Abstract
We use a pore-scale network model of three-phase flow to compute relative permeabilities, saturation paths and capillary pressures for a variety of displacement processes. The model is based on a random network of pores and throats with triangular, rectangular and circular cross-sections that represent the complex pore space observed in sandstones. We model wettability alteration after primary drainage and allow any values for the advancing and receding oil/water, gas/water and gas/oil contact angles. Multiple phases can be present in each pore, in wetting and spreading layers, as well as occupying the center of the pore space. In all, twenty different generic fluid configurations for two- and three-phase flow are analyzed.

With a network based on a description of Berea sandstone we can predict relative permeabilities for two-phase flow in a water-wet system and waterflood recoveries for mixed-wet media. We then predict the steady-state oil, water and gas three-phase relative permeabilities measured by Oak.1 We demonstrate that the predictions obtained by the network model compare favorably with those obtained using standard empirical relative permeability correlations.

We then study gas injection into media of different wettability and interpret the results in terms of pore-scale displacement processes.

Introduction
The simultaneous flow of three phases – oil, water and gas – in porous media occurs in a variety of reservoir and environmental engineering problems. During conventional two-phase mechanisms of oil recovery, such as waterflooding, oil recovery is typically 20 to 50% indicating that oil flows at relatively high saturations.2 In such circumstances uncertainties in the relative permeability often have little impact on recovery predictions. In contrast, during three-phase recovery processes such as gas injection, gas cap expansion, depressurization, solution gas drive and gravity drainage, to achieve a higher recovery oil may flow at very low saturations, implying that the oil relative permeability can also reach very low values. Direct measurement of relative permeabilities and capillary pressures in this regime is not only very difficult but also time consuming and inaccurate. Since two independent fluid saturations are required to define a three-phase system, there is an infinite number of possible fluid arrangements and displacement paths, making a comprehensive suite of experimental measurements for all three-phase displacements impossible. Consequently, numerical simulations of three-phase flow rely on empirical models to predict relative permeability and capillary pressure from measured two-phase values. However, these models may give predictions that vary as much as an order of magnitude from each other, or from direct measurements, since they have little or no physical basis.3 In such cases uncertainties in relative permeability, particularly at low oil saturation, may have a very significant impact on recovery predictions.

An appealing approach to predict three-phase properties accurately is to use physically-based pore-network modeling. The model should capture the pore-space geometry and the pertinent pore-scale displacement mechanisms and be tuned to, or be able to predict, any available single- or two-phase data. In recent years, several advances in pore-scale modeling have made this approach possible. Øren, Bakke and co-workers at Statoil4-6 have developed random network models based on the pore space geometry of the rock of interest. The model is derived either from a direct three-dimensional image of the pore space obtained from micro CT scanning, or from simulating the geological processes by which the rock was formed. Many other authors have also developed techniques to derive pore structures from a variety of measurements.7-24 While such approaches are
not routine, and the correct pore-space characterization of carbonates is very much an open question, for simple sandstones there are reliable methods from determining an equivalent network structure that attempts to mimic the properties of the real pore space. Second the pertinent displacement processes for three-phase flow have been observed and described using micromodel experiments.\textsuperscript{25-32} For water-wet media the pore-scale mechanisms of fluid displacement are now well established. The behavior of mixed-wet systems has a less firm experimental basis, but the work of Kovscek \textit{et al.}\textsuperscript{33} has provided a pore-level scenario for wettability alteration that has been used to describe fluid configurations for two- and three-phase displacements.\textsuperscript{34-38}

The network model described here combines three essential components: (1) a description of the pore space and its connectivity that mimics real systems; (2) a physically-based model of wettability alteration; and (3) a full description of fluid configurations for two- and three-phase flow. Many three-phase network models have been proposed previously.\textsuperscript{39-51} However, this is the first model that combines all three of the features above to predict three-phase relative permeability for media of arbitrary wettability using geologically realistic networks.

**Network model**

For two-phase flow the network model used here is based on the work of Øren, Patzek and co-workers.\textsuperscript{4-6,52} Rather than repeat their work, only a brief description of two-phase flow is provided. Further details are available in the Appendix.

**Geometry.** The model reads as input any two or three-dimensional network comprised of pores (nodes) connected by throats (links). Each pore or throat is assigned a total volume, an inscribed radius and a shape. The inscribed radius is used to assign a capillary entry pressure during multiphase flow. The pore and throats have a scalene triangular, square or circular cross-section. The cross-section has the same shape factor (ratio of cross-sectional area to perimeter squared) as the real system from which the network is derived. A clay volume is associated with the network. This represents a volume that remains water saturated. It can be adjusted to match the observed connate water saturation.\textsuperscript{4-6} In this paper a network based on a Berea sandstone, Fig. 1, will be used for all the results presented.

The network is a cube of volume 27 mm\textsuperscript{3} with 12,349 pores and 26,146 throats. The coordination number varies from 1 to 19 with an average of 4.19. The inscribed radius of the pores varies from 3.62 to 73.54 \(\mu\text{m}.\) The throat inscribed radius varies from 0.90 to 56.85 \(\mu\text{m}.\) The total porosity is 24.02\% with a clay volume of 5.7\%. 92.27\% of the pores and throats have a triangular cross-section, 6.51\% are square and 1.22\% are circular. The absolute permeability of the network is 2,600 mD.

**Primary drainage and wettability alteration.** The network is initially fully saturated with water and strongly water-wet, with the receding oil/water contact angle \(\theta_{owr} = 0^\circ.\) Oil then enters the network, and as the capillary pressure is increased step by step, it invades the pore or throat with the lowest capillary entry pressure in an invasion percolation process. Drainage is complete when a target capillary pressure or saturation has been reached, or when all pores and throats have been invaded by oil.

At the end of primary drainage, regions of the pore space in direct contact with oil may change their wettability. We assign an advancing oil/water contact angle \(\theta_{owa}\) to each oil-filled pore and throat after primary drainage. Different pores and throats may have different contact angles. Contact angles are assigned at random to oil-filled pores and throats, according to some specified distribution. In this paper, the contact angles will have a uniform distribution between some specified maximum and minimum values.

After primary drainage, the model simulates any sequence of water, gas and/or oil invasion.

