A 2D DYNAMIC PORE NETWORK MODEL FOR MODELING PRIMARY DRAINAGE
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Abstract
We present a dynamic network model to investigate the effects of capillary number and viscosity ratio on displacement patterns and fractional flow in primary drainage. This model predicts the events that are observed in the micro-model experiments, such as swelling of the wetting layers and the meniscus oscillation. Interfaces are tracked through the pore elements using a modified Poiseuille equation that involves a use of the equivalent hydraulic resistance of the fluids between the pore element centers. Snap-off in primary drainage is modeled by allowing the wetting layer conductance to change with changing local saturation. Initial computational results are presented for simulations of primary drainage.

1- Introduction

Understanding multiphase flow in porous media is important for a number of practical applications, including the implementation of improved recovery schemes from hydrocarbon reservoirs, contaminant clean-up and designing underground nuclear waste repositories. Fundamentally, the geometry of the void space of a porous medium and the interactions of the multiple phases with the solid determine macroscopic properties such as porosity, relative permeability, capillary pressure and resistivity index [1]. One approach to predicting these quantities is through pore-scale modeling that requires a detailed understanding of the physical processes occurring at the pore scale and a complete description of the morphology of the pore space.

The pore scale network is a representation of the void space of the reservoir rock. Wide void spaces are represented by pores that are connected by narrower regions called throats. Then using rules for determining fluid configurations and appropriate flow and transport equations, multiphase flow is computed in the network modeling. An excellent detailed review of several pore-scale processes and description of different types of pore-scale network model can be found in the classic text by Dullien [2], while more recent reviews are to be found in [3,4].

The majority of the existing pore network models are quasi-static [1,4-12] assuming that capillary forces alone control the fluid configuration in the pore space: capillary pressure is imposed on the network and the final, static position of all fluid-fluid interfaces is determined, ignoring the dynamic aspects of pressure propagation and interface dynamics due to viscous forces.

There are many important circumstances where the assumption of purely capillary-controlled displacement at the pore scale is not appropriate. The ratio of viscous to capillary pressure is conventionally defined using a capillary number $C_a$ [2]:

$$C_a = \frac{\mu q}{\sigma}$$

(1)
where $\mu$ is the viscosity of the displacing phase, $q$ is the total Darcy velocity (volume flowing per unit area per unit time) and $\sigma$ is the interfacial tension between the two fluid phases. For typical oil/water flows in reservoirs, $\mu$ is around $10^{-3}$ Pa.s, $\sigma$ is 0.05 Nm$^{-1}$ and $q$ is $10^{-5}$ m$^{-1}$s or lower, giving $C_a \approx 2 \times 10^{-7}$. Viscous forces become significant at the pore scale only when $C_a$ is the range $10^{-4} – 10^{-3}$ or larger [2]. Hence, quasi-static models are often accurate and can predict a variety of low-flow-rate experiments successfully [1,4]. However, if the viscosity is high (polymer flooding), the flow rate is very large (for instance, in fractures, gas reservoirs and near well-fores) or the interfacial tension is low (surfactant flooding, near-miscible gas injection and gas condensate reservoirs), viscous and capillary forces can be comparable at the pore scale. In these cases, a number of effects, such as the movement of disconnected ganglia of oil and the simultaneous filling of neighboring pores become significant [13].

Dynamic network models explicitly account for viscous forces: a specified inflow rate for one of the fluids is imposed and the subsequent transient pressure response and the associated interface positions are calculated. Koplik et al [14] simulated primary drainage in networks of spherical pores connected to cylindrical pore throats. Lenormand et al [15] and Blunt and King [16] used a simplification of the model of Koplik, assuming that the pores have volume but no resistance to flow and the throats have resistance to flow but no volume. Payatakes and co-workers [21-25] simulated flow in a network of spherical chambers connected through long cylindrical throats with a sinusoidally varying cross-section. We will use a similar geometry in our work. They concluded that a significant fraction of the oil flow at reservoir rates is accommodated through the movement of disconnected ganglia. More recently Hansen and co-workers [18-20] developed a dynamic network to study fluid movement in drainage and imbibition. They used a network of tubes that connected to each other through volumeless nodes. The capillary pressure within the tube was a function of the location of the interface. All these models only allow a single phase to be present in any cross-section through a throat – that is, they ignore contributions to flow through wetting layers that occupy the roughness and crevices of the pore space even when the center of the pore or throat is filled by non-wetting phase [4]. Blunt and Scher [26] and Hughes and Blunt [12] did accommodate wetting layer flow in a perturbative model where the wetting layers were all assigned a fixed conductance and where simultaneous filling of multiple pores was disallowed. They showed that rate effects can have a significant effect on the trapping of non-wetting phase (oil) during imbibition for capillary numbers as low as $10^{-4}$, since the ratio of the viscous to capillary force can be high, even at low flow rates, if flow occurs through low conductivity layers. Mogensen et al [17] also assumed a fixed conductance to the wetting layers in their dynamic model of imbibition. They concluded that the capillary number, aspect ratio (ratio of average pore diameter to average throat diameter) and contact angle all have a significant influence on the competition between piston-like advance (frontal movement of a phase displacing another in a throat) and snap-off (where wetting phase flows through layers and fills narrow regions of the pore space in advance of a connected front).

