Imperial College Consortium on Pore-Scale Modelling

Third Progress Report

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Executive Summary

This report describes progress from the Imperial College Consortium on Pore-Scale Modelling over the last year. We have made considerable progress and have succeeded in achieving the main goals of the original project, namely predictive pore-scale modelling for two- and three-phase flow and the development of a dual mesh upscaling method. The present project formally comes to an end in June 2003 and a final report will be prepared for that date. We would, however, like to continue our research in this area. In particular we have exciting plans to extend our work to verify a systematic procedure for predicting relative permeability, to study carbonate systems and to do further work on the fundamentals of three-phase flow. A separate document contains a proposal for the next phase of our research on pore-scale modelling. These plans will be discussed in more detail at our project meeting in January.

The first two chapters of this report are based on papers presented at the SPE Annual Meeting in San Antonio last September. The first paper is a study of pore-scale modelling and the impact of wettability on field-scale waterflood recovery. We use our network model to predict trends in recovery with wettability for core samples. Then using a North Sea field example we demonstrate how this behaviour can have a dramatic impact on field-scale recovery if a wettability transition above the oil/water contact is present. Such a trend in recovery is not captured by current relative permeability hysteresis models. The second paper outlines our work on three-phase network modelling and shows successful comparison of our results with steady-state three-phase relative permeability data.

The third chapter describes the use of pore-scale modelling to predict the non-Newtonian rheology of shear-thinning polymers in porous media. For four different experimental datasets we are able to predict the variation of apparent viscosity with flow rate in sand packs and sandstones using only easily measured experimental data to condition our network model.

The fourth chapter describes work that is due for presentation at the SPE Reservoir Simulation Symposium in February. We describe a dual mesh method in upscaling that retains fine grid information. We test the method on a wide variety of different cases.

The current researchers in the group are:

Martin Blunt, Professor of Petroleum Engineering – overall supervision and fundamental studies of three-phase flow.

Mohammad Piri, 3rd year PhD student – three-phase pore-scale modelling of mixed-wet systems.

Per Valvatne, 3rd year PhD student – predictive modelling of two- and three-phase flow.
Mohammed Al-Gharbi, 2nd year PhD student – modelling of rate dependent effects.

Xavier Lopez, 2nd year PhD student – pore-scale modelling of non-Newtonian flows.

Hirsohi Okabe, 2nd year PhD student – pore-scale modelling of carbonates.

I would like to thank you all for your continued support that allows us to fund so many students and without which this research would not be possible.

Martin Blunt
London, January 2003
Project Publications


Chapter 1

Prediction of wettability variation and its impact on waterflooding using pore- to reservoir-scale simulation

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Abstract
We describe a pore- to reservoir-scale investigation of wettability variation and its impact on waterflooding. Our aim is to demonstrate that network models incorporating realistic pore-space geometry and physics may be used as a tool to predict wettability variations and their impact on flow.

We successfully predict experimental data for water-wet and mixed-wet media using a three-dimensional pore-scale model of a Berea sandstone. We then focus upon the effect of variations in initial water saturation associated with capillary rise above the oil-water contact (OWC). This may lead to wettability variations with height, because the number of pore-walls which may be rendered oil-wet during primary drainage increases as the oil saturation increases.

We investigate the validity of empirical hysteresis models in which scanning curves are used to connect bounding waterflood and drainage curves for a given initial water saturation. If the wettability varies with initial water saturation, we demonstrate that the bounding waterflood curve, which is measured from the lowest water saturation (and hence the most oil-wet conditions), does not yield the correct scanning curves at higher water saturations.

We use network model derived relative permeabilities in a conventional field-scale simulation of waterflooding a typical North-Sea reservoir. If the wettability varies with height above the OWC, we demonstrate that the displacement efficiency is high. This is because the water relative permeability curves are generally low (characteristic of water-wet media), yet the residual oil saturation is also generally low (characteristic of oil-wet media). Assuming that the reservoir is uniformly water-wet or oil-wet, or using empirical hysteresis models, leads to an underestimate of recovery.

Using a physically-based pore-scale approach predicts behavior at both the core and field scales that is qualitatively different from that predicted by conventional relative permeability models. This work suggests that where there is a wettability trend in a reservoir, the relative permeabilities can have a first order impact on recovery predictions.

Introduction
The wettability of a crude oil/brine/rock system can have a significant impact on flow during oil recovery, and upon the volume and distribution of the residual oil. Wettability depends upon factors such as the mineralogy of the rock, the composition of the oil and water, the initial water saturation, and the temperature. Several studies have successfully used network models to investigate the effect of wettability variations on flow at the pore-scale. These models have progressively captured more of the physics of pore-scale displacement mechanisms; however, until recently, there has been less focus on capturing the complex pore-scale topology of a typical reservoir rock. Consequently, although these models have successfully interpreted phenomena observed experimentally, they have not been truly predictive.
The aim of this study is to investigate and predict the effect of wettability variations on flow at the reservoir-scale, using a pore-scale network model in conjunction with reservoir-scale conventional simulations. We use a three-dimensional (3-D) network model, which combines a physically-based pore-scale model of wettability alteration\textsuperscript{17} with a network representation of a Berea sandstone.\textsuperscript{22,23} The network is reconstructed directly from a sample of the sandstone, so the pore-size distribution and co-ordination number are fixed and are not ‘tuned’ to match experimental data. We successfully predict experimental relative permeability data for water-wet Berea sandstone,\textsuperscript{24} as well as waterflood recoveries for mixed-wet Berea.\textsuperscript{5} We study the effect of variations in initial water saturation associated with capillary rise above the oil-water contact (OWC) on wettability and relative permeability. Wettability variations with height have been observed in several reservoirs.\textsuperscript{25-28} We investigate empirical hysteresis models in which ‘scanning curves’ are used to connect bounding drainage and waterflood curves for a given initial water saturation.\textsuperscript{29,30} We find that if wettability varies with initial water saturation, then the scanning water relative permeability curves predicted by the empirical model are significantly higher than those predicted by the network model. This is because the empirical curves are generated using the bounding waterflood curve which is measured from the lowest initial water saturation (and hence most oil-wet conditions). We then use a conventional simulator to investigate the reservoir-scale impact of wettability variations on waterflooding. If the wettability varies with height above the OWC, we find that using the network model to generate scanning relative permeability curves yields a significantly higher prediction of recovery.

**Network model**

The 3-D network model is derived from a cube of volume 27\textsuperscript{3} reconstructed to represent a sample of Berea sandstone, as shown in Fig. 1. A topologically equivalent network of 12,349 pores and 26,146 throats is then generated.\textsuperscript{22,23} Each pore and throat is represented as a duct with a triangular, square or circular cross-section, characterized by an inscribed radius which controls the threshold capillary entry pressure, effective corner angles which control the amount of fluid held in wetting layers, and an effective volume which controls the mobile (non-clay bound) saturation.\textsuperscript{15-18,23,31-33} A clay volume is associated with the network. This represents a volume that remains water saturated. It can be adjusted to match the observed connate water saturation.\textsuperscript{22,23} Empirical formulae are used to compute the hydraulic conductance of each pore and throat.\textsuperscript{31} Two-phase flow is simulated for primary drainage and waterflooding assuming that capillary forces dominate, so the pores and throats are filled in order of increasing capillary entry pressure. This is a reasonable approximation for low capillary number flow.\textsuperscript{15,34}
The drainage cycle begins with the network fully saturated with water and strongly water-wet, with the receding contact angle $\theta_r = 0^\circ$. Oil then enters the network, and as the capillary pressure is increased step by step, it invades the pore or throat with the lowest capillary entry pressure in an invasion percolation process. Capillary entry pressures are calculated using the approach of Øren et al.\textsuperscript{23} At each step, water and oil saturations, relative permeabilities, and the capillary pressure are calculated subject to pressure boundary conditions at the inlet and outlet faces, and periodic boundaries on the other faces. To avoid end-effects, only a subset of the network model is included. Drainage is complete when a target capillary pressure or saturation has been reached, or when all pores and throats have been invaded by oil.

Wettability variations are modeled by changing the advancing contact angle $\theta_a$ assigned to each oil-filled pore after primary drainage.\textsuperscript{16,17} Different pores may have different contact angles. Depending upon the number of pores and throats invaded by oil, and the range of advancing contact angles, this approach allows us to model a mixed-wet system, in which only those pores invaded by oil become oil-wet, a fractionally-wet system, in which a fraction of the pores and throats invaded by oil become oil-wet, and a system which remains water-wet.

Water injection (waterflooding) is then simulated. There are three distinct types of water invasion: piston type, pore-body filling and snap-off. The capillary entry pressures for piston type and snap-off processes are taken from Øren et al.\textsuperscript{23} The capillary entry pressure for pore-body filling is given by a parametric model:\textsuperscript{17}

$$P_{cap} = \frac{2\gamma \cos \theta_a}{r} - \gamma \sum_{i=1}^{n} A_i x_i$$

where $n$ is the number of surrounding throats filled with oil, $r$ is the pore radius, $x_i$ are random numbers between 0 and 1, and $A_i$ are arbitrary parameters. Pore-body filling is a cooperative process\textsuperscript{15,36} favored when $n$ is small (many surrounding throats contain water) and suppressed for large $n$ (few surrounding throats filled with water). Eq. 1 accounts for the effect of converging/diverging pore geometries that can lead to
negative values of $P_{cap}$ even for $\theta_a < 90^\circ$.\textsuperscript{17} In this work we set the weighting parameters to be $A_1 = 0$, $A_2 = A_3 = A_4 = A_5 = 0.03 \, \mu\text{m}^{-1}$. There is no particular rationale for this choice of parameters, except that the values are similar to those used in previous work\textsuperscript{17} and is broadly consistent with calculated capillary pressures for square ducts.\textsuperscript{35,36} Piston-like advance leaves the center of the pore or throat filled with water, but if the advancing contact angle is large enough, water also remains in the corners with a layer of oil sandwiched in between. Oil can flow through these layers\textsuperscript{1,31} which are stable until the two oil/water interfaces meet.\textsuperscript{17} During water injection, water and oil saturations, relative permeabilities, and the capillary pressure are calculated in the same way as for drainage. Injection is complete when a target capillary pressure or saturation has been reached, or when all available pores and throats have been invaded by water. Further details of the network modeling approach can be found in Refs 23 and 33.

**Predicting experimental data**

**Water-wet data.** We begin by comparing the network model predictions with experimental data from water-wet Berea cores.\textsuperscript{24} During drainage, the receding contact angle is assumed to be $0^\circ$; there are no other parameters to adjust. Fig. 2 shows the match: it is good whether plotted on linear (Fig. 2a) or logarithmic (Fig. 2b) axis, although there is a slight tendency to overestimate the oil relative permeability at low water saturations. To predict the imbibition data we assumed a uniform distribution of advancing contact angles and adjusted the minimum and maximum values of contact angle to fit the data. The best match, shown in Fig. 3, was obtained using advancing contact angles drawn at random from a uniform distribution with a minimum of 50º and a maximum of 80º. The match is good for both the water and oil relative permeabilities, whether plotted on linear (Fig. 3a) or logarithmic (Fig. 3b) axis. Small changes in the contact angle distribution or the weighting parameters $A_i$ for pore filling do not adversely affect the match.

![Fig. 2—Drainage relative permeability predicted from the network model (lines) and measured on a water-wet Berea core (crosses; data from Oak\textsuperscript{24}). Plotted on linear scale in (a) and on logarithmic scale in (b).](image-url)
Given that the data was obtained from water-wet cores, it is perhaps surprising that the advancing contact angles that best match the data are quite large. However, it should be remembered that contact angles measured at a smooth flat surface may not resemble those found within the pore space of a typical reservoir rock, because of the additional complexities introduced by converging and diverging pore and throat geometries, surface roughness and heterogeneous mineralogy.37-39

Mixed-wet data. Jadhunandan and Morrow4 performed an exhaustive study of the effects of wettability on waterflood recovery for Berea sandstone, and we now compare the network model predictions with some of their experimental data. They waterflooded core samples that had been aged at different temperatures in different crude oils and brines, with different initial water saturations. Our hypothesis is that, for a given crude oil, brine and reservoir (or core) conditions, the distribution of contact angles in pores occupied by oil is similar regardless of the initial water saturation ($S_{wi}$). Wettability variations with $S_{wi}$ still occur because the number of pore-walls rendered oil-wet following primary drainage increases as $S_{wi}$ decreases.25-28 This hypothesis enables us to use data from a system at one value of $S_{wi}$ to predict wettability trends in a reservoir with varying $S_{wi}$; for example, associated with a transition zone above the OWC. Our assumption of a fixed distribution of contact angles for pores contacted by oil is not necessarily correct; for low $S_{wi}$ the initial capillary pressure is larger than for high $S_{wi}$. This will tend to favor the collapse of wetting films, making the pores invaded by oil more oil-wet.13 Furthermore, curvature effects may tend to make the smaller oil-filled pores more oil-wet13 which would also result in the distribution of contact angles becoming more oil-wet as $S_{wi}$ decreases.
We use the network model to match experimental waterflood data from one crude oil/brine system and aging temperature, where $S_w$ was varied from 7.8% (most oil-wet) to 31.1% (most water-wet; see Fig. 7 in Jadhunandan and Morrow). To reach the lowest $S_w$ of 7.8% during primary drainage, we adjusted the clay content of our network model. We then simulated primary drainage to the different values of $S_w$, followed by wettability alteration and then waterflooding. The network model predicted relative permeability curves and we used these in a semi-analytic Buckley-Leverett analysis, ignoring capillary pressure, to predict waterflood recovery. The matching parameter was the distribution of advancing oil/water contact angles. We assumed that the contact angle distribution after wettability alteration in pores invaded by oil was uniform between 180° and some lower limit. This reduced the match to a single parameter. Fig. 4 shows predicted and experimental results when the contact angle distribution varied uniformly between 110° and 180°. This is a plausible range based on the limited experimental data available for contact angles on flat surfaces where the crude oil induces an appreciable change. The reference ‘water-wet’ case (a core flood without wettability alteration) was simulated assuming a uniform contact angle distribution in the range 35° to 85°. This is similar to the distribution used to match the Oak data shown in Fig. 3.

The agreement between experiment and prediction is good bearing in mind that a single adjustable parameter was used to predict four recovery curves. The significant feature is that the trend in recovery is correctly predicted: the water-wet case gives a high recovery at breakthrough, but there is significant trapping of oil. With wettability alteration, recovery is higher with significant production of oil post-breakthrough. The most favorable recovery is found for an intermediate value of $S_w$ of around 20%, and the recovery is lower for both higher and lower values of $S_w$. This qualitative trend in recovery has been discussed previously in network modeling studies. However there are two differences with this work. First, we attempt a quantitative match with experiment. Second, hitherto the trend has been modelled in a network by keeping $S_w$ constant and changing the distribution of contact angles. In this study we keep the distribution of contact angles constant and vary $S_w$.

The pore-scale explanation for the results is as follows. For low values of $S_w$, most of the pores and throats are oil-wet and waterflooding is essentially a primary drainage type displacement. The water fills the larger pores and has a high relative permeability, resulting in rapid breakthrough. Recovery continues after breakthrough due to the drainage of oil through layers, but this is a very slow process. As $S_w$ increases, pore-level displacement can be initiated from pores and throats already filled with water. This leads to a more connected advance, as pores with a lower capillary entry pressure (essentially with the lowest contact angles) are filled first. This gives a delayed breakthrough and more favorable waterflood recoveries. For much larger values of $S_w$ recovery drops. This is because appreciable quantities of oil can be trapped: the large number of completely water-filled water-wet pores prevents the escape through oil layers of oil surrounded by water.

There are some significant differences between prediction and experiment and we tend to underestimate the impact of varying $S_w$ on waterflood recovery. It is possible to
improve the match by allowing the distribution of contact angles to be different for different values of $S_{wi}$. Fig. 5 shows these results. The range of contact angles is from $130^\circ$ to $180^\circ$ for $S_{wi} = 0.24$ and from $85^\circ$ to $180^\circ$ for $S_{wi} = 0.18$. The other cases are not adjusted. The agreement is now good in all cases, except that we over-predict recovery at later times for the lowest value of $S_{wi}$. The predicted results for this case are relatively insensitive to the contact angle distribution and so it is not easy to improve the match significantly.

The Berea network that we have used in the modeling studies has a larger permeability than the cores used in the experiments. Also, while the experimental results were generally reproducible, similar cores with similar values of $S_{wi}$ gave recoveries that varied by 0.1 or more, within the range of mismatch of our predictions. Further work is planned to tune the distribution of contact angles to match the Amott wettability indices and then predict waterflood recovery, enabling, in theory, relative permeabilities as a function of $S_{wi}$ to be predicted from a single wettability measurement.

The conclusion of this section is that it is possible to predict waterflood recovery for different values of $S_{wi}$ in mixed-wet systems within the range of experimental variation. We can now use the network model with some confidence to predict recovery in a reservoir with a wettability trend associated with a transition zone above the oil/water contact.

![Fig. 5](image)

Fig. 5—These results show the range of contact angles for different values of $S_{wi}$. The range is from $130^\circ$ to $180^\circ$ for $S_{wi} = 0.24$ and from $85^\circ$ to $180^\circ$ for $S_{wi} = 0.18$. The other cases are not adjusted. The agreement is now good in all cases, except that we over-predict recovery at later times for the lowest value of $S_{wi}$. The predicted results for this case are relatively insensitive to the contact angle distribution and so it is not easy to improve the match significantly.

![Fig. 4](image)

Fig. 4—Predictions of oil recovery (solid lines) using pore-scale modeling compared with experimental results from Jadhunandan and Morrow (points). In the network model we assumed that the pores contacted by oil had a uniform distribution of contact angles from $110^\circ$ to $180^\circ$, regardless of the initial water saturation $S_{wi}$. The water-wet reference case was simulated assuming a uniform distribution of contact angles from $35^\circ$ to $85^\circ$.

Relative permeability hysteresis

Relative permeability curves typically exhibit hysteresis during drainage and waterflooding, and several workers have argued that this can have a significant impact on oil recovery. Killough and Carlson presented empirical models for hysteresis in which relative permeabilities can vary between drainage and waterflood via intermediate ‘scanning’ curves (Fig. 6).
The drainage and waterflood curves which bound the scanning curves are determined experimentally, and the scanning curves are obtained by interpolating or re-mapping the bounding curves. Each scanning curve corresponds to a reversal in the direction of saturation change. The first set of scanning curves corresponds to a reversal from drainage to waterflooding, which occurs at the maximum non-wetting phase saturation obtained after drainage. These scanning curves correspond to waterflooding from different initial water saturations.