**Contact angles for three-phase flow.** In three-phase flow, the oil/water, gas/oil and gas/water contact angles are not independent. The relationship between them is:\textsuperscript{53-55}

\[
\sigma_{gw} \cos \theta_{gw} = \sigma_{go} \cos \theta_{go} + \sigma_{ow} \cos \theta_{ow} \tag{1}
\]
For oil invasion over surfaces previously contacted by oil, space that were not contacted by oil during primary drainage (displacing water) is zero if the oil contacts portions of the pore throats. For oil displacing gas, Eq. (2) with \( \cos \theta_{go} = \cos \theta_{gw} = 1 \) and from Eq. (2), \( \cos \theta_{gw} = (\sigma_{go} - \sigma_{ow})/\sigma_{gw} \). Van Dijke et al.\textsuperscript{46,50} suggested assigning the gas/oil and gas/water contact angles as a function of \( \theta_{go} \) by a linear interpolation between the water-wet and oil-wet limits:

\[
\cos \theta_{go} = \frac{1}{2} \left[ \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}} - 1 \right] \cos \theta_{ow} + 1 + \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}}
\]

(2)

\[
\cos \theta_{gw} = \frac{1}{2} \left[ 1 - \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{gw}} \right] \cos \theta_{ow} + 1 + \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{gw}}
\]

(3)

We use these expressions in this paper to find contact angles. To accommodate any type of displacement process, we assign both receding and advancing contact angles. The oil/water advancing contact angle \( \theta_{oww} \) (water displacing oil) is found as described before. The receding oil/water contact angle \( \theta_{owr} \) (oil displacing water) is zero if the oil contacts portions of the pore space that were not contacted by oil during primary drainage. For oil invasion over surfaces previously contacted by oil, contact angles are again assigned to different pores and throats but with the constraint that \( \theta_{owr} \leq \theta_{oww} \). For gas displacing oil (oil receding) \( \theta_{gor} \) is found from Eq. (2) using \( \theta_{owr} \) (water receding). For oil displacing gas, Eq. (2) with \( \theta_{oww} \) is used to find \( \theta_{gor} \). For gas displacing water (water receding) \( \theta_{gwr} \) is found from Eq. (3) using \( \theta_{owr} \). For water displacing gas, Eq. (3) with \( \theta_{oww} \) is used to find \( \theta_{gwr} \). In this work we only consider a spreading system \( C_{so} = 0 \), where \( \theta_{gor} = \theta_{gwr} = 0 \) for all pores and throats.

Three-phase fluid configurations. Fig. 2 shows all the possible configurations of one, two or three phases in a single pore or throat. Fluid arrangements for a single corner are illustrated – the whole pore or throat is composed of no (circle), three (triangular) or four (square) corners. For elements with a circular cross-section, only one phase may occupy the pore or throat at one time. For square and triangular cross-sections, the phase in the middle of the element must be the same for each corner, but different corners may have different configurations, depending on the corner angle. However, in a single pore or throat, the contact angles in each corner are the same. Table 2 in the Appendix lists the range of contact angles for which each configuration in Fig. 2 can occur. A capillary pressure between any two phases \( P_{cij} \) is defined as \( P_i - P_j \). We assume that the pores and throats have little curvature perpendicular to the cross-section. As a consequence, the radius of curvature \( r \) of the interface between two phases \( i \) and \( j \) in Fig. 2 is related to the capillary pressure by:

\[
P_{cij} = \frac{\sigma_{ij}}{r}
\]

(4)

Displacement calculations. The model assumes quasi-static displacement controlled, at the pore scale, entirely by capillary forces. The simulation proceeds as a series of displacements. A displacement is the change of the configuration in one pore or throat. This can represent the replacement of one phase by another in the center of the pore, or the collapse or formation of a layer in a single corner. There is a capillary pressure associated with the transition from one configuration to another.

We define pressures of water, oil and gas in the network. Initially when the network is water-filled all phase pressures are set to zero. As displacement proceeds, these pressures will increase. There are six possible types of displacement: oil into water (controlled by an increase in \( P_{cow} \)), water into oil (decrease in \( P_{cow} \)), gas into water (increase in \( P_{gwo} \)), gas into oil (decrease in \( P_{gwo} \)), gas into oil (increase in \( P_{ggo} \)), and oil into gas (decrease in \( P_{ggo} \)). Any displacement corresponds to one of these six processes. We rank the capillary pressures for all possible displacements for all pores and throats into six sorted lists, corresponding to the different displacement types.

The network model is designed to compute capillary pressure and relative permeability for a given input saturation path. A target final saturation of oil, water and gas is specified. The difference between the present saturations and the target saturations is computed. The phase with the most negative difference is the next to be injected. Imagine this is phase \( i \). This means that we consider a displacement event where the volume of \( i \) in an element increases. We assume that the pressure of the other two phases \( j \) and \( k \) are held fixed at a value specified at the last displacement where \( j \) and \( k \) were the invading phases, respectively. Since the pressures of phases \( j \) and \( k \) are fixed, \( P_{cjk} \) does not change. Phase \( i \) can invade either phase \( j \) or \( k \). The lowest pressure for phase \( i \) for a displacement event is found by comparing the most favorable capillary pressures from the sorted lists for \( i \) into \( j \) and \( i \) into \( k \). Imagine that the capillary pressures obtained for \( i \) into \( j \) is \( P_{cij} \) and for \( i \) into \( k \) is \( P_{cik} \). If:

\[
P_{cik} > P_{cij} + P_{cjk}
\]

then a displacement of \( i \) into \( j \) is considered, otherwise \( i \) into \( k \). If the event is allowed (see later for a discussion of this issue) then it takes place. If a displacement is not allowed, the event is taken off the sorted list and the most favorable elements of the \( i \) into \( j \) and \( i \) into \( k \) lists are again compared.
Fig. 2. Configurations of three phases in single corners of the pore space. All the different possible arrangements of oil (O), water (W) and gas (G) in a corner are shown. The bold solid line indicates regions of the pore space contacted by oil during primary drainage that have an altered wettability. The points show interfaces that are pinned with a contact angle greater than \( \pi/2 - \alpha \), where \( \alpha \) is the half angle of the corner. However, all the multiphase contact points may be pinned. This means that as the capillary pressure changes, the curvature of the interface changes but that the location of the interface/solid contact is fixed. A phase may be present in the center of the pore space or as a spreading layer, sandwiched between other phases. Water is always present in the corner. The network model simulates a sequence of displacement events that represent the change from one configuration to another. The tables in the Appendix list the capillary pressures for each displacement.
The phase pressure of \( i \) is updated whenever it reaches a new maximum. Displacement capillary pressures are now recomputed for the element filled and nearest neighbor elements, taking into account the new arrangements of fluid and capillary pressures and re-ordered in the sorted lists.