Despite this extensive literature on dynamic pore-scale modeling a number of key physical effects have yet to be captured accurately. In particular, previous work did not account for the swelling of wetting layers in both drainage and imbibition that allows snap-off, as observed in micromodel experiments [15]. Snap-off is the key
process by which the non-wetting phase becomes trapped, and determines, for instance, the amount of oil that is left unrecovered after waterflooding.

Related to this lack of physical realism is a controversy in the literature over the generic nature of multiphase flow in porous media. The conventional picture, based largely on quasi-static approaches to modeling, assumes that for typical reservoir displacements, each phase occupies its own connected sub-network through the porous medium. The hydraulic conductance of these sub-networks determines the multiphase flow properties, specifically the relative permeability [2,4]. Disconnected regions do not flow unless the capillary number is very high. In contrast, Payatakes et al. [21-25] suggest based on micromodel experiments and network modeling that the typical scenario for multiphase flow is significant transport via disconnected ganglia even at reservoir flow rates. However, their work can be criticized for not accommodating wetting layer flow and consequently substantially restricting the connectivity of the wetting phase.

In this work we introduce a conceptually simple dynamic model that explicitly simulates the dynamics of wetting layer swelling and snap-off. We are then able to address whether or not multiphase flow involves significant transport of disconnected non-wetting phase, even at typical reservoir flow rates, or whether this phenomenon is restricted to high capillary numbers.

2- Principles of the model

The model is based on three main principles:

- The amount of each phase in each pore or throat is known. The volume of each phase (with the contact angle) controls the configuration of fluids. This in turn determines the curvature of the oil-water interface and the pressure difference between these two fluids in each pore or throat.
- The system starts by injecting a certain volume of invading fluid over a period of one time step. Then by using equivalent networks of electrical resistors, the equivalent hydraulic resistances of the fluids between pore and throat centers are calculated and used in a volume balance equation to obtain the fluid pressures at pore and throat centers. Using the equivalent hydraulic resistance simplifies the problem and makes it no more complex than solving the material balance equations for single-phase flow.
- The pressure difference between the pore and throat centers and the phase conductances are used to move phases between pores and throats and hence to update the fluid volumes.

3- Network and pore geometry

The porous medium is represented by a square lattice of pores and throats. In cross-section each pore or throat has a scalene triangular cross-section. The inscribed radius of a pore or throat varies sinusoidally, as shown in Figure 1 [9]. Each pore is divided into several branches (equal to the number of connected throats), which are considered as extensions of the throats that connected to. All the pore branches meet at the center of the pore that is treaded as a volume-less joining point. This feature is introduced for two
reasons. First, we believe it is more realistic than assuming a straight channel (i.e. uniform inscribed radius along the length). Second, it allows us to assign a unique and continuously varying oil/water capillary pressure as the oil/water interface moves in a pore or throat. The inscribed radius between the pore and the connecting throat centers is given by:

\[
R = \left( \frac{R_P + R_t}{2} \right) \left( \frac{R_P - R_t}{2} \right) \cos \left( \frac{2\pi x}{l_P + l_t} \right)
\]  

(2)

Where, \( R_P, R_t \) are the pore and throat center radii respectively, \( l_P, l_t \) are the pore and throat lengths respectively and \( x \) is the distance between the throat center and the location of the interface, so that, \( x=0 \) at the throat center and \( x= (l_P+l_t)/2 \) at the pore center.

![Figure 1: Schematic diagram of the pore and throat geometries.](image)

**Table 1: parameters used for the distribution of generation the pore geometries.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{min} )</td>
<td>0.2( \mu m )</td>
</tr>
<tr>
<td>( R_{max} )</td>
<td>100( \mu m )</td>
</tr>
<tr>
<td>( l_{min} )</td>
<td>1( \mu m )</td>
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<tr>
<td>( l_{max} )</td>
<td>50( \mu m )</td>
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<td>Maximum aspect ratio</td>
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<tr>
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</tr>
<tr>
<td>( \delta )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.6</td>
</tr>
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</table>
3.1- SELECTION OF PORE AND THROAT SIZES.

The inscribed radius of the center of a throat is assigned at random according to a truncated Weibull distribution:

\[ R_t = (R_{t,\text{max}} - R_{t,\text{min}}) \left[ -\delta \ln \left( x \left( 1 - e^{\frac{1}{\gamma}} + e^{\frac{1}{\gamma}} \right) \right) \right] + R_{t,\text{min}} \]  

(3)

Where, \( x \) is a random number between zero and one. The parameters used in the distribution are shown in table 1.