Fig 5—Predictions of oil recovery (solid lines) using pore-scale modeling compared with experimental results from Jadhunandan and Morrow (points). In the network model we assumed that the pores contacted by oil had a uniform distribution of contact angles from 110° to 180°, for $S_{wi} = 0.311$ and $S_{wi} = 0.079$. The range of contact angles was 130° – 180° for $S_{wi} = 0.24$ and 85° to 180° for $S_{wi} = 0.18$. The water-wet reference case was simulated assuming a uniform distribution of contact angles from 35° to 85°.
Fig. 6—Hysteresis in relative permeability after Killough\textsuperscript{29}. Scanning curves are predicted based on experimentally measured drainage and waterflood relative permeabilities. See nomenclature for an explanation of the terms on the left-hand plot.

Comparison of network and hysteresis models

We now investigate the effect on the waterflood relative permeability curves of varying the water saturation obtained after drainage (the initial water saturation $S_{wi}$). Our aim was to reproduce the effect of variations in the initial water saturation observed in a transition zone above the OWC. We drained the network to water saturations ranging from connate ($S_{wi} = S_{wc} = 0.25$) to $S_{wi} = 0.9$, and then injected water until all the available pores and throats had been invaded. This yielded a suite of waterflood curves, each of which originates on the drainage curve at a different initial water saturation.

For comparison with the hysteresis models\textsuperscript{29,30} we assumed that network model results for waterflooding from connate water saturation gave the bounding waterflood curve. Normally this would be measured experimentally.

Initially, we assumed that pores and throats invaded by oil remain water-wet, with the same distribution of advancing contact angles as used to match the Oak data\textsuperscript{24} (Fig. 7). We then assumed that pores and throats invaded by oil become oil-wet, with the same range of advancing contact angles as used to match the Jadhunandan and Morrow\textsuperscript{4} data in Fig. 4 (Fig. 8). For convenience, we will refer to this system as ‘oil-wet’.

For the water-wet case (Fig. 7), the scanning relative permeability curves lie between the bounding drainage and waterflood curves, as predicted by the hysteresis model\textsuperscript{29,30}. However, for the oil-wet case the bounding waterflood curve for water lies above the drainage curve, while the scanning curves lie below the drainage curve (Fig. 8). The bounding curve is measured in the most oil-wet conditions above the transition zone, whereas the scanning curves reflect progressively less oil-wet conditions as the initial water saturation increases and the OWC is approached. For comparison in Fig. 9 we plot the scanning curves predicted by the Killough model for the bounding drainage and waterflood curves shown in Fig. 8. The left-hand plot (Fig. 9a) was calculated using the entire waterflood bounding curve. The minimum oil saturation obtained by the network model corresponds to a very small oil relative permeability, because oil can continue to flow very slowly through layers\textsuperscript{1,31}. Consequently, the residual oil saturation is much smaller than that which might be measured experimentally or
observed in a reservoir following waterflooding. The right-hand plot (Fig. 9b) was therefore obtained using the waterflood bounding curve truncated at a threshold oil relative permeability of $10^{-3}$. This is a typical experimental threshold.

The oil scanning curves predicted by the network and empirical models are similar (cf. Fig. 8 and Fig. 9), but the water scanning curves are rather different. Those predicted by the network model are lower if they originate at lower initial water saturations, and lie below the bounding drainage curve.

The water relative permeabilities predicted by the network model can be explained by considering the displacement sequence at the pore-scale. During waterflooding, water invades the largest oil-wet pores first. This results in a rapid increase in water saturation. However, the water relative permeability remains low until these pores form a connected network that spans the model. The water relative permeability then increases rapidly (Fig. 8; notice the shape of the curves originating at low initial water saturations). This behavior is not captured by the empirical hysteresis model.

It is clear that the scanning curves predicted by the network and empirical models are different. This suggests that the empirical model should not be used to predict hysteresis within a transition zone above the OWC if the wettability varies with height. However, it is not clear whether the differences are significant enough to affect waterflooding at the reservoir-scale. We address this issue in the next section.

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**Fig. 7**—Relative permeability (a) and capillary pressure (b) curves obtained from the network model. Crosses denote drainage curves, lines denote waterflood (imbibition) curves. The waterflood curves were calculated for initial water saturations ranging from connate ($S_{wi}=S_{wc}=0.25$) to $S_{wi}=0.9$. During waterflooding the advancing contact angle of pores and throats invaded by oil ranged from 50-80°.
Fig. 8—Relative permeability (a) and capillary pressure (b) curves obtained from the network model. Crosses denote drainage curves, lines denote waterflood curves. The waterflood curves were calculated for initial water saturations ranging from connate ($S_{wi} = S_{wc} = 0.25$) to $S_{wi} = 0.9$. During waterflooding the advancing contact angle of pores and throats invaded by oil ranged from 110-180°.

Fig. 9—Scanning curves predicted by the Killough hysteresis model using the bounding drainage and waterflood curves shown in Fig. 8 for the oil-wet case. The left-hand plot (a) shows scanning curves obtained using the entire waterflood water relative permeability curve. The right-hand plot (b) shows scanning curves obtained using the waterflood water relative permeability curve truncated at an oil relative permeability of $10^{-3}$, to mimic the curve which might be measured experimentally. (x) symbols denote drainage bounding curves; (+) symbols denote waterflooding bounding curves (cf. Fig. 8). To generate the scanning curves, best fit curves were matched to the network model bounding waterflood results.

**Impact of wettability variation on waterflooding**

Our aim was to investigate whether wettability variations associated with a transition zone above the OWC have a significant impact on waterflooding in the presence of realistic permeability heterogeneity. The reservoir model we used was based upon the
Maureen Field, which is located in Block 16/29a of the UK sector of the North Sea. The Maureen reservoir consists of Palaeocene sandstones which are interpreted to have been deposited as a broad submarine clastic fan across an extensive basin plain.\textsuperscript{44,45} The oil is trapped in a four way dip closure over a salt dome. The Maureen lithofacies is dominated by very clean, sand-rich, stacked massive sand sequences which characteristically show significant lateral facies variations and which are separated by shales of varying lateral extent. The sandstones range from 140 to 400 feet in thickness and have good reservoir properties, with porosities ranging from 0.18-0.25 and permeabilities ranging from 30-3000mD. The field originally contained undersaturated oil with no gas cap, and was produced by waterflooding, initially with 12 production and 7 water injection wells.

The reservoir model used in this study was constructed using stochastic techniques conditioned to nine of the operator’s exploration and development wells. The model was designed to capture the architecture of the shales that separate the sandstones, and the lateral and vertical variations in sandstone porosity and permeability. It was constructed on a grid containing 52x52x73 cells which was sufficiently refined to capture the significant permeability heterogeneities, yet was not too large to be used directly for flow simulation (\textbf{Fig. 10}). Consequently, the model was not upscaled. Shales were modeled as barriers to flow. The model was not history matched to Maureen production data as it was not intended to be a close representation of the Maureen field; it was intended simply to capture realistic permeability heterogeneity and be based upon real field data.

The initial distribution of water ($S_{wi}$) within the model was dictated by the drainage capillary pressure curve obtained from the network model and shown in Fig. 8. We were not concerned that the network model is a representation of Berea sandstone rather than Maureen sandstone as we were interested only in investigating the impact on waterflooding of wettability variations.

Oil and water densities were chosen to yield a transition zone in which the water saturation falls to connate ($S_{wc}$) at the top of the reservoir; the thickness of the transition zone is thus comparable to the thickness of the reservoir. Oil and water viscosities were taken from the Maureen Field; they are 0.7cP and 0.3cP respectively. The capillary pressure was scaled to different values of sandstone porosity and permeability using a J-Function approach, so the variation in $S_{wi}$ around the field reflects both the transition zone and the variation in sandstone reservoir quality.

Our waterflood simulations employed eight production wells located on the crest of the field, supported by four water injection wells on the flanks of the field. We simulated waterflooding for four different cases: (i) assuming that pores and throats invaded by oil remain water-wet and neglecting hysteresis, using only the bounding waterflood relative permeability curves shown in Fig. 7 (water-wet; no hysteresis); (ii) assuming that pores and throats invaded by oil become oil-wet and neglecting hysteresis, using only the bounding waterflood relative permeability curves shown in Fig. 8 (oil-wet; no hysteresis); (iii) assuming that pores and throats invaded by oil become oil-wet and attempting to include hysteresis by applying the Killough model implemented in Eclipse 100 (Schlumberger Geoquest, 2001), with the bounding drainage and waterflood curves
shown in Fig. 9. We apply the model with truncation of the bounding curves at an oil relative permeability of $10^{-3}$ (Fig. 9b); (iv) assuming that pores and throats invaded by oil become oil-wet and properly including hysteresis by generating a suite of relative permeability curves from the network model, which correspond to the initial water saturations within the simulation model (scanning curves derived from network model). A subset of this suite of curves is shown in Fig. 8.

The simulations yield the oil produced and watercut curves shown in Fig. 11. These demonstrate that recovery is significantly higher if hysteresis is included using scanning curves generated by the network model. Neglecting hysteresis and assuming that the reservoir is uniformly oil-wet yields the lowest recovery. Including hysteresis using the Killough model yields higher recovery than the purely oil-wet case, but it appears that the oil-wet character of the bounding imbibition curves has dominated (Fig. 9). Neglecting hysteresis and assuming that the reservoir remains water-wet yields higher recovery than the oil-wet and Killough cases, but recovery is still lower than if hysteresis is included using the network model.

Incorporating hysteresis in the oil-wet case by using the network model to generate scanning curves yields a significantly higher recovery after 1PV injected, because the scanning curves display both water-wet and oil-wet characteristics, depending upon the initial water saturation and hence their location within the model. Residual oil saturations are generally much lower than for the water-wet case, yet water relative permeabilities are generally much lower than for the oil-wet case (Fig. 8). This delays water breakthrough and yields a higher oil recovery.
Fig. 10—Reservoir model used in the simulations, showing the distribution of sandstone (light colours) and shale (dark colour). The areal model dimensions are approximately 6000x6000m. The OWC is a flat surface located at 8700 ft TVDSS. Production wells (P1 - P8) were located on the crest of the field; injection wells (I1 - I6) on the flanks.
Fig. 11-Pore volumes (PV) of oil produced (left-hand plot) and watercut (right-hand plot) as a function of pore volumes of water injected, for each of the simulations run.

Conclusions
We successfully predicted experimental relative permeability and waterflood recovery data for water-wet and mixed-wet Berea sandstone using a pore-scale network model. For the mixed-wet data we predicted the trend in waterflood recovery with initial water saturation ($S_{wi}$) by assuming that the distribution of contact angles in pores initially occupied by oil was independent of $S_{wi}$.

We investigated empirical hysteresis models in which ‘scanning curves’ are used to connect bounding drainage and waterflood curves for a given initial (minimum) water saturation. We found that for mixed-wet systems the scanning water relative permeability curves predicted by the empirical model are significantly higher than those predicted by the network model.

We then used a conventional simulator in conjunction with the relative permeability curves obtained from the network and empirical models to investigate the reservoir-scale impact of wettability variations on waterflood efficiency. If wettability varies with height above the OWC, we found that using the network model to generate scanning relative permeability curves yields a significantly higher recovery. This is because the scanning curves display both water-wet and oil-wet characteristics depending upon the initial water saturation. Residual oil saturations are generally much lower than for the water-wet case, yet water relative permeabilities are generally much lower than for the oil-wet case. This behavior is not predicted by conventional relative permeability models.
We suggest that network models based on the structure of real rocks and incorporating the pertinent pore-scale displacement processes may be used as a tool to predict wettability variations and their impact on fluid flow at the reservoir-scale.

**Nomenclature**

- $k_{ro} =$ oil relative permeability
- $k_{rw} =$ water relative permeability
- $k_{ro}^e =$ oil end-point relative permeability
- $k_{rw}^e =$ water end-point relative permeability
- $n =$ number of surrounding throats filled with oil
- $P_{cap} =$ capillary pressure, mL⁻¹t⁻², Pa
- $r =$ pore radius, L, m
- $S_{Nr} =$ residual non-wetting phase saturation
- $S_{Nf} =$ maximum non-wetting phase saturation for given scanning curve
- $S_{Nfr} =$ maximum residual non-wetting phase saturation
- $S_{wc} =$ connate water saturation
- $S_{wi} =$ initial water saturation
- $S_{or} =$ residual oil saturation
- $\theta_a =$ advancing contact angle, degrees
- $\theta_r =$ receding contact angle, degrees

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**References**


Chapter 2

Pore-scale modeling of three-phase flow in mixed-wet systems

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Abstract
We use a pore-scale network model of three-phase flow to compute relative permeabilities, saturation paths and capillary pressures for a variety of displacement processes. The model is based on a random network of pores and throats with triangular, rectangular and circular cross-sections that represent the complex pore space observed in sandstones. We model wettability alteration after primary drainage and allow any values for the advancing and receding oil/water, gas/water and gas/oil contact angles. Multiple phases can be present in each pore, in wetting and spreading layers, as well as occupying the center of the pore space. In all, twenty different generic fluid configurations for two- and three-phase flow are analyzed.

With a network based on a description of Berea sandstone we can predict relative permeabilities for two-phase flow in a water-wet system and waterflood recoveries for mixed-wet media. We then predict the steady-state oil, water and gas three-phase relative permeabilities measured by Oak. We demonstrate that the predictions obtained by the network model compare favorably with those obtained using standard empirical relative permeability correlations.

We then study gas injection into media of different wettability and interpret the results in terms of pore-scale displacement processes.

Introduction
The simultaneous flow of three phases – oil, water and gas – in porous media occurs in a variety of reservoir and environmental engineering problems. During conventional two-phase mechanisms of oil recovery, such as waterflooding, oil recovery is typically 20 to 50% indicating that oil flows at relatively high saturations. In such circumstances uncertainties in the relative permeability often have little impact on recovery predictions. In contrast, during three-phase recovery processes such as gas injection, gas cap expansion, depressurization, solution gas drive and gravity drainage, to achieve a higher recovery oil may flow at very low saturations, implying that the oil relative permeability can also reach very low values. Direct measurement of relative permeabilities and capillary pressures in this regime is not only very difficult but also time consuming and inaccurate. Since two independent fluid saturations are required to define a three-phase system, there is an infinite number of possible fluid arrangements and displacement paths, making a comprehensive suite of experimental measurements for all three-phase displacements impossible. Consequently, numerical simulations of three-phase flow rely on empirical models to predict relative permeability and capillary pressure from measured two-phase values. However, these models may give predictions that vary as much as an order of magnitude from each other, or from direct measurements, since they have little or no physical basis. In such cases uncertainties in relative permeability, particularly at low oil saturation, may have a very significant impact on recovery predictions.

An appealing approach to predict three-phase properties accurately is to use physically-based pore-network modeling. The model should capture the pore-space geometry and the pertinent pore-scale displacement mechanisms and be tuned to, or be able to predict, any available single- or two-phase data. In recent years, several advances in pore-scale modeling have made this approach possible. Øren, Bakke and co-workers at Statoil
have developed random network models based on the pore space geometry of the rock of interest. The model is derived either from a direct three-dimensional image of the pore space obtained from micro CT scanning, or from simulating the geological processes by which the rock was formed. Many other authors have also developed techniques to derive pore structures from a variety of measurements. While such approaches are not routine, and the correct pore-space characterization of carbonates is very much an open question, for simple sandstones there are reliable methods from determining an equivalent network structure that attempts to mimic the properties of the real pore space. Second the pertinent displacement processes for three-phase flow have been observed and described using micromodel experiments. For water-wet media the pore-scale mechanisms of fluid displacement are now well established. The behavior of mixed-wet systems has a less firm experimental basis, but the work of Kovscek et al. has provided a pore-level scenario for wettability alteration that has been used to describe fluid configurations for two- and three-phase displacements.

The network model described here combines three essential components: (1) a description of the pore space and its connectivity that mimics real systems; (2) a physically-based model of wettability alteration; and (3) a full description of fluid configurations for two- and three-phase flow. Many three-phase network models have been proposed previously. However, this is the first model that combines all three of the features above to predict three-phase relative permeability for media of arbitrary wettability using geologically realistic networks.

Network model
For two-phase flow the network model used here is based on the work of Øren, Patzek and co-workers. Rather than repeat their work, only a brief description of two-phase flow is provided. Further details are available in the Appendix

Geometry. The model reads as input any two or three-dimensional network comprised of pores (nodes) connected by throats (links). Each pore or throat is assigned a total volume, an inscribed radius and a shape. The inscribed radius is used to assign a capillary entry pressure during multiphase flow. The pore and throats have a scalene triangular, square or circular cross-section. The cross-section has the same shape factor (ratio of cross-sectional area to perimeter squared) as the real system from which the network is derived. A clay volume is associated with the network. This represents a volume that remains water saturated. It can be adjusted to match the observed connate water saturation. In this paper a network based on a Berea sandstone, Fig. 1, will be used for all the results presented.

The network is a cube of volume 27 mm$^3$ with 12,349 pores and 26,146 throats. The coordination number varies from 1 to 19 with an average of 4.19. The inscribed radius of the pores varies from 3.62 to 73.54 µm. The throat inscribed radius varies from 0.90 to 56.85 µm. The total porosity is 24.02% with a clay volume of 5.7%. 92.27% of the pores and throats have a triangular cross-section, 6.51% are square and 1.22% are circular. The absolute permeability of the network is 2,600mD.
**Primary drainage and wettability alteration.** The network is initially fully saturated with water and strongly water-wet, with the receding oil/water contact angle $\theta_{owr} = 0^\circ$. Oil then enters the network, and as the capillary pressure is increased step by step, it invades the pore or throat with the lowest capillary entry pressure in an invasion percolation process. Drainage is complete when a target capillary pressure or saturation has been reached, or when all pores and throats have been invaded by oil.

![Fig.1—An example Berea network. The network is a disordered lattice of pores connected by throats. The network topology, the radii of the pores and throats, their shapes and their volumes are all determined from a three-dimensional representation of the pore space of the system of interest. This network will be used to predict two- and three-phase relative permeabilities.](image)

At the end of primary drainage, regions of the pore space in direct contact with oil may change their wettability. We assign an advancing oil/water contact angle $\theta_{owa}$ to each oil-filled pore and throat after primary drainage. Different pores and throats may have different contact angles. Contact angles are assigned at random to oil-filled pores and throats, according to some specified distribution. In this paper, the contact angles will have a uniform distribution between some specified maximum and minimum values.

After primary drainage, the model simulates any sequence of water, gas and/or oil invasion.