Tables 3-5 in the Appendix list the expressions used to compute capillary pressure for every possible displacement. The capillary pressure depends on the contact angles, corner angles and radius of the element of interest, other capillary pressures and the configuration of nearest neighbor pores and throats. With twenty different configurations and six generic types of invasion, the list of possible displacements is rather daunting, as the three pages of tables in the Appendix indicates. However, each displacement can be considered as a two-phase process (one phase displaces another) and as a consequence are based on well-established two-phase expressions described by Øren, Patzek and others.\(^{4,6,52}\)

The new phase saturations are computed (how this is done is discussed in a later section) and the difference between this and the target saturation recomputed. Again the phase with the greatest negative difference is injected and the process continues until the saturation in the network and the target saturation agree to within some tolerance. If a new phase is injected then the sorted lists for displacement are re-ordered to account for changes in capillary pressures. Then the simulation ceases or a new target saturation is specified.

There is one subtlety associated with specifying capillary pressure. It is often the case that a local configuration change allows a subsequent displacement of the same phase at a more favorable pressure. An example of this is oil displacement into water a water-wet medium. \( P_{cow} \) may be high to allow oil to fill a throat, but oil fills an adjoining pore at a lower capillary pressure. If we used this new lower value of \( P_{cow} \) (corresponding to a lower phase pressure \( P_{oil} \)) to compute radii of curvature using Eq. (4), then we would find that in some elements the radius is inconsistent with the fluid configuration – for instance, water would be in a wetting layer too large to fit in a pore or throat. In strict capillary equilibrium, the fluid would rearrange throughout the network to give configurations consistent with the prevailing capillary pressure. This is, however, a very difficult task for a general three-phase model. Instead, phase pressures are defined as the maximum value ever reached during the whole simulation. Whenever a phase pressure reaches a new maximum, the fluid configuration is truly in a position of capillary equilibrium. In a simulation involving a complex displacement path, the phase pressures will continue to increase. However, since it is only pressure differences that control the displacement sequence, this is not a problem.

**Clustering and trapping.** To account for trapping we assign phases to clusters. A connected region of a phase defines a cluster. A phase can be located in a corner, layer or center of a pore or throat. Phase locations with the same phase in a neighboring pore and throat are connected to each other if the conditions listed in Table 1 in the Appendix are satisfied.

Every phase location in every element has a flag stating whether or not that location is connected to the inlet and outlet. A displacement is only allowed if the displaced phase is connected to the outlet and the displacing phase is connected to the inlet. After the displacement we update the clusters and the corresponding flags for each phase location in clusters connected to the element where the displacement has occurred and nearest neighbor elements. Fig. 13 in the Appendix shows a flowchart to define phase clusters – a burning algorithm is used to assign phase locations to clusters.

**Multiple displacements.** One unique feature of three-phase flow is multiple displacement.\(^{25,26,30,32,41,42,44}\) An invasion of phase \( j \) by phase \( i \) may be composed of a displacement of \( k \) by \( i \) followed by a displacement of \( j \) by \( k \). If all the phases are continuous, this is equivalent simply to two separate events. However, the intermediate phase in the displacement – \( k \) – may be trapped. In this case disconnected clusters of phase \( k \) can rearrange themselves in the pore space, and may reconnect, simply due to capillary forces, when one phase invades part of the cluster that in turn displaces the third phase. This is a double displacement process and has been observed in micromodel experiments\(^{25,26,30,32}\) and coded into network models.\(^{41,42,44,51}\) Multiple displacements, involving more than one intermediate stage, are also possible if two phases are trapped. A cascade of disconnected blobs nudge each other before a final displacement of a connected phase.\(^{51}\) The algorithm for considering such events is somewhat involved, since multiple events for all clusters need to be considered together with the conservation of volume when the cluster moves.\(^{51}\)

In this paper we ignore double and multiple displacements. We will only consider gas injection into relatively high oil saturations, with little or no oil initially trapped. In these cases double displacement is unlikely to be significant.

**Saturation computation.** Saturation is computed just before the phase pressure reaches a new maximum – this means that there are no possible displacements with the current phase pressures. If \( V_{ip} \) is the volume of phase \( p \) in element \( i \) (including the water volume in clay) then the saturation of phase \( p \) is given by:

\[
S_p = \frac{\sum_{i=1}^{n_p} V_{ip}}{n_p \sum_{i=1}^{n_e} V_{ip}}
\]  

where \( n_p \) is the number of phases (three) and \( n_e \) is the total number of pores and throats. The total volume, inscribed radius
and shape of each element is read in as input data. The volume of a phase in an element is the total volume multiplied by the fraction of the cross-sectional area occupied by that phase. Table 6 in the Appendix shows the expressions used to compute areas for the configurations shown in Fig. 2.

Relative permeability and capillary pressure. When saturation is computed, relative permeability and capillary pressure can also be found. This is not done after every saturation computation to save computer time. Typically, relative permeability is calculated around 20 – 40 times during a simulation. The capillary pressure is simply the difference between phase pressures. To compute absolute and relative permeability, conductances of each phase location connected to either inlet or outlet in each element are first specified. Normally exact analytic results are not possible, and empirical expressions are derived from solutions of the Stoke’s equation for flow in pores of different geometries and for different fluid configurations.4-6,38,52,57,58 Table 7 in the Appendix gives expressions for the conductances of each phase for the configurations shown in Fig. 2. Then the average conductance for each phase in the whole network is computed by explicitly calculating the flow through the network. From this absolute and relative permeability can be found. While the expressions for conductance may be involved, the procedure for computing relative permeability is straightforward and has been explained in detail by many authors.5,35,42,52

Predicting experimental data

Two-phase water-wet data. We compare the network model predictions with experimental two-phase data from water-wet Berea cores.1 During drainage, the receding contact angle is assumed to be 0º; there are no other parameters to adjust. Fig. 3 shows the match. To predict the imbibition data we assumed a uniform distribution of advancing contact angles with a minimum of 30º and adjusted the maximum value of contact angle to fit the data. The best match, shown in Fig. 4, was obtained using a maximum advancing contact angle of 80º. Small changes in the contact angle distribution did not adversely affect the match.