The pore radius at its center must be greater or equal to the maximum radius of the connecting throats. Therefore, the pore radius is given by the following expression:

\[ R_p = \max(R_{p,\{1,...,n\}}) \times \text{aspect ratio} \]

Where, \( n \) is the number of the connecting throats, and the aspect ratio is the ratio between the pore radius and the maximum radius of the connecting throats. It is obtained by using Equation 3 with replacing \( R_{t,\text{max}} \) and \( R_{t,\text{min}} \) by the maximum and minimum aspect ratios provided in Table 1. While that we use a topologically cubic lattice, we do allow the pore and throat lengths to vary – this corresponds physically to a distorted lattice, although we do not check if the network is physically realizable in three-dimensional space. We used the same parameters for selecting the pores and throats length, we reused Equation 3 replacing \( R_{t,\text{max}} \) and \( R_{t,\text{min}} \) by \( l_{\text{max}} \) and \( l_{\text{min}} \) provided in table 1. Once \( l_p \), \( l_t \), \( R_p \) and \( R_t \) are known. Equation 2 is used to determine the inscribed radius as a function of distance between the pore and throat centers.

4- Fluid flow through the network

Initially, the system is assumed to be completely filled with a defending, wetting fluid with viscosity \( \mu_1 \). The invading non-wetting fluid with viscosity \( \mu_2 \) is injected into the system from the inlet side with a constant injection rate. The fluids are assumed to be immiscible and incompressible.

4.1 – DETERMINATION OF FLUID CONFIGURATIONS

We start the model by assuming the volume of each phase in each pore or throat is known. Then, with known contact angles, the fluid configuration and the local capillary pressure can be computed.

The cross sectional area for an element (pore or throat) filled with a single phase is
\[ A = R^2 \sum_{i=1}^{n} \cot(\alpha_i) = CR^2 \]  

(4)

Where, \( n \) represents the number of the corners, \( \alpha \) is the corner half angle and \( C \) is a constant defined by Equation (4). Therefore, the volume can be given by the following equation:

\[ v = \frac{x^2}{x^1} CR^2 \frac{dx}{x^1} = C \int_{x^1}^{x^2} R^2 \frac{dx}{x^1} \]

(5)

Substituting Equation (2) in Equation (5), the volume equation becomes:

\[ v = C \int_{x^1}^{x^2} \left( \frac{R_p + R_i}{2} \right)^2 \left( \frac{R_p - R_i}{2} \cos \left( \frac{2\pi x}{l_p + l_i} \right) \right)^2 \frac{dx}{x^1} \]

\[ = C \left( \frac{R_p + R_i}{2} \right)^2 \left[ \frac{(R_p - R_i)^2}{8} \left( \frac{l_p + l_i}{4\pi} \right) \left( \frac{2\pi x}{l_p + l_i} \left( \frac{2\pi x}{l_p + l_i} \right) \right) \right]_{x^1}^{x^2} \]

(6)

Here, \( x_1 \) and \( x_2 \) are the limits of the integration that are determined by the location of the fluid interfaces. For example in Figure 2, the water fills the throat from the center up to the first oil-water interface, so here, \( x_1 = 0.0 \) and \( x_2 \) = the location of the interface.

Figure 2: Illustration of a fluid configuration: oil at the center of the pore/throat boundary.

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Figure 3: The equivalent electrical resistors diagram for Figure 2 used to compute hydraulic conductance.
4.2 – COMPUTING THE FLUID RESISTANCE

Calculating the fluid resistance is potentially a complicated problem, but using an equivalent electrical resistors diagram helps to simplify the computations. Before giving a detailed description of how the equivalent hydraulic resistance is obtained, it is worthwhile to state the expressions used in finding the phase resistance. A phase may occupy: the whole element cross-section, the corners with the other (non-wetting) phase in the center, or the center of the element with the other (wetting) phase in the corners. The conductance per unit length of a fluid occupying the whole cross-section is given by the following approximation based on Poiseuille’s law for flow in a circular cylinder [7]:

\[ G = \frac{\pi}{128} \left( \frac{A_t}{\pi} + R \right)^4 \]  

where, \( A_t \) is the cross-section area given by Equation (4) and \( R \) is the inscribed radius given by Equation (2). Therefore, the fluid resistance is:

\[ R_{\text{whole}} = \frac{\mu_{f,128}}{\pi} \left( \frac{1}{\sqrt{C^2 + 1}} \right)^4 \int_0^1 dx \frac{d x}{\left( \frac{R_p + R_w}{2} - \frac{R_p - R_w}{2} \right) \cos \left( \frac{2\pi x}{l_p + l_w} \right)} \]

\[ = \left[ \frac{dz}{4z^3} \int_0^{\beta \pi/2} \frac{B \sin(y)}{\sqrt{1 + B \cos(y)}} \left[ \frac{1}{2B^3 - 1} \left( \frac{2gB^2}{\sqrt{1 - B^2}} \tan^{-1} \left( \frac{1 - B}{1 + B \tan \left( \frac{y}{2} \right)} \right) \right] \right] \]

where, \( y = \frac{2\pi x}{l_p + l_w} \), \( B = \frac{R_p - R_w}{R_p + R_w}, d_1 = l_p + l_w, d_2 = 2\pi, d_3 = \frac{B^2 - 1}{A} = \frac{\mu_{f,128}}{\pi} \left( \frac{1}{\sqrt{C^2 + 1}} \right)^4 \)