**Contact angles for three-phase flow.** In three-phase flow, the oil/water, gas/oil and gas/water contact angles are not independent. The relationship between them is:53-55

$$\sigma_{gw} \cos \theta_{gw} = \sigma_{go} \cos \theta_{go} + \sigma_{ow} \cos \theta_{ow} \quad (1)$$

where $\sigma$ is the interfacial tension measured in thermodynamic equilibrium. This means that the spreading coefficient for oil $C_{3p} = \sigma_{gw} - \sigma_{ow} - \sigma_{go}$ is less than or equal to zero.54,56 For a strongly water-wet medium, $\cos \theta_{ow} = 0$ and from Eq. (2),

January 2003 28
\[
\cos \theta_{go} = \frac{(\sigma_{gw} - \sigma_{ow})}{\sigma_{go}}. \quad \text{For a strongly oil-wet medium, we assume that } \cos \theta_{ow} = -1 \text{ and } \cos \theta_{go} = 1 \text{ and from Eq. (2), } \cos \theta_{gw} = \frac{(\sigma_{go} - \sigma_{ow})}{\sigma_{gw}}. \quad \text{Van Dijke et al.}^{46,50} \text{ suggested assigning the gas/oil and gas/water contact angles as a function of } \theta_{ow} \text{ by a linear interpolation between the water-wet and oil-wet limits:}\n\]

\[
\cos \theta_{go} = \frac{1}{2} \left[ \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}} - 1 \right] \cos \theta_{ow} + 1 + \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}} \quad (2)
\]

\[
\cos \theta_{gw} = \frac{1}{2} \left[ 1 - \frac{\sigma_{go} - \sigma_{ow}}{\sigma_{gw}} \right] \cos \theta_{ow} + 1 + \frac{\sigma_{go} - \sigma_{ow}}{\sigma_{gw}} \quad (3)
\]

We use these expressions in this paper to find contact angles. To accommodate any type of displacement process, we assign both receding and advancing contact angles. The oil/water advancing contact angle \( \theta_{owa} \) (water displacing oil) is found as described before. The receding oil/water contact angle \( \theta_{owr} \) (oil displacing water) is zero if the oil contacts portions of the pore space that were not contacted by oil during primary drainage. For oil invasion over surfaces previously contacted by oil, contact angles are again assigned to different pores and throats but with the constraint that \( \theta_{owr} \leq \theta_{owa} \). For gas displacing oil (oil receding) \( \theta_{gor} \) is found from Eq. (2) using \( \theta_{owr} \) (water receding). For oil displacing gas, Eq. (2) with \( \theta_{owa} \) is used to find \( \theta_{goa} \). For gas displacing water (water receding) \( \theta_{gw} \) is found from Eq. (3) using \( \theta_{owr} \). For water displacing gas, Eq. (3) with \( \theta_{owa} \) is used to find \( \theta_{gwa} \). In this work we only consider a spreading system \( C_{so} = 0 \), where \( \theta_{gor} = \theta_{goa} = 0 \) for all pores and throats.

Three-phase fluid configurations. Fig. 2 shows all the possible configurations of one, two or three phases in a single pore or throat. Fluid arrangements for a single corner are illustrated – the whole pore or throat is composed of no (circle), three (triangular) or four (square) corners. For elements with a circular cross-section, only one phase may occupy the pore or throat at one time. For square and triangular cross-sections, the phase in the middle of the element must be the same for each corner, but different corners may have different configurations, depending on the corner angle. However, in a single pore or throat, the contact angles in each corner are the same. Table 2 in the Appendix lists the range of contact angles for which each configuration in Fig. 2 can occur. A capillary pressure between any two phases \( P_{ij} \) is defined as \( P_i - P_j \). We assume that the pores and throats have little curvature perpendicular to the cross-section. As a consequence, the radius of curvature \( r \) of the interface between two phases \( i \) and \( j \) in Fig. 2 is related to the capillary pressure by:

\[
P_{ij} = \frac{\sigma_{ij}}{r} \quad (4)
\]

Displacement calculations. The model assumes quasi-static displacement controlled, at the pore scale, entirely by capillary forces. The simulation proceeds as a series of displacements. A displacement is the change of the configuration in one pore or throat.
This can represent the replacement of one phase by another in the center of the pore, or the collapse or formation of a layer in a single corner. There is a capillary pressure associated with the transition from one configuration to another.

We define pressures of water, oil and gas in the network. Initially when the network is water-filled all phase pressures are set to zero. As displacement proceeds, these pressures will increase. There are six possible types of displacement: oil into water (controlled by an increase in $P_{cow}$), water into oil (decrease in $P_{cow}$), gas into water (increase in $P_{cgw}$), water into gas (decrease in $P_{cgw}$), gas into oil (increase in $P_{cgo}$), and oil into gas (decrease in $P_{cgo}$). Any displacement corresponds to one of these six processes. We rank the capillary pressures for all possible displacements for all pores and throats into six sorted lists, corresponding to the different displacement types.

The network model is designed to compute capillary pressure and relative permeability for a given input saturation path. A target final saturation of oil, water and gas is specified. The difference between the present saturations and the target saturations is computed. The phase with the most negative difference is the next to be injected. Imagine this is phase $i$. This means that we consider a displacement event where the volume of $i$ in an element increases. We assume that the pressure of the other two phases $j$ and $k$ are held fixed at a value specified at the last displacement where $j$ and $k$ were the invading phases, respectively. Since the pressures of phases $j$ and $k$ are fixed, $P_{jk}$ does not change. Phase $i$ can invade either phase $j$ or $k$. The lowest pressure for phase $i$ for a displacement event is found by comparing the most favorable capillary pressures from the sorted lists for $i$ into $j$ and $i$ into $k$. Imagine that the capillary pressures obtained for $i$ into $j$ is $P_{ij}$ and for $i$ into $k$ is $P_{ik}$. If:

$$P_{ik} > P_{ij} + P_{jk}$$

then a displacement of $i$ into $j$ is considered, otherwise $i$ into $k$. If the event is allowed (see later for a discussion of this issue) then it takes place. If a displacement is not allowed, the event is taken off the sorted list and the most favorable elements of the $i$ into $j$ and $i$ into $k$ lists are again compared.
Fig. 2. Configurations of three phases in single corners of the pore space. All the
different possible arrangements of oil (O), water (W) and gas (G) in a corner are shown.
The bold solid line indicates regions of the pore space contacted by oil during primary
drainage that have an altered wettability. The points show interfaces that are pinned
with a contact angle greater than \( \frac{\pi}{2} - \alpha \), where \( \alpha \) is the half angle of the corner.
However, all the multiphase contact points may be pinned. This means that as the
capillary pressure changes, the curvature of the interface changes but that the location
of the interface/solid contact is fixed. A phase may be present in the center of the pore
space or as a spreading layer, sandwiched between other phases. Water is always
present in the corner. The network model simulates a sequence of displacement events
that represent the change from one configuration to another. The tables in the Appendix
list the capillary pressures for each displacement.
The phase pressure of \( i \) is updated whenever it reaches a new maximum. Displacement capillary pressures are now recomputed for the element filled and nearest neighbor elements, taking into account the new arrangements of fluid and capillary pressures and re-ordered in the sorted lists.

Tables 3-5 in the Appendix list the expressions used to compute capillary pressure for every possible displacement. The capillary pressure depends on the contact angles, corner angles and radius of the element of interest, other capillary pressures and the configuration of nearest neighbor pores and throats. With twenty different configurations and six generic types of invasion, the list of possible displacements is rather daunting, as the three pages of tables in the Appendix indicates. However, each displacement can be considered as a two-phase process (one phase displaces another) and as a consequence are based on well-established two-phase expressions described by Øren, Patzek and others.\(^4\)\(^-\)\(^6\),\(^52\)

The new phase saturations are computed (how this is done is discussed in a later section) and the difference between this and the target saturation recomputed. Again the phase with the greatest negative difference is injected and the process continues until the saturation in the network and the target saturation agree to within some tolerance. If a new phase is injected then the sorted lists for displacement are re-ordered to account for changes in capillary pressures. Then the simulation ceases or a new target saturation is specified.

There is one subtlety associated with specifying capillary pressure. It is often the case that a local configuration change allows a subsequent displacement of the same phase at a more favorable pressure. An example of this is oil displacement into water a water-wet medium. \( P_{cow} \) may be high to allow oil to fill a throat, but oil fills an adjoining pore at a lower capillary pressure. If we used this new lower value of \( P_{cow} \) (corresponding to a lower phase pressure \( P_o \)) to compute radii of curvature using Eq. (4), then we would find that in some elements the radius is inconsistent with the fluid configuration – for instance, water would be in a wetting layer too large to fit in a pore or throat. In strict capillary equilibrium, the fluid would rearrange throughout the network to give configurations consistent with the prevailing capillary pressure. This is, however, a very difficult task for a general three-phase model. Instead, phase pressures are defined as the maximum value ever reached during the whole simulation. Whenever a phase pressure reaches a new maximum, the fluid configuration is truly in a position of capillary equilibrium. In a simulation involving a complex displacement path, the phase pressures will continue to increase. However, since it is only pressure differences that control the displacement sequence, this is not a problem.

**Clustering and trapping.** To account for trapping we assign phases to clusters. A connected region of a phase defines a cluster. A phase can be located in a corner, layer or center of a pore or throat. Phase locations with the same phase in a neighboring pore and throat are connected to each other if the conditions listed in Table 1 in the Appendix are satisfied.

Every phase location in every element has a flag stating whether or not that location is connected to the inlet and outlet. A displacement is only allowed if the displaced phase
is connected to the outlet and the displacing phase is connected to the inlet. After the 
displacement we update the clusters and the corresponding flags for each phase location 
in clusters connected to the element where the displacement has occurred and nearest 
neighbor elements. Fig. 13 in the Appendix shows a flowchart to define phase clusters – a 
burning algorithm is used to assign phase locations to clusters.

**Multiple displacements.** One unique feature of three-phase flow is multiple 
displacement.\(^{25,26,30,32,41,42,44}\) An invasion of phase \(j\) by phase \(i\) may be composed of a 
displacement of \(k\) by \(i\) followed by a displacement of \(j\) by \(k\). If all the phases are 
continuous, this is equivalent simply to two separate events. However, the intermediate 
phase in the displacement – \(k\) – may be trapped. In this case disconnected clusters of 
phase \(k\) can rearrange themselves in the pore space, and may reconnect, simply due to 
capillary forces, when one phase invades part of the cluster that in turn displaces the 
third phase. This is a double displacement process and has been observed in 
micromodel experiments\(^{25,26,30,32}\) and coded into network models.\(^{41,42,44,51}\) Multiple 
displacements, involving more than one intermediate stage, are also possible if two 
phases are trapped. A cascade of disconnected blobs nudge each other before a final 
displacement of a connected phase.\(^{51}\) The algorithm for considering such events is 
somewhat involved, since multiple events for all clusters need to be considered together 
with the conservation of volume when the cluster moves.\(^{51}\)

In this paper we ignore double and multiple displacements. We will only consider gas 
innjection into relatively high oil saturations, with little or no oil initially trapped. In 
these cases double displacement is unlikely to be significant.

**Saturation computation.** Saturation is computed just before the phase pressure 
reaches a new maximum – this means that there are no possible displacements with the 
current phase pressures. If \(V_{ip}\) is the volume of phase \(p\) in element \(i\) (including the water 
volume in clay) then the saturation of phase \(p\) is given by:

\[
S_p = \frac{\sum_{i=1}^{n_p} V_{ip}}{\sum_{i=1}^{n_p} \sum_{p=1}^{n_p} V_{ip}}
\]

where \(n_p\) is the number of phases (three) and \(n_e\) is the total number of pores and throats. 
The total volume, inscribed radius and shape of each element is read in as input data. 
The volume of a phase in an element is the total volume multiplied by the fraction of the 
cross-sectional area occupied by that phase. Table 6 in the Appendix shows the 
expressions used to compute areas for the configurations shown in Fig. 2.

**Relative permeability and capillary pressure.** When saturation is computed, relative 
permeability and capillary pressure can also be found. This is not done after every 
saturation computation to save computer time. Typically, relative permeability is 
calculated around 20 – 40 times during a simulation. The capillary pressure is simply 
the difference between phase pressures. To compute absolute and relative permeability, 
conductances of each phase location connected to either inlet or outlet in each element 
are first specified. Normally exact analytic results are not possible, and empirical
expressions are derived from solutions of the Stoke’s equation for flow in pores of different geometries and for different fluid configurations.\textsuperscript{4-6,38,52,57,58} Table 7 in the Appendix gives expressions for the conductances of each phase for the configurations shown in Fig. 2. Then the average conductance for each phase in the whole network is computed by explicitly calculating the flow through the network. From this absolute and relative permeability can be found. While the expressions for conductance may be involved, the procedure for computing relative permeability is straightforward and has been explained in detail by many authors.\textsuperscript{5,35,42,52}

**Predicting experimental data**

**Two-phase water-wet data.** We compare the network model predictions with experimental two-phase data from water-wet Berea cores.\textsuperscript{1} During drainage, the receding contact angle is assumed to be 0\degree; there are no other parameters to adjust. Fig. 3 shows the match. To predict the imbibition data we assumed a uniform distribution of advancing contact angles with a minimum of 30\degree and adjusted the maximum value of contact angle to fit the data. The best match, shown in Fig. 4, was obtained using a maximum advancing contact angle of 80\degree. Small changes in the contact angle distribution did not adversely affect the match.

**Two-phase mixed-wet data.** Jadhunandan and Morrow\textsuperscript{60} performed an exhaustive study of the effects of wettability on waterflood recovery for Berea sandstone. In a companion paper\textsuperscript{61} we predict waterflood recovery curves as a function of initial water saturation. Our hypothesis is that, for a given crude oil, brine and reservoir (or core) conditions, the distribution of contact angles in pores occupied by oil is similar regardless of the initial water saturation ($S_{wi}$). Wettability variations with $S_{wi}$ still occur because the number of pore-walls rendered oil-wet following primary drainage increases as $S_{wi}$ decreases. We matched the recovery curves assuming that the advancing oil/water contact angle distribution varied uniformly between 110\degree and 180\degree. Tuning of the network model to match a single recovery curve is sufficient to predict successfully waterflood recovery for experiments performed with different values of initial water saturation, representing, for instance, the transition zone of an oil reservoir near the oil/water contact. In ref. 61 we demonstrate how properly incorporating physically-based network model predictions of relative permeability has a huge impact on estimated field-scale recovery, even for two-phase flow.
Fig. 3—Drainage relative permeability predicted by the network model (lines) and measured on a water-wet Berea core (points; data from Oak¹).

Fig. 4—Imbibition relative permeability predicted from the network model (lines) and measured on a water-wet Berea core (points; data from Oak¹). To match the data we assumed a uniform distribution of contact angles from $30^\circ$ to $80^\circ$.

**Three-phase data.** We now return to the work of Oak¹ and attempt to predict his three-phase steady-state experimental data. Here the systems were water-wet and we used the distribution of advancing oil/water contact angles that matched the two-phase waterflood data (Fig. 4). We then used Eqs. (2) and (3) to assign gas/oil and gas/water contact angles. We simulated the same flooding sequence as in the experiments, including tracking the changes of two independent saturations. Lerdahl et al.⁴⁵ also showed successful network model predictions for this dataset. The difference here is that we honor saturation paths, show a point-by-point comparison of data and make predictions for mixed-wet systems.

Fig. 5 shows the saturation paths for the experiments. The network model successfully tracks almost identical paths. We model gas injection into oil with a low water saturation. In these circumstances little or no oil is trapped at the beginning of gas injection and neglecting multiple displacements is unlikely to affect the results.
Fig. 5—Experimental (points) and network model (lines) saturation paths for the four three-phase flow experiments considered. Note that the network model almost exactly tracks the path followed in the experiments. The numbers refer to the numbers of Oak’s experiments.¹

Fig. 6 shows the predicted and measured oil relative permeabilities. The predictions are good at high oil saturation, but tend to over-estimate $k_{ro}$ at low saturation. This could be because we have assigned too much connectivity and conductance to oil layers. The predictions of gas relative permeability shown in Fig. 7 all plot on a single curve and are an almost exact match for two of the experiments. Because gas is the most non-wetting phase we expect gas to occupy the largest pores and throats regardless of the oil and water saturations, resulting in $k_{rg}$ being a function of $S_g$ only (see, for instance⁴¹). The large scatter in the experimental results is thus surprising and may be due to experimental problems.¹ The predictions for the water relative permeability, Fig. 8, are excellent. The experimental two-phase (Figs 3 and 4) and three-phase water relative permeabilities are not exactly the same, but using saturation tracking we are able to predict these differences.
Fig. 6—Experimental (points) and network model (lines) oil relative permeabilities for the four three-phase flow experiments considered.

Fig. 7—Experimental (points) and network model (lines) gas relative permeabilities for the four three-phase flow experiments considered.
Fig. 8—Experimental (points) and network model (lines) water relative permeabilities for two three-phase flow experiments. The water saturation did not change for the other two experiments.

Fig. 9 shows a comparison of predicted and measured oil relative permeabilities. Also included are comparisons with conventional three-phase relative permeability correlations: Stone 1, and saturation-weighted interpolation. See Ref. 3 for details of how the comparisons were made. Experiment 20 of Oak represents two-phase gas injection into oil with a fixed water saturation. The oil relative permeability for experiment 20 and the oil/water relative permeability shown in Fig. 3 were used as input to the empirical three-phase models. Thus by construction they obtain a perfect prediction for experiment 20. Experiment 10 also has a fixed water saturation and is not considered. For the other two experiments, the water saturation is low, and the empirical models predict relative permeabilities close to those obtained for experiment 20. This, unsurprisingly, gives reasonable predictions. The network model makes predictions of similar accuracy, although it does tend to overestimate \( k_{ro} \) at low values of the oil saturation (as discussed above). However, the network model does not require any two-phase curves as input. Moreover, as we show in the next section, it can be used to compute relative permeabilities outside the range of saturations and wettabilities tested experimentally. The empirical model predictions for Oak’s data are considerably worse at high water saturations. A network model test in these cases would require the incorporation of multiple displacements that will be the subject of future work.
Fig. 9—Comparison of measured and predicted oil relative permeabilities. The predictions of the network model are compared with those from empirical models: Stone 1 and saturation-weighted interpolation. The predictions have similar accuracy, but the network model predicts two- and three-phase data, while the empirical models assume two-phase gas/oil and oil/water data as input.