Two-phase mixed-wet data. Jadhunandan and Morrow60 performed an exhaustive study of the effects of wettability on waterflood recovery for Berea sandstone. In a companion paper61 we predict waterflood recovery curves as a function of initial water saturation. Our hypothesis is that, for a given crude oil, brine and reservoir (or core) conditions, the distribution of contact angles in pores occupied by oil is similar regardless of the initial water saturation (Swi). Wettability variations with Swi still occur because the number of pore-walls rendered oil-wet following primary drainage increases as Swi decreases. We matched the recovery curves assuming that the advancing oil/water contact angle distribution varied uniformly between 110º and 180º. Tuning of the network model to match a single recovery curve is sufficient to predict successfully waterflood recovery for experiments performed with different values of initial water saturation, representing, for instance, the transition zone of an oil reservoir near the oil/water contact. In ref. 61 we demonstrate how properly incorporating physically-based network model predictions of relative permeability has a huge impact on estimated field-scale recovery, even for two-phase flow.

Three-phase data. We now return to the work of Oak1 and attempt to predict his three-phase steady-state experimental data. Here the systems were water-wet and we used the distribution of advancing oil/water contact angles that matched the two-phase waterflood data (Fig. 4). We then used Eqs. (2) and (3) to assign gas/oil and gas/water contact angles. We simulated the same
flooding sequence as in the experiments, including tracking the changes of two independent saturations. Lerdahl et al. also showed successful network model predictions for this dataset. The difference here is that we honor saturation paths, show a point-by-point comparison of data and make predictions for mixed-wet systems.

Fig. 5 shows the saturation paths for the experiments. The network model successfully tracks almost identical paths. We model gas injection into oil with a low water saturation. In these circumstances little or no oil is trapped at the beginning of gas injection and neglecting multiple displacements is unlikely to affect the results.

![Fig. 5](image)

**Fig. 5—Experimental (points) and network model (lines) saturation paths for the four three-phase flow experiments considered. Note that the network model almost exactly tracks the path followed in the experiments. The numbers refer to the numbers of Oak's experiments.**

Fig. 6 shows the predicted and measured oil relative permeabilities. The predictions are good at high oil saturation, but tend to over-estimate $k_{ro}$ at low saturation. This could be because we have assigned too much connectivity and conductance to oil layers. The predictions of gas relative permeability shown in Fig. 7 all plot on a single curve and are an almost exact match for two of the experiments. Because gas is the most non-wetting phase we expect gas to occupy the largest pores and throats regardless of the oil and water saturations, resulting in $k_{rg}$ being a function of $S_g$ only (see, for instance ). The large scatter in the experimental results is thus surprising and may be due to experimental problems. The predictions for the water relative permeability, Fig. 8, are excellent. Water is the most wetting phase and resides in the smallest pores and the three-phase relative permeability is similar to the two-phase curves shown in Figs. 3 and 4.

![Fig. 6](image)

**Fig. 6—Experimental (points) and network model (lines) oil relative permeabilities for the four three-phase flow experiments considered.**

![Fig. 7](image)

**Fig. 7—Experimental (points) and network model (lines) gas relative permeabilities for the four three-phase flow experiments considered.**
Fig. 8—Experimental (points) and network model (lines) water relative permeabilities for two three-phase flow experiments. The water saturation did not change for the other two experiments.

Fig. 9 shows a comparison of predicted and measured oil relative permeabilities. Also included are comparisons with conventional three-phase relative permeability correlations: Stone 1, and saturation-weighted interpolation. See Ref. 3 for details of how the comparisons were made. Experiment 20 of Oak represents two-phase gas injection into oil with a fixed water saturation. The oil relative permeability for experiment 20 and the oil/water relative permeability shown in Fig. 3 were used as input to the empirical three-phase models. Thus by construction they obtain a perfect prediction for experiment 20. Experiment 10 also has a fixed water saturation and is not considered. For the other two experiments, the water saturation is low, and the empirical models predict relative permeabilities close to those obtained for experiment 20. This, unsurprisingly, gives reasonable predictions. The network model makes predictions of similar accuracy, although it does tend to overestimate $k_{ro}$ at low values of the oil saturation (as discussed above). However, the network model does not require any two-phase curves as input. Moreover, as we show in the next section, it can be used to compute relative permeabilities outside the range of saturations and wettibilities tested experimentally. The empirical model predictions for Oak’s data are considerably worse at high water saturations. A network model test in these cases would require the incorporation of multiple displacements that will be the subject of future work.

Mixed-wet three-phase relative permeability
We will now use our model to predict relative permeability for three-phase mixed-wet systems. In this case we do not have experimental data to compare against, but the success of our predictions for mixed-wet two-phase systems and water-wet two- and three-phase systems gives us some confidence that our results are valid. We consider primary drainage to connate water saturation followed by waterflooding to $S_{w} = 0.4$ ($S_{oi} = 0.6$). Then gas is injected at a constant oil/water capillary pressure. We study two cases: water-wet, where the distribution of advancing oil/water contact angles is the same as used to predict the two- and three-phase data (uniform from $30^\circ$ – $80^\circ$); and oil-wet where the distribution of advancing contact angles ($110^\circ$ – $180^\circ$) is the same as used to predict mixed-wet waterflood data.

Fig. 10 shows the oil relative permeability. In the oil-wet case, oil remains in smaller pores and throats after waterflooding than for a water-wet medium, giving a lower relative permeability. This is seen in Fig. 10 for oil saturations larger than approximately 0.4. At lower oil saturations, gas has invaded most of the oil-filled elements. The relative permeability is limited by the connectivity of the oil phase. In the water-wet medium, oil layers collapse during gas injection, whereas oil layers remain stable throughout the displacement for the oil-wet case. Thus the oil-wet relative permeability is larger at low oil saturation.
The water-wet gas relative permeability in Fig. 11 is larger than the oil-wet case. In a water-wet medium, the gas always occupies the largest pores and throats. For an oil-wet system, gas is not the most non-wetting phase in the presence of water and will displace water from some of the smaller pores, resulting in poorer connectivity and conductance. This effect of wettability has been discussed previously and is well established experimentally. This result is a direct consequence on the constraint on contact angles, Eq. (1). The water relative permeability in Fig. 12 is at first sight surprising. One might expect that the water relative permeability for the oil-wet case to be higher than for the water-wet medium, since water can be non-wetting to both oil and gas in oil-wet systems, occupying the larger pore spaces. The explanation is that during waterflooding water invades the larger pores and throats in the oil-wet medium. This results in an increase in water saturation, but the oil-wet water-filled elements fail to span the network, meaning that the water relative permeability remains very low. This effect is discussed in Ref. 61. During gas injection, since gas displaces water, the water relative permeability can only decrease from its already negligible value.
Conclusions
We have described a quasi-static pore-network model for three-phase flow in mixed-wet systems that uses networks with a variable topology and different pore shapes, incorporating the current state-of-the-art in the description of pore-scale displacement mechanisms and the effects of wettability.