In the case where fluid occupies the corners with \( \theta + \beta < \pi / 2 \) (where \( \theta \) is the oil/water contact angle measured through the water), the conductance per unit length is given by the following approximation [27]:

\[ G = \sum_{i=1}^{n} \left[ A_i (1 - \sin \phi_i)^2 \sin \theta \right] \]

where, \( \phi_i = \frac{\pi}{2} - \beta_i - \theta, \phi_2 = \cot \beta_i \cos \theta - \sin \theta, \phi_3 = \left( \frac{\pi}{2} - \beta_1 \right) \tan \beta_1 \)

The corner area, \( A_c \) is:

\[ A_c = r^2 \left( \cos \theta (\cot \beta \cos \theta - \sin \theta) + \theta + \beta - \pi \right) \]
where \( r \) is the radius of the curvature of the wetting layer. We assume that the curvature of the wetting layer is constant in a single element, although it varies between pores and throats. Thus we can write the wetting layer flow resistance as:

\[
R_{\text{wet, layers}} = \frac{\mu_f}{G} \frac{(x_2 - x_1)}{G} \tag{11}
\]

In the case where fluid occupies the center of an element with wetting phase in the corners, the conductance per unit length is given by an expression similar to Eq. (7):

\[
G = \frac{\pi}{128} \left( \frac{A_{\text{cen}} + R}{\pi} \right)^4 \tag{12}
\]

where, \( A_{\text{cen}} = A_t - \sum_{j=1}^{n} A_{ij}, A_t \) is the corner area, \( n \) represents the number of the corners.

Then, the resistance of the fluid can be given as follows:

\[
R_{\text{cen}} = \frac{128 \mu_f}{\pi} \int \frac{dx}{\left( \frac{CR^2(x) - \sum_{i=1}^{n} A_{ij}}{\pi} + R(x) \right)^4} \tag{13}
\]

where, \( R(x) \) is the inscribed radius given by Equation 2. This integration cannot be evaluated analytically. Hence numerical integration has been used.

Using these formulae enables the model to handle any number of fluid interfaces between pore and throat centers. The use of the electrical resistors diagram and equivalent hydraulic resistance simplifies and clarifies this approach. For example, Figure 2 shows a fluid configuration where oil occupies the center of pore/throat boundary. Its equivalent electrical resistors diagram is given in Figure 3. Thus, the equivalent hydraulic resistance of the fluids between the interfaces is given by the following equation:

\[
\frac{1}{R_{\text{eq, interface}}} = \frac{1}{R_{P-\text{corn}}^w + R_{P-\text{corn}}^o} + \frac{1}{R_{T-\text{corn}}^w + R_{T-\text{corn}}^o} \tag{14}
\]

where \( R_{P-\text{corn}}^w \) is the pore wetting layers resistance, \( R_{T-\text{corn}}^w \) is the throat wetting layers resistance, \( R_{P-\text{corn}}^o \) is the pore oil resistance and \( R_{T-\text{corn}}^o \) is throat oil resistance. Then, the equivalent hydraulic resistance for the fluids between pore and the throat centers can be obtained through the following expression:

\[
R_{\text{eq}} = R_{P-\text{cent}}^w + \left( \frac{1}{R_{P-\text{cent}}^w + R_{T-\text{cent}}^o} \right) + R_{T-\text{cent}}^o \tag{15}
\]

Where, \( R_{P-\text{cent}}^w \) is the pore water resistance and \( R_{T-\text{cent}}^w \) is the throat water resistance. This is only one example of how the equivalent hydraulic resistance is computed. Appendix A lists the expressions used for the different fluid configurations.

5- Solving for the fluid pressure

Since the equivalent hydraulic resistance is used in the volume conservation equations, the model can be considered as solving a single phase flow problem in which the phase
conductance between the pore and throat centers is found from the equivalent hydraulic resistance calculated in the previous step. Therefore, the total flow rate between the pore center and throat center will be:

$$Q_{\text{total}} = \frac{P_p - P_t + P_c}{R_{eq}}$$

(16)

where, $P_p$ and $P_t$ are the pore center pressure and throat center pressures respectively, $P_c$ is the sum of the capillary pressures of the menisci that are between the pore and throat centers. The absolute value of the capillary pressure at any meniscus is given by the following expression:

$$P_c = \frac{2\sigma \cos(\theta + \phi)}{R(x)}$$

(17)

Where, $\theta$ is the contact angle, $R(x)$ is the throat inscribed radius given by equation 1 and $\phi$ is the inclination angle where $\tan \phi = \frac{dR(x)}{dx}$.

The sign of the capillary pressure at any meniscus depends on the location of the non-wetting fluid, if it is on the right of the meniscus, the sing is positive. For a system of $m$ throats and $n$ pores, we have $(m+n)$ unknowns. These unknowns are determined by applying volume conservation for both the pores and throats. The conservation equation for pore $j$ with $n$ connecting throats, labeled $i$ is:

$$\sum_{i=1}^{n} Q_{ij}^p = \sum_{i=1}^{n} \frac{P_p^i - P_t^i + P_C}{R_{eq}^i} = 0$$

(18)

For a throat $i$, where $R$ and $L$ label the left and right pores:

$$\frac{P_p^R - P_t^R}{R_{eq}^R} + \frac{P_p^L - P_t^L + P_C}{R_{eq}^L} = 0$$

(19)

Equations (18) and (19) define a series of simultaneous linear equations for the unknown pore and throat pressures that are solved using a standard iterative matrix solver (http://rene.ma.utexas.edu/CNA/ITPACK/manuals/user2c/).