Mixed-wet three-phase relative permeability

We will now use our model to predict relative permeability for three-phase mixed-wet systems. In this case we do not have experimental data to compare against, but the success of our predictions for mixed-wet two-phase systems and water-wet two- and three-phase systems gives us some confidence that our results are valid. We consider primary drainage to connate water saturation followed by waterflooding to $S_w=0.4$ ($S_{oi}=0.6$). Then gas is injected at a constant oil/water capillary pressure. We study two cases: water-wet, where the distribution of advancing oil/water contact angles is the same as used to predict the two-and three-phase data (uniform from $30^\circ$ – $80^\circ$); and oil-wet where the distribution of advancing contact angles ($110^\circ$ – $180^\circ$) is the same as used to predict mixed-wet waterflood data.61

Fig. 10 shows the oil relative permeability. In the oil-wet case, oil remains in smaller pores and throats after waterflooding than for a water-wet medium, giving a lower relative permeability. This is seen in Fig. 10 for oil saturations larger than approximately 0.4. At lower oil saturations, gas has invaded most of the oil-filled elements. The relative permeability is limited by the connectivity of the oil phase. In the water-wet medium, oil layers collapse during gas injection, whereas oil layers remain stable throughout the displacement for the oil-wet case. Thus the oil-wet relative permeability is larger at low oil saturation.
The water-wet gas relative permeability in Fig. 11 is larger than the oil-wet case. In a water-wet medium, the gas always occupies the largest pores and throats. For an oil-wet system, gas is not the most non-wetting phase in the presence of water and will displace water from some of the smaller pores, resulting in poorer connectivity and conductance. This effect of wettability has been discussed previously and is well established experimentally. This result is a direct consequence on the constraint on contact angles, Eq. (1). The water relative permeability in Fig. 12 is at first sight surprising. One might expect that the water relative permeability for the oil-wet case to be higher than for the water-wet medium, since water can be non-wetting to both oil and gas in oil-wet systems, occupying the larger pore spaces. The explanation is that during waterflooding water invades the larger pores and throats in the oil-wet medium. This results in an increase in water saturation, but the oil-wet water-filled elements fail to span the network, meaning that the water relative permeability remains very low. This effect is discussed in Ref. 61. During gas injection, since gas displaces water, the water relative permeability can only decrease from its already negligible value.

Fig. 10—Three-phase oil relative permeability predicted using network modeling for gas injection into oil and water. Curves for a water-wet and an oil-wet case are shown.
Fig. 11—Three-phase gas relative permeability for gas injection into oil and water. Curves for a water-wet and an oil-wet case are shown.

Fig. 12—Three-phase water relative permeability for gas injection into oil and water. Curves for a water-wet and an oil-wet case are shown.
Conclusions
We have described a quasi-static pore-network model for three-phase flow in mixed-wet systems that uses networks with a variable topology and different pore shapes, incorporating the current state-of-the-art in the description of pore-scale displacement mechanisms and the effects of wettability.

The model successfully predicts experimental two-phase relative permeability and waterflood recovery data for water-wet and mixed-wet Berea sandstone, and three-phase water-wet data.

We used the model to predict three-phase relative permeability for a mixed-wet Berea. We discussed the differences between water-wet and mixed-wet behavior in terms of pore-scale fluid arrangements.

Nomenclature
\[ A = \text{area, } L^2, m^2 \]
\[ b = \text{length of water-wet surface in the corner, } L, m \]
\[ C_{so} = \text{oil spreading coefficient, } L^{-1}t^2, Nm^{-1} \]
\[ g = \text{conductance, } ML^3t^{-1}, Nm^2s \]
\[ G = \text{shape factor} \]
\[ k_r = \text{relative permeability} \]
\[ n = \text{number of surrounding throats filled with oil} \]
\[ P_c = \text{capillary pressure, } mL^{-1}t^2, \text{Pa} \]
\[ r = \text{radius of curvature, } L, m \]
\[ R = \text{inscribed radius of pore or throat, } L, m \]
\[ S = \text{saturation} \]
\[ V = \text{volume, } L^3, m^3 \]
\[ \alpha = \text{half angle of corner, radians} \]
\[ \mu = \text{viscosity, } ML^{-1}t^{-1}, \text{Pa.s} \]
\[ \sigma = \text{interfacial tension, } L^{-1}t^2, \text{Nm}^{-1} \]
\[ \theta = \text{contact angle, radians} \]

Subscripts
\[ a = \text{advancing} \]
\[ c = \text{corner} \]
\[ g = \text{gas} \]
\[ h = \text{hinging} \]
\[ i,j,k = \text{phase or element labels} \]
\[ o = \text{oil} \]
\[ p = \text{phase} \]
\[ r = \text{receding} \]
\[ t = \text{total} \]
\[ w = \text{water} \]

Acknowledgements
The members of the Imperial College Consortium on Pore-Scale Modelling (BHP, Enterprise Oil, Gaz de France, JNOC, PDVSA-Intevep, Schlumberger, Shell, Statoil, the U.K. Department of Trade and Industry and the EPSRC) are thanked for their
financial support. We also thank Pål-Eric Øren (Statoil) for sharing his Berea network data with us.

Appendix

In this appendix we provide details of the equations used to compute capillary pressure, fluid areas (and volumes) and conductance, and we explain how we define connectivity of clusters.

Connectivity. Table 1 lists the circumstances under which phase locations in adjacent pores and throats are considered connected. Note that we assume that within a single pore or throat all corners are connected. This definition of connectivity is used to define clusters of each phase using the flow chart illustrated in Fig. 13.

<table>
<thead>
<tr>
<th>Phase location(1)</th>
<th>Phase location(2)</th>
<th>Connected?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner</td>
<td>Corner</td>
<td>YES</td>
</tr>
<tr>
<td>Corner</td>
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</tr>
<tr>
<td>Corner</td>
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<td>NO</td>
</tr>
<tr>
<td>Layer</td>
<td>Layer</td>
<td>YES</td>
</tr>
<tr>
<td>Layer</td>
<td>Center</td>
<td>YES</td>
</tr>
<tr>
<td>Center</td>
<td>Center</td>
<td>YES</td>
</tr>
</tbody>
</table>

Note: Phase location (1) and (2) are located in two nearest neighbor pore and throats.

Table 1—How connectivity is defined. A phase location is the location of a phase in a pore or throat.

Configurations in three-phase flow and capillary pressure. Table 2 lists the range of contact angles for which the configurations in Fig. 2 are allowed. Tables 3-5 indicate the equations used to compute capillary pressure for different displacements: table 3 is for oil/water, gas/oil and gas/water displacements, while table 4 is for water/oil, water/gas and oil/gas displacements. Table 5 lists the equations for layer collapse. Layer formation is accounted for simply by considering layer collapse in reverse. Imagine that we are considering the displacement of phase $j$ by phase $i$. The configuration change can be of two types: either a configuration with $j$ in the center is replaced by one with $i$ in the center (tables 3 and 4), or a layer collapses or forms (table 5). The tables list all the relevant expressions to use. The contact angles and the nature of the displacement allow a unique configuration to be determined after the displacement.

Areas. Table 6 lists the equations used to compute cross-sectional area for the different configurations in Fig. 2. These areas are used to compute saturation, Eq. (6), and to find conductances.

Conductance. Table 7 lists the expressions used to compute phase conductance for each configuration. These are used to find the relative permeabilities.

Area Calculation. The equations used to compute areas and referred to in the tables are listed below.

\[ A_c = \pi R^2 \]  
Circular cross-section \hspace{1cm} (A1a)

\[ A_s = 4R^2 \]  
Square cross-section \hspace{1cm} (A1b)
$A_t = \frac{R^2}{4G}$ Triangular cross-section \hspace{1cm} (A1c)

$A_c = r^2 \left[ \cos \theta \left( \cot \alpha \cos \theta - \sin \theta \right) + \theta + \alpha - \frac{\pi}{2} \right]$ \hspace{1cm} (A2a)

$A_c = \left[ r \frac{\cos(\theta + \alpha)}{\sin \alpha} \right]^2 \sin \alpha \cos \alpha \text{ if } \alpha + \theta = \pi/2$ \hspace{1cm} (A2b)

**Conductance.** The equations for conductance calculations are:

$g = \frac{0.5GA^2}{\mu}$ Circular cross-section \hspace{1cm} (A3)

$g = \frac{0.5623GA^2}{\mu}$ Square cross-section \hspace{1cm} (A4)

$g = \frac{3R^2A}{20\mu}$ Triangular cross-section \hspace{1cm} (A5)

$g = \frac{A_t^3(1 - \sin \alpha)^2 \tan \alpha \varphi_3^2}{12 A_c \sin^2 \alpha (1 - \varphi_3) \left[ 1 + f_1 \varphi_3 - (1 - f_2 \varphi_3) \sqrt{\frac{A_t}{A_c}} \right]}$ \hspace{1cm} (A6)

$\varphi_3 = \left( \frac{\pi}{2} - \alpha \right) \tan \alpha$ \hspace{1cm} (A7)

$g = \frac{A_t^2 \tan \alpha \left(1 - \sin \alpha\right)^2 \varphi_3^2}{12 \sin^2 \alpha (1 - \varphi_3)(1 + f\varphi_3)^2}$ \hspace{1cm} (A8)

$g = \frac{A_t^2 \left( 1 - \sin \alpha \right)^2 (\varphi_2 \cos \theta - \varphi_1) \varphi_3^2}{12 \sin^2 \alpha (1 - \varphi_3)^2 (\varphi_2 + f\varphi_1)^2}$ \hspace{1cm} (A9a)

$\varphi_1 = \frac{\pi}{2} - \alpha - \theta$ \hspace{1cm} (A9b)

$\varphi_2 = \cot \alpha \cos \theta - \sin \theta$ \hspace{1cm} (A9c)

**Capillary Pressure.** The equations used to compute capillary pressure are:

$P_c = \frac{2\sigma \cos \theta}{R}$ \hspace{1cm} (A10)
\[ P_c = \frac{\sigma (1 + 2\sqrt{\pi G}) \cos \theta}{\left[ R_p + \sum_{i=1}^{n} e_i R_i x_i \right]} \]  
\quad (A11)

\[ e_1 = 0, \ e_2 = 0.5, \ e_3 = 1, \ e_4 = 2, \ e_5 = 5, \ e_6 = 10. \]  
This is an empirical model for pore filling. The sum runs over throats adjoining the pore.

\[ P_c = \frac{\sigma}{R} \left[ \cos \theta_2 + \sqrt{\frac{\tan \alpha}{2} \left( \sin 2\theta_2 - 2\theta_2 - 2\alpha + \pi \right)} \right] \]  
\quad (A12)

\[ \theta_3 = \cos^{-1} \left[ -r \sin^{-1} \left( \frac{b \sin \alpha}{r} \right) \right] \]  
\quad (A13)

Note in the following equations, \( \alpha_i \leq \alpha_2 \leq \alpha_1 \) for a triangular cross-section.

\[ b_i = r_{\text{extreme}} \frac{\cos(\theta_i + \alpha_i)}{\sin \alpha_i}, \quad (r_{\text{extreme}} = \sigma / P_{c,\text{extreme}}) \]  
\quad (A14a)

if the interface has not moved yet. If the interface has moved:

\[ b_i = r_p \frac{\cos(\theta_2 + \alpha_i)}{\sin \alpha_i} \]  
\quad (A14b)

\[ \theta_{h,i} = \cos^{-1} \left( r_{\text{extreme}} \frac{\cos(\theta_i + \alpha_i)}{r_p} \right) - \alpha_i \]  
\quad (A15a)

if the interface has not moved yet. If the interface has moved:

\[ \theta_{h,i} = \theta_2 \]  
\quad (A15b)

\[ a_i = \sin^{-1} \left( \frac{b_i \sin \alpha_i}{r_p} \right) \]  
\quad (A16a)

if the interface has not moved yet. If the interface has moved:

\[ a_i = (\pi / 2) - \theta_2 - \alpha_i \]  
\quad (A16b)

\[ r_p = \frac{R^2 - r_p b \cos \theta_h + r_p^2 \left( \frac{\pi}{2} - \theta_h - \alpha \right)}{2r_p a + 2[R - b] \cos \theta_2} \]  
\quad (A17)

\[ r_p = \frac{R^2 - r_p \sum_{i=1}^{n} b_i \cos \theta_{h,i} + r_p^2 \sum_{i=1}^{n} \left( \frac{\pi}{2} - \theta_{h,i} - \alpha_i \right) - A}{2r_p \sum_{i=1}^{n} a_i + \left[ \frac{r}{2G} - 2 \sum_{i=1}^{3} b_i \right] \cos \theta_2} \]  
\quad (A18)

\( n = \) number of contributing corners
\[ A = \text{summation of corner areas (only those corners that do not have layers or corners of the invading phase.)} \]

\[ P_c = \frac{\sigma}{R} \left[ \cos \theta_2 - \sqrt{\tan \alpha \left( -\sin 2\theta_2 + 2\theta_2 - 2\alpha - \pi \right)} \right] \quad (A19) \]

\[ P_c = \frac{\sigma}{R} \left[ \cot \alpha \cos \theta_2 - \sin \theta_2 \right] \quad (A20) \]

\[ P_c = \frac{\sigma}{r_{\text{extreme}}} \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_1 + \alpha)} \quad \theta_2 \leq \pi - \alpha \quad (A21) \]

\[ P_c = \frac{\sigma}{r_{\text{extreme}}} \frac{-1}{\cos(\theta_1 + \alpha)} \quad \theta_2 \geq \pi - \alpha \quad (A22) \]

Note: In equations (A21) and (A22) \( \alpha \) is for the sharpest corner if a layer of filled corner of the invading phase is present in more than one corner.

\[ P_c = \frac{\sigma}{b} \left[ \frac{3 \sin^2 \alpha + 4 \sin \alpha \cos \theta + \cos^2 \theta}{\sin^2 \alpha \sqrt{4 \cos^2 \alpha - 3 - \cos^2 \theta - 4 \sin \alpha \cos \theta}} \right] \quad (A23) \]

\[ R_c = \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_1 + \alpha)} \quad \theta_2 \geq \theta_1 \quad (A24) \]

\[ R_c = \frac{\cos \theta_2 - \sin \alpha}{\cos \theta_1 - \sin \alpha} \quad \theta_2 < \theta_1 \quad (A25) \]

\[ P_c = \frac{\sigma_1 P_c}{\sigma_2 R_C} \quad (A26) \]

\[ P_c = \frac{\sigma_2 P_c R_C}{\sigma_1} \quad (A27) \]

\[ P_c = \frac{\sigma(1 + 2\sqrt{\pi G}) \cos \theta_1}{R} F_d(\theta_2, G) \quad (A28) \]

\[ F_d = \frac{1 + \sqrt{1 - 4GC_1 \cos^2 \theta_2}}{1 + 2\sqrt{\pi G}} \quad (A29) \]
\[ C_i = \sum_{i=1}^{3} \cos \theta_2 \frac{\cos(\theta_2 + \alpha_i)}{\sin \alpha_i} - \left[ \frac{\pi}{2} - \theta_2 - \alpha_i \right] \]  
(A30)

\[ \theta_3 = \cos^{-1} \left[ \frac{-2r \sum_{i=1}^{n} \sin^{-1} \left( \frac{b_i \sin \alpha_i}{r} \right)}{R - 2 \sum_{i=1}^{n} b_i} \right] \]  
(A31)

\( n \) = number of corners containing layers or filled corners of the invading phase.

\[ P_c = \frac{\sigma}{R} \left( \cos \theta_2 - \frac{2 \sin \theta_2}{\cot \alpha_1 + \cot \alpha_2} \right) \]  
(A32)

\[ P_c = \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2) - b_2} \right) \]  
(A33)

\[ P_c = \frac{\sigma}{R} \left( \cos \theta_2 - \frac{2 \sin \theta_2}{\cot \alpha_1 + \cot \alpha_3} \right) \]  
(A34)

\[ P_c = \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_3) - b_3} \right) \]  
(A35)

\[ P_c = \frac{\sigma}{R} \left( \cos \theta_2 - \frac{2 \sin \theta_2}{\cot \alpha_2 + \cot \alpha_3} \right) \]  
(A36)

\[ P_c = \left( \frac{\sigma(\cot \alpha_2 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3) - b_3} \right) \]  
(A37)

\[ P_c \approx \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2)} \right) \]  
(A38)

if only corner 1 has a layer or filled corner of the invading phase.

\[ P_c \approx \left( \frac{\sigma(\cot \alpha_1 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_1 + \cot \alpha_2)} \right) \]  
(A39)

if only corner 1 has a layer or filled corner of the invading phase.

\[ P_c \approx \left( \frac{\sigma(\cot \alpha_2 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]  
(A40)

if only corner 2 has a layer or filled corner of the invading phase.

\[ P_c \approx \left( \frac{\sigma(\cot \alpha_2 \cos \theta_2 - \sin \theta_2)}{R(\cot \alpha_2 + \cot \alpha_3)} \right) \]  
(A41)

if only corner 2 has a layer or filled corner of the invading phase.
$$P_e \approx \left( \frac{\sigma (\cot \alpha_3 \cos \theta_2 - \sin \theta_2)}{R (\cot \alpha_2 + \cot \alpha_3)} \right)$$ \hspace{1cm} (A42)$$

if only corner 3 has a layer or filled corner of the invading phase.