The model successfully predicts experimental two-phase relative permeability and waterflood recovery data for water-wet and mixed-wet Berea sandstone, and three-phase water-wet data.

We used the model to predict three-phase relative permeability for a mixed-wet Berea. We discussed the differences between water-wet and mixed-wet behavior in terms of pore-scale fluid arrangements.

Nomenclature

- $A$ = area, $L^2$, $m^2$
- $b$ = length of water-wet surface in the corner, $L$, m
- $C_{so}$ = oil spreading coefficient, $L^{-1}t^{-2}$, $Nm^{-1}$
- $g$ = conductance, $ML^3t^{-1}$, $Nm^2s$
- $G$ = shape factor
- $k_r$ = relative permeability
- $n$ = number of surrounding throats filled with oil
- $P_c$ = capillary pressure, $mL^{-1}t^{-2}$, Pa
- $r$ = radius of curvature, $L$, m
- $R$ = inscribed radius of pore or throat, $L$, m
- $S$ = saturation
- $V$ = volume, $L^3$, $m^3$
- $\alpha$ = half angle of corner, radians
- $\mu$ = viscosity, $ML^{-1}t^{-1}$, Pa.s
- $\sigma$ = interfacial tension, $L^{-1}t^{-2}$, $Nm^{-1}$
- $\theta$ = contact angle, radians

Subscripts

- $a$ = advancing
- $c$ = corner
- $g$ = gas
- $h$ = hinging
- $i,j,k$ = phase or element labels
- $o$ = oil
- $p$ = phase
- $r$ = receding
- $t$ = total
- $w$ = water

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Appendix
In this appendix we provide details of the equations used to compute capillary pressure, fluid areas (and volumes) and conductance, and we explain how we define connectivity of clusters.

Connectivity. Table 1 lists the circumstances under which phase locations in adjacent pores and throats are considered connected. Note that we assume that within a single pore or throat all corners are connected. This definition of connectivity is used to define clusters of each phase using the flow chart illustrated in Fig. 13.

<table>
<thead>
<tr>
<th>Phase location(1)</th>
<th>Phase location(2)</th>
<th>Connected?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner</td>
<td>Corner</td>
<td>YES</td>
</tr>
<tr>
<td>Corner</td>
<td>Layer</td>
<td>NO</td>
</tr>
<tr>
<td>Corner</td>
<td>Center</td>
<td>NO</td>
</tr>
<tr>
<td>Layer</td>
<td>Layer</td>
<td>YES</td>
</tr>
<tr>
<td>Layer</td>
<td>Center</td>
<td>YES</td>
</tr>
<tr>
<td>Center</td>
<td>Center</td>
<td>YES</td>
</tr>
</tbody>
</table>

Note: Phase location (1) and (2) are located in two nearest neighbor pore and throats.

Table 1—How connectivity is defined. A phase location is the location of a phase in a pore or throat.

Configurations in three-phase flow and capillary pressure. Table 2 lists the range of contact angles for which the configurations in Fig. 2 are allowed. Tables 3-5 indicate the equations used to compute capillary pressure for different displacements: table 3 is for oil/water, gas/oil and gas/water displacements, while table 4 is for water/oil, water/gas and oil/gas displacements. Table 5 lists the equations for layer collapse. Layer formation is accounted for simply by considering layer collapse in reverse. Imagine that we are considering the displacement of phase $j$ by phase $i$. The configuration change can be of two types: either a configuration with $j$ in the center is replaced by one with $i$ in the center (tables 3 and 4), or a layer collapses or forms (table 5). The tables list all the relevant expressions to use. The contact angles and the nature of the displacement allow a unique configuration to be determined after the displacement.

Areas. Table 6 lists the equations used to compute cross-sectional area for the different configurations in Fig. 2. These areas are used to compute saturation, Eq. (6), and to find conductances.

Conductance. Table 7 lists the expressions used to compute phase conductance for each configuration. These are used to find the relative permeabilities.
Area Calculation. The equations used to compute areas and referred to in the tables are listed below.

\[ A_i = \pi R^2 \]  
Circular cross-section \hspace{1cm} (A1a)

\[ A_i = 4R^2 \]  
Square cross-section \hspace{1cm} (A1b)

\[ A_i = \frac{R^2}{4G} \]  
Triangular cross-section \hspace{1cm} (A1c)

\[ A_e = \frac{r^2}{2} \left[ \cos \theta \left( \cot \alpha \cos \theta - \sin \theta \right) + \theta + \alpha - \frac{\pi}{2} \right] \]  
(A2a)

[Note in the following equations, \( \alpha_1 \leq \alpha_2 \leq \alpha_3 \) for a triangular cross-section.]

Conductance. The equations for conductance calculations are:

\[ g = \frac{0.5G A_i^2}{\mu} \]  
Circular cross-section \hspace{1cm} (A3)

\[ g = \frac{0.5623G A_i^2}{\mu} \]  
Square cross-section \hspace{1cm} (A4)

\[ g = \frac{3R^2 A_i}{20\mu} \]  
Triangular cross-section \hspace{1cm} (A5)

\[ g = \frac{A_i^3}{12 A_e} \frac{1 - \sin \alpha}{\tan \alpha \varphi_3^2} \left[ 1 + f_1 \varphi_3 - (1 - f_2 \varphi_3) \frac{A_2}{A_c} \right]^2 \]  
(A6)

\[ \varphi_3 = \left( \frac{\pi}{2} - \alpha \right) \tan \alpha \]  
(A7)

[Note in the following equations, \( \alpha_1 \leq \alpha_2 \leq \alpha_3 \) for a triangular cross-section.]

