} (1969)
Hunter & Riahi \textit{J. Fluid Mech.} \textbf{72} (1975)

\[ \text{Nu} \]

\[ Ta = 0 \]

\[ x + = \text{top wall} \]
\[ o ◊ = \text{bottom wall} \]

Rossby (\( \sigma = 6.8 \))
experiment

Hunter & Riahi (\( \sigma = \infty \))
upper bound \( \sim Ta^{-2} \)
Conclusions — DNS

- Rotation causes columnar flow-structuring

- Under strong rotation vortical plumes cover nearly all vertical transport

- Enhanced heat transfer at moderate rotation rates → Ekman pumping

- Strong geostrophic damping of vertical motion at higher rotation rates

Outline

- Direct Numerical Simulations
- Laboratory experiments

Large Scale Recirculation and background rotation; emergence of vortical regime; modification of structure functions by background rotation.
Experimental setup

Cylindrical convection cell of diameter and height 23 cm placed on rotating table

Measurement technique
→ Stereo-PIV
Parameter range

Prandtl — Working fluid is water: \( \sigma \approx 7 \)

Rayleigh — Temperature difference \( \Delta T \) up to 5 K: \( Ra = 0 \ldots 10^9 \)

Taylor — Centrifugal effects are small
   Example: if \( \Omega^2 r / g < 0.1 \rightarrow Ta = 0 \ldots 10^{11} \)

Rossby — At \( Ra = 10^9 \): \( Ro = \infty \ldots 0.039 \)
Stereo-PIV

- 2 cameras at different viewing angles
- Laser sheet
- Seeded water

2 views at different angles
3rd velocity component through geometric reconstruction

3 components of velocity in 2D cross-section of domain
Stereo-PIV
Stereo-PIV

Re_\lambda \sim 200 \ , \ H=5 \ cm

\Omega=0: \ stationary, \ reproducible, \ and \ (u')^2 \sim (v')^2 \sim (w')^2.

Characterization of rotating turbulence at several heights in the rotating fluid; \ \Omega=1, \ 5, \ 10 \ rad/s.
Measurements

At $Ra = 1.1 \times 10^9$, $\sigma = 6.4$

(1) Effect of rotation on well-known large-scale circulation cell of nonrotating case

$Ta = 0$, $Ro = \infty$

$Ta = 1.3 \times 10^6 \ldots 8.4 \times 10^7$, $Ro = 11.5 \ldots 1.4$

(2) Flow behaviour at larger rotation rates $\rightarrow$ towards vortical regime

$Ta = 3.4 \times 10^8 \ldots 2.2 \times 10^{10}$, $Ro = 0.72 \ldots 0.090$
No rotation ($Ro = \infty$)

Large-scale circulation (LSC) across cylinder domain, azimuthal oscillation
Oscillation of LSC \((Ro = \infty)\)

“Centroid for \(w > 0\)” (O) and “centroid for \(w < 0\)” (X)

From autocorrelation \(R\):
oscillation period \(\tau_0 = 140\) s

(In agreement with Xi et al., PRE 73, 056312 (2006).)
Ro=5.8  
Large-scale circulation cell remains intact

Ro=2.9  
Break-up of LSC, some vorticity is apparent

Ro=0.09  
Vertical transport mostly inside tiny vortices
Vorticity animation (\(Ro = 0.090\))

\[ \omega_z \text{ (1/s)} \]

Vortices of both signs are present; quasi-2D vortex interactions
**Bolgiano-Obukhov (BO) scaling**

Structure function: \( S_w^p(r) = \langle |w(x + r) - w(x)|^p \rangle \)

In buoyancy-dominated convection scaling determined by \((g\alpha), N, r\).

\[ N = \kappa \langle |\nabla T|^2 \rangle \iff \epsilon = \nu \langle |\nabla u|^2 \rangle \]

Dimensional analysis gives:

\[ S_w^p(r) \sim (g\alpha)^{2p/5} N^{p/5} r^{3p/5} \]

BO scaling valid for \( r > L_B = \frac{\epsilon^{5/4}}{(g\alpha)^{3/2} N^{3/4}} \)

Estimate from other work: \( L_B = 6.2 \text{ mm} \)
Spatial SFs at $\Omega = 0$ ($Ro = \infty$)

Second-order SF of vertical velocity

Open symbols: calculated in $y$ direction

Closed symbols: calculated in $x$ direction
Temporal SFs and Bolgiano length/time

From experiments: time series $w(t)$

Temporal SF: $S_{w}^{p}(\tau) = \langle |w(t + \tau) - w(t)|^{p} \rangle$

Taylor's hypothesis: $r$ can be replaced by $U\tau$ for turbulence with an effective `sweeping' velocity $U$

Temporal SF scaling: $S_{w}^{p}(\tau) \sim \tau^{3p/5}$

From model of LSC [Villermaux *Phys. Rev. Lett.* 75 (1995)]: $U = 2H/\tau_{0}$

Estimated “Bolgiano time” $\tau_{B} = L_{B}/U = 1.9$ s
Temporal SFs at $\Omega = 0 \ (Ro = \infty)$

Indication of BO scaling in temporal SFs
Temporal SFs at different $\Omega$

Scaling range ends at time scale dependent on $\Omega$
Steepening at moderate $\Omega$ (compared to BO)

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Summary — Experiment

- Stereo-PIV measurements in cylindrical convection cell
- Effects of rotation on LSC studied
- At higher rotation rates the vortical state is found
- Structure functions give indications of BO scaling without rotation; rotation modifies scaling

*R.P.J. Kunnen, H.J.H. Clercx, B.J. Geurts, L.J.A. van Bokhoven, and R.A.D. Akkermans, to be submitted to PRE.*
Outlook

Investigation of vortical plumes and relation with heat transfer

DNS on a cylindrical domain → comparison with experiment

Experiments using Laser Induced Fluorescence → local temperature measurement in 2D cross-sections