Fig. 13—Flow chart used to determine clusters of each phase.
\[ P_e \approx \left( \frac{\sigma (\cot \alpha \cos \theta_2 - \sin \theta_2)}{R (\cot \alpha_1 + \cot \alpha_3)} \right) \]  \hspace{1cm} (A43)

if only corner 3 has a layer or filled corner of the invading phase.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \theta_{ow} )</th>
<th>( \theta_{gw} )</th>
<th>( \theta_{go} )</th>
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<tbody>
<tr>
<td>A</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>B</td>
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<td>---</td>
<td>---</td>
</tr>
<tr>
<td>C</td>
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<tr>
<td>D</td>
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<td>---</td>
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<td>F</td>
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<tr>
<td>G</td>
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<td>(\geq (\pi/2) + \alpha )</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 2—Showing the range of contact angles for which each of the configurations in Fig. 2 are allowed.
Oil to water, gas to water, gas to oil – piston type displacements with layers of the invading phase

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_j &lt; (\pi/2) - \alpha_1$</td>
<td>A(28)-A(30)</td>
<td>$\sigma_{ij}$</td>
<td>$\theta_j$</td>
<td>$\theta_j$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_j &lt; \theta_{extreme} &amp; \theta_j \geq (\pi/2) - \alpha_1$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_j$</td>
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</tr>
<tr>
<td>Triangular</td>
<td>$\theta_j \geq \theta_{extreme} &amp; \theta_j \geq (\pi/2) - \alpha_1$</td>
<td>A(14)-A(16),A(18)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\pi - \theta_j$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j &lt; (\pi/4)$</td>
<td>A(12)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_j$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j &lt; \theta_{extreme} &amp; \theta_j \geq (\pi/4)$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_j$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j \geq \theta_{extreme} &amp; \theta_j \geq (\pi/4)$</td>
<td>A(14)-A(17)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\pi - \theta_j$</td>
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<td>Iterative</td>
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<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_j$</td>
<td>---</td>
</tr>
</tbody>
</table>

Note (1): $\theta_j = \theta_{ij}$, $ij=ow, gw, so$

Note (2): $\theta_{extreme} = \pi - \theta_3$, where $\theta_3$ is calculated from equations (A31) for triangular cross-sections and (A13) for square cross-sections with $r = \sigma_{ij} |P_{ij}$|

Note (3): It can be either of $\pi - \theta_1$ or $\theta_1$ based on the type of interface that $\theta_1$ belongs to ($\theta_1$ is the contact angle that interface moved with last time)

Note (4): $r_{extreme} = \sigma |P_{extreme}$, where $\sigma$ & $P_{extreme}$ belong to the last move of the interface

Note (5): $\theta_{ow} = (\theta_{ow})_{PD}$ for the displacement of water by oil during primary drainage

Oil to water, gas to water, gas to oil – snap-off displacements

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_j \geq (\pi/2) + \alpha_1$</td>
<td>A(32)-A(43)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\pi - \theta_j$</td>
<td>Min. $P_{ij}$</td>
<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_j \geq \alpha_1 &amp; \theta_j \leq (\pi/2) + \alpha_1$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\pi - \theta_j$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_j &lt; \alpha_1$</td>
<td>A(22)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j &gt; (3\pi/4)$</td>
<td>A(20)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\pi - \theta_j$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j \geq (\pi/4) &amp; \theta_j \leq (3\pi/4)$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\pi - \theta_j$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_j &lt; (\pi/4)$</td>
<td>A(22)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>

Note (1): It can be either of $\pi - \theta_1$ or $\theta_1$ based on the type of interface that $\theta_1$ belongs to ($\theta_1$ is the contact angle that interface moved with last time)

Table 3—Listing of the expressions used to compute capillary pressure for all possible oil/water, gas/water and gas/oil displacements.
Water to oil, water to gas, oil to gas – piston type displacements with no layers or filled corners of the invading phase

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} &gt; (\pi/2) + \alpha_1$</td>
<td>A(28)-A(30)</td>
<td>$\sigma_{ij}$</td>
<td>$\theta_{ij}$</td>
<td>$\pi - \theta_{ij}$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} \geq \theta_{extreme}$ &amp; $\theta_{ij} \leq (\pi/2) + \alpha_1$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} &lt; \theta_{extreme}$ &amp; $\theta_{ij} \leq (\pi/2) + \alpha_1$</td>
<td>A(14)-A(16),A(18)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} &gt; (3\pi/4)$</td>
<td>A(19)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} \geq \theta_{extreme}$ &amp; $\theta_{ij} \leq (3\pi/4)$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} &lt; \theta_{extreme}$</td>
<td>A(14)-A(17)</td>
<td>$\sigma_{ij}$</td>
<td>Note (3)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>Circular</td>
<td>$0 \leq \theta_{ij} \leq \pi$</td>
<td>A(10)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
</tr>
</tbody>
</table>

Note (1): $\theta_{ij} = \theta_{ij}^{ow}, \theta_{ij} = \theta_{ij}^{gw}, \theta_{ij} = \theta_{ij}^{go}$

Note (2): $\theta_{extreme} = \theta_{ij}^{extreme}$, where $\theta_{ij}^{extreme}$ is calculated from equations (A31) for triangular cross-sections and (A13) for square cross-sections with $r = \sigma_{ij} / P_{c,ij}$

Note (3): It can be either of $\pi - \theta_{ij}$ or $\theta_{ij}$ based on the type of interface that $\theta_{ij}$ belongs to ($\theta_{ij}$ is the contact angle that interface moved with last time)

Note (4): $r_{extreme} = \sigma / P_{c,extreme}$, where $\sigma$ & $P_{c,extreme}$ belong to the last move of the interface

Water to oil, water to gas, oil to gas – snap-off displacements

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Contact Angle Range</th>
<th>Equation(s)</th>
<th>$\sigma$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} \leq (\pi/2) - \alpha_1$</td>
<td>A(32)-A(43)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>Max. $P_{c,ij}$</td>
<td>---</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} \leq \pi - \alpha_1$ &amp; $\theta_{ij} &gt; (\pi/2) - \alpha_1$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>$\theta_{ij} &gt; \pi - \alpha_1$</td>
<td>A(22)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} \leq (\pi/4)$</td>
<td>A(20)</td>
<td>$\sigma_{ij}$</td>
<td>---</td>
<td>$\theta_{ij}$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} &gt; (\pi/4)$ &amp; $\theta_{ij} \leq (3\pi/4)$</td>
<td>A(21)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>$\theta_{ij}$</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$\theta_{ij} &gt; (3\pi/4)$</td>
<td>A(22)</td>
<td>$\sigma_{ij}$</td>
<td>Note (1)</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>

Note (1): It can be either of $\pi - \theta_{ij}$ or $\theta_{ij}$ based on the type of interface that $\theta_{ij}$ belongs to ($\theta_{ij}$ is the contact angle that interface moved with last time)

Table 4—Listing of the expressions used to compute capillary pressure for all possible water/oil, water/gas and oil/gas displacements.
Threshold capillary pressures for layer collapse events

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Layer</th>
<th>Displacing Phase</th>
<th>Equation</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>I, Q</td>
<td>Oil</td>
<td>Water</td>
<td>(A23)</td>
<td>$\sigma_{ow}$</td>
<td>$\theta_{owh}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>J, T</td>
<td>Gas</td>
<td>Water</td>
<td>(A23)</td>
<td>$\sigma_{gw}$</td>
<td>$\theta_{gwh}$</td>
<td>Iterative</td>
</tr>
</tbody>
</table>

Note: $b$ is calculated for the corner interface and $\theta_{owh}$ is for the layer interface

Threshold capillary pressures for layer collapse events

<table>
<thead>
<tr>
<th>Configuration(s)</th>
<th>Layer</th>
<th>Displacing Phase</th>
<th>Equations</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>K, L, M, S</td>
<td>Oil</td>
<td>Water</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{owh}$</td>
<td>$\theta_{goh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>K, L, M, S</td>
<td>Oil</td>
<td>Gas</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{owh}$</td>
<td>$\theta_{goh}$</td>
<td>$P_{cow}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>N, O, P, R</td>
<td>Gas</td>
<td>Water</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{gwh}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>N, O, P, R</td>
<td>Gas</td>
<td>Oil</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{go}$</td>
<td>$\theta_{goh}$</td>
<td>$P_{cgw}$</td>
<td>---</td>
<td>Iterative</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Water</td>
<td>Oil</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{gw}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$\theta_{gwh}$</td>
<td>---</td>
<td>$P_{cgw}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>Q</td>
<td>Water</td>
<td>Gas</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{ow}$</td>
<td>$\sigma_{gw}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$\theta_{gwh}$</td>
<td>$P_{cow}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>T</td>
<td>Water</td>
<td>Gas</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{ow}$</td>
<td>$\theta_{gwh}$</td>
<td>$\theta_{owh}$</td>
<td>---</td>
<td>$P_{cgw}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>T</td>
<td>Water</td>
<td>Oil</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{gw}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{gwh}$</td>
<td>$\theta_{owh}$</td>
<td>$P_{cgw}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
<tr>
<td>S</td>
<td>Gas</td>
<td>Oil</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{gw}$</td>
<td>$\theta_{goh}$</td>
<td>$\pi - \theta_{gwh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>R</td>
<td>Oil</td>
<td>Gas</td>
<td>(A24)-(A26)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>---</td>
<td>$P_{cgo}$</td>
<td>Iterative</td>
</tr>
<tr>
<td>R</td>
<td>Oil</td>
<td>Water</td>
<td>(A24)-(A25), (A27)</td>
<td>$\sigma_{go}$</td>
<td>$\sigma_{ow}$</td>
<td>$\pi - \theta_{goh}$</td>
<td>$\pi - \theta_{owh}$</td>
<td>$P_{cgo}$</td>
<td>---</td>
<td>Iterative</td>
</tr>
</tbody>
</table>

Table 5—Listing of the expressions used to compute capillary pressure for all possible displacements involving layer collapse. The capillary pressures for layer formation are found by considering the layer collapse events in reverse.
Table 6—Expressions used to compute the cross-sectional areas of each phase for each of the 20 configurations shown in Fig. 2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Area</th>
<th>Equation</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sw}$</td>
<td>$\sigma_{sw}/P_{sw}$</td>
</tr>
<tr>
<td>D</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{gr}$</td>
<td>$\sigma_{gr}/P_{gr}$</td>
</tr>
<tr>
<td>E</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>F</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{gh}$</td>
<td>$\sigma_{gh}/P_{gh}$</td>
</tr>
<tr>
<td>G</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>H</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{gh}$</td>
<td>$\sigma_{gh}/P_{gh}$</td>
</tr>
<tr>
<td>I</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>J</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\pi-\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>K</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>L</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{gh}$</td>
<td>$\sigma_{gh}/P_{gh}$</td>
</tr>
<tr>
<td>M</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>N</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>O</td>
<td>$A_w$</td>
<td>A(2)</td>
<td>$\theta_{gh}$</td>
<td>$\sigma_{gh}/P_{gh}$</td>
</tr>
<tr>
<td>P</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>Q</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>R</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>S</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
<tr>
<td>T</td>
<td>$A_w=A_s+A_w$</td>
<td>A(2)</td>
<td>$\theta_{sh}$</td>
<td>$\sigma_{sh}/P_{sh}$</td>
</tr>
</tbody>
</table>

Notes: (1) Area at the center is total area minus the corner areas. (2) Total areas for different cross-sections are calculated using equation A(1)
Table 7—Expressions used to compute the conductance of each phase for each of the 20 configurations shown in Fig. 2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Equation</th>
<th>Conductance</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_c$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{vac}$</td>
</tr>
<tr>
<td>D A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>$\theta_{vac}$</td>
</tr>
<tr>
<td>E A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{geh}$</td>
</tr>
<tr>
<td>F A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>$\theta_{geh}$</td>
</tr>
<tr>
<td>G A(7),A(8)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{vac}$</td>
</tr>
<tr>
<td>H A(7),A(8)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>$\theta_{geh}$</td>
</tr>
<tr>
<td>I A(7),A(8)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{vac}$</td>
</tr>
<tr>
<td>J A(6),A(7)</td>
<td>$g_{vw}$</td>
<td>$A_y$ $A_w$</td>
<td>$A_y$</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{vac}$</td>
<td></td>
</tr>
<tr>
<td>K A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{geh}$</td>
</tr>
<tr>
<td>K A(6),A(7)</td>
<td>$g_{vw}$</td>
<td>$A_y$ $A_w$</td>
<td>$A_y$</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>$\theta_{geh}$</td>
<td></td>
</tr>
<tr>
<td>L A(7),A(9)</td>
<td>$g_{vw}$</td>
<td>--- ---</td>
<td>---</td>
<td>---</td>
<td>$A_w$</td>
<td>---</td>
<td>---</td>
<td>1</td>
<td>$\theta_{geh}$</td>
</tr>
<tr>
<td>L A(6),A(7)</td>
<td>$g_{vw}$</td>
<td>$A_y$ $A_w$</td>
<td>$A_y$</td>
<td>$A_w$</td>
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</table>

Note: Conductance through the center of a pore or throat is calculated from equations A(3)-A(5) using the center area which is total area minus corner areas.
References


Chapter 3

Predictive Pore-Scale Network Modeling of Single-Phase Non-Newtonian Flow

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Abstract
The majority of complex fluids used in oilfield applications are polymeric solutions exhibiting shear-thinning (pseudoplastic) behavior in solution. We study the flow of power-law fluids in porous media using network modeling. We use realistic representations of porous media that capture the geometry and topology of sands and sandstones as input to our flow model. The bulk rheology (variation of viscosity with shear rate) is used to define a relationship between pressure drop and average flow velocity in each pore. We then compute the variation of average velocity with apparent viscosity for the whole network. We successfully predict the results of four experiments from different sources in the literature. The predictions are superior to those using simplified capillary bundle models and involve no adjustable parameters.

Keywords: Power law, Non-Newtonian rheology in porous media, Network modeling

Introduction
The flow of complex fluids, such as polymer gels and surfactants, in porous media is of particular interest to the oil industry. For instance, during water flooding, polymeric solutions are often added to the aqueous phase to increase its viscosity, resulting in a more stable displacement of oil by the injected water. Polymers in solution can also be used in oil production wells to block excessive water production from a watered-out high permeability layer. In water injection wells, using polymers in treatment fluids helps to initialize and stabilize fractures hence increasing injectivity (9,16,23,25). The presence of surfactant in the water phase also lowers the surface tension between the aqueous and oil phases, lowering the residual oil saturation and increasing oil recovery (23,25).

These polymeric fluids exhibit complex behavior such as time and shear rate dependency of their macroscopic properties and reactions with oil, water or the porous medium itself. However, most of the current understanding of flow in porous media has been developed for purely Newtonian fluids, where the viscosity is independent of shear rate.

Many researchers have attempted to relate the bulk properties of complex fluids to their behavior in a porous medium (1,7-9,12-14,24-30). Since measuring bulk properties of polymeric solutions is a standard and reliable experimental procedure, quite naturally then, efforts have been made to extend the laws of motion for purely Newtonian fluids (Darcy’s law) to rheologically complex ones using easily measurable properties such as the shear-rate/viscosity behavior. To do this, it is necessary to have a convenient and appropriate representation of the porous medium that captures its macroscopic and microscopic properties. A common approach consists of representing the medium by a bundle of parallel capillary tubes. This allows the definition of an average radius that is dependent on macroscopic properties of the medium such as porosity, absolute permeability and some measure of tortuosity $\tau$. As a result, analytical expressions of flow properties can be derived from Stokes’ law (2,22,23,25). Then, using a mathematical description of the fluid rheology (such as power law, Carreau or Cross models), several authors (2,7-9,12-14,16,24-30) have derived expressions to define an apparent shear rate experienced by the fluid in the porous medium from the Darcy
velocity. In practice, apparent viscosity ($\mu_{app}$) and Darcy velocity ($q$) are often the measured quantities. Experimental results suggest that the overall shape of the $\mu_{app}(q)$ curve is similar to that in the bulk $\mu(\gamma)$. Using dimensional analysis there is a length that relates velocity to shear rate. Physically this length is related to the pore size. One estimate of this length is the square root of the absolute permeability times the porosity, $K\phi$ (7-9,12-14,24-30). This allows the determination of in situ rheograms from the bulk measured $\mu(\gamma)$: $\mu_{app}(q) = \mu(\gamma = q/\sqrt{K\phi \gamma_{pm}})$. Many authors have remarked that this method leads to in situ rheograms that are shifted from the bulk curve by a constant factor, $\alpha$; i.e.:

$$\mu_{app} (q) = \mu(\gamma = \alpha q / \sqrt{K\phi})$$  [1]

Reported values for $\alpha$ vary depending on the approach chosen, but experimental results suggest it generally lies in the range 1 to 15. Pearson & Tardy (22) reviewed the different mathematical approaches used to describe non-Newtonian flow in porous media. They concluded that none of the present continuum models give accurate estimates of macroscopic transport properties: $\alpha$ is a function of both the bulk rheology and the pore structure and currently there is no theory that can predict its value reliably.

An appealing approach to predict the rheological behavior of complex fluids is to use network models that represent the topological features of porous media. These models have been extensively used to analyze and understand multiphase transport phenomena (3-6,15,18-20). The porous medium is represented as a collection of interconnected elements (pores and throats), where each element is assigned some idealized geometry with effective properties that capture the characteristics of real rocks. Macroscopic properties such as capillary pressure, electrical resistivity and relative permeability can be computed. Blunt et al. (3,4) provide recent reviews on how to describe and represent multiphase flow through topologically complex porous media. Although these network models were originally developed to study multiphase flow of purely Newtonian fluids, the approach has been extended to non-Newtonian phenomena. Pearson & Tardy (22) considered the flow of a power-law fluid in a square capillary bundle network. They concluded that the tortuosity of the medium significantly affected transport properties. Sorbie et al. (24) investigated the rheology of Xanthan using a two-dimensional network of capillary tubes on a regular rectangular lattice with randomly distributed radii. Using a capillary bundle model, they found that although the approach gave a reasonable estimate of the flow rate/pressure drop relationship in a single bond, an empirical correction factor was needed to predict their simulation results. They also highlighted the importance of cooperative effects within a network and more generally within a complex medium. Both these modeling studies (22,24) used topologically simple networks and so were not able to make predictions of properties in natural porous materials.

Some authors have used experimental results to investigate the validity of some of the theoretical approaches in the literature. Teeuw & Hesselink (26) investigated the flow behavior of a biopolymer in Bentheim sandstone cores. From their experiments they inferred that for large polymer molecules the exclusion of the polymer from the pore
walls was likely to lead to an apparent slip effect whereby the molecules travel at a higher flow rate through the medium than the solvent. Greaves & Patel (12) conducted polymer-flooding experiments on high permeability Elginshire sandstone rock. They observed a higher polymer effective viscosity in the porous medium than in the beaker at low flow rates. They suggested that the interaction of polymer molecules with the solid could explain their results. Likewise, studies performed by Cannella et al (7) on Berea cores seemed to indicate in situ effective viscosity values higher than the bulk ones for high concentration polymer solutions. All the authors above, as well as many others (11,13,14,16,27-30), could only reproduce their experimental findings using the capillary bundle approach through the introduction of an empirical correction factor (varying for each author), hence limiting the predicting capability of the methods undertaken.

We present here a new approach that uses networks based on real porous media with variable topology that does not require any fitting parameters to reproduce experimental data from several different sources.

**Fluid modeling and bulk rheology**

The two main polymers used in the oil industry for hydrocarbon recovery are synthetic polycrylamide (in its partially hydrolyzed form, HPAM) and Xanthan biopolymer (23). Although a majority of applications are performed with HPAM, our model was based on Xanthan properties since most experimental results available in the literature are based on Xanthan solutions (7-9,12-14,26,27,29,30). Several phenomenological effects can influence the flow of polymeric solutions in rocks. Most flexible polymers such as HPAM exhibit viscoelastic behavior when flowing in porous media. This is due to the series of contractions and expansions in the porous medium through which the polymer molecules are flowing, and usually results in an additional pressure drop. Because of its structure, the molecule of Xanthan can be to a good approximation treated as a “rigid rod”. This implies that viscoelastic effects associated with the flow of Xanthan in porous media are somewhat less significant than for HPAM, which makes it more suitable for simulation studies. In this text, we will neglect any viscoelastic effects and assume the Xanthan solution only exhibits a shear-dependent viscosity.