Capillary Pressure. The equations used to compute capillary pressure are:

\[ P_c = \frac{2\sigma \cos \theta}{R} \]  
(A10)

\[ P_c = \frac{\sigma(1 + 2\sqrt{\pi G}) \cos \theta}{\left[ R_p + \sum_{i=1}^{n} e_i R_{i,j} x_i \right]} \]  
(A11)

\[ e_1 = 0, \ e_2 = 0.5, \ e_3 = 1, \ e_4 = 2, \ e_5 = 5, \ e_6 = 10. \]  
This is an empirical model for pore filling. The sum runs over throats adjoining the pore.

\[ P_c = \frac{\sigma}{R} \left[ \cos \theta + \tan \alpha \frac{\sin 2\theta_2 - 2\theta_2 - 2\alpha + \pi}{2} \right] \]  
(A12)

\[ \theta_3 = \cos^{-1} \left[ \frac{-r \sin^{-1} \left( \frac{b \sin \alpha}{R} \right)}{R - b} \right] \]  
(A13)

[Note in the following equations, \( \alpha_1 \leq \alpha_2 \leq \alpha_3 \) for a triangular cross-section.]

If the interface has not moved yet. If the interface has moved:

\[ b_i = r_{extreme} \frac{\cos(\theta_i + \alpha_i)}{\sin \alpha_i} \]  
\( r_{extreme} = \sigma / P_{c,extreme} \) \hspace{1cm} (A14a)

\[ \theta_{h,i} = \cos^{-1} \left( \frac{r_{extreme} \cos(\theta_i + \alpha_i)}{r_p} \right) - \alpha_i \]  
(A15a)

If the interface has not moved yet. If the interface has moved:

\[ \theta_{h,i} = \theta_2 \]  
(A15b)

\[ a_i = \sin^{-1} \left( \frac{b_i \sin \alpha_i}{r_p} \right) \]  
(A16a)

If the interface has not moved yet. If the interface has moved:

\[ a_i = \pi / 2 - \theta_2 - \alpha_i \]  
(A16b)

\[ r_p = \frac{R^2 - r_p b \cos \theta_2 + r_p^2 (\theta_2 - \alpha_i)}{2r_p a + 2[R - b \cos \theta_2]} \]  
(A17)
\[ r_p = \frac{R^2 - r_p \sum_{i=1}^{n} b_i \cos \theta_{h,i} + r_p^2 \sum_{i=1}^{n} \left( \frac{\pi}{2} - \theta_{h,i} - \alpha_i \right) - A}{2r_p \sum_{i=1}^{n} \alpha_i + \left[ \frac{r}{2} - 2 \sum_{i=1}^{n} b_i \right] \cos \theta_2} \]

\[ n = \text{number of contributing corners} \]
\[ A = \text{summation of corner areas (only those corners that do not have layers or corners of the invading phase.)} \]

\[ P_c = \frac{\sigma}{R} \left[ \cos \theta_2 - \frac{\tan \alpha}{2} \left( -\sin 2\theta_2 + 2\theta_2 - 2\alpha - \pi \right) \right] \]

\[ P_c = \frac{\sigma}{r_{\text{extreme}}} \cos(\theta_2 + \alpha) \quad \theta_2 \leq \pi - \alpha \]  

\[ P_c = \frac{\sigma}{r_{\text{extreme}}} \frac{-1}{\cos(\theta_2 + \alpha)} \quad \theta_2 \geq \pi - \alpha \]

Note: In equations (A21) and (A22) \( \alpha \) is for the sharpest corner if a layer of filled corner of the invading phase is present in more than one corner.

\[ P_c = \frac{\sigma}{R} \left( 3\sin^2 \alpha + 4\sin \alpha \cos \theta + \cos^2 \theta \right) \]

\[ b \left[ \cos \alpha \sin \alpha \left( 2\sin \alpha + \cos \theta \right) + \sin^2 \alpha \sqrt{4 \cos^2 \alpha - 3 - \cos^2 \theta - 4 \sin \alpha \cos \theta} \right] \]

\[ R_c = \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_1 + \alpha)} \quad \theta_2 \geq \theta_1 \]

\[ R_c = \frac{\cos \theta_2 - \sin \alpha}{\cos \theta_1 - \sin \alpha} \quad \theta_2 < \theta_1 \]

\[ P_c = \frac{\sigma P_2}{\sigma_2 R_c} \]

\[ P_c = \frac{\sigma_1 P_1 R_c}{\sigma_1} \]

\[ P_c = \frac{\sigma(1 + 2\sqrt{\pi G}) \cos \theta_i}{R} F_d(\theta_2, G) \]

\[ F_d = \frac{1 + \sqrt{1 - 4GC_1 \cos^2 \theta_2}}{1 + 2\sqrt{\pi G}} \]

\[ C_1 = \sum_{i=1}^{n} \cos \theta_2 \cos(\theta_2 + \alpha_i) - \left[ \frac{\pi}{2} - \theta_2 - \alpha_i \right] \]

\[ \theta_3 = \cos^{-1} \left[ \frac{-2r \sum_{i=1}^{n} \sin^{-1} b_i \sin \alpha_i}{2G - 2 \sum_{i=1}^{n} b_i} \right] \]

\[ n = \text{number of corners containing layers or filled corners of the invading phase.} \]

\[ P_c = \frac{\sigma}{R} \left( \cos \theta_2 - \frac{2\sin \theta_2}{\cot \alpha_1 + \cot \alpha_2} \right) \]

\[ P_c = \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2) - b_1} \right) \]

\[ P_c = \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2) - b_2} \right) \]

\[ P_c = \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2) - b_3} \right) \]

\[ P_c \approx \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]

\[ P_c \approx \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]

if only corner 1 has a layer or filled corner of the invading phase.

\[ P_c \approx \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]

if only corner 1 has a layer or filled corner of the invading phase.

\[ P_c \approx \frac{\sigma}{R} \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]

if only corner 2 has a layer of filled corner of the invading phase.
if only corner 2 has a layer or filled corner of the invading phase.

\[
P_c \approx \left( \frac{\sigma (\cot \alpha_2 \cos \theta_2 - \sin \theta_2)}{R (\cot \alpha_2 + \cot \alpha_1)} \right) \quad (A41)
\]

if only corner 3 has a layer or filled corner of the invading phase.

\[
P_c \approx \left( \frac{\sigma (\cot \alpha_3 \cos \theta_2 - \sin \theta_2)}{R (\cot \alpha_2 + \cot \alpha_3)} \right) \quad (A42)
\]

\[
P_c \approx \left( \frac{\sigma (\cot \alpha_3 \cos \theta_2 - \sin \theta_2)}{R (\cot \alpha_2 + \cot \alpha_3)} \right) \quad (A43)
\]

if only corner 3 has a layer or filled corner of the invading phase.