The pressures are used to compute the phase flow rate across the pore/throat boundaries. For example, from the equivalent resistors diagram of figure 2, it is clear that the water flow rate across the pore/throat boundary is the wetting layer flow rate and it is given through the following equation:

$$Q_{\text{water}} = \frac{(P_p - P_t + P_c)}{R_{eq}} \left( \frac{R_{p^R_{\text{pore}}} + R_{p^R_{\text{cor}}}^{\text{corn}}}{R_{eq} - \left( R_{p^R_{\text{pore}}} + R_{p^R_{\text{cor}}}^{\text{corn}} \right)} \right)$$

(20)

The oil flow rate then becomes:

$$Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}}$$

(21)

6- Selection of the time step

We choose a time step $\Delta t$ according to the following formula:

$$\Delta t = \min \left[ 0.00005 \text{ s}, \min \left( \frac{v_i}{Q_i}, i = 1, n \right) \right]$$

(22)

Where, $n$ is the number of the elements in the pore network, $v_i$ is the $i^{th}$ element volume (i.e. the maximum amount of fluid that can be held in the $i^{th}$ element), $Q_i$ is the total flow
rate into the \( i \)th element. Eq. (22) ensures that an element cannot be completely filled in a single time step. The time step value of 0.00005 s ensures that in most cases only a small fraction of a volume of a pore or throat is filled with invading fluid in a time step.

7- Updating fluid volumes

Once the fluid volumes are determined, the configuration of the phases in the elements is adjusted. There are two steps in this process. First, the pressure computation only determines the total flow of oil and water between pore and throat centers – we need to use the fluid configuration to determine the flow rates of each phase from pore to throat. Having done this the water volume in each element is updated. From the new fluid volume the configuration of each phase in each element is determined and a new total fluid resistance can be found, the pressure recomputed and the simulation continues. For example, consider the fluid configuration shown in figure 2. By assuming the total flow rate across the pore/throat boundary given by Eq. (16) and the flow direction is towards the throat, the total displaced volume from the pore can be given by the following expression:

\[
v_{\text{displaced}} = \frac{P_p - P_w + P_r \Delta t}{R_{eq}}
\]

(23)

where \( \Delta t \) is time step.

Knowing that the flow direction is towards the throat, it becomes obvious that the water-oil interface is pushed towards the pore/throat boundary by an amount equivalent to the displaced volume. The mathematical analysis of this approach is a bit tricky, where first, the current water volume in the water filled portion of the pore is obtained through Eq. (5) with \( x_2 = \frac{l_p + l_t}{2} \) and \( x_1 \) = the current location of the interface. Then the new water volume is given through the following expression:

\[
v_{\text{water}}^{\text{new}} = v_{\text{water}}^{\text{current}} + v_{\text{displaced}}
\]

(24)

Then, Eq. (6) is reused to find the new location of the interface, with volume = \( v_{\text{water}}^{\text{new}} \) and \( x_2 = \frac{l_p + l_t}{2} \). It is clear that the interface location cannot be obtained from a direct substitution, and so an iterative method must be used to obtain an interface location consistent with the imposed change in volume.

8- Micro-flow mechanisms of primary drainage

Since the pore center in our model is treated as a joining point between two or more sinusoidal flow channels, the fluid displacement mechanisms can be divided into two main types: piston-like and snap-off mechanisms. The purpose of this section is to explain how the above description of the model can be used to model the micro-flow mechanisms that have been seen in the micro-model experiments.
8.1- PISTON-LIKE MECHANISM

This is the mechanism in which the displacing fluid pressure is high enough to allow the displacing fluid to enter the bulk of the pore element by pushing the displaced fluid in front of it. In our model, we use this mechanism to describe oil invasion of a single pore branch or throat side.

This type of invasion occurs if the pressure of the element that contains the displacing fluid is higher than the pressure of the connected element that contains the displaced fluid. For example, the diagram shown in figure 4a shows a pore branch that is completely filled with oil and a throat that is fully water saturated. Now, if the pore pressure is higher than the throat pressure, the oil then advances towards the throat center (Figure 4b). The amount of oil that enters the throat is controlled by the pressure difference between the pore and the throat and the pore-throat geometries. The oil continues moving in until it passes the throat center and starts filling the other side of the throat (Figure 4c).

In our model, this mechanism is modeled by changing the fluid configurations according to the location of the oil/water interface. Figure 5 shows a simplified flow chart of our technique of modeling this mechanism. From Figure 4, it is clear that this mechanism is a combination of two fluid configurations (one is between the first pore and the throat centers, and the second is between the throat and the following pore centers). The equations of the fluid equivalent resistance, phase fluxes and field pressures for both configurations are provided in Appendix A. The volume of oil that enters the system (displaced volume) is obtained by finding the difference between the water fluxes of the two configurations. If this volume is positive, the oil will enter the throat, otherwise, the fluid configuration remains unchanged. If oil enters the throat, the volume of the oil in the throat and the new location of the oil/water interface are determined according to the procedure explained in section 7. If the location of the new interface can be located within the current fluid configuration, there will be no change in the fluid configuration, otherwise, the fluid configuration will be changed to one in which the location of the new interface has moved beyond the center of the throat, as seen in Figure 5.
Figure 4: Invasion of a single pore branch or throat side. a) Oil is filling the pore branch. b) Oil is entering the throat side. c) Oil is passing the throat center.