There are two other effects that may be of importance when considering the flow of Xanthan. The first one tends to exclude the polymer molecules from the surface of the pores hence creating an apparent slip effect. This is usually attributed to entropic exclusion (23-25) near the rock surface resulting in part of the pore volume being unavailable to flow. Next to the wall, the molecules cannot rotate freely and as a result are forced into flowing towards the center of the pore. This phenomenon may be of importance when pore sizes approach that of the polymer molecules. The second mechanism is polymer adsorption. Due to interactions between the surface and the molecules, part of the polymer is bound to the pore wall (23). This results in fewer polymer molecules flowing in the bulk of the solution, as opposed to the excluded volume effect. For simplicity, these phenomena are not considered in this work.

Escudier et al. (10) performed a set of experiments on Xanthan gum that provides a basis for modeling the bulk rheology of the non-Newtonian phase. These experiments used solutions with a wide range of conditions (temperature, pH, solvent salinity). Fig.
Fig. 1 shows the viscosity/shear rate results they obtained from seven Xanthan solutions at a concentration of 2500ppm.

![Fig. 1: Viscometric behavior of Xanthan gum solutions together with Carreau-Yasuda (—), Cross (— — — —) model fits and experimental data (from Escudier et al. (10))](image)

Fig. 1 also presents the predicted results using two constitutive models for pseudoplastic (shear-thinning) fluids, namely Carreau-Yasuda and Cross models. The Cross model is a good fit to the data with the added advantage over the Carreau-Yasuda model of having only four free parameters: the viscosity at low shear rate, $\mu_0$, the viscosity at high shear rate, $\mu_\infty$, a time constant $\lambda$ and an exponential parameter $m$:

$$
\mu_{\text{eff}} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + \left(\frac{\lambda \gamma}{m}\right)^m} 
$$

[2]

However, the Cross model [2] does not provide a direct analytical solution to relate pressure drop to effective viscosity in a single capillary, as shall be discussed later. For simplicity, a truncated power law model was used to match the data.

$$
\mu_{\text{eff}} = \text{Max}[\mu_\infty; \text{Min}(C \gamma^{n-1}; \mu_0)]
$$

[3]

This model not only ensures the right boundary conditions are respected, but also provides satisfactory results when compared to the experimental data and the Cross model (Fig. 2). For this specific example, the fit of the model was obtained using the values showed in Table 1.
Table 1: Truncated power law parameters used to fit the experimental data.

<table>
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<tr>
<th>Experiment</th>
<th>$C$</th>
<th>$n$</th>
<th>$\mu_0$ (Pa.s)</th>
<th>$\mu_\infty$ (Pa.s)</th>
<th>$\phi$</th>
<th>$K$ (D)</th>
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<td>0.42</td>
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<td>-</td>
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<td>0.418</td>
<td>0.5</td>
<td>0.0015</td>
<td>0.34</td>
<td>0.525</td>
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<td>0.45</td>
<td>0.1</td>
<td>0.0015</td>
<td>0.5</td>
<td>5</td>
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<td>Greaves &amp; Patel (12)</td>
<td>0.25</td>
<td>0.53</td>
<td>0.09</td>
<td>0.0015</td>
<td>0.2</td>
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<tr>
<td>Cannella et al (7)</td>
<td>0.195</td>
<td>0.48</td>
<td>0.102</td>
<td>0.0015</td>
<td>0.2</td>
<td>0.264</td>
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</table>

Fig. 2: Comparison between rheogram predictions using Cross model and a truncated power law

Network model

In order to simulate accurately transport phenomena in porous media, a detailed description of the pore space is needed (3,4,18-20,22,23). We use networks derived from two porous media in this study: a sand-pack and a Berea sandstone. A digital three-dimensional representation of the pore space is constructed by modeling sedimentation and compaction of the grains making up the medium. From this, a topologically equivalent network model is built (Fig. 3) with pore sizes, shapes and connectivity based on the three-dimensional representation. How exactly this is done is described by other authors (19,20): in this work we simply read in the network as input to our flow model. Table 2 lists the main properties of the two networks used.

Pores and throats are modeled as having a square, circular or triangular cross section depending on their assigned shape factors. This shape factor, the ratio of cross-sectional area to perimeter squared, as well as volumes, connection numbers and cross sectional areas are calculated from the 3D pore-space representation. In multiphase flow
modeling studies, angular pore shapes allow wetting phase to reside in the corners, while the non-wetting phase fills the centers. We will use the same pore shapes in this single-phase study. This approach to representing porous media has proved very successful in predicting single and multiphase flow properties of porous media, including permeability (5), relative permeability and capillary pressure (6,18,19,20). In this paper, we demonstrate that the same approach can be used to predict the rheology of power-law fluids.

Fig. 3: Network generation process for sandstone. a) A three-dimensional image of the pore space is obtained by simulating the sedimentary processes by which the rock was formed (19,20). b) A topologically equivalent network of pores connected by throats is then constructed (19,20). Each pore and throat has a volume, shape and cross-sectional area derived from the three-dimensional image. Fluid flow through this network is then simulated.

As mentioned earlier, the advantage of using a truncated power-law model, instead of Carreau-Yasuda model for instance, is that it allows the development of an analytical solution relating effective viscosity to pressure drop based on single-phase non-Newtonian flow in a circular capillary. Several authors (2,24) have showed that for a power-law fluid, the effective viscosity can be related to the pressure drop. For a purely Newtonian fluid of constant viscosity $\mu$ flowing in a circular capillary of radius $R$, the Hagen-Poiseuille law applies:

$$Q_N = \frac{\pi \Delta P R^4}{8 L \mu} = \frac{G}{\mu} \Delta P$$  \[4\]

where $Q_N$ is the volumetric flow rate due to a pressure drop $\Delta P$ experienced along the length $L$. $G$ is defined as the hydraulic conductance.

For a power law fluid [2], the equivalent equation (2,23,24) relates the volumetric flow rate to the rheological parameters ($C$ and $n$) and capillary tube dimensions ($R$ and $L$).

$$Q_{NN} = \frac{\pi n R \left(\frac{3 n + 1}{n}\right)}{3 n + 1} \left(\frac{\Delta P}{2 CL}\right)^{\frac{1}{n}}$$  \[5\]
By defining the effective viscosity through a pseudo Hagen-Poiseuille law [4] using $Q_{NN}$ as defined in [5], one can obtain the following equation for the viscosity of a power-law fluid flowing in a circular tube (23,24).

$$\mu_{\text{eff}} = C \left( \frac{3n + 1}{4n} \right) \left( \frac{R \Delta P}{2CL} \right)^{\frac{n-1}{n}} \quad [6]$$

Equation [6] applies to a pure power law fluid, with $n$ less than 1, and therefore extrapolates to zero for infinite pressure drops and to infinity when $\Delta P$ approaches zero. Equation [6] assumes that power-law behavior is observed over the entire pore or throat cross-section. A truncation to $\mu_{\text{eff}}$ in equation [6] is applied whenever the viscosity goes beyond the low and high flow rate limiting viscosities (equation [3]). This is an assumption – a more accurate analysis would apply the viscosity limits within the capillary. However, we use equation [6] because of its simplicity.

Our network models are mainly composed of irregular triangular-shaped pores and throats, while equation [6] was derived for a circular cylinder. To account for non-circular pore shapes we replace $R$ in equation [6] with an appropriately defined equivalent radius, $R_{\text{equiv}}$. We use an empirical approach to define $R_{\text{equiv}}$ based the conductance of the pore or throat. This equivalent radius is determined using Poiseuille’s law (equation [4]).

$$R_{\text{equiv}} = \left( \frac{8G}{\pi} \right)^{1/4} \quad [7]$$

Note that for circular elements equation [7] reduces to equation [4]. For non-circular pores and throats (mainly triangular shaped in our network) the expressions for $G$ are more complex (3,4,15,20). See the Appendix for the expressions used for $G$ in non-circular pores and throats. Once $G$ is computed, equation [7] is used to find the equivalent radius and this is used in equation [6] to find the effective viscosity.

In a network of pores and throats we do not know each pressure drop $\Delta P$ a priori. Hence to compute the flow and effective viscosities requires an iterative approach (24). An initial guess is made for the effective viscosity in each network element. The choice of this initial value is rather arbitrary but does influence the rate of convergence, although not the final results. As a general rule, when one is interested in solving for only one flow rate across the network, the initial viscosity guess can be taken as the limiting boundary condition, $\mu_0$ (i.e. the viscosity at very low shear rates). However, when trying to explore results for a range of increasing flow rates, the convergence process can be significantly speeded up by retaining the last solved solution for viscosity.

Once each pore and throat has been assigned an effective viscosity and conductance, the relationship between pressure drop and flow rate across each element can be found.
By invoking conservation of volume in each pore with appropriate inlet and outlet boundary conditions (constant pressure), the pressure field is solved across the entire network using standard techniques (see, for instance, (4)). As a result the pressure drop in each network element is now known, assuming the initial guess for viscosity. Then, using equation [6], the effective viscosity of each pore and throat is updated and the pressure recomputed. The method is repeated until satisfactory convergence is achieved. In our case, convergence must be achieved simultaneously in all the network elements. The tolerance used for the simulation studies was set such that the pressure would be recomputed if the flow rate in any pore or throat changed by more than 1% between iterations. The total flow rate across the network $Q_t$ is then computed and an apparent viscosity is defined as follows:

$$\mu_{app} = \mu_N \frac{Q_N}{Q_t}$$  \[9\]

where $Q_N$ is the total flow rate for a simulation with the same pressure drop with a fixed Newtonian viscosity $\mu_N$. The Darcy velocity is obtained from $q = Q_v/A$, where $A$ is the cross-sectional area of the network.

There are a number of assumptions in the method. First the rheology of the non-Newtonian phase is described by a truncated power-law model. Effects of adsorption, surface exclusion and elasticity were not considered. Second, the expressions for the hydraulic conductivity of the non-Newtonian phase in irregular-shaped elements used an empirical expression for effective radius and were based on solutions for a circular cylinder with power-law behavior throughout.

**Results**

We now validate the model by predicting several different experimental results. First the model predictions are tested against two experiments in sand-packs and then against two sandstone datasets.

Hejri et al [13] studied the flow of a Flocon® 4800 solution (Xanthan biopolymer) in various unconsolidated sand-packs covering a wide range of permeability values. They conducted single-phase displacement experiments for three polymer concentrations. The shear rate dependence of the viscosity of Flocon® 4800 is shown below (Fig. 4).
The rheological parameters determined experimentally by Hejri et al. were used as input in our model and are summarized in Table 1. Note, however, that their experiments only covered the shear-thinning region, which means that our assignment of high and low viscosity cut-offs is somewhat arbitrary. In order to reproduce experimental conditions as closely as possible, we used a network based on a sand-pack. The main characteristics of this network are presented in Table 2. Simulations were run covering a range of flow rates corresponding to the experimental conditions.

<table>
<thead>
<tr>
<th>Network</th>
<th>$\phi$</th>
<th>$K$ (D)</th>
<th>Pore size Range (10^{-6} m)</th>
<th>Throat size Range (10^{-6} m)</th>
<th>Average Coordination number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand-pack</td>
<td>0.34</td>
<td>101.8</td>
<td>Grain sizes 100 - 425</td>
<td>-</td>
<td>5.46</td>
</tr>
<tr>
<td>Berea</td>
<td>0.19</td>
<td>3.148</td>
<td>7.24 – 147.08</td>
<td>1.8 – 113.70</td>
<td>4.19</td>
</tr>
</tbody>
</table>

Table 2: Network characteristics

Fig. 5 compares the rheology measured in the porous medium with a prediction based on equation [1] with a best fit to the data obtained for $\alpha = 0.98$. Notice two problems associated with this empirical approach. First, as discussed before, $\alpha$ cannot be predicted a priori. Second, equation [1] cannot match the low velocity behavior accurately – this is because the results are affected by the low-rate cut-off in the viscosity that affects some of the smaller pores in the sand-pack, even though it is not evident in the beaker properties.
We predicted the apparent viscosity using our sand-pack network and the results are also shown in Fig. 5. The shape of the curve is correct, but it is shifted to higher flow rates. The reason for this is obvious: the network sand-pack has a much higher permeability (101D) than the experimental system (0.893D). This means that in the network the pores are, on average, larger with lower shear rates for a given velocity, resulting in higher apparent viscosities. We can easily account for this effect by realizing that simply re-scaling the network size will result in a porous medium of identical topological structure, but different permeability. We generated a new network with all lengths scaled by a factor $\sqrt{K^{\text{exp}}/K^{\text{net}}}$, where the superscripts $\text{exp}$ and $\text{net}$ stand for experimental and network, respectively, and re-run the simulations. By construction the re-scaled network now has the same permeability as the experimental sand-pack, but otherwise has the same structure as before. The results now almost perfectly overlie the experimental data (Fig. 5). Note that this is not an ad-hoc procedure since the scaling factor is based on the experimentally measured permeability.

![Fig. 5: Comparison between network simulations, the experimental data of Hejri et al. (13) and an empirical relation using bulk rheology and an empirical scaling factor.](image)

The re-scaled network results are identical, and match the experimental data, if all the lengths are multiplied by a constant factor to match the experimental permeability, or if the flow rate is re-scaled using equation [10].

While the length re-scaling works well, it is somewhat cumbersome, since it involves the generation of a new network for each case considered. Another approach is to re-scale the $\mu_{\text{app}}(q)$ curve using equation [1]. If equation [1] were valid, then to obtain the correct apparent viscosity for a medium with a different permeability and porosity than the network, we need to re-scale the network flow rates as follows:
Applying this post-priori re-scaling gives identical results to re-adjusting all the lengths in the network (Fig. 5). This is despite the fact that equation [1] is not strictly valid in this case, since the shapes of the bulk and porous medium viscosity curves are different. The introduction of a porosity ratio in equation [10] has negligible effect, since the experimental and network porosities are similar. Since the scaling in equation [10] is easy to apply and gives the same results as re-scaling the network, we will use equation [10] for all our subsequent predictions. Note again that the scaling factor is based on experimentally measured parameters and involves no adjustable constants.

Vogel & Pusch (27) conducted flow experiments on sand-packs using three different types of polymer: a polysaccharide solution, a hydroxyethylcellulose (biological origin) solution and a polyacrylamide solution (HPAM). As before, a network based on a sand-pack (Table 2) was used to predict the flow of a polysaccharide solution. As for Hejri et al, the bulk rheological parameters were the same as those measured by the authors (Table 1).

![Comparison between network simulations and Vogel & Pusch experiments](image)

**Fig. 6.: Comparison between network simulations and Vogel & Pusch experiments (27).**

After re-scaling using equation [10], good agreement was found between the experimental results and the model simulations (Fig. 6). We will now seek to reproduce results from two polymeric displacements in sandstone.

Greaves and Patel (12) studied the displacement of a Flocon™ 4800C solution through high permeability Elginshire sandstone cores. Again, use was made of the rheological

\[
q^\text{re-scaled} = q^\text{net} \sqrt{\frac{K^\text{exp}}{K^\text{net}}} \phi^\text{exp} \phi^\text{net} \]

[10]
parameters (Fig. 7) determined experimentally by the authors and network simulations were run in order to predict the relationship between apparent viscosity and flow rate.

![Graph of rheology](image)

**Fig. 7: Rheology of Flocon 4800C solution, after Greaves & Patel (12).**

The sand-pack network was first used to address the need for using a more elaborate porous medium representation. Simulation results (Fig. 8) showed that even after re-scaling the model was not able to reproduce the experimental results. The apparent viscosity was underestimated by the network. The sand-pack fails to capture the complexity of a highly tortuous consolidated sandstone. When we used a Berea network (properties in Table 2) the results were excellent (Fig. 8). This emphasizes the importance of capturing both the pore-scale physics and the correct pore structure to make accurate predictions. However, the network simulations did not match the experimental data at very low flow rates. The experiments suggested a higher viscosity at low flow rates in the porous medium than in the rheometer. This is possibly due to pore blocking by adsorption, which we do not account for in our model.
Cannella et al (7) investigated the flow behavior of Xanthan in porous media for different polymer concentrations, rock lithologies, residual oil saturations and rock permeability. They used experimental results and effective medium theory to relate Darcy velocity to \textit{in situ} shear rate (equation [A.7]). In order to analyze their findings, a similar fluid rheology was used in the network simulations (see Table 2), corresponding to the power law parameters they used for their 1200ppm Xanthan solution, (Fig. 9).

They provided us with a very good case for comparison, since they used the same type of fluid in the same type of rock (Berea sandstone) as our simulations. Fig. 10 shows...
their experimental results for the flow of 1200ppm Xanthan solution through Berea cores, and the network predictions, which are a good match in the power-law region.

From Fig. 10 it can be seen that, for low velocity, the apparent viscosity does not reach a maximum value, as shown in Fig. 9. These experimental results are consistent with those from Greaves & Patel (12), which suggests that, in real rocks, pore blocking phenomena are of importance at low velocity. Again, as for the previous example, our model predictions were found to be in very good agreement for the shear-thinning region. For very high Darcy velocities, the model predictions deviated moderately from the experimental points, although the appropriate limiting conditions were reached.

**Fig. 10: Comparison between network simulations and Cannella et al experiments (7).**

**Discussion**

As discussed in the Introduction, the traditional approach in the literature is to introduce a length scaling to relate the viscosity/shear rate relationship to one between viscosity and Darcy velocity in a porous medium. At first sight it appears that we have done something similar, in introducing a permeability, or rather a length scaling to our network model results. However, there is a significant difference: we have been able to match four experimental datasets on different systems without any arbitrary adjustable parameters – the permeability scaling comes from easily obtained independent experimental data. In contrast, all previous, more empirical, approaches have needed to introduce an extra factor that varies with different porous media and fluid systems. To emphasize this, Table 3 shows $\alpha$ factors (Equation [1]) and constants $\beta$ (Appendix) necessary to match the four datasets presented here. Notice that even for the same porous medium, the values necessary to obtain a match vary. Using the network model, that captures correctly the topological complexity of the porous media and its non-linear
Conclusions
We have used a pore-network model to predict the relationship between apparent viscosity and flow rate in porous media. The model uses topologically disordered networks of irregularly shaped pores connected by throats that represent different porous media of interest. Empirical expressions were used to find the effective viscosity in each pore and throat as a function of pressure drop, assuming a power-law shear thinning fluid. An iterative procedure was used to find the flow throughout the network and from this an apparent viscosity could be defined. We applied a length rescaling to predict results for porous media with different permeabilities than the network.