Fig. 13—Flow chart used to determine clusters of each phase.
Table 2—Showing the range of contact angles for which each of the configurations in Fig. 2 are allowed.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\theta_{ow}$</th>
<th>$\theta_{gw}$</th>
<th>$\theta_{go}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>B</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>C</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>D</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
</tr>
<tr>
<td>E</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>F</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
</tr>
<tr>
<td>G</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>H</td>
<td>---</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>---</td>
</tr>
<tr>
<td>I</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>J</td>
<td>---</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
<td>---</td>
</tr>
<tr>
<td>K</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
</tr>
<tr>
<td>L</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
</tr>
<tr>
<td>M</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
</tr>
<tr>
<td>N</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
</tr>
<tr>
<td>O</td>
<td>---</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
</tr>
<tr>
<td>P</td>
<td>---</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
</tr>
<tr>
<td>Q</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>---</td>
</tr>
<tr>
<td>R</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
</tr>
<tr>
<td>S</td>
<td>$\geq \left(\pi/2\right) - \alpha$</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
</tr>
<tr>
<td>T</td>
<td>$&lt; \left(\pi/2\right) - \alpha$</td>
<td>$\geq \left(\pi/2\right) + \alpha$</td>
<td>---</td>
</tr>
</tbody>
</table>
Oil to water, gas to water, gas to oil – piston type displacements with layers of the invading phase

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>σ</th>
<th>θ₁</th>
<th>θ₂</th>
<th>θ</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; (π/2) - α₁</td>
<td>A(28)-A(30)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; θ&lt;sub&gt;extreme&lt;/sub&gt; &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≥ (π/2) - α₁</td>
<td>A(10)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; ≥ θ&lt;sub&gt;extreme&lt;/sub&gt; &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≥ (π/2) - α₁</td>
<td>A(14)-A(16),A(18)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (3) π - θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>Iterative</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; (π/4)</td>
<td>A(12)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; θ&lt;sub&gt;extreme&lt;/sub&gt; &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≥ (π/4)</td>
<td>A(10)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; ≥ θ&lt;sub&gt;extreme&lt;/sub&gt; &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≥ (π/4)</td>
<td>A(14)-A(17)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (3) π - θ&lt;sub&gt;ij&lt;/sub&gt;</td>
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<td>Iterative</td>
<td></td>
</tr>
<tr>
<td>Circular</td>
<td>0 ≤ θ&lt;sub&gt;ij&lt;/sub&gt; ≤ π</td>
<td>A(10)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
</tr>
</tbody>
</table>

Note (1): θ<sub>ij</sub> = θ<sub>ij</sub><sub>r</sub>, i,j=ow, gw, go

Note (2): θ<sub>extreme</sub> = π - θ₃, where θ₃ is calculated from equations (A31) for triangular cross-sections and (A13) for square cross-sections with r = σ<sub>ij</sub>/P<sub>ciej</sub>

Note (3): It can be either of π - θ₄ or θ₃ based on the type of interface that θ₄ belongs to (θ₄ is the contact angle that interface moved with last time)

Note (4): r<sub>extreme</sub> = σ/P<sub>c,extreme</sub>, where σ & P<sub>c,extreme</sub> belong to the last move of the interface

Note (5): θ<sub>on</sub> = (θ<sub>onw</sub>)<sub>PD</sub> for the displacement of water by oil during primary drainage

Oil to water, gas to water, gas to oil – snap-off displacements

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>σ</th>
<th>θ₁</th>
<th>θ₂</th>
<th>θ</th>
<th>Comment</th>
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</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; (π/2) + α₁</td>
<td>A(28)-A(30)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; ≥ α₁ &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≤ (π/2) + α₁</td>
<td>A(21)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (1) π - θ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>---</td>
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<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; α₁</td>
<td>A(22)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (1)</td>
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<tr>
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<td>A(20)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
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<td>π - θ&lt;sub&gt;ij&lt;/sub&gt;</td>
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<td></td>
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<tr>
<td>Square</td>
<td>θ&lt;sub&gt;ij&lt;/sub&gt; ≥ (π/4) &amp; θ&lt;sub&gt;ij&lt;/sub&gt; ≤ (3π/4)</td>
<td>A(21)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (1) π - θ&lt;sub&gt;ij&lt;/sub&gt;</td>
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<td>θ&lt;sub&gt;ij&lt;/sub&gt; &lt; (π/4)</td>
<td>A(22)</td>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Note (1)</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Note (1): It can be either of π - θ₄ or θ₃ based on the type of interface that θ₄ belongs to (θ₄ is the contact angle that interface moved with last time)

Table 3—Listing of the expressions used to compute capillary pressure for all possible oil/water, gas/water and gas/oil displacements.
### Water to oil, water to gas, oil to gas – piston type displacements with no layers or filled corners of the invading phase

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_y &gt; (\pi/2) + \alpha_1$</td>
<td>A(28)-A(30)</td>
<td>$\sigma_{ij}$</td>
<td>$\theta_{ij}$</td>
<td>$\pi - \theta_{ij}$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_y \geq \theta_{\text{extreme}}$ &amp; $\theta_y \leq (\pi/2) + \alpha_1$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_y$</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_y &lt; \theta_{\text{extreme}}$ &amp; $\theta_y \leq (\pi/2) + \alpha_1$</td>
<td>A(14)-A(16),A(18)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\theta_y$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y &gt; (3\pi/4)$</td>
<td>A(19)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_y$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y \geq \theta_{\text{extreme}}$ &amp; $\theta_y \leq (3\pi/4)$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_y$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y &lt; \theta_{\text{extreme}}$</td>
<td>A(14)-A(17)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\theta_y$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Circular</td>
<td>$0 \leq \theta_y \leq \pi$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_y$</td>
<td>---</td>
</tr>
</tbody>
</table>