Figure 5: Flow chart of oil invasion of a single throat side shown in figure 4.
8.2- SNAP-OFF

Snap-off is a mechanism that is controlled by wetting layer flow. Water accumulates in these layers until some critical point, when it snaps at the center of the pore element separating the oil into two droplets. The accumulation of the water in the wetting layers is function of contact angle and pore/throat aspect ratio.

In our model, there are two types of snap-off. Snap-off that occurs to the oil that invades a fully water saturated throat will be called snap-off in drainage. The second snap-off occurs in a pore element that is already filled with oil. Here, the water starts accumulating in the wetting layers as a result of a drop in the capillary pressure at the front of displacement. Many authors have described this snap-off during imbibition (see for instance, Blunt [4] and Mogensen et al [17]). However, there is nothing preventing it from happening in drainage as mentioned by Toledo et al [28]. This type of snap-off will be called snap-off in imbibition, since it is common in the imbibition process.

8.2.1- Snap-off in Drainage
Theoretically, as the non-wetting phase invades a fully wetting phase saturated pore, the wetting phase will be pinned in the corners and crevices of the pore wall to form wetting layers. As the non-wetting fluid passes the narrowest section of the pore, the radius of the curvature of the wetting layers will be at its smallest value. Then, with increasing the non-wetting phase saturation, the wetting phase in the layers starts to accumulate at the narrowest section of the pore (see Figure 7) as the capillary pressure decreases. This process will continue until a point where a very low capillary pressure is reached. Then the wetting fluid cannot be held in the wetting layers any more, and so it will snap at the narrowest region of the pore separating the oil into two droplets.

If the volume of water flowing into the throat is more than the water flowing out in figure 7c, the volume of water in the center of the throat will increase and after snap-off water will fill the whole throat cross-section at its narrowest point. This means that oil will enter the throat but it cannot penetrate it. After snap-off, if the water flow into the throat is less than that flowing out, the volume of water in the center will shrink until the two oil menisci meet, reconnecting the oil and the oil then continues flowing to the next pore.

8.2.2- Snap-off in Imbibition
When water in the wetting layers invades an oil filled throat, it will start flowing through the wetting layers filling. If the water flowing in through the wetting layers is more than that flowing out of the throat, water starts accumulating in the wetting layers (see figure 8a). This accumulation process might eventually lead to snap-off (figure 8b).

Our strategy for modeling this type of snap-off is illustrated in figure 8c. Snap-off occurs when the volume of water at the narrowest part of the throat is too large to be accommodated in layers. At this point water spontaneously and instantly fills the center of the throat and the oil is separated into two.
Figure 7: Snap-off in drainage. (a) Oil is approaching the narrowest region of the throat. (b) Water in the wetting layers starts accumulating at the narrowest part of the throat. (c) Water snaps off at the center of the throat separating the oil into two droplets.

Figure 8: Modeling snap-off in imbibition. a) Water accumulates in the wetting layers. b) Water snaps-off at the center of the throat. c) Water volume as a function of capillary pressure. When the water volume in the throat is sufficiently large, the water can only be accommodated by having the center of the throat completely filled with water – this represents snap-off.
9- Simulations

The flow is characterized by two dimensionless numbers: the capillary number \((Ca)\), Eq. (1) and the viscosity ratio \((M)\), which is defined as the ratio between the invading fluid viscosity \(\mu_2\) and the defending fluid viscosity \(\mu_1\).

\[
M = \frac{\mu_2}{\mu_1}
\]

In this work, we divide our results under three categories: the influence of capillary number on the dynamic fluid movements, the relation between the capillary number (flow rate) and the snap-off phenomenon and the influence of the viscosity ratio on the fluid movement.

9.1- INFLUENCE OF CAPILLARY NUMBER ON THE DYNAMIC FLUID MOVEMENT

In this subsection, we change the capillary number by varying the injection flow rate. The general understanding of oil invasion is that at high capillary numbers the oil fills all the pore elements (in all direction) and moves towards the outlet face in a piston like fashion [15]. However, at low capillary number, the oil flows through a pathway of larger pores and throats with the lowest capillary pressure. This leads to a ramified, invasion percolation-like displacement that can be simulated readily with quasi-static models [15,16]. In this section we will test whether or not our model reduces to the quasi-static limit as the flow rate is decreased.