We used a sand-pack and a Berea network to predict four different datasets in the literature. Our input data was the permeability and porosity of the system and its bulk rheological behavior. We then predicted the apparent viscosity of the fluid in the porous medium as a function of flow rate. In all cases the agreement between simulation and experiment was excellent without resorting to any arbitrary adjustable parameters. In contrast, we showed that traditional empirical approaches would not have been able to predict the data without resort to an ad hoc scaling parameter that varied with the fluid system and porous medium.

This work offers the possibility to predict reliably the behavior of non-Newtonian fluids in porous media. Further work to include the effects of polymer adsorption, excluded volume and viscoelastic effects is planned.
Appendix

Conductance calculations

The flow rate $q$ between two adjacent pores $i$ and $j$ is given by

$$ q_{p,ij} = \frac{G_{ij} (P_i - P_j)}{\mu L_{ij}} $$

[A.1]

where $G$ is the fluid conductance, $L$ is the total length and $P$ is the pressure.

The conductance between two pore bodies is taken to be the harmonic mean of each individual conductance – the two pores, $I, j$, and the connecting throat, $t$ (see Fig. 11 below).

Fig. 11: Schematic of a pore-throat-pore element used to define fluid conductance.

$$ \frac{L_{ij}}{G_{ij}} = \frac{L_i}{G_i} + \frac{L_t}{G_t} + \frac{L_j}{G_j} $$

[A.2]

For a circular tube the conductance $G$ is given analytically by Poiseuille’s law

$$ G = \lambda A^2 S = \frac{1}{2} A^2 S $$

[A.3]

where $A$ is the cross-sectional area of the pore or throat and $S$ is the shape factor (ratio of cross-sectional area to perimeter squared). Analytical expressions for equilateral triangles and squares are also available (20) with $\lambda$ being 3/5 and 0.5623 respectively. From numerical simulations Øren et al (20) found that the conductance for an irregular triangle was closely approximated by the above equation, using the same constant $\lambda$ as for an equilateral triangle, 3/5.
Correction factors
We used equation [1] to find values of the correction factor $\alpha$ that gave the best match to the four experiments we studied. We also followed the approach of Cannella et al (7). Based on effective medium theory and on their experimental results, they proposed the apparent porous media viscosity in the shear-thinning region to be:

$$\gamma_{\text{app}} = \beta \left( \frac{3n + 1}{4n} \right)^{\frac{n}{n-1}} \cdot \frac{q}{\sqrt{K\varphi}} \quad [A.4]$$

For their experiments, they found $\beta$ to be equal to 6. $\alpha$ and $\beta$ can be related as follows:

$$\alpha = \beta \left( \frac{3n + 1}{4n} \right)^{\frac{n}{n-1}} \quad [A.8]$$

Values for $\alpha$ and $\beta$ are presented in Table 3 below. Note that both factors vary with the porous medium and the fluid properties, and so cannot be treated as universal constants.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Medium</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hejri et al [13]</td>
<td>Sand-pack</td>
<td>0.98</td>
<td>1.23</td>
</tr>
<tr>
<td>Vogel &amp; Pusch [27]</td>
<td>Sand-pack</td>
<td>1.34</td>
<td>1.67</td>
</tr>
<tr>
<td>Greaves &amp; Patel [12]</td>
<td>Sandstone</td>
<td>7.6</td>
<td>9.57</td>
</tr>
<tr>
<td>Cannella et al [7]</td>
<td>Sandstone</td>
<td>4.8</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table 3: Calculated $\alpha$ and $\beta$ values for the four experiments studied.
Nomenclature

\( \phi \): Porosity
\( \gamma \): Shear rate (s\(^{-1}\))
\( \lambda \): Time constant in the Cross model (s)
\( \mu \): Constant Newtonian viscosity (Pa.s)
\( \mu_{\text{app}} \): Apparent viscosity (Pa.s)
\( \mu_{\text{pm app}} \): Apparent porous media viscosity (Pa.s)
\( \mu_{\text{app, re-scaled}} \): Re-scaled apparent viscosity (Pa.s)
\( \mu_{\text{eff}} \): Effective viscosity (Pa.s)
\( \mu_{\infty} \): Lower Newtonian effective viscosity (Pa.s)
\( \mu_{0} \): Higher Newtonian effective viscosity (Pa.s)
\( C \): Consistency index in the power-law model (Pa.s\(^{2-n}\))
\( n \): Exponential parameter in the power-law model
\( A \): Cross sectional area (m)
\( G \): Fluid conductance (m\(^4\))
\( K \): Absolute permeability (m\(^2\); D)
\( L \): Length (m)
\( m \): Exponential parameter in the Cross model
\( \Delta P \): Pressure drop (Pa)
\( q \): Darcy velocity (m.s\(^{-1}\))
\( Q_{N} \): Flow rate (m\(^3\).s\(^{-1}\))
\( R \): Radius of capillary tube (m)
\( S \): Shape factor (dimensionless)
\( R_{\text{equ}} \): Equivalent radius based on conductance (m)
\( \alpha, \beta \): dimensionless scaling parameters

Subscripts: net: network, exp: experimental
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Chapter 4

Dual Mesh Method in Upscaling

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Abstract
We develop and test a dual mesh approach to upscaling. A fine grid model is divided into coarse blocks, each of which contains many smaller cells. Pressure is computed first on the coarse grid using upscaled average transmissivities. These pressures are then used to compute the pressure at the small scale within each coarse block. In this way an approximation for the pressure is obtained without ever having to solve the full fine-grid problem, saving both CPU time and memory. The velocity is everywhere continuous and used to transport fluids on the fine grid - this step is no different from a conventional finite-difference approach once the pressure field is known.

We develop the method for multi-well two-phase incompressible flow with gravity in three dimensions - the method is an extension of the work of Verdière et al., and uses ideas developed by Gautier et al. in the context of streamline-based simulation. We discuss how this approach can be the platform for a fully decoupled dual mesh approach, where the fine grid transport problem is also decoupled into a series of coarse cell problems, resulting in a rapid but accurate method that can be applied in principle to grids with any number of cells.

We test the approach on two and three-dimensional problems with gravity. One of the test cases is a model from the SPE 10th Comparative Solution Project on upscaling. The dual mesh method gives results that are very close to the fine grid results, but uses less CPU time and memory. The results are good even when a very coarse mesh is used and the method is more accurate than using standard single-phase upscaling techniques.

Introduction
Modern reservoir characterization tools can routinely generate very refined reservoir models containing millions of gridblocks. However, it is normally impossible to perform a flow simulation on such a fine grid using conventional approaches and a coarser grid simulation is performed. Different upscaling techniques can be used for calculating effective parameters for each coarse grid block.

However, upscaling techniques either require a fine grid solution to derive coarse grid properties, or make assumptions about large-scale boundary conditions that introduce poorly constrained errors into the methods. The almost universal practice in reservoir engineering is to limit upscaling to single phase properties only (see Christie and Blunt as an example).

An alternative approach to upscaling is to solve the pressure equation on the coarse scale using upscaled properties while fine scale information is introduced during the saturation update by using either a pressure or flux refinement. This dual mesh technique achieves a considerable improvement in the precision of fluid recovery estimates compared to conventional coarse grid simulation, while using considerably less CPU time and memory than a full fine scale simulation.

Ramé and Killough were the first to implement a dual grid method in the context of single-phase miscible flow. They presented two-dimensional (2D) examples using a finite element method to solve for the pressure field on the coarse scale. A spline scheme was used to interpolate the coarse scale simulation to the fine grid. A finite
difference scheme was used for the conservation equation that was solved on the fine grid.

Guérillot and Verdière\textsuperscript{1, 2} introduced a dual mesh method similar to the implementation in this paper. The velocity field was estimated within each coarse grid block by solving for the pressure with approximate boundary conditions\textsuperscript{2}. Water injection in 2D was simulated using simplified injection-production boundary conditions. Geometric averaging of the transmissivity was used for multiphase flow upscaling.

Guedes and Schiozer\textsuperscript{9} used a similar approach and included gravity effects and well boundary conditions as source or sinks in one grid block. The refining technique developed in Hermitte and Guérillot\textsuperscript{10} was used to obtain a continuous velocity field within each coarse grid block.

In the context of streamline-based simulation\textsuperscript{11}, the dual mesh method has been implemented in three dimensions (3D) considering well boundary conditions and gravity effects\textsuperscript{3}. Transmissibilities were upscaled using the pressure solve method\textsuperscript{12}.

Arbogast and Bryant\textsuperscript{13} developed an alternative approach in the context of a mixed finite element formulation. The pressure field was solved implicitly on a coarse mesh and Green functions were used to derive the fine scale solution from the coarse scale simulation. Considering capillary and gravity effects, examples in 2D and 3D were presented.

Hou \textit{et al.}\textsuperscript{14, 15} developed a multiscale finite element method where finite element basis functions are calculated on a coarse mesh incorporating the fine scale description of the reservoir. These basis functions were then used to solve the elliptic problem on the coarse mesh for different types of boundary condition and to reconstruct the velocity field on the fine mesh. A 2D example of water injection was provided without considering capillary or gravity effects.

In this paper, an extension of the dual mesh method\textsuperscript{2} is presented in 3D considering gravity effects and multi-well completions. Well boundary conditions will be treated as in Gautiet \textit{et al.}\textsuperscript{3} For each time step, the pressure field is solved on a coarse mesh. The velocity field is recomputed inside each coarse grid block at the fine scale using flux boundary conditions from the coarse simulation. This process ensures continuity for the velocity field. Finally, saturations are updated with an explicit scheme using a conventional IMPES approach.

A formulation of a fully decoupled dual mesh approach is discussed and proposed as an extension of the method. The conservation equations (saturation update) for each coarse grid block are solved separately. The main advantage would be a more flexible time stepping for the simulation ensuring a faster run time.

Test cases results are presented in 2D and 3D for the dual mesh method. The 10\textsuperscript{th} SPE Comparative Solution Project\textsuperscript{4} is used to show the limits of the method. The dual mesh method can improve considerably the fluid recovery estimates in comparison with standard single-phase upscaling techniques.
The Dual Mesh Formulation

After deriving the multiphase fluid flow equations solved for this study, a description of the dual mesh method implementation is presented. Immiscible flow with gravity is considered. At the end of this section, a formulation of the fully decoupled approach is proposed and discussed as an extension of the dual mesh method.

Multiphase Flow Equation

For incompressible, immiscible fluid displacement, the Darcy flow \( v_p \) for each phase \( p \) ignoring capillary pressure is:

\[
K_v \rho_p \mu_p = -\nabla P - \rho_p g
\]  

(1)

\( K \) is the absolute permeability, \( k_{rp} \) is the relative permeability for each phase \( p \), \( P \) the fluid pressure, \( g \) the acceleration due to gravity and \( \rho_p \) and \( \mu_p \) are the phase density and the viscosity, respectively.

Considering two phases (water, oil), the conservation equation is solved for the water phase only. Hence,

\[
\phi \frac{\partial S_w}{\partial t} + \nabla \cdot v_w = Q
\]  

(2)

where \( \phi \) is the porosity, \( S_w \) water saturation, \( Q \) the well source term and \( t \) the time. The water velocity \( v_w \) can be calculated from the total velocity \( v_t \) by:

\[
v_w = f_w v_t + K \frac{\lambda_p}{\lambda_t} (\rho_w - \rho_o) g
\]  

(3)

with \( \lambda_p \) and \( \lambda_t \) the phase and the total mobilities respectively defined by:

\[
\lambda_p = \frac{k_p}{\mu_p}, \lambda_t = \lambda_w + \lambda.
\]  

(4)

\( f_w \) is the water fractional flow:

\[
f_w = \frac{\lambda_w}{\lambda_t}
\]  

(5)

and the total velocity \( v_t \) is:

\[
v_t = v_w + v_o.
\]  

(6)

For each time step, the velocity field is deduced from the pressure field calculated from Eq. (1) solved with an implicit scheme. The water saturation is updated using Eq. (2) with an explicit scheme where the water velocity is found from the total velocity, Eq. (3).
Conventional IMPES Finite Difference Formulation

Using a block centered Cartesian grid to represent the reservoir, the continuous domain is discretized into \( n \) cells with a constant size \( \Delta x \), \( \Delta y \) and \( \Delta z \) in the \( x \), \( y \) and \( z \) direction respectively.

Using a seven point stencil, the pressure \( P_i \) of each block \( i \) is deduced from the classical matrix system\(^{16}\):

\[
TP = G + Q \tag{7}
\]

where the vector \( P \) contains the unknown pressure values of each block \( P_i \), the matrix \( T \) is the inter block transmissivity matrix, \( G \) is the inter block gravity transmissivity vector and \( Q \) the vector containing the well source terms as defined in Peaceman\(^{17}\).

For a direction \( d \) (\( d=x, y \) or \( z \)), the inter block transmissivity defined between a node \( i \) and \( i+1 \) is the harmonic average of the adjacent block transmissivities in this direction, hence:

\[
T_{\frac{x}{2}} = \frac{2}{T_{x,i} + T_{x,i+1}} \tag{8}
\]

with

\[
T_{x,i} = \frac{\Delta y \Delta z}{\Delta x} \lambda_{x,i,k} \tag{9}
\]

Each term of the vector \( G \) is the difference of the depth \( \Delta D \) between the considered cell and its neighbors multiplied by the harmonic average of the adjacent block gravity transmissivities in the \( z \) direction only, as the grid is Cartesian:

\[
G_{\frac{z}{2}} = \frac{2}{G_{z,i} + G_{z,i+1}} \tag{10}
\]

with

\[
G_{z,i} = \frac{\Delta y \Delta x}{\Delta z} \lambda_{z,i,k} \tag{11}
\]

and

\[
\lambda_{z} = \left( \frac{\rho_w k_w + \rho_o k_o}{\mu_w} \right) g \tag{12}
\]

The vector solution \( P \) is obtained from Eq. (7) by inverting the matrix \( T \). This is the most time consuming part of the simulation. The total velocity field can then be calculated from \( P \) and used to update the water saturation. Using the orientation of the phase (water) velocity and an appropriate time step constrained by the CFL conditions, saturations are updated with a conventional upstream weighting technique\(^{16}\).
The Dual Mesh Algorithm

Fig. 1- Five steps for the dual mesh algorithm: a reservoir description (step 1) is upscaled to the coarse mesh (step 2) using conventional methods. The pressure and total velocity are computed on the coarse mesh (step 3). The total velocity is used as a boundary condition to compute the pressures within each coarse block (step 4). The fluxes are weighted by the fine mesh transmissivities across the face. The fine-scale velocity field is used to update saturation (step 5).

By using a coarser mesh to represent the reservoir, the size of the system of linear equations (7) becomes smaller and faster to solve. Different upscaling techniques can be used to calculate effective transmissivities for each coarse grid block. The purpose of the dual mesh approach is to retain the fine scale heterogeneity by using it for the saturation update while maintaining a fast simulation by solving the pressure field on the coarse mesh. The way to implement this algorithm can be decomposed in five steps. These five steps have to be performed for each timestep.

Step 1. Initial boundary conditions such as reservoir properties and well completion are updated.

Step 2 The second step consists of building the coarse mesh and calculating effective properties using two different methods in this study: (i) a geometric average of the fine cell transmissivities, and (ii) the pressure solve method.
As in Gautier et al.\textsuperscript{3}, a coarse transmissivity component $T_d^{coarse}$ and a coarse gravity transmissivity component $G_z^{coarse}$ is calculated using the pressure solve method for each direction $d$ ($d=x$, $y$ or $z$). This can be done by applying a pressure drop $\Delta P_d$ along the $d$ direction between two opposite faces of the coarse cell and no flow boundary conditions on the other faces (Fig. 2) and ignoring gravity. Computing the pressure field and the total exit flow in the direction $d$, $Q_d^{coarse}$ the associated effective transmissivity is:

$$T_d^{coarse} = \frac{Q_d^{coarse}}{\Delta P_d}$$

By repeating this procedure for all three directions, the diagonal effective transmissivity tensor can be calculated for the coarse grid block.

![Diagram](Image)

**Fig. 2-** The pressure solve method for upscaling transmissibilities in each coarse grid block: in this figure the coarse grid block contains 3 x 3 x 3 fine grid blocks. By applying a pressure drop $\Delta P_d$ in one direction $d$ and no flux boundary conditions on the others, an effective transmissibility $T_d^{coarse}$ is found from the calculated flow $Q_d^{coarse}$. A similar calculation is performed in all three directions producing a diagonal effective transmissibility tensor.

The effective gravity transmissivity component is obtained by imposing a constant pressure drop equal to zero in the vertical direction only so that only gravity effects will produce the exit flow.

$$G_z^{coarse} = \frac{Q_z^{coarse}}{\Delta Z^{coarse}}$$

with $\Delta Z^{coarse}$ being the coarse grid block size in the vertical direction.

**Step 3.** The system is solved and the pressure and the total velocity field are determined for the coarse mesh.

**Step 4.** The total velocity field on the fine grid is obtained from the fluxes $Q_d^{coarse}$ determined in step 3, and from the fine scale heterogeneity. To refine the coarse fluxes
to the fine scale, the following problem is solved for each coarse grid block separately. First, the flux boundary conditions at the faces of the coarse cell are defined at the fine scale by weighting the coarse flux associated with the considered face by the fine scale transmissivity across the face. Hence, the boundary condition along the \( x \) direction for the fine grid block \( i \) attached to the face \( f \) will be the flux \( Q_{x,\text{fine}} \) defined by:

\[
Q_{x,\text{fine}} = \sum_{s' \in \{s\}} T_{x,\text{fine}}^{s,s' \rightarrow i} Q_{x,\text{coarse}}^{s\rightarrow i}
\]  

(15)

The pressure at the center of the grid block is assumed to be equal to the coarse pressure \( P_{\text{coarse}} \) of the coarse cell.

**Step 5.** This step corresponds to the conventional saturation updating performed on the whole fine grid using the total velocity field deduced from the previous step. This total velocity field is continuous everywhere in the grid so that volume is conserved exactly. If the fine grid block is not connected to the next coarse grid block the velocity is found from the pressure gradient between its neighbors (thin arrow in Fig. 1 step 5). If the fine grid block is connected to the next coarse grid block then the flux boundary condition (15) is used to define the total velocity (thick arrow in Fig. 1 step 5). Finally, a standard single-point upstream weighting method is used to update the saturations on the whole fine grid.