**Note (1):** $\theta_{ij} = \theta_{ij}, i,j=ow, gw, go$

**Note (2):** $\theta_{\text{extreme}} = \theta_3$, where $\theta_3$ is calculated from equations (A31) for triangular cross-sections and (A13) for square cross-sections with $r = \sigma_{ij}/P_{ij}$

**Note (3):** It can be either of $\pi - \theta_1$ or $\theta_1$ based on the type of interface that $\theta_1$ belongs to ($\theta_1$ is the contact angle that interface moved with last time)

### Water to oil, water to gas, oil to gas – snap-off displacements

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_y \leq (\pi/2) - \alpha_1$</td>
<td>A(32)-A(43)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>Max. $P_{ij}$</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_y \leq \pi - \alpha_1$ &amp; $\theta_y &gt; (\pi/2) - \alpha_1$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y \leq (\pi/4)$</td>
<td>A(20)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y &gt; (\pi/4)$ &amp; $\theta_y \leq (3\pi/4)$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_y &gt; (3\pi/4)$</td>
<td>A(22)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
</tbody>
</table>

**Note (1):** It can be either of $\pi - \theta_1$ or $\theta_1$ based on the type of interface that $\theta_1$ belongs to ($\theta_1$ is the contact angle that interface moved with last time)

### Table 4—Listing of the expressions used to compute capillary pressure for all possible water/oil, water/gas and oil/gas displacements.

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**PIRI, M. AND BLUNT, M.J.**

SPE 77726
Threshold capillary pressures for layer collapse events

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Layer</th>
<th>Displacing Phase</th>
<th>Equation</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>I, Q</td>
<td>Oil</td>
<td>Water</td>
<td>(A23)</td>
<td>$\sigma_{ow}$</td>
<td>$\theta_{owh}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>J, T</td>
<td>Gas</td>
<td>Water</td>
<td>(A23)</td>
<td>$\sigma_{gw}$</td>
<td>$\theta_{gwh}$</td>
<td>Iterative</td>
</tr>
</tbody>
</table>

Note: $b$ is calculated for the corner interface and $\theta_{owh}$ is for the layer interface

Threshold capillary pressures for layer collapse events

<table>
<thead>
<tr>
<th>Configuration(s)</th>
<th>Layer</th>
<th>Displacing Phase</th>
<th>Equations</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>K, L, M, S</td>
<td>Oil</td>
<td>Water</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{owh}$</td>
<td>$\theta_{goh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>K, L, M, S</td>
<td>Oil</td>
<td>Gas</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{owh}$</td>
<td>$\theta_{goh}$</td>
<td>$P_{cow}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>N, O, P, R</td>
<td>Gas</td>
<td>Water</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{gwh}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>N, O, P, R</td>
<td>Gas</td>
<td>Oil</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{gwh}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>$P_{cgw}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Q</td>
<td>Water</td>
<td>Oil</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{gw}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$\theta_{gwh}$</td>
<td>---</td>
<td>$P_{cgw}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>Q</td>
<td>Water</td>
<td>Gas</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{gw}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$\theta_{gwh}$</td>
<td>$P_{cow}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>T</td>
<td>Water</td>
<td>Oil</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{gwh}$</td>
<td>$\theta_{owh}$</td>
<td>$P_{cgw}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>S</td>
<td>Gas</td>
<td>Oil</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{gw}$</td>
<td>$\theta_{goh}$</td>
<td>$\pi - \theta_{gwh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>S</td>
<td>Gas</td>
<td>Water</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{gw}$</td>
<td>$\theta_{goh}$</td>
<td>$\pi - \theta_{gwh}$</td>
<td>$P_{cgo}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>R</td>
<td>Oil</td>
<td>Gas</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>---</td>
<td>$P_{cow}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>R</td>
<td>Oil</td>
<td>Water</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$P_{cgo}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
</tbody>
</table>

Table 5—Listing of the expressions used to compute capillary pressure for all possible displacements involving layer collapse. The capillary pressures for layer formation are found by considering the layer collapse events in reverse.
Table 6—Expressions used to compute the cross-sectional areas of each phase for each of the 20 configurations shown in Fig. 2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Area</th>
<th>Equation</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{cwr}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>D</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{gw}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>E</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{cw}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>F</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>G</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{cwh}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>H</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>I</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{cwh}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>J</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\pi - \theta_{cwh}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>K</td>
<td>$A_u$</td>
<td>A(2)</td>
<td>$\theta_{cwr}$</td>
<td>$\sigma_{ww}/P_{con}$</td>
</tr>
<tr>
<td>L</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>M</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>N</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>O</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>P</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>Q</td>
<td>$A_u + A_w + A_{wl}$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>R</td>
<td>$A_u + A_w$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>S</td>
<td>$A_u + A_w + A_{wl}$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
<tr>
<td>T</td>
<td>$A_u + A_w + A_{wl}$</td>
<td>A(2)</td>
<td>$\theta_{gwh}$</td>
<td>$\sigma_{gw}/P_{con}$</td>
</tr>
</tbody>
</table>

Notes: (1) Area at the center is total area minus the corner areas.  (2) Total areas for different cross-sections are calculated using equation A(1)
Table 7—Expressions used to compute the conductance of each phase for each of the 20 configurations shown in Fig. 2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Equation</th>
<th>Conductance</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_c )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>A(7),A(9)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>D</td>
<td>A(7),A(9)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>E</td>
<td>A(7),A(9)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>F</td>
<td>A(7),A(9)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>G</td>
<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
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<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
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<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>I</td>
<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>J</td>
<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>K</td>
<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>L</td>
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<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
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<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
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<tr>
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<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
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<td>( \theta_{mr} )</td>
</tr>
<tr>
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<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
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<td>( \theta_{mr} )</td>
</tr>
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<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
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<tr>
<td>Q</td>
<td>A(7),A(8)</td>
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<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
<td>R</td>
<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 1 )</td>
<td>( \theta_{mr} )</td>
</tr>
<tr>
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<td>A(7),A(8)</td>
<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
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<tr>
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<td>( g_w )</td>
<td>( A_w )</td>
<td>( A_r )</td>
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<td>( f )</td>
<td>( f )</td>
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<td>( A_w )</td>
<td>( A_r )</td>
<td>( A_w )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
<td>( \theta_{mr} )</td>
</tr>
</tbody>
</table>

Note: Conductance through the center of a pore or throat is calculated from equations A(3)-A(5) using the center area which is total area minus corner areas.
References