Figure 9 shows the fluid distribution for simulations at different capillary number (oil is shown in red and water in blue). In all the simulations, we used a 2D network of 9x9 pores, a unit viscosity ratio (fluid viscosity of 1 Pa.s) and 0.05 N/m interfacial tension. The runs were stopped at first oil breakthrough. The Darcy oil velocity is obtained by dividing the oil injection rate by the inlet cross-sectional area. The oil injection rate is the sum of the oil flow rates between the inlet throats and the connecting pore branches. The inlet cross-sectional area is the product of the length of the inlet and the mean thickness of the lattice due to the average radius of the pores. It is 52.1\(\mu\)m². Each simulation was performed at a constant injection rate. In the pictures shown in figure 9, each pore is connected to four throats and a small white line is used to distinguish between the pore branch and the throat. In addition, when snap-off occurs there are throats having two interfaces (water occupies the throat center). Figure 9a shows the results of a quasi-static model for the same network (9x9 pores) to compare it with the dynamic one at the lowest capillary number \(Ca = 3.07 \times 10^{-5}\) (figure 9b).

As the capillary number is increased, the oil flows through more of the inlet pores and sweeps more of the network, although there is an increasing frequency of snap-off. At the lowest capillary numbers, the displacement is dendritic and the sequence of pores and throats are filled is largely controlled by their entry capillary pressure (determined by the inscribed radius). The displacement patterns for \(Ca = 3.07 \times 10^{-5}\) and \(Ca = 0\) are similar, although not identical, since the perturbative effect of viscous forces does affect the exact pathway of filled pores and throats, especially away from the inlet. As the flow rate increases, viscous forces become more significant and small pores near the inlet may be filled in preference to larger pores or throats near the outlet, because of the significant pressure drop across the network. Furthermore, dynamic events, such as
snap-off, become more common, and the oil is not necessarily connected to the inlet, although it is still flowing. At the highest capillary number studied, 0.33, oil moves largely indiscriminately through pores and throats of any size as a train of generally disconnected ganglia.
Figure 9: Fluid distributions for simulations of primary drainage at different capillary number $C_a$. In this and subsequent figures, the water in the centers of pores and throats is shown in blue and oil is shown in red. The fine white lines separate pores and throats.

a) A quasi-static model, representing the limit of $C_a=0$. b) A run for $C_a=3.07 \times 10^{-5}$. c) $C_a=3.8 \times 10^{-4}$. d) $C_a=0.3.3$. 
9.2- THE INFLUENCE OF VISCOSITY RATIO ON FLUID MOVEMENT

In this subsection, we study the effect of viscosity ratio on displacement patterns and the degree of snap-off. For illustrative purposes we ran simulations with a capillary number of $8.3 \times 10^{-2}$ on the same 9x9 network as before. We also ran a series of simulations on a statistically similar 30x30 network with a capillary number of $1.05 \times 10^{-2}$.

At high flow rates and viscosity ratios greater than 1 (oil more viscous than water), it would be expected that the displacement of oil by water would be stable, with a relatively float front progressing through the system. Figure 10a illustrates a displacement for $M=10$ that confirms this. However, there is a significant amount of snap-off and a significant proportion of the oil moves as disconnected ganglia. The reason for large amounts of snap-off can be explained from studying the fluid resistance of a pore element that contains oil in the center and water in wetting layers. As explained in section 4.2, the phase resistance is a combination of the geometry (i.e. whether the phase occupies the center, layer or whole pore) and viscosity. Therefore, with an oil viscosity ten times higher than the water viscosity, in the large pores, the oil resistance will be of the same order of magnitude as the wetting layers. This means that the oil flow is relatively slow compared to the accumulation of water in wetting layers. Thus water has sufficient time to accumulate and snap at the center, leading the generation of the oil ganglia seen in figure 10a.

For a viscosity ratio less than one ($M=0.1$), the oil fingers through the water (figure 10b), since the displacement is now unstable. In addition, the wetting layer flow is less significant compared to oil flow in the pore centers, since the water is more viscous, and as a consequence there is less snap-off.

![Diagram of oil and water displacement patterns](a)
Figure 11 shows the total oil fractional flow and the oil fractional flow considering only the movement of oil that is connected to the inlet as function of oil saturation for a 2D network of size of 30 x 30, for (a) $M = 10$; and (b) $M = 0.1$. For $M = 10$, snap-off is common and noticeable amount of produced oil is coming from disconnected oil (figure 11a). In the case of $M = 0.1$, the oil fingers into the water through the larger pores and throats, which means a high oil fractional flow is reached at low oil saturation. In addition, there is not much difference between the total oil fractional flow curve and the continuous one which indicates less occurrence of snap-off.
10- Conclusions

A new dynamic pore network model for simulating two-phase flow in porous media has been developed that accounts for flow in wetting layers. This model has the capability of predicting the events that are observed in the micro-model experiments, such as swelling of the wetting layers and meniscus oscillations. The model is based on an idealization of the pore space as a network of triangular cross section pores in which the cross section varies sinusoidally. The transient pressures were computed at each pore element center (pores & throats) and the locations of the interfaces were updated by using a modified Pouiseuille equation in which an equivalent hydraulic resistance between the pore and throat centers was used. In addition, the model simulates the snap-off, as seen in micro-model experiments [15].

Numerical results are presented for primary drainage. From these results we conclude:

- At low capillary number, the inlet pressure is low so that oil tends to flow through the larger pores that have the smallest capillary entry pressures. With increasing the capillary number, the oil can enter all pores and the displacement is less ramified.
- With a viscosity ratio greater than one (oil more viscous than water) and high flow rates, a flat frontal displacement is observed but a large number of oil ganglia. These ganglia are formed by snap-off, which is favored due to the comparatively low flow resistance in wetting layers.
- For a viscosity ratio less than one the oil fingers through the water and there is less snap-off.