**3D Implementation**

Two different steps need to be considered for the implementation of the dual mesh method in 3D: the gravity segregation for the saturation update and the well boundary conditions.

**Gravity Segregation.** The difference in density of oil and water induces a vertical displacement described by the second term of Eq. Error! Reference source not found. for the water velocity. Usually, using an IMPES scheme, saturations are updated by determining the upstream direction from each phase velocity. This direction is defined by the pressure gradient between two adjacent grid blocks (Eq. Error! Reference source not found.). However, as defined in step 5 of the dual mesh algorithm, in order to respect mass balance, the dual mesh approach will provide for each time step a continuous total velocity field, while the pressure will be discontinuous across the coarse cell faces. This will result in an erroneous determination of the upstream direction in some cases. To determine the correct upstream direction for the water velocity without using the pressure gradient direction, we compute mobilities using an average saturation \( S_{\text{av}} \) of two adjacent grid blocks (1 and 2 in Fig. 3) \( S_{\text{av}} = (S_1 + S_2)/2 \) and use them in Eq. Error! Reference source not found. to determine the sign of \( v_w \) between these two blocks.
Fig. 3- To determine an upstream direction for each phase at a grid face, phase mobilities are calculated using the average saturation of the two adjoining grid blocks. Then Eq. Error! Reference source not found. is used to determine the sign of $v_w$ and therefore the upstream direction.

**Well Boundary Conditions.** We used the same implementation as Gautier et al.\textsuperscript{3} for the well boundary conditions. Hence, for a fine grid block $i$ perforated by a well, the well source term $Q_{well,\text{fine}}$ is deduced from the coarse scale well source term $Q_{well,\text{coarse}}$ weighted by a well transmissibility $T_{i,\text{well}}$ defined by Peaceman\textsuperscript{17}. Hence,

$$Q_{i,\text{well}}^{\text{fine}} = \frac{T_{i,\text{well}}}{\sum_i T_{i,\text{well}}} Q_{\text{well,coarse}}$$ \hspace{1cm} (16)

This technique ensures an enhanced flow from the well through high transmissivity layers.

**Numerical Test Cases**

In this section, we present numerical results that demonstrate the application of the method described above in 2D and 3D. The ability of the dual mesh method is assessed by comparing results with the fine scale simulation referenced as the fine model in the following. Two different upscaling techniques are used to calculate effective transmissibilities with the dual mesh method: the pressure solve method ($psm$ model), and geometric averaging ($ga$ model). The coarse model corresponds to the simulation performed on the coarse mesh only. This model corresponds to the conventional approach used in upscaling. A geometric average of the absolute permeability is used to define coarse block properties.

**2D Horizontal**

The first example is a 2D horizontal reservoir (one layer in the vertical direction). The fine mesh contains 30 x 30 grid blocks while the coarse mesh is composed of 10 x 10 cells (Fig. 4). The permeability consists of a high permeability zone of 100 mD (in white) and a low permeability zone of 1 mD (in black). The percentage of the black facies is 80%. The porosity is constant and equal to 0.25. Water is injected at one corner of the grid with a constant rate of 5 m$^3$ per day while the production occurs at a constant pressure of 76 bars from the opposite corner. We used Corey-type functions for the relative permeability\textsuperscript{18}. Hence, for each phase $p$, the relative permeability $k_{rp}$ is written:
\[ k_{rp} = k_{r_{\text{max}}} \left( \frac{S_p - S_{rp}}{S_{\text{max}_p} - S_{rp}} \right)^{n_p} \]  

(17)

with \( k_{r_{\text{max}}^o} = 1 \), \( k_{r_{\text{max}}^w} = 0.4 \), \( S_{\text{max}_o} = S_{\text{max}_w} = 0.8 \), \( S_{ro} = S_{rw} = 0.2 \), \( n_w = 1.5 \) and \( n_o = 2 \). The oil and water viscosities are 0.8 and 1 centipoise, respectively.

Watercut at the production well has been compared for the four different models (fine, coarse, ga and psm) (Fig. 5). The coarse mesh gives poor predictions of recovery. The dual mesh method gives results very similar to the fine grid, especially for the psm model. The ga model corresponds approximately to the numerical results presented by Verdière and Guérillot².
Fig. 5- Watercut comparisons for a 2D horizontal test case between simulations performed on the initial fine mesh (fine), the coarse mesh only (coarse) and using the dual mesh method with geometric averaging (ga) and the pressure solve method (psm) for upscaling.

Fig. 6 compares the saturation fields for the different methods after 300 days of injection. It can be seen that the dual mesh method captures the effects of the fine scale heterogeneity that are lost using coarse grid simulation.
Fig. 6- Comparison of the saturation field simulated after 300 days of injection for the fine mesh (fine), the coarse mesh (coarse), and the dual mesh method using the pressure solve method (psm) and geometric averaging (ga).

2D Vertical

In this example, the reservoir is represented by a 2D grid with 60 layers in the vertical direction and 60 in the horizontal direction (Fig. 7). The size of the reservoir is 900 m by 600 m by 60 m in the x, y and z directions respectively. The permeability field is once again composed of one zone with a high permeability of 100 mD (in white) and a low permeability zone of 0.1 mD (in black). The coarse mesh is 10 by 10 cells. The water is injected at a constant rate of 100 m$^3$/day and the fluid is produced at a constant bottom hole pressure of 80 bars.

The same gravity number $N_g$ as defined by Gautier et al.$^3$ will be used to quantify impact of gravity:

$$N_g = \frac{\Delta \rho g L^2}{\Delta P h H}$$  \hspace{1cm} (18)

$\Delta \rho$ is the difference in density between water and oil, $\Delta P_h$ and $L$ are the pressure difference and the distance between the two wells respectively and $H$ is the height of the reservoir. In this simulation, $N_g = 0.3$, typical of reservoir displacements$^3$.

In this case, the dual mesh approach using the pressure solve method for upscaling is virtually identical to the fine mesh simulation and much more accurate than using the geometric average (Fig. 8).
Fig. 7 - a) Permeability field for a 2D case with gravity effects composed of a high permeability zone of 100 mD in white and a low permeability zone of 0.1 mD in black. b) Saturation field comparisons after 1000 days of injection between simulations performed on the fine mesh (fine), on the coarse mesh (coarse) and with the dual mesh approach using the pressure solve method (psm) and geometric averaging (ga) for upscaling.

Fig. 8 – Watercut comparison for the 2D vertical case between simulations performed on the initial fine mesh (fine), the coarse mesh (coarse), and the dual mesh method using the pressure solve method (psm) and geometric averaging (ga) for upscaling.
Once again the comparison of the saturation fields between the four different models show that the dual mesh method captures the fine scale heterogeneity defined by the original permeability field much better than the coarse scale simulation (Fig. 7 b).

3D Simulations
In this section, the dual mesh method is assessed using three different scenarios generated from the 10th SPE Comparative Project on upscaling\(^4\). In order to compare simulations with the same code, the original size of the grid (approximately one million grid blocks) has been reduced to 93,500 cells (20x55x85).

The three different scenarios differ in their fine scale permeability and porosity heterogeneity and well completions (Fig. 9). For scenario 1, the porosity is constant and equal to 0.2 while the permeability field is composed of two facies: 80% at 100 mD (in grey) and 20% at 1 mD (in black). The water is injected at one corner of the grid and the fluid is produced at the opposite corner. This scenario corresponds approximately to the cases performed by Gautier et al.\(^3\).

**Fig. 9 – The three different scenarios created for testing the dual mesh method in 3D. a) Scenario 1: constant porosity of 0.2 with a dual facies permeability (100 mD in black and 1 mD in grey), one injector and one producer well at the two opposite corners of the grid. b) Scenario 2: same as a) but with one central injector and four producers at each corner of the grid. c) An upscaled version of the 10th SPE Comparative Project 2001\(^4\) with one central injector and four producers.**
Scenario 2 has the same reservoir properties as scenario 1. There is one central injector and four producers located at each corner of the grid.

Scenario 3 is an upscaled version of the 10th SPE Comparative Project 2001\(^4\). The permeability is upscaled using geometric averaging.

We used the same well boundary conditions as Christie and Blunt\(^4\). Hence, all the wells are vertical and completed throughout the whole formation and the water is injected at 795 m\(^3\)/day while the fluid is produced at a constant bottom hole pressure of 275 bars. The fine mesh is a 20x55x85 (93,500 cells) grid while the coarse mesh is 5x11x5 grid (275 cells). The dimension of the fine grid block is 18.288 m by 12.192 m by 0.6096 m in the x, y and z directions, respectively. Corey-type relative permeability functions have been used with \(n_o = n_w = 2\), \(k_{rw} = k_{ro} = 0.2\) and \(k_{rmaxo} = k_{rmaxw} = 1\). The density is 1000 kg/m\(^3\) for the water and 850 kg/m\(^3\) for the oil. The viscosity is 3.0 cp for oil and 0.3 cp for the water. \(N_g\) varies from 1.2 to 0.8 during the simulation.

**Watercut comparison**

For scenario 1, the dual mesh method accurately reproduces the fine mesh simulation (Fig. 10).

![Fig. 10- Watercut comparison for scenario 1 between simulations performed on the fine mesh (fine), the coarse mesh only (coarse) and with dual mesh method using the pressure solve method (psm) and geometric averaging (ga) for upscaling.](image)

For scenario 2, the results are similar: the psm model gives the best fit to the fine mesh simulation while the coarse model gives poor results (Fig. 11).

For scenario 3, the results are similar: the psm model gives the best fit to the fine mesh simulation while the coarse model gives poor results (Fig. 12).

Scenario 3 provides rather different results (Fig. 12). For producers 3 and 4 the psm model is still the closest to the fine mesh simulation. On the other hand, for wells 1 and 2, the psm model gives predictions worse than the coarse model. Geometric averaging always produces results that are superior to the coarse cell simulation.
The performance of the four models is different if we look at the cumulative oil production for producer 1 (Fig. 13). We compare the results with those from the upscaling study on the same field\(^1\) where single-phase upscaling using the pressure solve method was applied for different coarse grid sizes. Our fine case (20x55x85) provides results approximately identical with the initial fine mesh simulation (60x220x85) performed by Christie and Blunt\(^4\). The worst prediction is provided by the 5x11x6 coarse mesh using the single-phase pressure solve approach. Using geometric averaging for single phase upscaling on a 5x11x5 mesh (our coarse model) improves the result but is still far from the fine mesh simulation.

**Fig. 11-** Watercut comparison, for each producer, for scenario 2 between simulations performed on the fine mesh (fine), the coarse mesh only (coarse) and the dual mesh method using the pressure solve method (psm) and geometric averaging (ga) for upscaling.
Fig. 12 - Watercut comparison, for each producer, for scenario 3 between simulations performed on the fine mesh (fine), the coarse mesh only (coarse) and the dual mesh method using the pressure solve method (psm) and geometric averaging (ga) for upscaling.

In this case, the dual mesh method (psm and ga models) is slightly better than the coarse simulation for the cumulative oil production estimates. Note that the dual mesh method performed worse for the watercut. The reason for this apparent discrepancy is that the dual mesh method better captures the total mobility of the system than the coarse grid simulation.

Fig. 13 - Cumulative oil production comparison for producer 1 in scenario 3 with different grid sizes using the single phase upscaling pressure method. Our fine case (20x55x85) is close to the initial fine mesh (60x220x85). The results are compared to our coarse case (5x11x5, using geometric averaging) and the dual mesh method using the pressure solve method (psm) or geometric averaging (ga).

This example demonstrates that the dual mesh method performance is dependent on the complexity of the heterogeneity of the reservoir. Scenarios 1 and 2 have a “simpler” heterogeneity (constant porosity, high percentage of the high permeability zone) than
scenario 3 (heterogeneous porosity field and channels). From the watercut comparison, scenario 3 (Fig. 12) shows that the dual mesh method using a geometric averaging (ga model) matches quite well the fine model for early times, but it seems to converge to the coarse model at late times. However, it must be emphasized that we have used a very coarse grid for our comparisons — although the dual mesh method is inaccurate, it does provide better results than using standard single-phase upscaling techniques.

Discussion
Comparing the CPU time between the four models, the dual mesh method is generally faster than the fine mesh simulation. The speed-up factor is defined as the ratio between the time needed for the fine grid simulation and the studied case (ga, psm and coarse) (Table 1).

<table>
<thead>
<tr>
<th>Model</th>
<th>2Dhorizontal</th>
<th>2Dvertical</th>
<th>Scenario 3</th>
</tr>
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<tr>
<td>Fine mesh</td>
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<td>60x60</td>
<td>20x55x85</td>
</tr>
<tr>
<td>Coarse mesh</td>
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<td>10x10</td>
<td>5x11x5</td>
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<td>3.9</td>
</tr>
<tr>
<td>Psm</td>
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<td>1.3</td>
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<tr>
<td>Coarse</td>
<td>3.6</td>
<td>28</td>
<td>432</td>
</tr>
</tbody>
</table>

Table 1- Speed-up factor comparison between the different test cases run in this study: 2D horizontal, 2D vertical and scenario 3 that corresponds to an upscaled version of the 10th SPE Comparative Project 20014.

The speed-up factor depends on the size of the model and the case considered. If the fine mesh simulation can be run quickly, the dual mesh method could be even slower (case 2D horizontal). But, obviously there is no point in using the dual mesh method for such a case. The most complex case performed in this study (scenario 3, 3D with gravity) gives a speed up factor of around 4 which corresponds approximately to the values measured by other authors. However, as discussed in Arbogast and Bryant, this value depends on so many different parameters (the mesh, the nature of the displacement and the efficiency of the code) that it is difficult to predict a speed-up factor from first principles.
A Fully Decoupled Approach
The time savings from the dual mesh method, although significant, are insufficient to allow the technique to be applied to grids of essentially arbitrary size. There are two reasons for this. First, solving the saturation equation on the fine grid using an IMPES approach introduces a severe time-step limitation that makes the code slow for large problems. Even though each recomputation of the pressure field is faster than for the fine grid, the number of pressure solves is approximately the same. This limitation can be mitigated by a number of techniques, such as the use of streamline-based simulation\textsuperscript{3, 11}. The second problem is that a large number of coarse cell problems need to be solved. Imagine that we have $n$ fine grid blocks and $m$ coarse blocks each containing $n/m$ cells. Let $T(k)$ be the computer time necessary to solve for the pressure for a system with $k$ grid blocks. The fine grid simulation time per timestep is $T(n)$ (ignoring the time taken to update the saturation which is generally shorter). The dual mesh method takes $5mT(n/m)+T(m)$. The factor of 5 represents the time taken to compute the average transmissivity in the three flow directions, the gravity mobility, and the pressure field within each coarse cell. The second term is the computation of pressure on the coarse grid. Modern multigrid methods now achieve almost linear performance\textsuperscript{19}, meaning that $T(k) \sim k$. This means that per timestep, the dual mesh and fine grid methods use approximately the same time, and that apparent savings are due to grid-size dependent prefactors in the scaling of $T$.

A way to overcome both the limitations mentioned above is to develop a fully decoupled method. Both the pressure field and the saturation update would be solved on the coarse grid. At the coarse scale, average flux boundary conditions would be used to update the saturation. Within each coarse block, the fine scale saturation distribution would then be updated using effective flux boundary conditions for water at the cell faces. The time-step would be chosen so that the average saturation reached matched that in the coarse cell simulation. Then new effective transmissivities and average water fluxes would be computed and these would be used for the next step of the simulation at the coarse level. In this way the fine scale and coarse scale simulations are decoupled – they only exchange information through flux boundary conditions and effective transmissivities. This method would remove the time-step constraint inherent for very fine grid simulation studies. However, it would introduce further approximations and would not overcome the scaling of run time limitation. To make the method more efficient, explicit fine-scale computation of the saturation update could be limited to a sub-set of coarse cells, where the saturation was changing rapidly. In other cells, average properties computed from similar cells, or using single-phase upscaling only, could be applied. This would be equivalent to adaptive mesh refinement\textsuperscript{20}.

Conclusions
We have presented an extension of the dual mesh method\textsuperscript{2} to 3D with multi-well completions and gravity. We used a variety of test cases to demonstrate that the method gives superior accuracy than coarse grid simulation while being faster than a simulation on the fine grid.
We have proposed a fully decoupled dual mesh approach that could lead to more substantial time savings, although it would introduce further approximations into the method.

**Nomenclature**

- $D$ = depth, [L], m
- $f_p$ = phase fractional flow
- $g$ = acceleration due to gravity, [LT$^{-2}$], m s$^{-2}$
- $G$ = gravity transmissivity, [L$^{2}$T$^{-1}$], m$^2$s$^{-1}$
- $G$ = gravity vector, [L$^{3}$T$^{-1}$], m$^3$s$^{-1}$
- IMPES = Implicit Pressure Explicit Saturation scheme name
- $K$ = absolute permeability tensor, [L$^2$], m$^2$, mD
- $k_{rp}$ = phase relative permeability
- $k_{r_{max}}$ = maximum phase relative permeability
- $n_p$ = phase exponent for Corey type relative permeability model
- $P$ = pressure, [ML$^{-1}$T$^{-2}$], Pa
- $Q$ = source vector, [L$^3$T$^{-1}$], m$^3$s$^{-1}$
- $Q$ = source term, [L$^3$T$^{-1}$], m$^3$s$^{-1}$
- $Q_d$ = flux in direction d, [L$^3$T$^{-1}$], m$^3$s$^{-1}$
- $S_p$ = phase saturation
- $S_{r_{max}}$ = maximum phase saturation
- $S_{rp}$ = residual phase saturation
- $t$ = time, [T], s
- $T$ = transmissivity, [L$^4$TM$^{-1}$], m$^4$skg$^{-1}$
- $T$ = transmissivity matrix, [L$^4$TM$^{-1}$], m$^4$skg$^{-1}$
- $v_p$ = phase velocity vector, [LT$^{-1}$], m s$^{-1}$
- $v_t$ = total velocity vector, [LT$^{-1}$], m s$^{-1}$

**Greek Letters**

- $\lambda_p$ = phase mobility, [LTM$^{-1}$], mskg$^{-1}$
- $\mu_p$ = phase viscosity, [ML$^{-1}$T$^{-1}$], Pa s
- $\nabla$ = gradient operator
- $\phi$ = porosity
- $\rho_p$ = phase density, [ML$^{-3}$], kg m$^{-3}$

**Subscripts**

- $i$ = fine grid block index
- $g$ = gas
- $n$ = number total of fine grid block
- $o$ = oil
- $p$ = phase index (w for water, o for oil, g for gas)
- $t$ = total
- $w$ = water
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References