Figure 11: Influence of viscosity ratio on fluid movement, (a) 2D results of The total oil fractional flow and connected oil fractional flow curves for $M=10$. b) 2D results of The total oil fractional flow and connected oil fractional flow curves for $M=0.1$. 

![Graph](image.png)
Appendix A: Fluid Configurations

In all the following figures:

- Throat centre, P = Pore, T = Throat, W = water, O = oil.

P/T = Pore/Throat boundary, Corn = corners, Cent = centre, P_C = Capillary pressure.

1- Fluid configuration (a):

![Fluid configuration (a)](image)

Figure A.1: a) fluid configuration a: all water, b) The equivalent electrical resistors diagram.

\[ R_{eq} = R_P^W + R_T^W, \quad R_{eq}^W = R_{eq}, \quad R_{eq}^O = 0.0 \]  

\[ Q_{total} = \frac{(P_P - P_T)}{R_{eq}}, \quad Q_{water} = Q_{total}, \quad Q_{oil} = 0.0 \]  

2- Fluid configuration (b):

![Fluid configuration (b)](image)

Figure 2: a) fluid configuration b: One oil/water interface (in the pore), b) The equivalent electrical resistors diagram.

\[ R_{eq} = (R_{P, corn}^w / R_{P, cent}) + R_P^w + R_T^W, \quad R_{eq}^W = R_{P, corn}^w + R_P^w + R_T^W, \quad R_{eq}^O = R_{P, cent}^o \]
\[ Q_{\text{total}} = \frac{(P_T - P_T + P_e)}{R_{eq}} , \quad Q_{\text{water}} = Q_{\text{total}} \left( \frac{P_{eq}}{R_{eq}} \right) , \quad Q_{\text{oil}} = 0.0 \] \quad (2.2)

3- Fluid configuration (c):

Figure 3: a) fluid configuration c: One oil/water interface (at the pore/throat boundary). b) The equivalent electrical resistors diagram.

\[ R_{eq} = \left( \frac{R_{eq}}{R_{P\text{-cor}}} \right) + R_{eq} , \quad R_{eq} = R_{P\text{-cor}} + R_{eq} , \quad R_{eq} = R_{P\text{-cent}} \] \quad (3.1)

\[ Q_{\text{total}} = \frac{(P_T - P_T + P_e)}{R_{eq}} , \quad Q_{\text{water}} = \left( P_T - P_T + P_e \right) \left( \frac{1 - R_{eq}}{R_{eq}} \right) , \quad Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}} \] \quad (3.2)

4- Fluid configuration (d):

Figure 4: a) fluid configuration d: One oil/water interface (in the throat). b) The equivalent electrical resistors diagram.

\[ R_{eq} = \left( \frac{R_{eq}}{R_{eq}} \right) + R_{eq} , \quad R_{eq} = R_{eq} + R_{eq} , \quad R_{eq} = R_{eq} \] \quad (4.1)

\[ Q_{\text{total}} = \frac{(P_T - P_T + P_e)}{R_{eq}} , \quad Q_{\text{oil}} = \left( \frac{R_{eq} + R_{eq}}{R_{eq} + R_{eq}} \right) , \quad Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}} \] \quad (4.2)
5- Fluid configuration (e):

Figure 5: a) Fluid configuration e: Water in the corners, oil in the centre. b) The equivalent electrical resistors diagram.

\[
R_{\text{eq}} = \frac{R_{\text{W-corn}} + R_{\text{P-corn}}}{R_{\text{W-corn}} + R_{\text{P-corn}}} \quad R_{\text{eq}} = R_{\text{W-corn}} + R_{\text{P-corn}} \\
Q_{\text{total}} = \frac{P_T - P_L + P_L}{R_{\text{eq}}} \quad Q_{\text{water}} = \frac{P_T - P_L + P_L}{R_{\text{eq}}} \\
Q_{\text{oil}} = Q_{\text{total}} - Q_{\text{water}}
\]  

\(5.1\)  

6- Fluid configuration (f):

Figure 6: a) Fluid configuration f: One oil/water interface (in the throat). b) The equivalent electrical resistors diagram.

\[
R_{\text{eq}} = R_{\text{W}} + R_{\text{P}} \quad R_{\text{eq}} = R_{\text{W}} + R_{\text{P}} + R_{\text{W-corn}} + R_{\text{P-corn}} \\
R_{\text{eq}} = R_{\text{W}} + R_{\text{P}} + R_{\text{W-corn}} + R_{\text{P-corn}} \quad R_{\text{eq}} = R_{\text{W}} + R_{\text{P}} + R_{\text{W-corn}} + R_{\text{P-corn}} \\
Q_{\text{total}} = \frac{P_T - P_L + P_L}{R_{\text{eq}}} \quad Q_{\text{water}} = Q_{\text{total}} \quad Q_{\text{oil}} = 0.0
\]  

\(6.1\)
References:


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